

# Equivalence classes of future paths for sensor allocation and threat analysis

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## ABSTRACT

Sensor allocation and threat analysis are difficult fusion problem that can sometimes be approximately solved using simulations of the future movement of adversary units. In addition to requiring detailed motion models, such simulation also requires large amounts of computational resources, since a large number of possibilities must be examined. In this paper, we extend our previously introduced framework for doing such simulations more efficiently. The framework is based on defining equivalence classes of future paths of a set of units. In the simplest case, two paths are considered equivalent if they give rise to the same set of observations.

For sensor management, each considered sensor plan thus entails an equivalence relation on the set of future paths. This can be used to significantly reduce the number of "alternative futures" that need to be considered for the simulation.

For threat analysis, the equivalence relation can instead be based on the perceived threat against own units. We describe how the equivalence classes induced by such relations could be used to improve the visualization of threat analysis systems. User interaction can also be used to refine the equivalence classes; we argue that such interaction will be essential for international operations where it is difficult to define actors and targets.

**Keywords:** Sensor management, resource allocation, threat analysis

## 1. INTRODUCTION

Sensor resource management and threat/impact analysis (level 3 and 4 fusion in the JDL model<sup>1</sup>) are integral parts of future information fusion systems. In the kind of situations that face us in international operations other than war (OOTW), we need fast and reliable such systems that work even if we have less detailed doctrinal knowledge on the enemy than in traditional warfare.

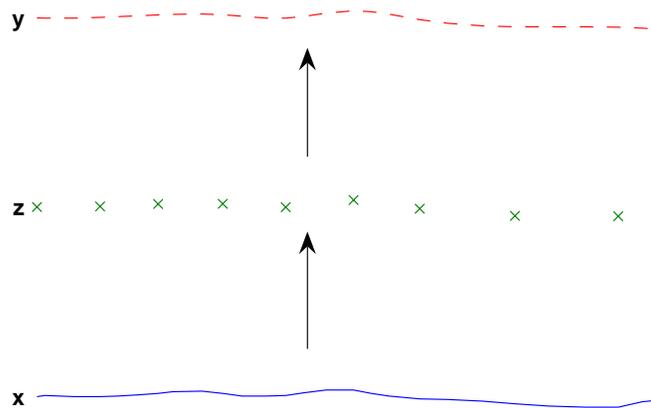
In this kind of operations, we will face adversaries that organize themselves in an ad hoc manner in order to achieve their goals. The presence of civilians in the areas of operations also adds additional challenges: the enemy may try to hide among them, splitting into small groups that take separate routes to their destination. It is also likely that we will need to process and fuse different kinds of sensor observation reports than before: in urban situations where the enemy is hard to distinguish from the civilians, HUMINT from observers or from automatic processing of, *e.g.*, mass media reports will be increasingly used. New small target tracking and fusion methods with new kinds of sensor observations are needed in order to track and predict what such small groups are planning and achieve a satisfactory situation awareness.

In this paper, we describe how the concept of equivalence classes of future multi-target paths could be used in level 3 and 4 fusion systems. In a previous paper,<sup>2</sup> we discussed equivalence classes of future multi-target paths for sensor policy evaluation and presented results from simulations of the method. In this paper, we extend the concept to threat and impact analysis, and discuss possible connections to plan recognition.

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**Figure 1.** Example of equivalence classes of future paths for single-target sensor management. A simulated future  $x$  and a sensor scheme  $s$  to generate observations  $z$  which are input into a filter giving a track  $y$  which can be compared to  $x$  in order to evaluate  $s$ .

Briefly, an equivalence class of future multi-target paths is a set of paths whose members we cannot distinguish from each other. The simplest example is to consider paths of a target that generate the same sensor observations, but it is also possible to consider several paths or courses of actions to be equivalent if they give rise to the same effects or high-level impact. In OOTW operations where there will be many different actors whose actions may or may not be coordinated, we can achieve a significant reduction in the number of alternatives that need to be considered by the user. For example, in a scenario with both peaceful protesters and militant rioters, we can consider all futures that threaten, *e.g.*, the TV station equivalent. In that case, high-level planning need not care about which of the adversary groups that threaten it. (Note that for low-level planning and prevention of threats, it will of course be necessary to include the identity and previous actions of the threatening group.)

In principle, all simulation-based sensor management and threat analysis system could be enhanced by partitioning the set of possible futures in such a way. The idea could also be used to cut down on the number of possible (future or current) situation pictures that need to be displayed to a specific user. By considering two such situation pictures to be equivalent *for a specific user* if their differences are irrelevant for that user, the system could select which alternatives to display for different users. It would also be possible to pre-process the global situation picture and partition its alternatives in equivalence classes before doing local processing on them.

