Entropy-Based Counter-Deception in Information Fusion

Johan Schubert(✉)

Department of Decision Support Systems,
Division of Defence and Security, Systems and Technology,
Swedish Defence Research Agency, Stockholm, Sweden
schubert@foi.se
http://www.foi.se/fusion

Abstract. In this paper, we develop an entropy-based degree of falsity and combine it with a previously developed conflict-based degree of falsity in order to grade all belief functions. The new entropy-based degree of falsity is based on observing changes in entropy that are not consistent with combining only truthful information. With this measure, we can identify deliberately deceptive information and exclude it from the information fusion process.

Keywords: Deception · Counter-deception · Information fusion · Entropy · Conflict · Belief function · Dempster-Shafer theory

1 Introduction

Managing false and possibly deliberately deceptive information is, in general, an important issue within an information fusion process. If false and deceptive information is not actively managed, it becomes impossible to trust any conclusions that is based on combining information from several different sources without knowing if one is deceptive. Conclusions that are drawn based on a combination of information from all sources may become degraded or false when truthful information is combined with deceptive information that supports untrue possibilities.

We previously developed methods within the theory of belief functions [1–6] for clustering information regarding several different subproblems that should be managed separately when the information regarding different subproblems might be mixed up [7–11]. When we know that all information concerns only one problem at hand, this method could be used to identify false pieces of information and allow us to calculate a conflict-based degree of falsity for each piece of evidence [12]. These approaches use a function of the conflict [13, 14] in Dempster's rule [2] as criterion function.

Smets [15] developed a methodology for managing a special case of deception where a deceiver may observe a truthful report and send the complement of a truthful belief function as deception instead of the truthful report itself. Pichon *et al.* [16] later developed a correction scheme that generalizes Shafer's discounting rule [4] by taking into account uncertain meta-knowledge regarding the source relevance and truthfulness. This model now subsumes Smets' model. Furthermore, they recently introduced a contextual correction mechanism [17] for [16].

J. Vejnarová and V. Kratochvíl (Eds.): BELIEF 2016, LNAI 9861, pp. 174–181, 2016.
DOI: 10.1007/978-3-319-45559-4_18

However, the approach taken by Smets is a special case where the deceiver always sends the complement of what is observed from a truthful source. We think that this is not a realistic strategy by the deceiver, as it is easily countered by the counter-deception technique developed in Smets' approach [15]. Instead, we would allow the deceiver to act in any way it chooses and assume it might want to deceive us by supporting some focal elements of the frame of discernment that are wrong but we already somewhat believe. We think that this might be a more realistic approach.

In this paper, we develop an entropy-based measure of degree of falsity based on the change in entropy when truthful belief functions are combined with a deceptive belief function. The aim is that this new approach should be able to manage more generic types of deception than Smets' approach. As we have previously developed a conflict-based measure of degree of falsity [12] we will here combine these two approaches into one method for recognizing and managing deceptive information.

In Sect. 2, we discuss approaches to analyzing belief functions for their likelihood of being false due to deception. In Sect. 3, we review a previous approach to grading pieces of evidence for their degree of falsity based on their contribution to the conflict [13, 14] received from Dempster's rule [2]. We then develop a new complimentary approach for grading pieces of evidence based on such changes in entropy that are not consistent with adding truthful evidence into the combination of all belief functions (Sect. 4). In Sect. 5, we combine the previously developed conflict-based degree of falsity with the new entropy-based degree of falsity into a combined degree of falsity. We use this approach to reason about which pieces of evidence might be false and should be either discounted or eliminated from the combination of information from all sources. Finally, in Sect. 6, we present the study's conclusions.

2 Analyzing Belief Functions

A belief function that is constructed to be deliberately false may be discovered in two different ways. Such a belief function is aimed to change the conclusion when analyzing the combination of all belief functions. Thus, it must be different from truthful belief functions.

One way to find this is by observing the conflict when combining a new belief function with all previous belief functions. For each belief function at hand, we may observe the change in conflict if we remove this particular belief function from the entire set of all available belief functions [7, 18]. This will either lower the conflict or leave it unchanged. From the change in conflict, we can derive a degree of falsity for the belief function in question and, for example, use that to discount this particular belief function [12].

