Analysis and assessment of effects-based plans*

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ABSTRACT

In this paper we present how a cross impact matrix may be used in effects-based planning and effects-based assessment for plan evaluation, plan refinement, generation of alternative plans, and subjective assessment of plans and plan elements. The purpose of using a cross impact matrix within the effects-based planning process is to find inconsistencies and decisive influences within developed plans. The cross impact matrix represents the impact between all activities, supporting effects, decisive conditions, and military end state of the plan. We develop morphological methods for analyzing activities, evaluating and refining plans, and sensitivity based methods using Dempster-Shafer theory to find the decisive influences. For the effects-based assessment process we develop a method that takes subjective assessments regarding the activities of a plan as inputs. From these assessments and the cross impact matrix we calculate assessments for all other plan elements. The method is based on belief functions and their combination under a new generalization of the discounting operation. The methods are implemented in a Collaboration Synchronization Management Tool (CSMT).

1 INTRODUCTION

A *cross impact matrix* (CIM) [3, 4] can be used for *morphological analysis* [5] on the operational command level by the staff of a joint task force headquarter in an *Effects-Based Approach to Operations* [6] during planning, execution and assessment of an operation. In morphological analysis we break down the plan into essential sub-concepts, each concept representing a dimension in the CIM. The purpose of using morphological analysis is to find inconsistencies in plans developed within the *effects-based planning* (EBP) [7] process. The CIM consists of all *activities* (A), *supporting effects* (SE), *decisive conditions* (DC) and *military end state* (MES) of the plan. In this paper we use British concepts [8]. It is created by a broad working group which must assess how each activity impacts every other activity and supporting effect, how each supporting effect impacts every decisive condition (and possibly other supporting effects), and how every decisive condition impacts the military end state (and possibly other decisive conditions). In this paper we present how a CIM may be used in EBP for plan evaluation, plan refinement and generation of alternative plans. We develop methods for analyzing activities and evaluating and refining plans within EBP, and develop a subjective method for *effects-based assessment* (EBA) based on Dempster-Shafer theory of belief functions [9–14] and the CIM.

The cross impact will aid the planning staff to find and exploit synergies by making all identified relationships between planned activities and their impact upon the supporting effects, etc. explicit. The values entered in the CIM during planning can be continuously updated during execution of the plan as the staff increases its knowledge of the current operational environment. Together with other information about the operation the explicit values in the CIM can therefore aid decision makers in gaining a more

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similar understanding of the situation, possibly leading to better decisions. The CIM can also be used during assessment of the operation as it should contain the most current view of what impact all supporting effects have on the decisive conditions and what impact all decisive conditions have on the military end state.

Accepting human subjective assessments regarding the successful outcome of activities of the plan, we can use the impacts between plan elements as described by the CIM to calculate similar subjective assessments of all desired supporting effects, decisive conditions and the military end state. Using this methodology we get an early assessment of all plan elements during *effects-based execution* (EBE) and may early on observe if activities and desired effects are developing according to plan. By observing the change over time of these subjective assessments of effects and conditions as assessments of activities are updated, we notice if trends are moving in the right direction as more activities are further executed.

The methods are implemented in a Collaboration Synchronization Management Tool (CSMT) [15].

In Sec. 2 we describe the construction of a CIM and analysis of plan elements using the CIM. We continue to analyze and refine alternative plans. In Sec. 3 we develop an algorithm for assessment of plan elements using the CIM, and show how this may be used for subjective assessment of all desired effects. Finally, in Sec. 4 conclusions are drawn.

2 THE CREATION OF THE CROSS IMPACT MATRIX

The cross impact matrix will initially be created during the planning process. It should be created by a working group containing key subject matter experts as required by the type of operation planned. The working group will first need to enter all planned activities into the CIM, and it is important that all activities are well defined. They will then have to decide which positive or negative impact each activity will have on every other activity. It is important to note that even if activity A_1 has a positive impact on activity A_2 then A_2 could have a negative impact on A_1. In the next step the working group must decide what impact all activities have on the supporting effects, what impact all supporting effects have on the decisive conditions and what impact the decisive conditions have on the military end state.

It is important to note that the CIM will not be able to handle the effects of synergy. If the combined effect of performing activities A_1, A_2 and A_3 simultaneously is higher than the sum of performing each one separately, this can not be modeled within standard CIM analysis. However, it can be managed if A_1, A_2 and A_3 are combined into one activity with several alternatives.

The CIM can be introduced in EBP and used for evaluation of the plan and generation of alternative plans. The work with CIM in EBP may be conducted using the following tasks.

2.1 Form a plan

Before the CIM is constructed, a plan must be formed according to EBP, see Figure 1. A plan is formed top-down from the MES, where the MES is broken down into effects and actions that should lead to the desired MES. This work is outside the scope of this article.