This paper is outlined in the following way. Section 2 formalizes the notion of equivalence classes of multi-target paths and reviews its use for sensor management. In sections 3 and 4, we describe how the concept could be used for threat analysis and situation awareness, including global versus local short-time prediction.

## 2. EQUIVALENCE CLASSES

Figure 1 (from reference<sup>2</sup>), shows a conceptual view of simulation-based fusion systems. Given the current situation, we simulate a possible future (bottom of figure). By simulating sensors and observers, we get a set of sensor reports describing this (simulated) future, shown in the middle of the figure. These are input into a fusion system which gives the output shown at the top of the figure. The figure shows how this would look like for a single-target tracking system; extensions to other systems are discussed below.

Note that while the figure shows sensor observations in the middle, we could use a more general “view” operator to select those components of the simulated future that are interested from our point of view or given our knowledge on what effects the enemy are seeking.

This simulation-based fusion system can also be described mathematically. We assume that we are given a density vector  $x_0$ , which describes the positions of the units of interest at time  $t = 0$ , and that  $V$  describes the set of possible view operators.

Three different random sets<sup>3,4</sup> are used:

1.  $\mathbf{X}(t)$  denotes the situation picture at time  $t$ , conditioned on it being equal to  $\mathbf{x}_0$  at time 0. It can be seen as representing a simulation of ground truth: the instance  $\hat{x}(t)$  of  $\mathbf{X}(t)$  occurs with probability  $P[\mathbf{X}(t) = \hat{x}(t) | \mathbf{X}(0) = x_0]$ . For simplicity of notation, the conditioning on  $x_0$  is not explicitly shown in the following.
2. For each view operator  $v(t) \in V$  and instance  $\hat{x}(t)$  of the future ground truth, a set of possible observations  $\mathbf{Z}(x(t), s(t), t)$  is calculated at time  $t$ .  $\mathbf{Z}$  is also a random set; note that it depends on the simulated ground truth as well as on  $v$ . In the case that  $v$  does not represent a sensor allocation scheme,  $\mathbf{Z}$  represents the aspects of  $x(t)$  that are interesting for the operator or system that implements the method. This could, for instance, be based on spatial distances or targets of interests.
3. Finally, we feed the observations  $\mathbf{Z}$  to a fusion system, in order to determine what the future situation picture would be. This gives rise to the final random set,  $\mathbf{Y}(t)$ .  $\mathbf{Y}(t)$  is our fusion system's approximation of the (simulated) ground truth  $\hat{x}(t)$  using the observations  $\mathbf{Z}$ .

Note that all of the random sets introduced are explicitly time-dependent. Here, an expression like  $P[\mathbf{X}(t)]$  denotes the probability of the entire time-evolution of  $\mathbf{X}(t)$ , not just the probability at a specified time.  $P[\cdot]$  can thus be seen as a "probability density functional" in the space of all explicitly time-dependent random sets.

We have previously<sup>2</sup> used this approach to evaluate possible sensor allocation schemes. For that purpose, the set  $V$  was a set of sensor allocations  $S$ ,  $\mathbf{Z}$  was a set of simulated sensor observations and  $y$  was the output of a simulated particle filter.

Which sensor allocation scheme to use was determined by comparing the assumed ground truth  $\hat{x}(t)$  to the fusion system's simulated view  $\hat{y}(t)$ , using a fitness measure  $h(x(t), y(t))$ . By averaging over the ensemble of observations  $\mathbf{Z}$  and simulated fusion process output  $\mathbf{Y}$ , a fitness is determined for each scheme

$$H(\hat{x}(t), s) = \int P[\mathbf{Z}(t) = \hat{z}(t) | \mathbf{X}(t) = \hat{x}(t), s] \times P[\mathbf{Y}(t) = \hat{y}(t) | \mathbf{Z}(t) = \hat{z}(t)] \times h(\hat{x}(t), \hat{y}(t)) d\hat{z}(t) d\hat{y}(t). \quad (1)$$

If we average also over the random set  $\mathbf{X}(t)$ , we can get the best sensor allocation scheme as

$$s^{\text{best}} = \arg \min_{s(t) \in S} \int P[\mathbf{X}(t) = \hat{x}(t)] H(\hat{x}(t), s(t)) dx(t) \quad (2)$$

Implementing equations (1) and (2) is computationally infeasible for all but the smallest problems, so some sort of approximation is needed.

