Target tracking in archipelagic ASW: a not-so-impossible proposition ?

Karsten Jöred and Per Svensson

Department of Information System Technology
Defence Research Establishment,
SE-172 90 Stockholm, Sweden

Abstract

This paper deals with the problem of tracking the motion of a submarine in shallow waters using adaptive planning for the positioning of passive sonobuoys. A computational model and an interactive simulation system were developed and validated. The system is capable of simulating a submarine hunt using multi-sensor data fusion of signals from passive sonobuoys and of computing a near-optimal placement of the next buoy to be deployed.
Keywords: Submarine tracking, Kalman filter, Position prediction, Sonar equation, Adaptive planning.

Introduction

The goal of this study is to determine the possible benefits of using multi-sensor data fusion and adaptive planning when tackling the difficult problem of determining and tracking the position of a submarine in archipelagic anti-submarine warfare (AASW). Based on a simplified gaming scenario, we investigate effects of employing multisensor data fusion and optimal sensor allocation in shallow-water target tracking using passive, non-directional sonobuoys (5): a single target moving along a predetermined two-dimensional path is to be followed as long as possible, given a limited supply of sonobuoys and a prespecified tracking performance.

During the game, either the user or the system itself may place buoys at optional locations within the area. The information acquired from the sonobuoys is used to calculate the position of the submarine. This is done by solving a linear least-squares problem, arising from an average-sensor-position-centered formulation of the time differences of arrival (TDOA) equations Blixt (1), Wahlstedt et al (6). The signal-to-noise ratios at the positions of the sonobuoys are calculated by use of the sonar equation Burdic (2). The sonobuoy-position uncertainties are taken into account to calculate an elliptical confidence region for the location of the submarine (6).

A Kalman filter-based prediction method Bar-Shalom and Li (3), Sorenson (4) for the near future position of the target was developed. This method models the kinematics of a "generic" submarine and fuses this apriori knowledge with the sensor measurements to obtain an optimal estimate of the submarine's position at each point in time. To enable effective automatic buoy deployment an algorithm was designed and implemented, which calculates a near-optimal position of the next sonobuoy to be deployed.

We are grateful to Erland Sangfelt, Staffan Harling and Sven-Lennart Wirkander of the Defence Research Establishment, and to Mats Nordin, Department of Marine Technology at Chalmers University of Technology, for their expert advice during our validation of the model.

Simulation Model for Position Estimation from Sonobuoy Signals

For each discrete time step, the signal-to-noise ratio at the locations of the sonobuoys is computed. If the ratio is considered to be sufficiently high, and if the integration time has been reached, the information from that sonobuoy is taken into account. Next, the TDOA of the sound reaching the buoys is calculated. Using this the most likely target position and its confidence region are estimated.

The source level of the submarine target is dependent on its speed. The local sound level decreases monotonically with the distance from the source, as described by the sonar equation.

Since in our concept the distance between target and sensor is much shorter than is usually assumed in sonar detection models, in the order of a few hundred meters, it was decided that neither sound refraction nor reflections from features in the environment need to be considered.

The submarine follows a predefined polygonal path with a predefined speed. The game starts with a first detection of the submarine being presented to the player.

The player may interactively specify the location of each sonobuoy. Sonobuoys can not be retrieved for reuse during a game. Only a limited (preset) number of buoys are available.

Sonar Equation

The sonar equation (2) states the relationship between the emitted sound level, the received sound level, the environment conditions, and the sonar equipment. The equation is:

$$(SL - TL) - (NL - DI) = DT \ ,$$

where SL is the Source Level, TL the Transmission Loss between source and sonobuoy, NL the ambient noise level, DI the Directivity Index, modelling a noise reduction capability of certain types of sonobuoy, and DT the Detection Threshold, all expressed in dB. The calculation of DT will be described below, while for a discussion of the remaining parameters we refer to Johansson and Svensson (7) and (2).

Detection Threshold

This is a measure of the minimum sound level at the position of the buoy required to detect the signal. Assuming the target is a modern submarine without discernible resonances, the broadband detection threshold needs to be used. From (2):

$$DT = 5\log d - 5\log T\beta \qquad (1)$$

Here T denotes the integration time of the signal and β denotes the bandwidth. The choice of integration time is crucial to the behaviour of the system. If it is set too short, detection becomes impossible and if it is set too long tracking will not be possible.