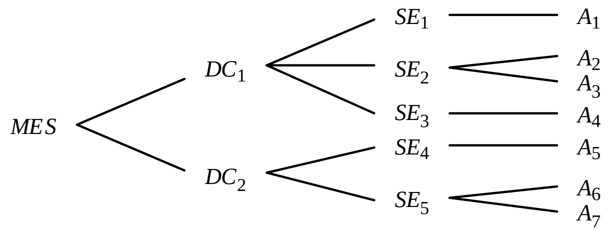

Figure 1: Effects-based planning: *MES* = military end state, *DC* = decisive condition, *SE* = supporting effect, *A* = activity.

2.2 Construct the CIM based on the plan

The plan consists of a military end state, decisive conditions, supporting effects and activities. The number of these elements is denoted *n*. Construct a CIM with *n*-1 rows and *n* columns. Listing these elements, except the military end state, to the left of the CIM and list the elements, including the military end state, above the CIM, see Figure 2. The CIM consists of values ranging from -9 to 9, where -9 denotes large negative influence, 0 means no influence and 9 denotes high positive influence. For example, an impact value of 8, i.e., “high positive influence”, might be assigned between the activity of “securing an area” and the activity of “transporting through that area”. How much the element of row *i* influences the element of column *j* is stored in *cell*(*i*, *j*) in the CIM (for example the activity A_2 influence the activity A_4 in a positive way with a factor 2, but A_4 influence A_2 in a negative way by a factor of -2).

	Military End State	DC_1	DC_2	SE_1	SE_2	A_1	A_2	A_3	A_4
DC_1	5	0	6	0	0	0	0	0	0
DC_2	8	6	0	0	0	0	0	0	0
SE_1	0	5	2	0	0	0	0	0	0
SE_2	0	5	8	0	0	0	0	0	0
A_1	0	0	0	3	3	0	4	-2	-3
A_2	0	0	0	3	-2	3	0	0	2
A_3	0	0	0	3	6	0	8	0	0
A_4	0	0	0	4	-2	-7	-2	0	0

Figure 2: The CIM contains military end state, decisive conditions, supporting effects and activities (dark gray cells always contain zeros).

It is important to separate between direct and indirect influence. Only direct influence should be stated in the CIM. Also, one should be very careful not to assign any direct influences between two activities if these more properly concern influences between each of the two activities and the supporting effect.

At this initial stage of the construction of the CIM we include the basic elements of the plan meaning that each element usually has only one alternative. Thus, all activities should be performed and all supporting effects and decisive conditions should be reached. In Figure 3 an implementation of the CIM is shown.