A second approach is to observe the change in entropy when receiving a new belief function. If we receive a good belief function about the problem at hand we should assume that it will further reduce both the scattering and the nonspecificity of the basic belief by focusing it on small focal sets containing the ground truth. Thus, the belief of the ground truth will gradually become more believed and the entropy of the combined belief function will approach zero. On the other hand, if we receive a false belief function that incrementally changes the belief function a small step towards a uniform mass function, then the entropy of the combined belief function will increase. A very strong false belief function may swap the preferred order of the focal sets and leave the entropy unchanged or increased.

We will use both of these approaches to identify which belief functions may be deceptive in order to manage or eliminate them completely from the combination.

3 Conflict-Based Degree of Falsity

We interpret the conflict received when combining a set of basic belief assignments (bbas) χ, as if there is at least one bba in χ that violates the representation of the frame of discernment Θ. Such a bba is interpreted as if it does *not* belong to the evidence that refer to the problem at hand [18].

A conflict when combining all bbas may thus be interpreted as a piece of evidence on a metalevel stating that at least one bba does not belong to χ.

We have,

$$\begin{aligned} m_\chi\left(\exists j.e_j \notin \chi\right) &= c_0, \\ m_\chi(\Theta) &= 1 - c_0, \end{aligned} \tag{1}$$

where χ is the set of all bbas, c_0 is the conflict when combining all bbas, e_j is bba number j, and Θ is the frame of discernment.

Let us study one particular piece of evidence e_q in χ. If e_q is removed from χ, the conflict when combining all remaining bbas in χ decreases from c_0 to c_q. This decrease is interpreted as if there exists some evidence on the metalevel indicating that e_q does not belong to χ [12],

$$\begin{aligned} &m_{\Delta\chi}\left(e_q \notin \chi\right), \\ &m_{\Delta\chi}(\Theta), \end{aligned} \tag{2}$$

where $\Delta\chi$ is a label for this piece of evidence.

The conflict that remains c_q after e_q has been removed from χ is interpreted as evidence on the metalevel that there is at least one other bba e_j, $j \neq q$, that does not belong to $\chi - \{e_q\}$.

We have,

$$\begin{aligned} m_{\chi-\{e_q\}}\left(\exists j \neq q.e_j \notin \left(\chi - \{e_q\}\right)\right) &= c_q, \\ m_{\chi-\{e_q\}}(\Theta) &= 1 - c_q. \end{aligned} \tag{3}$$

Using Eqs. (1) and (3), we can derive Eq. (2) by stating that the belief in the proposition that there is at least one bba that does not belong to χ, $\exists j.e_j \notin \chi$, must be equal, regardless of whether we base that belief on (1) before e_q is taken out from χ, or on the combination of (2) and (3) after e_q is taken out from χ.

That is,

$$\mathrm{Bel}_{\chi}\left(\exists j.e_j \notin \chi\right) = \mathrm{Bel}_{\Delta\chi\oplus\left(\chi-\{e_q\}\right)}\left(\exists j.e_j \notin \chi\right). \tag{4}$$

On the left hand side (LHS) of Eq. (4) we have,

$$\mathrm{Bel}_{\chi}\left(\exists j.e_j \notin \chi\right) = m_{\chi}\left(\exists j.e_j \notin \chi\right) = c_0 \tag{5}$$

and, on the right hand side (RHS) Eq. (4) we have,

$$\mathrm{Bel}_{\Delta\chi\oplus\left(\chi-\{e_q\}\right)}\left(\exists j.e_j \notin \chi\right) = c_q + m_{\Delta\chi}\left(e_q \notin \chi\right)\left(1 - c_q\right). \tag{6}$$

By stating that LHS = RHS, we derive the basic belief number (bbn) of Eq. (2) as,

$$\begin{aligned} m_{\Delta\chi}\left(e_q \notin \chi\right) &= \frac{c_0 - c_q}{1 - c_q}, \\ m_{\Delta\chi}(\Theta) &= \frac{1 - c_0}{1 - c_q}. \end{aligned} \tag{7}$$

We call this the conflict-based degree of falsity of e_q. For additional details, see [12].