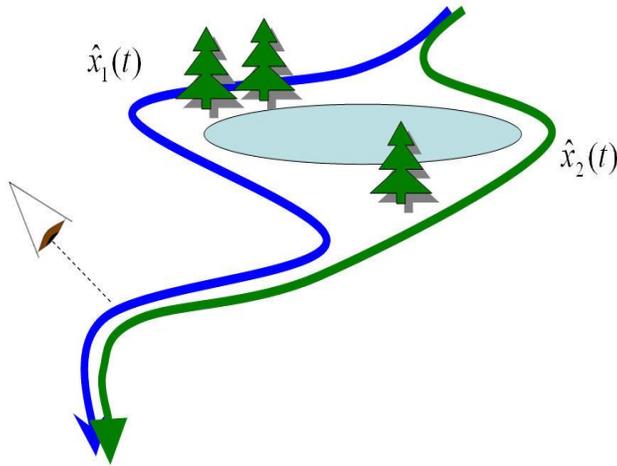
A different approach to computing these equations is to first rewrite them in terms of equivalence classes of future multi-target paths. Consider two alternative future paths  $\hat{x}_1(t)$  and  $\hat{x}_2(t)$ . If  $\hat{x}_1(t)$  and  $\hat{x}_2(t)$  are very similar, it might not be necessary to simulate them both. Consider, for example, the two sets of paths shown in figure 2. If the sensor is located as shown, it will not be possible for it to distinguish between the two paths. (We are here assuming that the sensor detection probability  $p_D = 1$ ; see below.)

The concept can be formalised mathematically by saying that define two possible future ground truths  $\hat{x}_1(t)$  and  $\hat{x}_2(t)$  are equivalent with respect to the sensor allocation scheme  $s(t)$  if

$$\mathbf{z}(\hat{x}_1(t), s(t), t) = \mathbf{z}(\hat{x}_2(t), s(t), t) \quad (3)$$

for all  $t$ , and write that  $\hat{x}_1(t) \sim_s \hat{x}_2(t)$ . Note that the sensor allocation scheme  $s$  controls several sensors, not just one.

We exploit the equivalence classes by rewriting equation 2 as



**Figure 2.** This figure illustrates the concept of equivalent paths. Given the sensor location shown, it is impossible to distinguish between the two paths; hence they belong to the same equivalence class.

$$s^{\text{best}} = \arg \min_{s(t) \in S} \int_{\hat{x}(t) \in X_s} P[\hat{x}(t)] H(\hat{x}(t), s(t)) d\hat{x}(t) \quad (4)$$

where  $X_s$  denotes the partition of  $X$  induced by the equivalence relation  $\sim_s$  defined above.

By removing the restriction that equation 3 should hold for all  $t$ , we can adapt the method for short-time sensor management: in this case, the number of distinct equivalence classes will be fewer, leading to fast implementations.

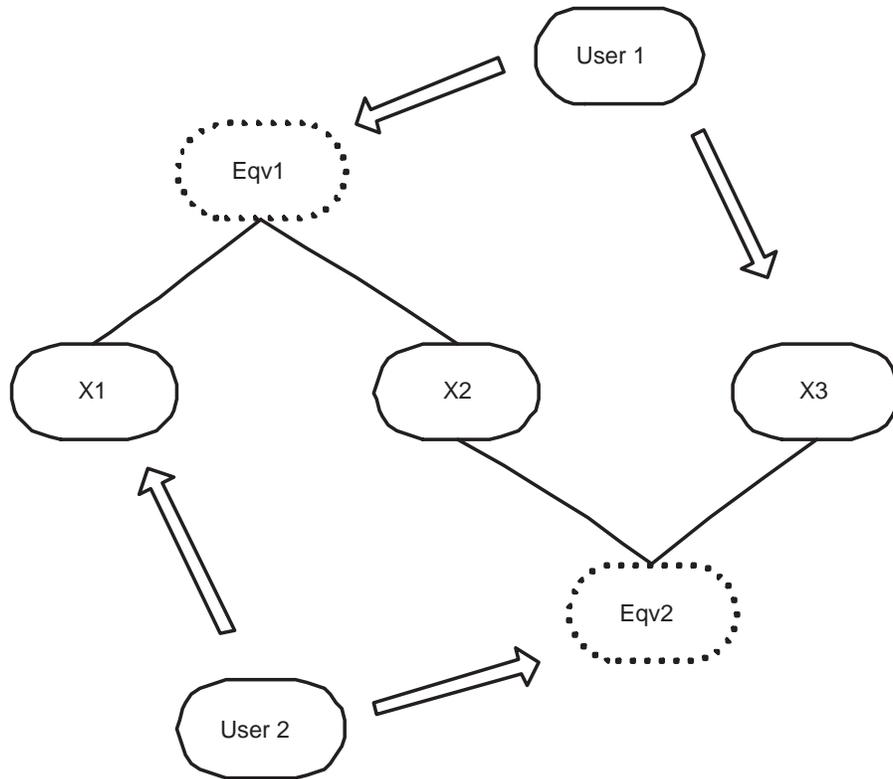
Formally, the definition of equivalence classes as given above is only correct for sensors with detection probability  $p_D = 1$ . The cause of the problem is that when comparing the sensor observations generated by two simulated future ground-states, a dependency on the stochastic process used to generate the observations is induced. This means that we should amend the definition of an equivalence class to also include this stochastic process, for example by including the seed of the random number generator used in computer simulations. Another approach to defining proper equivalence classes would be consider a two-layer structure of random sets.