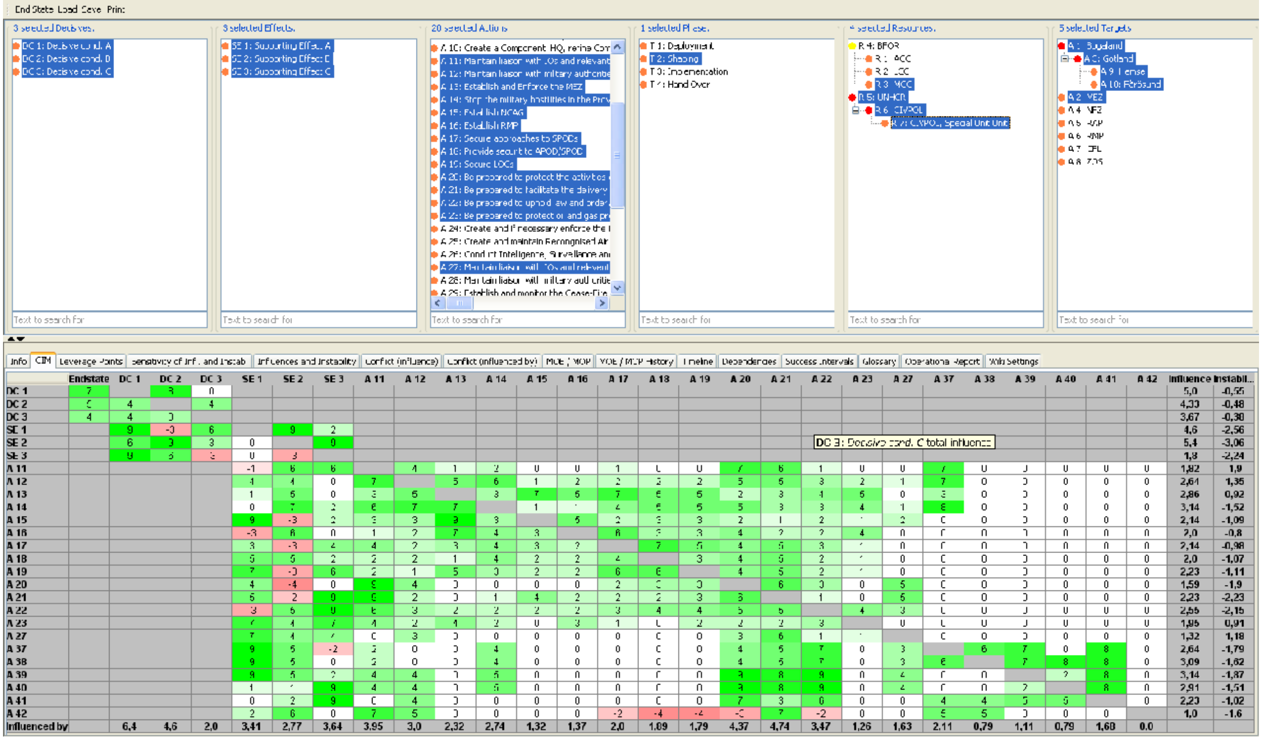

Figure 3: A complete CIM with activities, supporting effects, decisive conditions and military end state.

2.3 Opportunities: Create new alternatives

It may be possible to state some alternative decisive conditions, supporting effects or activities. For instance, we may have two different activities we have to decide between. They could describe different things to do, or they could do the same thing at different times or places. Then we would have two different instances of the plan.

We calculate consistency and stability for each element of the plan (activities, supporting effects and decisive conditions) relative all other elements. When calculating for each row we obtain how much each element influences other elements (by column how much it is influenced by other elements). For each row we have

$$\text{AltConsistency}(Alt_i) = \sum_j \text{impact}(i, j) \tag{1}$$

were $\text{impact}(i, j)$ is the impact value in the CIM, $Alt_i \in \{DC_1, DC_2, SE_1, SE_2, A_1, A_2, A_3, A_4\}$, Figure 4, and

$$\text{AltStability}(Alt_i) = \prod_j \frac{\min[\text{CV}(i,j), \text{CV}(j,i)]}{\max[\text{CV}(i,j), \text{CV}(j,i)]} \tag{2}$$

where the coefficient value CV(i, j) is calculated as

$$\text{CV}(i,j) = \begin{cases} \text{impact}(i,j) + 1, & \text{impact}(i,j) \geq 0 \\ \dfrac{1}{1 - \text{impact}(i,j)}, & \text{impact}(i,j) < 0 \end{cases} \tag{3}$$

For the sake of legibility we present the consistency values normalized and the stability values normalized and logarithmized according to

$$\text{NormAltConsistency}(Alt_i) = \frac{\text{AltConsistency}(Alt_i)}{|\{Alt_j | \forall j\}|} \tag{4}$$

and

$$\text{NormAltStability}(Alt_i) = 9\frac{\log_{10}[\text{AltStability}(Alt_i)]}{2[|\{Alt_j | \forall j\}| - 1]} \tag{5}$$