To determine the signal-to-noise ratio d we have to decide values of Pfa, the false alarm probability, and PD, the detection probability. Once this has been done the corresponding value of d can be obtained from the ROC curve (2). We have chosen Pfa =0.0001, PD =0.99, β=1000 Hz and T=4 sec. Thus we get d=36 and finally, DT=-10.

The Least Squares Fix Method

The Hyperbolic Fix Method (6) is commonly used to calculate target positions from sonobuoy signals, based on TDOA measurements from two pairs of sonobuoys and resulting in a generally non-unique solution of a non-linear system of equations. To efficiently utilize information from a larger number of buoys, when $n \geq 4$ sonobuoys hear the target, an overdetermined, non-linear system of equations can be set up whose solution is an estimate of the unknown position of the target. This system can be transformed into a linear system of n-1 equations, plus one equation of the original type (1), (6). The linear system can be solved using a standard least squares method. We will call this technique the Least Squares Fix Method.

Uncertainty Ellipse Computation

The least squares solution for the position of the target provides an estimate of the mean value of the statistical distribution for the submarine's position. To model also the variance, or mean square error, of the position, the uncertainties of the sonobuoy positions and of the time differences of arrival need to be represented as stochastic variables. The uncertainty is modelled by assuming that these variables are Gaussian-distributed with zero mean. For a derivation of the parameters of the uncertainty ellipse the reader is referred to (1) and (6).

Predicting the Position of the Submarine

The buoy configuration delivers a position estimate and the uncertainty in this estimate for every time step. To reduce the uncertainty and to enable the motion of the submarine to be predicted, a Kalman filter was developed, whose target kinematics submodel is given by the first order stochastic differential equation:

$$\dot{x}(t) = f(x(t)) + w_c(t) \qquad (2)$$

Here x is the state vector, describing the kinematic properties of the target. f is the system state function. w_c is the system noise and represents unpredictable events in the system.

Since if possible we want to keep our model linear, all nonlinearities are placed in the noise vector. This gives us the following state function and noise vectors:

$$f(x(t)) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} = AX$$
$$w_c(t) = \begin{bmatrix} 0 \\ 0 \\ \ddot{x} \\ \ddot{y} \end{bmatrix} \qquad (3)$$

The parameters that need to be estimated are the variances of the acceleration components σ_p^2 and σ_n^2. For a single-propeller submarine operating under sound emission limitations, the practical maximum values of the acceleration components have been assumed to be a_p=0.02 m/s^2 and a_n=0.04 m/s^2. By choosing the variances as a_p^2 and a_n^2 the filter can make good predictions of realistic manoeuvres.

Normally in a Kalman filter the measurement covariance matrix is constant, given a priori by the characteristics of the measuring instrument, but here it will change over time. One could picture this situation as a succession of measurements with different instruments.

The Buoy Deployment Problem

The tracking problem would be trivial if one had sonobuoys enough to cover the entire possible area (volume in case a two-dimensional view of the problem is inadequate) with a sufficiently dense grid of sensors, but assuming there are too few sonobuoys to achieve such coverage, *when and where* should the available buoys be allocated ?

Estimation of the buoy range

Both when estimating the uncertainty in a future position estimation of the target and when calculating an optimal buoy position, it is important to be able to estimate the buoy range as accurately as possible. This range is dependent on the speed and type of the target as well as on the sea conditions. In order for the algorithms described in this paper to work accurately in practice, the sensitivity of each buoy would need to be calibrated with respect to an absolute normal.

After the speed of the submarine has been estimated, the corresponding source level *SL* can be interpolated from a table. Using the sonar equation (2), the buoy range *BR* can be calculated as:

$BR = 10^{(SL - NL - DT)/TL}$, where *NL* is the noise level and *TL* the transmission loss.

Deciding when to deploy a buoy

For each time step the buoy configuration delivers an estimated position of the target and the uncertainty in this estimate (1), (8). These values could be used as the final estimate of the system state, but then one would not use all available information, i. e., our apriori knowledge of the target's dynamic limitations. By using the Kalman filter however, the information from every new measurement can be fused with all information gathered until current time.