In future work, we plan to extend this work to generate and evaluate sensor allocation schemes in cooperation with a user for high-level resource management problems (level 5 fusion<sup>5</sup>). It would also be interesting to implement the ideas for low-level platform or sensor control. While such systems may not need to process as many different possible futures as the higher-level planning systems envisioned here, they face higher demands for faster than real-time response. This means that the use of algorithms that directly generate equivalence classes (as described in section 5 below) instead of enumerating all possible futures will be needed. There are many existing approaches to sensor and resource management that could be extended to use the ideas presented here.<sup>6,7</sup>

Kadar et al<sup>8</sup> have recently described a system that predicts a future enemy path and uses this to determine figures-of-merit for dynamic sensor planning. In addition to our use of equivalence classes of multi-target paths, another major difference between that work and the method presented here is that we average over many predicted paths as well as over many realizations of the observations and of the emulated fusion module.

### 3. SITUATION AWARENESS

The equivalence relation introduced in section 2 can also be used to filter what information to display for different users. By constructing different view operators that correspond to a specific user's interests, the decision support system can select to display only those alternative hypothesis on the current or future situation that are distinct to them, as indicated in figure 3.



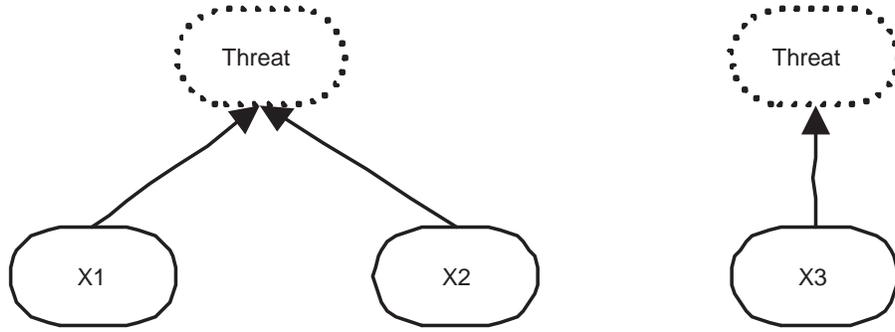
**Figure 3.** Equivalence classes for situation awareness. Three different alternative situation pictures are shown. For user 1,  $x_1$  and  $x_2$  are equivalent, while for user 2 it is  $x_2$  and  $x_3$  that cannot be distinguished. When forming their situation awareness and planning their response, the users only need to take account of the equivalence classes of alternatives.

Note that this is not the same as displaying only those alternatives that are relevant or interesting to a user: the entire situation-picture or prediction would be displayed. But the number of alternatives displayed would be reduced. In addition to reducing the cognitive load of the users, this also enables the decision support system to do more local processing, like short-term prediction for planning. By organizing the view operators for different command levels hierarchically, it might also be possible to combine the local processing of several nodes in the system.

It would be interesting to use the situation-picture comparison concepts introduced in<sup>9,10</sup> to determine equivalence classes of situation-pictures. This should be combined with user input to prune the number of alternatives that need to be considered. Since the methods for comparing situation-pictures are somewhat coarse, it would be necessary to determine threshold values for when to consider two situation-pictures sufficiently similar. (Note that this would mean that the partitioning would not be in strict equivalence classes, since transitivity would not hold if we allowed a finite difference between two situation-pictures.) One use of this could for instance be as pre-processing before high-level decisions are made: automatic or semi-automatic low-level systems will probably produce a large number of hypotheses on the current situation-picture as well as short-time predictions. When such hypotheses are combined at, *e.g.*, an FHQ, there is a need to cut down on the number of alternatives presented to the decision-makers. By presenting (approximate) equivalence classes instead, it is easier to determine the high-level plan to follow.