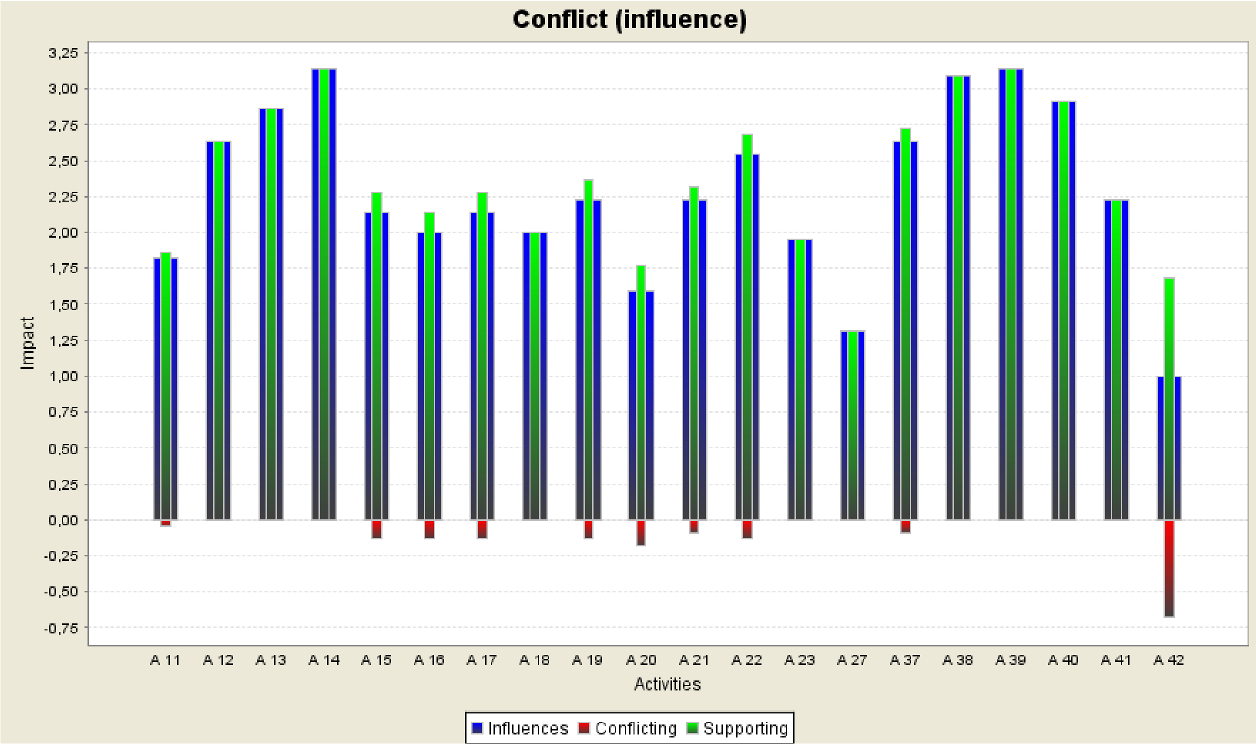

Figure 4: The figure shows how different activities influence other activities. For example, activity A_{39} influence others strongly positive, while A_{42} influence some in a positive manner (green) and others in a negative way (red). Average in blue.

In Figure 5 we observe the influence and stability for all activities.

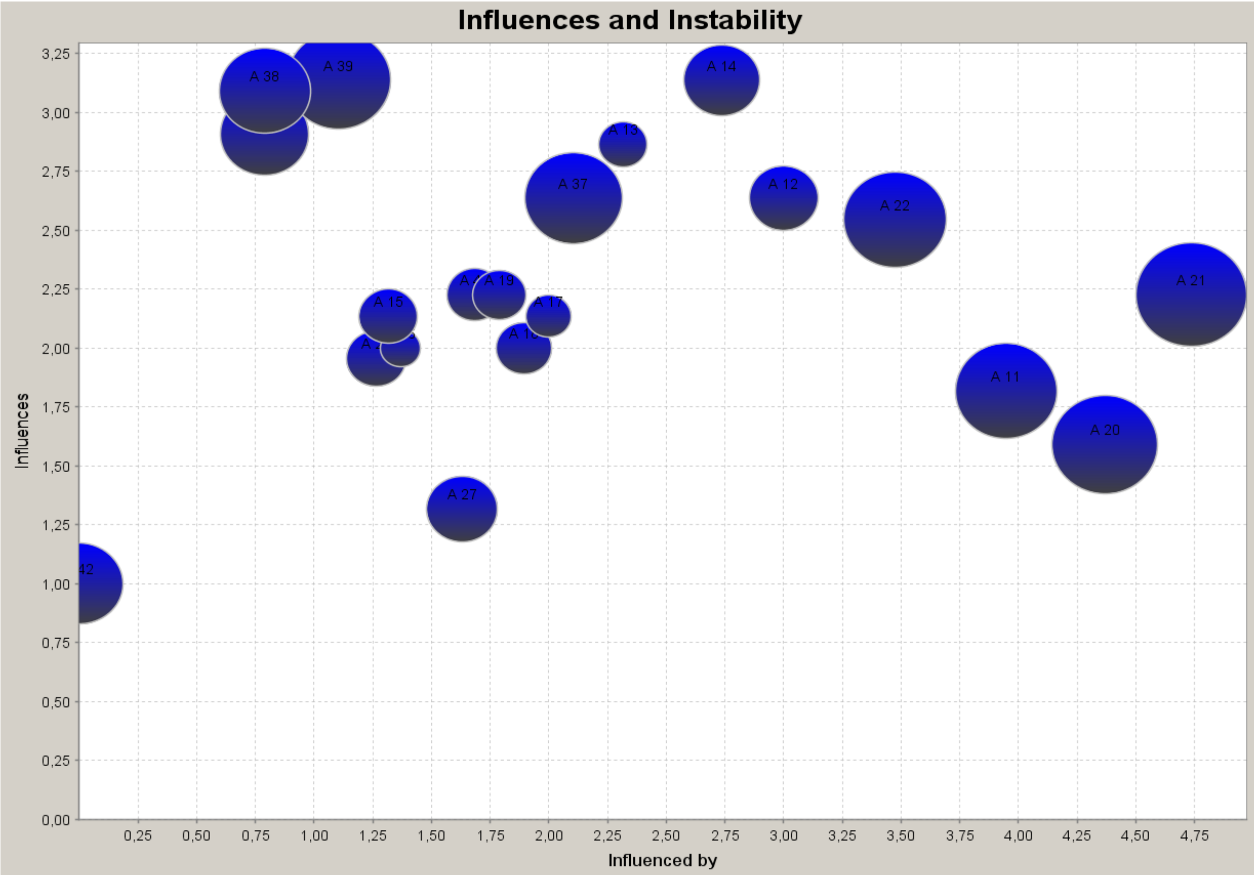

Figure 5: Top view contains all elements of the plan. Bottom view contains all activities ranked by how much they influence and are influenced by other activities. Circle size correspond to instability (large circles implies high instability).

