Constructing Multiple Frames of Discernment for Multiple Subproblems*

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Abstract. In this paper we extend a methodology for constructing a frame of discernment from belief functions for one problem, into a methodology for constructing multiple frames of discernment for several different subproblems. The most appropriate frames of discernment are those that let our evidence interact in an interesting way without exhibit too much internal conflict. A function measuring overall frame appropriateness is mapped onto a Potts spin neural network in order to find the partition of all belief functions that yields the most appropriate frames.

Keywords: Dempster-Shafer theory, belief function, representation, frame of discernment, clustering, Potts spin, conflict, simulated annealing.

1 Introduction

In this paper we extend a methodology for constructing a frame of discernment for *one* problem [1] into a methodology for constructing multiple frames of discernment from a set of belief functions [2, 3] for *several* different subproblems. These belief functions are assumed to concern different subproblems that should be handled separately. Previously, we have developed methods for clustering belief functions that are mixed-up [4–8] based on their pairwise conflicts. These methods were developed to manage simple support and consonant belief functions. The case with nonconsonant belief functions can be handled by decomposition into simple support functions followed by clustering of the decomposed parts [9]. If the number of clusters K (in Fig. 4.) is unknown, it can be estimated by observing the change in the logarithm of a meta frame appropriateness function (*MFA*) for different number of clusters [10, p. 90], or inferred using specification [5] and *a priori* information [11], or managed by particle filtering methods [12].

The methodology for constructing a frame of discernment is extended by adopting a measure of frame appropriateness for a single problem into handling multiple subproblems. This new function is mapped onto a Potts spin neural network. We reuse a previously developed methodology for clustering large amounts of belief function in such a manner as to find the best frames of discernment for these subproblems. When

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the mixed-up belief functions are partitioned into subsets corresponding to the different subproblems, a frame of discernment is constructed within each subproblem using the methodology developed for a single problem [1].

In Sec. 2 we study the problem of construction alternative frames of discernment. In Sec. 3 we extend this methodology to multiple subproblems. In Sec. 4 we review Potts spin theory. We then put everything together by mapping the multiple frame construction problem onto Potts spin (in Sec. 5). In Sec. 6 we present an algorithm for constructing multiple frames. Finally, in Sec. 7 conclusions are drawn.

2 Constructing Alternative Frames of Discernment

Let us assume we have a set of evidence $\chi = \{m_i\}$ that originates from *one* problem with yet undetermined representation. The focal elements of each belief function m_i contain pieces of that representation. Our task is to find the most appropriate frame of discernment that lets our evidence "interact in an interesting way" without "exhibit too much internal conflict" in the words of Glenn Shafer [3, p. 280].

This will usually not be the union of all cores of m_i as different cores may hold non-exclusive elements. For example, one belief function may assign support to a focal element "Red" in relation to the color of a car. Another belief function may assign support to a focal element "Fast" in relation to speed of that car. Obviously, "Red" and "Fast" are not both elements of the frame of discernment as they are not exclusive. However, the "(Red, Fast)" pair might be an element of the frame.

Our task of finding the most appropriate frame of discernment becomes finding the most appropriate cross product of different unions of cores. Let us begin by introducing the representation needed to construct a frame of discernment from input data.

For an example of the material in Sec. 2 see [1].

2.1 Representation

Assume we have a set of evidence χ. We observe the core C_i of each available belief function m_i. We assume that the core of each belief function is a subset of exclusive but not exhaustive elements of a so far unconstructed frame of discernment.

Let $C = \{C_i\}$ be the set of all cores of χ, where C_i is the core of m_i, the ith piece of evidence. We have

$$C_i = \bigcup_j \{A_j | m_i(A_j) > 0\} \tag{1}$$

where A_j is a focal element of m_i.

Let $\Omega = \{\Omega_k\}$ be the set of all possible set partitions of C (the set of all cores), where Ω_k is the kth possible partition of C. We have

$$\Omega_k = \{\omega_l\} \tag{2}$$

where the ω_l's are disjoint subset of C, i.e., $\forall l.\ \omega_l \subseteq C$ such that

$$\bigcup_l \omega_l = \{C_i\} \equiv C \tag{3}$$

and $\omega_m \cap \omega_n = \varnothing$ whenever $m \neq n$.