Using the Kalman filter to predict the position of the submarine, we obtain for each time step t_j a confidence ellipse $E(t_j + \Delta t)$ within which the submarine will be located with given probability *p* at time $t_j + \Delta t$. Wherever in this confidence ellipse the submarine may be, we want to be able to measure its position with an error less than δ. If and only if this is possible, there is no need to deploy another buoy.

Deciding where to position the buoy

In every simulation step we want to gain as much information as possible from the buoy configuration. Therefore the position for the next buoy deployment should satisfy two requirements:

- There should be no other buoy position which would enable the buoy configuration to add more information to the system in the next simulation step, whatever may be the future target position within the predicted area (*local* optimization problem).
- In order to save buoys, the position should be chosen so that the buoy can be of use for as long time as possible (*global* optimization problem).

These requirements are in conflict and a balance between them has to be found. To solve the first optimization problem, we should choose the position for the next buoy so that the information from our next measurement will have as small uncertainty as possible. Thus, a buoy must be deployed at time t_{j+1}. The submarine will then be in the known confidence region $E(t_{j+1})$ with probability *p*.

Let $r_i \in R_j$, r_i denoting the position of buoy *i* and R_j the area where buoy *i* can be deployed so as to cover $E(t_{j+1})$.

Further, let $\gamma(r_i, \hat{\rho}_{j+1})$ denote the length of the major axis of the uncertainty ellipse associated with the position measurement of a target located at $\hat{\rho}_{j+1}$ with buoy *i* deployed in position r_i. Then:

$$G(r_i) = \max_{\rho \in E(t_{i+1})} \{\gamma(r_i, \rho)\} \qquad (4)$$

will estimate the largest uncertainty associated with a measurement in

$E(t_{j+1})$.

Now we can formally state our first optimization problem:

$$\text{Minimize } G(r_i) \text{ subject to the constraint } r_i \in R_j \qquad (5)$$

If we were allowed to consider this requirement only, we would place the buoys as far away in the direction of motion as possible. But then the buoys would eventually end up in a row, leading to a singular system matrix, and the resulting confidence ellipse would grow indefinitely. To achieve a balance between these different requirements the following strategy was chosen:

Fix a tolerance δ for $G(r_i)$. We consider the first requirement to be fulfilled if a buoy position results in $G(r_i) < \delta$. If the position for r_i which maximizes the distance e along the direction of motion covered by the buoy i is chosen, we have a candidate for the optimal position but this position is still only optimal in a local sense. Figure 1 illustrates the effect of the buoy position on the accuracy of the target position estimate. The position in the (x,y)-plane for which we have a minimum in the $G(r_i)$ direction corresponds to the first requirement above. A buoy position in the grey area, corresponding to points in the (x,y)-plane with $G(r_i) < \delta$, with maximal e is a candidate for the optimal position.

Regaining Contact

The game will start from a point in time and a position for an initial observation of the submarine. At the later time for the tracking to start, the travelling distance for the submarine can be estimated given its speed. The speed is unknown but is expected to be low since the submarine is operating under sound emission limits. Let:

- dt = time difference between the initial observation and the start of tracking
- s_i = the assumed speed of the target in the first attempt
- b_i = the approximate buoy range when the target is travelling with speed s_i
- $r_i = s_i d_i$ is the travelling range of the submarine given its speed s_i

The simulator uses the following strategy: On a radius = r_i around the observation the buoys are deployed at distances < $2b_i$. This continues until one of the buoys indicates a detection. Around this buoy a few more buoys are deployed in a circle so as to get at least four hearing buoys. If no detection was made, the procedure is repeated assuming the speed $s_{i+1}=2s_i$.

The critical parameter in this strategy is the time difference dt. If it is too long, the assumption of constant direction of motion and speed of the submarine will be unrealistic. The error in the speed estimation will also increasingly deteriorate the estimated travelling range as dt increases.

Conclusions

We have developed a simplified two dimensional model without islands and bottom structure and *all conclusions below are related to the model and not to reality.* On the other hand, the model is based on the application of simple but well-established physical theories, and the substitution of numerical values for the model parameters has been done in cooperation with domain experts. Thus, we expect our main conclusions to be valid if the model were applied to real sonobuoy data.