We may now create new alternatives, mostly alternative activities to realize some supporting effect, but it is also possible to consider new alternative supporting effects or decisive conditions to reach the intended military end state. For each new alternative it is important to note which activity, supporting effect or decisive condition it belongs to, Figure 6.

		Military End State	DC_1		DC_2	SE_1		SE_2	A_1		A_2	A_3	A_4
			DC_{11}	DC_{12}		SE_{11}	SE_{12}		A_{11}	A_{12}			
DC_1	DC_{11}	5	0	0	6	0	0	0	0	0	0	0	0
	DC_{12}	6	0	0	6	0	0	0	0	0	0	0	0
DC_2		8	6	8	0	0	0	0	0	0	0	0	0
SE_1	SE_{11}	0	5	4	2	0	0	0	0	0	0	0	-3
	SE_{12}	0	6	3	2	0	0	2	0	0	0	0	2
SE_2		0	5	5	8	0	0	0	0	0	0	0	0
A_1	A_{11}	0	0	0	0	3	8	3	0	0	4	-2	-3
	A_{12}	0	0	0	0	7	9	1	0	0	2	2	3
A_2		0	0	0	0	3	1	-2	3	1	0	0	2
A_3		0	0	0	0	3	5	6	0	0	8	0	0
A_4		0	0	0	0	4	7	-2	-7	-1	-2	0	0

Figure 6: The CIM now contains alternatives for many decisive conditions, supporting effects and activities.

These new alternative are introduced into the CIM and all new matrix values must be assigned. After this is done new consistency and stabilities can be calculated. This procedure can be repeated until satisfaction is reached and a suitable set of alternatives are at hand. If a new alternative gives good consistency and stability for some element of the plan this may be found satisfying and work may continue on finding new alternatives for other elements. However, if the new alternative gives poor values we must try to find further alternatives for the same element. When this process has been repeated until satisfaction is reached for all elements of the plan, we have a CIM with several alternatives for many of the activities, supporting effects and decisive conditions.

The CIM is now expanded with several alternatives activities, supporting effects and decisive conditions (e.g., A_{12}, SE_{12} and DC_{12}). The alternative activity may for example be a change in timing or intensity of an activity in order to improve on the plan. When this is done we may evaluate the plan with different alternative activities.

2.4 Leverage Points: Decisive influence from activities

We can calculate which activities that provides a decisive influence on a particular supporting effect, decisive condition or on the military end state by performing a sensitivity analysis using Dempster-Shafer theory. In this analysis we assume simple frames of discernment for each supporting effect, decisive condition and the military end state with only two possible outcomes, $\Theta = \{\mathrm{AdP}, \neg\mathrm{AdP}\}$ on each hierarchical level of the plan, where AdP means an *Adequate Plan*. Either the desired supporting effect, decisive condition or the military end state is achieved or it is not. The calculation is made by first, for a certain activity A_k, calculating the support for the requested $m_{SE_j}(\mathrm{AdP})$, $m_{DC_j}(\mathrm{AdP})$ or $m_{MES}(\mathrm{AdP})$ with $m_{A_i}(\mathrm{AdP}) = 1 \quad \forall i$ and then recalculating the same with $m_{A_k}(\mathrm{AdP}) = 0.99$ and $m_{A_i}(\mathrm{AdP}) = 1 \quad \forall i \neq k$. Here, $m_{A_i}(\neg\mathrm{AdP}) = 0 \quad \forall i$. By selecting these mass functions as input data we will be able to perform numerical differentiation of all supporting effects, decisive conditions and the military end state with respect to each individual activity. The value of these derivatives shows the influence of the individual activities on these effects, conditions and end state.

If we are only interested in which activities have a decisive influence on some particular supporting effect or decisive condition then we may choose to calculate only these values, but if we are interested in which activities have a decisive influence on the plan at large, then we must perform the calculation for the military end state level.

Before combining the mass functions we discount them using the impact values of the CIM. This ensures that each activity influences the supporting effect to its proper degree. We have

$$m_{A_i}^{\alpha_{kj}}(X): \begin{cases} \alpha_{kj} m_{A_i}(\mathrm{AdP}), & X = \mathrm{AdP} \\ \alpha_{kj} m_{A_i}(\neg\mathrm{AdP}), & X = \neg\mathrm{AdP} \\ 1 - \alpha_{kj} m_{A_i}(\mathrm{AdP}) - \alpha_{kj} m_{A_i}(\neg\mathrm{AdP}), & X = \Theta \end{cases} \tag{6}$$

where the discounting factor

$$\alpha_{kj} = \frac{\mathrm{impact}(k, j)}{10} . \tag{7}$$

This is a generalization where discounting factors may assume values less than 0, i.e., α_{kj}= {0.9, 0.8, 0.7, ..., 0.9}. These discounted mass functions are combined using Dempster's rule.