Let $\Theta = \{\Theta_k\}$ be the set of all possible cross products relating to Ω, such that Θ_k is the cross product of all unions of elements of the partition Ω_k, (2). We have

$$\Theta_k = \times \{\theta_l\} \tag{4}$$

where θ_l is the union of the elements in ω_l, $\omega_l \in \Omega_k$, and θ_l must be an exclusive set of elements. We have

$$\forall l.\ \theta_l = \bigcup \omega_l = \bigcup_i \{C_i | C_i \in \omega_l\} \tag{5}$$

such that

$$\bigcup_l \theta_l = \bigcup_l \{\bigcup \omega_l\} = \bigcup_i \{C_i\} = \bigcup C \tag{6}$$

where all θ_l's observe two different crucial type conditions:

Type Condition 1. No element of any θ_p may belong to any other cross product elements θ_q, i.e.,

$$\theta_p \cap \theta_q = \varnothing , \tag{7}$$

whenever $p \neq q$.

This will eliminate any frame that obviously distributes elements of the same type over different cross product elements.

Type Condition 2. Every cross product element θ_l must be an exclusive set, i.e.,

$$e_m \cap e_n = \varnothing , \tag{8}$$

whenever $\forall m,n \exists l.\ e_m, e_n \in \theta_l$.

2.2 Abridgment

For all possible frames of discernment $\{\Theta_k\}$, where $|\Theta_k| > 1$, we may include further assumptions that make the frames tighter. This may lead to more interesting interaction between the belief functions and lead to firmer conclusions provided that the conflict does not increase in any significant way. Every frame is based on assumptions. The frame we begin with is based on the assumption that the elements of that frame are all disjunct possible alternatives, and that no other possibilities exists. Whether a tighter or looser frame is to be preferred is a matter of appropriateness. Most often this will be a point of balance where meaningful interaction is weighted against too much conflict.

Let us study one particular frame of discernment Θ_i from the remaining set of possible frames Θ that observe both type condition 1 and 2, (7) and (8), respectively. We have $\Theta_i = \times \{\theta_l\}$. At least one cross product element θ_l must be abridged to construct a new abridged frame of Θ_i. We have a set of all possible abridgments of Θ_i,

$$\Theta'_i = \{\Theta'_{ij}\}_j = \{\times \{\theta'_{lj}\}\}_j \tag{9}$$

where $\theta'_{lj} \in 2^{\theta_l}$ and 2^{θ_l} is the power set of θ_l, $\theta'_{lj} \neq \varnothing$, and $\exists j.\ \theta'_{lj} \neq \theta_l$.

2.3 Enlargement

We may make enlargements to any frame of discernment $\{\Theta_k\}$. The only enlargement we can perform is to enlarge a particular cross product element θ_l with an element of unstated meaning. Let us denote these elements Λ_l, one for each θ_l.

Let us again look at $\Theta_i = \times \{\theta_l\}$. For each cross product element θ_l there is one possible enlargement: enlarging θ_l by Λ_l. At least one cross product element θ_l must be enlarged to construct a new enlarged frame of Θ_i. The set of all possible enlargements of Θ_i becomes $\Theta_i'' = \{\Theta_{ij}''\}_j = \{\times\{\theta_{lj}''\}\}_j$ where $\theta_{lj}'' \in \{\theta_l, \theta_l + \{\Lambda_l\}\}$ and $\exists j.\ \theta_{lj}'' \neq \theta_l$.

2.4 Appropriate Representation

We evaluate the alternative frames of discernment on the grounds of being appropriate for yielding interesting interactions among the available belief functions without exhibiting too much internal conflict. A measure of frame appropriateness was defined in [1]. This measure gives an equal weight to both conditions were both must be appropriate simultaneously (see [1]).

Definition 1. Let Θ_k be a frame of discernment and let $\{m_j\}$ be a set of all available belief functions defined on Θ_k. We define a measure of frame appropriateness of Θ_k, denoted as $FA(\Theta_k)$, by

$$FA(\Theta_k|\{m_j\}) = \left[1 - Con(\oplus\{m_j|\Theta_k\})\right]\left[1 - \frac{AU(\oplus\{m_j|\Theta_k\})}{\log_2|\Theta_k|}\right], \quad (10)$$

where Con is the conflict in Dempster's rule and AU is the functional called the aggregated uncertainty. We have $Con \in [0, 1]$, $AU \in [0, \log_2|\Theta_k|]$ and $FA \in [0, 1]$.