4 Entropy-Based Degree of Falsity

Let us measure the change in entropy by observing the change in the aggregated uncertainty functional (AU) of the combination of all belief functions, both with and without the particular belief function in question e_q.

4.1 Aggregated Uncertainty Functional

The aggregated uncertainty functional AU was discovered by several authors around the same time [19–21]. AU is defined as

$$AU(\mathrm{Bel}) = \max_{\{p_x\}_{x\in\Theta}}\left\{-\sum_{x\in\Theta} p(x)\log_2 p(x)\right\} \tag{8}$$

where $\{p_x\}_{x\in\Theta}$ is the set of all probability distributions such that $p_x \in [0,1]$ for all $x \in \Theta$,

$$\sum_{x\in\Theta} p(x) = 1 \tag{9}$$

and

$$\mathrm{Bel}(A) \leq \sum_{x \in A} p(x) \tag{10}$$

for all $A \subseteq \Theta$. For an overview, see [22]. The AU measure corresponds to measures of nonspecificity and scattering that generalize Hartley information [23] and Shannon entropy [24].

An algorithm for numeric computation of AU was found by Meyerowitz *et al.* [25]. See [26] for implementation.

We define the entropy as a normalization of AU [27, 28],

$$Ent\left(\{m_j\}\right) = \frac{AU\left(\bigoplus\{m_j\}\right)}{\log_2|\Theta|} \tag{11}$$

where m_j is the set of all bbas under combination, $AU \in [0, \log_2|\Theta|]$ and $Ent \in [0, 1]$.

Using Ent and AU, we may define an entropy-based degree of falsity for a deceptive piece of evidence as

$$\begin{aligned} m_{\Delta Ent}\left(e_q \notin \chi\right) &= Ent_q\left(\left\{m_j | j \neq q\right\}_j\right) - Ent_0\left(\left\{m_j\right\}_j\right), \\ m_{\Delta Ent}(\Theta) &= 1 - m_{\Delta Ent}\left(e_q \notin \chi\right), \end{aligned} \tag{12}$$

where Ent_0 is the entropy with e_q included in the combination, and Ent_q is the entropy without e_q, under the assumption that $m_{\Delta Ent}\left(e_q \notin \chi\right) \geq 0$. Provided that the difference in Eq. (12) is positive and that there is no change in the bbn of the top focal element, this may serve as an adequate measure of falsity for a deceptive piece of evidence based on change of entropy. For a deceptive piece of evidence that changes the order of focal elements we may have a negative difference. For truthful evidence we expect a negative difference and would like to define the degree of falsity as zero. For a general and incremental approach that takes these situations into account see Sect. 4.2.

4.2 Incremental Steps of Entropy Change

Let us focus on e_q, which we want to evaluate by changes in entropy Ent. Because the entropy might increase when we remove e_q we will study a series of incremental changes. We will discount mass function m_q at different rates and observe the incremental changes in entropy. We have [2],

$$m_q(A) = \begin{cases} \alpha m_q(A), & A \subset \Theta \\ 1 - \alpha + \alpha m_q(A), & A = \Theta \end{cases} \tag{13}$$

where $0 \leq \alpha \leq 1$. Let α be defined as

$$\alpha = \frac{i}{n}, \tag{14}$$

where n is a parameter of choice with $0 \leq i \leq n$.

We have,

$$m_q^i(A) = \begin{cases} \frac{i}{n} m_q(A), & A \subset \Theta \\ 1 - \frac{i}{n} + \frac{i}{n} m_q(A), & A = \Theta \end{cases}. \tag{15}$$

Let $\Delta Ent_q^{k+1,k}$ be the incremental change in entropy between two situations using m_q^{k+1} and m_q^k, respectively, in the calculation of $\Delta Ent_q^{k+1,k}$.

We have,

$$\Delta Ent_q^{k+1,k} = Ent_q\left(\left\{m_q^{k+1}, m_j | j \neq q\right\}_j\right) - Ent_q\left(\left\{m_q^{k}, m_j | j \neq q\right\}_j\right). \tag{16}$$

We may extend Eq. (12) to define an incremental entropy-based degree of falsity as

$$\begin{aligned} m_{\Delta Ent}(e_q \notin \chi) &= \frac{1}{2}\sum_{k=0}^{n-1} \begin{cases} 0, & \forall 0 \leq l \leq k.\ \Delta Ent_q^{l+1,l} \leq 0 \\ \left|\Delta Ent_q^{k+1,k}\right|, & \text{otherwise} \end{cases}, \\ m_{\Delta Ent}(\Theta) &= 1 - m_{\Delta Ent}(e_q \notin \chi), \end{aligned} \tag{17}$$

using Eq. (16).