For each activity A_k and every supporting effect SE_j we can calculate

$$\mathrm{DecisiveInfluence}(A_k \to SE_j) = \left\{ \left[m_{SE_j}(\mathrm{AdP}) \middle| \begin{array}{l} m_{A_i}(\mathrm{AdP}) = 1 \quad \forall i \\ m_{A_i}(\neg\mathrm{AdP}) = 0 \quad \forall i \end{array} \right] - \left[m_{SE_j}(\mathrm{AdP}) \middle| \begin{array}{l} m_{A_k}(\mathrm{AdP}) = 0.99 \\ m_{A_i}(\mathrm{AdP}) = 1 \quad \forall i \neq k \\ m_{A_i}(\neg\mathrm{AdP}) = 0 \quad \forall i \end{array} \right] \right\} \tag{8}$$

where

$$m_{SE_j}(\text{AdP}) = \max\left\{0,\ 1 - \prod_k \left[1 - \frac{\text{impact}(k,j)}{10} \cdot m_{A_k}(\text{AdP})\right]\right\} \tag{9}$$

and $0 \le m_{SE_j}(\text{AdP}) \le 1$.

We have chosen to cap the value of $m_{SE_j}(\text{AdP}) \ge 0$ and not handle the case where $m_{SE_j}(\text{AdP}) < 0$.

By substituting $\{m_{SE_j} \to m_{DC_j}\}$ in Eq. (8) we calculate for each activity A_k and each decisive condition DC_j which influence this activity has on this decisive condition,

$$\text{DecisiveInfluence}(A_k \to DC_j) = \left\{\left[m_{DC_j}(\text{AdP}) \middle| \begin{array}{ll} m_{A_i}(\text{AdP}) = 1 & \forall i \\ m_{A_i}(\neg\text{AdP}) = 0 & \forall i \end{array}\right] - \left[m_{DC_j}(\text{AdP}) \middle| \begin{array}{ll} m_{A_k}(\text{AdP}) = 0.99 & \\ m_{A_i}(\text{AdP}) = 1 & \forall i \ne k \\ m_{A_i}(\neg\text{AdP}) = 0 & \forall i \end{array}\right]\right\} \tag{10}$$

where

$$m_{DC_j}(\text{AdP}) = \max\left\{0,\ 1 - \prod_k \left[1 - \frac{\text{impact}(k,j)}{10} \cdot m_{SE_k}(\text{AdP})\right]\right\} \tag{11}$$

However, most interesting is perhaps the influences the different activities have on the plan at large, i.e., the military end state. By substituting $\{m_{SE_j} \to m_{MES}\}$ in Eq. (8) we calculate for each activity which influence it has on the military end state, $\text{DecisiveInfluence}\,(A_k \to MES)$. Since we only have one military end state we get one value for each activity and may thus rank these by the calculated

$$\text{DecisiveInfluence}(A_k \to MES) = \left\{\left[m_{DC_j}(\text{AdP}) \middle| \begin{array}{ll} m_{A_i}(\text{AdP}) = 1 & \forall i \\ m_{A_i}(\neg\text{AdP}) = 0 & \forall i \end{array}\right] - \left[m_{DC_j}(\text{AdP}) \middle| \begin{array}{ll} m_{A_k}(\text{AdP}) = 0.99 & \\ m_{A_i}(\text{AdP}) = 1 & \forall i \ne k \\ m_{A_i}(\neg\text{AdP}) = 0 & \forall i \end{array}\right]\right\} \tag{12}$$

where

$$m_{MES}(\text{AdP}) = \max\left\{0,\ 1 - \prod_k \left[1 - \frac{\text{impact}(k,j)}{10} \cdot m_{DC_k}(\text{AdP})\right]\right\}. \tag{13}$$

These calculations can be made both with the initial CIM where each activity has only one alternative and with the later CIM where some activities have two or more alternatives. If the calculations are made for the later CIM then we must carry out the calculation separately for each alternative i, e.g., for activity A_k and military end state

$$\forall i. \qquad \text{DecisiveInfluence}\,(A_{ki} \to MES) \tag{14}$$

after which the decisive influence by activity A_k on the military end state is calculated as

$$\text{DecisiveInfluence}\,(A_k \to MES) \;=\; \max_i \{\,\text{DecisiveInfluence}\,(A_{ki} \to MES)\,\}. \tag{15}$$

An example of decisive influence on the military end state is shown in Figure 7.