The aggregated uncertainty functional AU [13−15] is defined as

$$AU(\text{Bel}) = \max_{\{p_x\}_{x \in \Theta}} \left\{ -\sum_{x \in \Theta} p(x)\log_2 p(x) \right\}, \quad (11)$$

where $\{p_x\}_{x \in \Theta}$ are all probability distributions such that $p_x \in [0, 1]$ for all $x \in \Theta$,

$$\sum_{x \in \Theta} p(x) = 1 \quad (12)$$

and

$$\text{Bel}(A) \leq \sum_{x \in A} p(x) \quad (13)$$

for all $A \subseteq \Theta$.

2.5 An Algorithm for Computing *AU*

An algorithm for computing AU was found by Meyerowitz *et al.* [16]. For the sake of completeness we cite the algorithm here, in the way it is described by Harmanec *et al.* [17], Fig. 1. The computational time complexity of AU is $O(2^{|\Theta|})$.

Input: a frame of discernment X, a belief function Bel on X.
Output: $AU(Bel)$, $\{p_x\}_{x\in X}$ such that $AU(Bel) = -\sum_{x\in X} p_x \log_2 p_x$, $p_i \geq 0$, $\sum_{x\in X} p_x = 1$, and $Bel(A) \leq \sum_{x\in X} p_x$ for all $\emptyset \neq A \subseteq X$.
Step 1. Find a non-empty set $A \subseteq X$, such that $Bel(A) / |A|$ is maximal. If there are more than such sets A than one, take the one with maximal cardinality.
Step 2. For $x \in A$, put $p_x = Bel(A) / |A|$.
Step 3. For each $B \subseteq X - A$, put $Bel(B) = Bel(B \cup A) - Bel(A)$.
Step 4. Put $X = X - A$.
Step 5. If $X \neq \emptyset$ and $Bel(X) > 0$, then go to Step 1.
Step 6. If $Bel(X) = 0$ and $X \neq \emptyset$, then put $p_x = 0$ for all $x \in X$.
Step 7. Calculate $AU(Bel) = -\sum_{x\in X} p_x \log_2 p_x$.

Fig. 1. An algorithm for computing $AU(Bel)$

2.6 An Algorithm for Constructing a Frame of Discernment

Using the results of the preceding sections we develop an algorithm for constructing and evaluating all possible frames of discernment. This algorithm will first generate the possible frames using different partitions of the set of all cores. From these possible frames we generate abridgments and enlargements. The frames are evaluated using *FA*, (10), in Fig. 2. The most appropriate frame that maximizes *FA* is then selected.

Input: a set of belief functions χ.
Output: Possible frames of discernment $\{\Theta_i\}$, $\{\Theta'_{ij}\}$, $\{\Theta''_{ij}\}$. Frame appropriateness $\forall ij.\ FA(\Theta_i|\chi)$, $FA(\Theta'_{ij}|\chi)$, $FA(\Theta''_{ij}|\chi)$.
Step 1. $\forall i.$ generate C_i using (1). Set $C = \{C_i\}$.
Step 2. $\forall k.$ generate Ω_k using (2). Set $\Omega = \{\Omega_k\}$.
Step 3. $\forall k.$ generate Θ_k using (4). Set $\Theta = \{\Theta_k\}$.
Step 4. $\forall ij.$ generate $\{\Theta'_{ij} \mid \forall kl.Con(\oplus\{m_j|\Theta'_{kl}\}) < 1,\ \Theta'_{kl} \supset \Theta'_{ij}\}$ using (9).
Step 5. $\forall k.$ If $Con(\oplus\{m_j|\Theta_k\}) > 0$ then $\forall j.$ generate Θ''_{ij}. Set $\Theta''_i = \{\Theta''_{ij}\}_j$.
Step 6. Compute frame appropriateness $\forall ij.\ FA(\Theta_i|\chi)$, $FA(\Theta'_{ij}|\chi)$, $FA(\Theta''_{ij}|\chi)$ using (10).