As long as we receive a sequence of negative incremental changes, we consider m_q to be true. However, if there is a positive incremental change this is interpreted (to a degree) that this piece of evidence is false. The sequential inclusion of m_q may eventually cause a flip in the preferred focal element, followed by a series of negative incremental changes that must be counted towards the degree of falsity when the distribution becomes more and more focused around false focal elements.

This information, $m_{\Delta Ent}(e_q \notin \chi)$, can serve as an indication that m_q might be deliberately false, and may function as an indication even if the direct conflict with the main body of truthful evidence is low.

5 Combine Degree of Falsity with Change of Entropy

In order to find which pieces of evidence might be false, we combine $m_{\Delta\chi}(e_q \notin \chi)$ with $m_{\Delta Ent}(e_q \notin \chi)$ by Dempster's rule; i.e., $m_{\Delta\chi}(e_q \notin \chi) \bigoplus m_{\Delta Ent}(e_q \notin \chi)$. This is a conflict-free combination as both mass functions have the same foci.

We get,

$$\begin{aligned} m_{\Delta\chi \bigoplus \Delta Ent}(e_q \notin \chi) &= m_{\Delta\chi}(e_q \notin \chi) + m_{\Delta Ent}(e_q \notin \chi) \\ &\quad - m_{\Delta\chi}(e_q \notin \chi) \cdot m_{\Delta Ent}(e_q \notin \chi), \\ m_{\Delta\chi \bigoplus \Delta Ent}(\Theta) &= 1 - m_{\Delta\chi \bigoplus \Delta Ent}(e_q \notin \chi), \end{aligned} \tag{18}$$

by using Eq. (7) and Eqs. (11), (15)–(17) and the algorithm in [26] to compute Eq. (8).

Based on this results (of Eq. (18)) we can manage all m_q ($\forall q$) in one of several different ways:

1. We may discount all m_q based on $m_{\Delta\chi \bigoplus \Delta Ent}(e_q \notin \chi)$ using Eq. (13) with $\alpha = 1 - m_{\Delta\chi \bigoplus \Delta Ent}(e_q \notin \chi)$. Evidence with a high degree of combined conflict-based and entropy-based falsity will be discounted to its degree with a low α. Subsequently, we handle all evidence with whatever mass remains after discounting as if it is true. This approach is somewhat crude and may not be the most preferable way to manage all evidence.
2. A more refined approach is to perform sequential incremental discounts using increments of $\alpha = 1 - m_{\Delta\chi \bigoplus \Delta Ent}(e_q \notin \chi)$ as was suggested in [12]. With that approach it is possible to manage the conflict by appropriate discounts that bring the conflict down to an acceptable level.
3. A third approach is to evaluate and rank all m_q based on $m_{\Delta\chi \bigoplus \Delta Ent}(e_q \notin \chi)$ and if there is a natural partition of all m_q into two groups (corresponding to true and false reports) we eliminate the false group from the combination.

We think that managing all evidence in an interactive and incremental way using Eq. (18) and Approach 3 above whenever possible is a good way to find and manage deceptive information in an information fusion process.

6 Conclusions

We have developed an approach for counter-deception in information fusion. This method combines the study of conflict in Dempster's rule with observation of changes in entropy to determine which belief functions are deceptive. With this methodology, we can prevent deceptive information from being included in the information fusion process.