In summary, in the model’s world, it is possible to track a hostile submarine in shallow-water environments using only passive, non-directional sonobuoys; with four or more sonobuoys in suitable positions the submarine’s position can be estimated and a confidence ellipse for this position can be calculated; from a Kalman filter not only a prediction of the motion is achieved, but also a reduction of the uncertainty in the same. The more one knows about the dynamic properties of the target the larger reduction can be obtained; by determining the uncertainty of future measurement one can position the buoy in a suitable way. A critical factor in the simulation is the time span from the initial observation until contact has been regained.

Before the technique can be tested in a real submarine tracking scenario, one needs to obtain practical solutions to the following problems: (1) computing on-line the TDOA for each interesting pair of sonobuoys and (2) determining on-line good estimates for the location of each sonobuoy, at least for testing purposes. The latter problem could be solved for example by measuring the sound transmission time to each sonobuoy from three or four underwater sonic beacons whose positions are known. In our simulations, a standard deviation of 15 m in buoy position allows tracking down to a buoy hearing range around 100 m. For practical application, it is desirable, perhaps required, that an all-passive, non-detectable system can be utilized and that the tracking principle can be used also in disturbed conditions such as presence of intense surface traffic, multiple targets, bad weather conditions, etc.

This study does not attempt to solve all these problems, nor are we proposing or analyzing a detailed design of a future underwater surveillance system. We find it encouraging and thought-provoking, however, that the notoriously difficult problem of tracking a submarine in an archipelagic environment might lend itself to a solution which requires neither sophisticated and inherently uncertain modelling of long-range signal propagation patterns nor a vast network of static sensor elements.

References

(1) Catarina Blixt: Felanalys vid lokalisering av hydroakustiska transienter, 1991, FOA Rapport C 20814-2.2, Defence Research Establishment, Stockholm.

(2) William S. Burdic: Underwater acoustic system analysis, 1991, Prentice Hall Inc.

(3) Yaakov Bar-Shalom, Xiao-Rong Li: Estimation and tracking: Principles, Techniques and Software, 1993, Artech House, Inc.

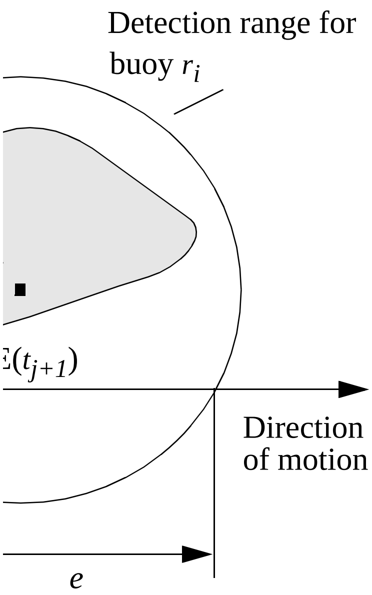
(4) Harold W. Sorenson: Kalman Filtering: Theory and Application, 1985, IEEE Press.

(5) Navy International, September 1990, 307-312.

(6) Alexander Wahlstedt, Jesper Fredriksson, Karsten Jöred and Per Svensson: Submarine Tracking by Means of Passive Sonobuoys. I. Design of a Simulation Model and System, 1996, FOA-R--96-00386-505--SE, Defence Research Establishment, Stockholm.

(7) Kristian Johansson and Per Svensson: Submarine Tracking by Means of Passive Sonobuoys. II. Position Estimation and Buoy Deployment Planning, 1997, FOA-R--97-00440-505--SE, Defence Research Establishment, Stockholm.

(8) Kristian Johansson, Karsten Jöred and Per Svensson: Submarine tracking using multi-sensor fusion and reactive planning for the positioning of passive sonobuoys, in:
N. Hörnqvist and E. Norrbrand (eds): Hydroakustik 97, Proceedings, 1997, FOA-R--97-00552-409--SE, Defence Research Establishment, Stockholm.



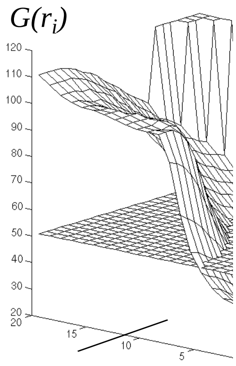


Fig. 1 An optimal buoy position has maximal *e* and stays within the grey area.

(9)