Fig. 2. An algorithm for generating and evaluating appropriate frames of discernment

Brute force implementation of *FA* has a computational time complexity of $O(|\chi|^{|\chi|}2^{|\Theta|})$. Implementing step 2–4 in an iterative way may reduce the term $|\chi|^{|\chi|}$ of the time complexity.

3 Constructing Multiple Frames of Discernment

In this section we extend the methodology from Sec. 2 into a new methodology for constructing several multiple frames of discernment for different subproblems. This is done by extending *FA* (10) to several subsets. Let us define such a function of overall frame appropriateness.

Definition 2. Let the meta frame appropriateness function,

$$MFA(\{\chi_a\}_a) \triangleq \prod_a FA_a, \tag{14}$$

over several subproblems χ_a be the product of the frame appropriateness functions FA_a for these subproblems χ_a.

In order to find the best frames of discernment for these subproblems we maximize $MFA(\{\chi_a\}_a)$,

$$\max MFA(\{\chi_a\}_a) = \max \prod_a FA_a. \tag{15}$$

For computational reasons the actual maximization of *MFA* is done in several steps.

First, let us map *MFA* onto a Potts spin neural network that will cluster all belief functions into subsets using an approximation of *MFA* as a distance measure in such a manner that *MFA* is maximized. This will partition the belief functions into subsets that should be handle separately is such a way that it gives us the best overall frames of discernment for the subproblems.

Secondly, for each subproblem separately, a frame of discernment is constructed using the algorithm for constructing an appropriate frame of discernment, Fig. 2. With these frames of discernment each subproblem can be solved separately by combining all belief functions in the subset.

4 Potts Spin Theory

The Potts spin problem [18] consists of minimizing an energy function

$$E = \frac{1}{2}\sum_{i,j=1}^{N}\sum_{a=1}^{q} J_{ij}S_{ia}S_{ja} \tag{16}$$

by changing the states of the spins S_{ia}'s, where $S_{ia} \in \{0, 1\}$ and $S_{ia} = 1$ means that belief function i is in cluster a. This model serves as a clustering method if J_{ij} is used as a penalty factor when report i and j are in the same cluster.

The minimization is carried out by simulated annealing. In simulated annealing temperature is an important parameter. The process starts at a high and continues by gradually lowering the temperature. As the temperature is lowered the spins gradually become more influenced by the interactions J_{ij}'s so that a minimum of the energy function (16) is reached. This gives us the best partition of all belief functions into the clusters with minimal energy function.

For computational reasons we use a mean field model, where spins are deterministic with $V_{ia} = \langle S_{ia}\rangle$, $V_{ia} \in [0, 1]$. The Potts mean field equations are formulated [19] as

$$V_{ia} = \frac{e^{-H_{ia}[V]/T}}{\sum_{b=1}^{K} e^{-H_{ib}[V]/T}} \tag{17}$$

where

$$H_{ia}[V] = \sum_{j=1}^{N} J_{ij}V_{ja} - \gamma V_{ia}\,. \tag{18}$$

In order to minimize the energy function, (17) and (18) are iterated until a stationary equilibrium state has been reached for each temperature. Then, the temperature is lowered step–by–step by a constant factor until $\forall i, a.\ V_{ia} \approx 0, 1$ in the stationary equilibrium state.

The time complexity of Potts spin clustering was shown to be $O(N^2K^2)$ in terms of the number of belief functions N (= $|\chi|$) and the number of clusters K [7].

5 Mapping a Multiple Frame Construction Problem onto Potts Spin

In order to map the meta appropriateness function *MFA* onto a Potts spin network we need to rewrite *MFA* as a sum of terms similar to the energy function being minimized in (16). To find the best set of frames of discernment we maximize $MFA(\{\chi_a\}_a)$. This can be rewritten as a sum of terms over the subsets χ_a

$$\max MFA(\{\chi_a\}_a) = \max \prod_a FA_a, \tag{19}$$

$\Leftrightarrow$

$$\begin{aligned}\max \log \prod_a FA_a &= \max \log \prod_a \left[1 - Con(\oplus\{m_j|\Theta_a\})\right]\left[1 - \frac{AU(\oplus\{m_j|\Theta_a\})}{\log_2|\Theta_a|}\right] \\ &= \max \sum_a \log\left[1 - Con(\oplus\{m_j|\Theta_a\})\right] + \log\left[1 - \frac{AU(\oplus\{m_j|\Theta_a\})}{\log_2|\Theta_a|}\right] \\ &= \min \sum_a -\log\left[1 - Con(\oplus\{m_j|\Theta_a\})\right] - \log\left[1 - \frac{AU(\oplus\{m_j|\Theta_a\})}{\log_2|\Theta_a|}\right].\end{aligned} \tag{20}$$

Furthermore, we must also rewrite *MFA* as a sum of terms over pairwise simple support functions as all interactions in Potts spin are pairwise.