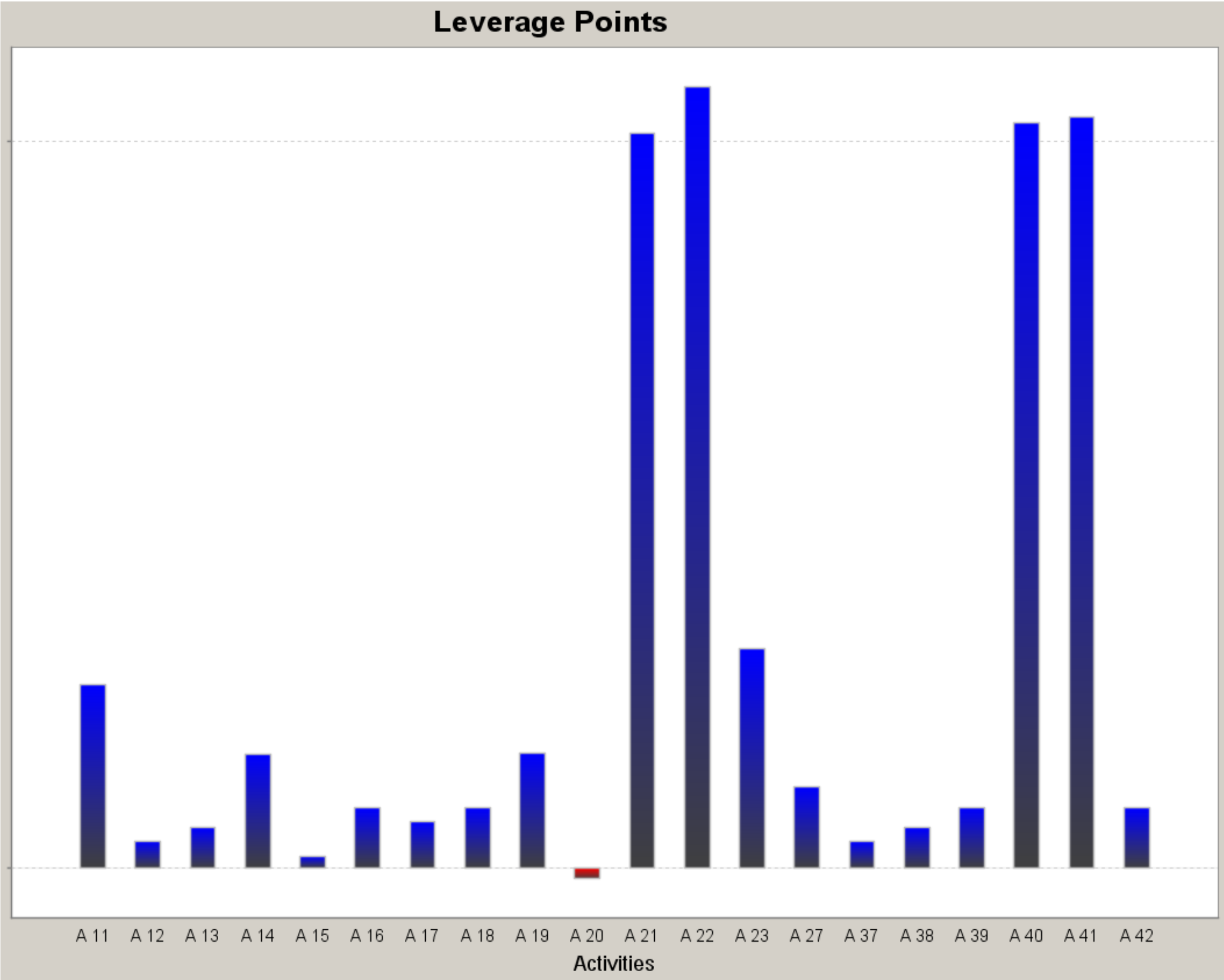

Figure 7: Leverage points show the impact of success of each activity on the success of the military end state. Activities A_{21}, A_{22}, A_{40} and A_{41} have high impact.

2.5 Plan refinement

We may now evaluate the current plan and propose incremental changes to the plan by performing a CIM analysis, or make a complete CIM analysis to obtain the optimal plan according to the given CIM. These alternative modes of procedure are based on the same analysis and only represent different ways to sort evaluated instances (I) of the plan. In each mode of procedure a complete CIM analysis is performed.

The evaluation is performed by calculating consistency and stability for each possible instance of the plan, according to

$$\text{Consistency}(I) = \sum_{i \in I} \sum_{j \in I} \text{impact}(i, j) \tag{16}$$

and

$$\text{Stability}(I) = \prod_{i \in I} \prod_{\substack{j \in I \\ j > i}} \frac{\min[\text{CV}(i, j), \text{CV}(j, i)]}{\max[\text{CV}(i, j), \text{CV}(j, i)]} \tag{17}$$

In Figure 8 the eight alternative plans of Figure 6 are evaluated by consistency and stability[1].