#### 4. THREAT ANALYSIS

An interesting use of equivalence classes is to consider view operators that correspond to the predicted threat that different opposing actors pose. A simple example is shown schematically in figure 4. Here, three possible futures are shown. A threat analysis system predicts that the effects of  $x_1$  and  $x_2$  will be the same. A planning system that helps the human operator to plan possible responses thus does not need to display or process both those futures.



**Figure 4.** In this figure, the future situation-pictures  $x_1$  and  $x_2$  are equivalent, since the impact analysis module predicts the same threat for them.

We could also base equivalence classes on perceived enemy plans. After recognizing plans based either on enemy movement (as implemented in<sup>11-13</sup>) or based on what capabilities the enemy has (as suggested in<sup>14</sup>) we can partition them into equivalence classes based on their total or local effects. As in the case for situation-pictures, here too there would be a need to consider the perspectives of different users. One could also partition a plan into its constituent parts, creating an abstract representation of the enemy's goals.

Often, the plan recognition modules will not work perfectly, *i.e.*, we will not attain perfect knowledge of what the enemy is planning to do. In such cases, it would be useful to first find the equivalence class of plans that represents the most interesting or threatening future actions and then try to guide sensor platforms in such a way that we can distinguish between the different futures contained in the equivalence class, in order to prevent the enemy from carrying out their plan.

## 5. ALGORITHMS

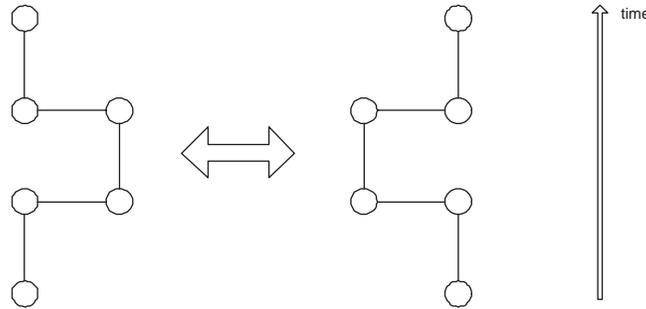
In order to make use of equivalence classes, they must be incorporated in fusion algorithms. There are two different ways this can be done. One alternative is to simulate several future situation-pictures and then partition them into equivalence classes, while the other would generate the equivalence classes directly, without first wasting time in generating more than necessary futures. In a previous paper,<sup>2</sup> we implemented such an algorithm for evaluating sensor allocation schemes.

This approach to developing algorithms can be compared to continuous-time Monte Carlo algorithms.<sup>15</sup> In standard Monte Carlo algorithms, moves are generated and then either accepted or rejected in each time-step. The acceptance probability depends on the properties of the system and should be chosen so that the process approximates the physical process one wishes to model. In contrast, in continuous-time Monte Carlo simulations the proposed move is *always* accepted. However, simulation-time is updated by a timestep  $\Delta t$  which is not necessarily 1.  $\Delta t$  is chosen to reflect the mean amount of time that would have been needed to accept the move in a standard Monte Carlo simulation, *i.e.*, it is given by  $\Delta t = e^{\Delta F}$ , where  $\Delta F$  is the free energy barrier between the current and the proposed states. The method is very useful in studying systems with large entropic barriers; it would be interesting to apply it to high-level threat analysis using particle filters.

Another possible way to develop more efficient algorithms for generating equivalence classes is to use ideas from simulations of polymers and random walks. A single-target path can be compared to a directed polymer. As shown in figure 5, polymer Monte Carlo simulations are performed by proposing moves of parts of the polymer chain.<sup>16</sup> In a similar way, the currently considered future single-target path could be changed at certain (future) times, while keeping the position at time 0 fixed. For multi-target future paths, moves could be added that allow distinct chains to exchange parts. In such simulations the proposal distribution of moves should be adapted so that moves are only accepted if they lead to a state that belongs to a different equivalence class of futures.

## 6. DISCUSSION

In this paper, we presented some ideas for how to extend the concept of equivalence classes of future multi-target paths to threat and situation analysis. There are many opportunities for future work in this area. In addition to developing algorithms along the lines outlined in section 5, we also need to extend the methods for plan recognition referenced in section 4. It



**Figure 5.** This figure shows one possible move when simulating directed polymers. Similar moves could be used to simulate equivalence classes; multi-target paths could be considered a set of polymers with time as the restricted direction.

would be interesting to define equivalence classes of the graph structures of inferred Bayesian networks for situation and threat analysis. Finding the best structure for analysing situations is an important and difficult optimisation problem. In OOTW operations, it will be necessary to quickly adapt the Bayesian networks used to deal with new situations. Using equivalence classes of structures would make it easier to search through the space of all possible structures.

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