It was shown in [20] that minimizing a sum of $-\log(1 - s_k s_l)$ terms is an approximation correct to leading order, i.e., all second order terms in $\{s_i\}$ are unchanged in this approximation. The first term in the last line of (20) can be rewritten as

$$\sum_a -\log\left[1 - Con(\oplus\{m_j|\Theta_a\})\right] = \sum_a -\log\left[1 - \left(\sum_{\substack{k,l \\ S_k, S_l \in \chi_a}} s_k s_l - X\right)\right], \tag{21}$$

while the actual function being minimized in the neural network is

$$\begin{aligned}\sum_a \sum_{\substack{k,l \\ S_k, S_l \in \chi_a}} -\log(1 - s_k s_l) &= \sum_a -\log\left[\prod_{\substack{k,l \\ S_k, S_l \in \chi_a}} (1 - s_k s_l)\right] \\ &= \sum_a -\log\left[1 - \left(\sum_{\substack{k,l \\ S_k, S_l \in \chi_a}} s_k s_l - Y\right)\right],\end{aligned} \tag{22}$$

where X and Y are higher order terms.

The second term in the last line of (20) can be rewritten as

$$\sum_a -\log\left[1 - \frac{AU(\oplus\{m_j|\Theta_a\})}{\log_2|\Theta_a|}\right] \approx \sum_a -\log\left[1 - \left(\sum_{\substack{k,l \\ m_k, m_l \in \chi_a}} \frac{AU(\oplus\{m_k, m_l|\Theta_a\})}{(|\chi_a| - 1) \cdot \log_2(|\Theta_a|)} - W\right)\right] \tag{23}$$

when $|\chi_a| \geq 2$, and where *W* are higher order terms in *AU* (i.e., $(AU)^p,\ p \geq 2$).

When calculating $AU(\oplus\{m_j|\Theta_a\})$ and $AU(\oplus\{m_k, m_l|\Theta_a\})$ in (23) each leading term comes in twice in *AU*. However, since $AU(\oplus\{m_k, m_l|\Theta_a\})$ is summed up for all pairs m_k, m_l in χ_a the leading terms come in $2 \cdot (|\chi_a| - 1)$ times in the equation. To compensate for this multiple counting we must include a factor $1/(|\chi_a| - 1)$. This approximation is correct in its first order terms.

Thus, the function being minimized is

$$\sum_{a} \sum_{\substack{k,l \\ m_k, m_l \in \chi_a}} -\log\left(1 - \frac{AU(\oplus\{m_k, m_l | \Theta_a\})}{(|\chi_a| - 1) \cdot \log_2(|\Theta_a|)}\right) = \sum_{a} -\log\left[\prod_{\substack{k,l \\ m_k, m_l \in \chi_a}} \left(1 - \frac{AU(\oplus\{m_k, m_l | \Theta_a\})}{(|\chi_a| - 1) \cdot \log_2(|\Theta_a|)}\right)\right]$$
$$= \sum_{a} -\log\left[1 - \left(\sum_{\substack{k,l \\ m_k, m_l \in \chi_a}} \frac{AU(\oplus\{m_k, m_l | \Theta_a\})}{(|\chi_a| - 1) \cdot \log_2(|\Theta_a|)} - Z\right)\right] \tag{24}$$

which is identical in its first order terms to (23).

Thus, maximizing the meta frame appropriateness function $MFA(\{\chi_a\}_a)$ (15), is equivalent in its first order terms to minimizing

$$\min \sum_{a} \sum_{\substack{k,l \\ S_k, S_l \in \chi_a}} \left[-\log(1 - s_k s_l) - \log\left(1 - \frac{AU(\oplus\{m_k, m_l | \Theta_a\})}{(|\chi_a| - 1) \cdot \log_2(|\Theta_a|)}\right)\right]. \tag{25}$$

An algorithm for minimizing (25) is shown in Fig. 4. This is an adoption from [7]. Here the interactions J_{kl} are identical to (25). All parameters of Fig. 4. are immediate except for the number of clusters K. Its determination is domain dependent and can be found in several different ways as discussed in Sec. 1, e.g., using the method of [10, p. 90].