References

1. Dempster, A.P.: Upper and lower probabilities induced by a multivalued mapping. Ann. Math. Stat. **38**, 325–339 (1967)
2. Dempster, A.P.: A generalization of Bayesian inference. J. R. Stat. Soc. Ser. B **30**, 205–247 (1968)
3. Dempster, A.P.: The Dempster-Shafer calculus for statisticians. Int. J. Approx. Reason. **48**, 365–377 (2008)
4. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press, Princeton (1976)
5. Yager, R.Y., Fedrizzi, M., Kacprzyk, J. (eds.): Advances in the Dempster-Shafer Theory of Evidence. Wiley, New York (1994)
6. Smets, P., Kennes, R.: The transferable belief model. Artif. Intell. **66**, 191–234 (1994)
7. Schubert, J.: On nonspecific evidence. Int. J. Intell. Syst. **8**, 711–725 (1993)
8. Schubert, J.: Managing inconsistent intelligence. In: 3rd International Conference on Information Fusion, pp. TuB4/10-16 (2000)

9. Schubert, J.: Clustering belief functions based on attracting and conflicting metalevel evidence using potts spin mean field theory. Inf. Fusion **5**, 309–318 (2004)
10. Schubert, J.: Clustering decomposed belief functions using generalized weights of conflicts. Int. J. Approx. Reason. **48**, 466–480 (2008)
11. Schubert, J., Sidenbladh, H.: Sequential clustering with particle filtering - estimating the number of clusters from data. In: 8th International Conference on Information Fusion, paper A4-3 (2005)
12. Schubert, J.: Conflict management in Dempster-Shafer theory using the degree of falsity. Int. J. Approx. Reason. **52**, 449–460 (2011)
13. Smets, P.: Analyzing the combination of conflicting belief functions. Inf. Fusion **8**, 387–412 (2007)
14. Schubert, J.: The internal conflict of a belief function. In: Denoeux, T., Masson, M.-H. (eds.) Belief Function: Theory and Applications. AISC, vol. 164, pp. 169–177. Springer, Heidelberg (2012)
15. Smets, P.: Managing deceitful reports with the transferable belief model. In: 8th International Conference on Information Fusion, pp. 893–899 (2005)
16. Pichon, F., Dubois, D., Denœux, T.: Relevance and truthfulness in information correction and fusion. Int. J. Approx. Reason. **53**, 159–175 (2012)
17. Pichon, F., Mercier, D., Lefèvre, É., Delmotte, F.: Proposition and learning of some belief function contextual correction mechanisms. Int. J. Approx. Reason. **72**, 4–42 (2016)
18. Schubert, J.: Specifying nonspecific evidence. Int. J. Intell. Syst. **11**, 525–563 (1996)
19. Chau, C.W.R., Lingras, P., Wong, S.K.M.: Upper and lower entropies of belief functions using compatible probability functions. In: Komorowski, J., Raś, Z.W. (eds.) ISMIS 1993. LNCS, vol. 689, pp. 306–315. Springer, Heidelberg (1993)
20. Maeda, Y., Ichihashi, H.: An uncertainty measure with monotonicity under the random set inclusion. Int. J. Gen Syst **21**, 379–392 (1993)
21. Harmanec, D., Klir, G.J.: Measuring total uncertainty in Dempster-Shafer theory: a novel approach. Int. J. Gen Syst **22**, 405–419 (1994)
22. Klir, G.J.: Uncertainty and Information: Foundation of Generalized Information Theory. Wiley, Hoboken (2006)
23. Hartley, R.V.L.: Transmission of information. Bell Syst. Tech. J. **7**, 535–563 (1928)
24. Shannon, C.E.: A mathematical theory of communication. Bell Syst. Tech. J. **27**(379–423), 623–656 (1948)
25. Meyerowitz, A., Richman, F., Walker, E.: Calculating maximum-entropy probability for belief functions. Int. J. Uncertain. Fuzziness Knowl.-Based Syst. **2**, 377–389 (1994)
26. Harmanec, D., Resconi, G., Klir, G.J., Pan, Y.: On the computation of uncertainty measure in Dempster-Shafer theory. Int. J. Gen Syst **25**, 153–163 (1996)
27. Schubert, J.: Constructing multiple frames of discernment for multiple subproblems. In: Hüllermeier, E., Kruse, R., Hoffmann, F. (eds.) IPMU 2010. CCIS, vol. 80, pp. 189–198. Springer, Heidelberg (2010)
28. Schubert, J.: Constructing and evaluating alternative frames of discernment. Int. J. Approx. Reason. **53**, 176–189 (2012)