6 An algorithm for constructing multiple frames

Let us describe an algorithm for constructing the best frames of discernment for several multiple subproblems, χ_a, Fig. 3.

Input: A set χ of simple support functions or consonant belief functions.
Output: Frames of discernment $\{\Theta_a\}_a$ for all subproblems χ_a.
Step 1. Instantiate all interactions J_{ij} (in Fig. 4.) between all pairs in χ, using (10) in Fig. 1.
Step 2. Partition χ by minimizing *MFA* (14) using the Potts spin clustering algorithm, Fig. 4.
Step 3. For each subproblem χ_a use the algorithm to construct the most appropriate frame of discernment Θ_a, Fig. 2. Return $\{\Theta_a\}_a$.

Fig. 3. An algorithm for constructing multiple frames of discernment for multiple subproblems

Using this algorithm will construct a set of frames of discernment $\{\Theta_a\}_a$ for several subproblems χ_a that should be handled separately. This set of frames is best in terms of minimizing the overall frame appropriateness *MFA* over all subproblems.

7 Conclusions

We have extended a methodology for constructing a frame of discernment from incoming belief functions for *one* problem, into a methodology for constructing multiple frames of discernment for *several* different subproblems. This lets our evidence interact in an interesting way within each subproblem without exhibit too much internal conflict. This dual task is achieved simultaneously for all subproblems by maximizing a function of overall frame appropriateness over all subproblems.

INITIALIZE

K (number of clusters); N (= $|\chi|$) (number of simple support functions);

$$J_{ij} = \left[-\log(1 - s_i s_j)\delta_{|A_i \cap A_j|} - \log\left(1 - \frac{\mathrm{AU}(\oplus\{m_i, m_j|\Theta_a\})}{(|\chi_a| - 1) \cdot \log_2(|\Theta_a|)}\right)\right] \quad \forall i, j \text{, where}$$

$|\chi_a| = \frac{N}{K}$ and $|\Theta_a| = |C_i \cup C_j|$; $s = 0$; $t = 0$; $\varepsilon = 0.001$; $\tau = 0.9$; $\gamma = 0.5$;

$T^0 = T_c$ (a critical temperature) $= \frac{1}{K} \cdot max(-\lambda_{min}, \lambda_{max})$, where λ_{min} and λ_{max} are the extreme eigenvalues of M, where $M_{ij} = J_{ij} - \gamma\delta_{ij}$;

$V_{ia}^0 = \frac{1}{K} + \varepsilon \cdot rand[0,1] \quad \forall i, a$;

REPEAT–1

- REPEAT–2

  $\forall i$ Do:

  - $|\chi_a| = \max\left\{2, \sum_{i=1}^{N} V_{ia}^s\right\}$;
  - $H_{ia}^s = \sum_{j=1}^{N} J_{ij} V_{ja}^{\begin{cases} s+1, & j < i \\ s, & j \geq i \end{cases}} - \gamma V_{ia}^s \quad \forall a$;
  - $F_i^s = \sum_{a=1}^{K} e^{-H_{ia}^s / T^t}$;
  - $V_{ia}^{s+1} = \frac{e^{-H_{ia}^s / T^t}}{F_i^s} + \varepsilon \cdot rand[0,1] \quad \forall a$;
  - $s = s + 1$;

  UNTIL–2

  $\frac{1}{N}\sum_{i,a}\left|V_{ia}^s - V_{ia}^{s-1}\right| \leq 0.01$;

- $T^{t+1} = \tau \cdot T^t$;
- $t = t + 1$;

UNTIL–1

$\frac{1}{N}\sum_{i,a}(V_{ia}^s)^2 \geq 0.99$;

RETURN

$\left\{\chi_a \middle| \forall S_i \in \chi_a.\ \forall b \neq a\ V_{ia}^s > V_{ib}^s\right\}$;

Fig. 4. Clustering algorithm

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