	DC_1	DC_2	SE_1	SE_2	A_1	A_2	A_3	A_4	Con	Sta
Plan 1	DC_{11}	DC_2	SE_{11}	SE_2	A_{11}	A_2	A_3	A_4	63	-3,43
Plan 2	DC_{11}	DC_2	SE_{11}	SE_2	A_{12}	A_2	A_3	A_4	77	-3,47
Plan 3	DC_{11}	DC_2	SE_{12}	SE_2	A_{11}	A_2	A_3	A_4	79	-3,44
Plan 4	DC_{11}	DC_2	SE_{12}	SE_2	A_{12}	A_2	A_3	A_4	90	-3,34
Plan 5	DC_{12}	DC_2	SE_{11}	SE_2	A_{11}	A_2	A_3	A_4	65	-3,45
Plan 6	DC_{12}	DC_2	SE_{11}	SE_2	A_{12}	A_2	A_3	A_4	79	-3,50
Plan 7	DC_{12}	DC_2	SE_{12}	SE_2	A_{11}	A_2	A_3	A_4	79	-3,30
Plan 8	DC_{12}	DC_2	SE_{12}	SE_2	A_{12}	A_2	A_3	A_4	90	-3,20

Figure 8: A list over the plans with consistency (Con) and stability (Sta) values. Both plan 4 and plan 8 have high consistency (= 90). However, plan 8 has the higher stability, making this the preferred plan. [The stability values are logarithmized and normalized (≤ 0,00); The elements are from the CIM in Figure 6].

3 ASSESSMENT OF PLAN ELEMENTS

The CIM is a model of influence between elements of the plan. In assessment, our interest is on the impact between activities on the lowest level and supporting effects on the next level, and so forth. We receive subjective assessments regarding activities as user input. These are in the form of basic belief assignments (bbas) that express support for and against the success of that activity, encoded as AdP and ¬AdP, respectively.

[1] Eq. (16) and Eq. (17) were derived through reverse engineering by the author in 1995 (unpublished at the time).

3.1 Combining assessments

In this problem we have the same simple frame of discernment as in Sec. 2,

$$\Theta = \{\mathrm{AdP}, \neg\mathrm{AdP}\} \quad (18)$$

on each hierarchical level of the plan.

We have a set of n bbas each with three bodies of evidence, i.e., $\{\{(\mathrm{AdP}, m_i(\mathrm{AdP}))$, $(\neg\mathrm{AdP}, m_i(\neg\mathrm{AdP}))$, $(\Theta, 1 - m_i(\mathrm{AdP}) - m_i(\neg\mathrm{AdP}))\}\}_{i=1}^{n}$, where, e.g., $(\mathrm{AdP}, m_i(\mathrm{AdP}))$ is the first body of evidence of the ith bba giving support to AdP. Thus, for the ith bba we have,

$$m_i(A) = \begin{cases} m_i(\mathrm{AdP}), & A = \mathrm{AdP} \\ m_i(\neg\mathrm{AdP}), & A = \neg\mathrm{AdP} \\ 1 - m_i(\mathrm{AdP}) - m_i(\neg\mathrm{AdP}), & A = \Theta \end{cases} . \quad (19)$$

The CIM contains all information regarding the impact of each activity on all supporting effects. When the impact on a particular supporting effect SE_j is less than full we discount the bba m_i in relation to its degree of impact on SE_j

$$m_i^{\alpha_{ij}}(A) = \begin{cases} \alpha_{ij} m_i(\mathrm{AdP}), & A = \mathrm{AdP} \\ \alpha_{ij} m_i(\neg\mathrm{AdP}), & A = \neg\mathrm{AdP} \\ 1 - \alpha_{ij} m_i(\mathrm{AdP}) - \alpha_{ij} m_i(\neg\mathrm{AdP}), & A = \Theta \end{cases} \quad (20)$$

Combining all $m_i^{\alpha_{ij}}$, we get

$$m^{\alpha_{ij}}_{\oplus\{m_i\}_{i=1}^{n}}(A) = \begin{cases} K\left\{\prod_i [1 - \alpha_{ij} m_i(\neg\mathrm{AdP})] - \prod_i [1 - \alpha_{ij} m_i(\mathrm{AdP}) - \alpha_{ij} m_i(\neg\mathrm{AdP})]\right\}, & A = \mathrm{AdP} \\ K\left\{\prod_i [1 - \alpha_{ij} m_i(\mathrm{AdP})] - \prod_i [1 - \alpha_{ij} m_i(\mathrm{AdP}) - \alpha_{ij} m_i(\neg\mathrm{AdP})]\right\}, & A = \neg\mathrm{AdP} \\ K\prod_i [1 - \alpha_{ij} m_i(\mathrm{AdP}) - \alpha_{ij} m_i(\neg\mathrm{AdP})], & A = \Theta \end{cases} \quad (21)$$

where

$$m^{\alpha_{ij}}_{\oplus\{m_i\}_{i=1}^{n}}(\mathrm{AdP}) + m^{\alpha_{ij}}_{\oplus\{m_i\}_{i=1}^{n}}(\neg\mathrm{AdP}) + m^{\alpha_{ij}}_{\oplus\{m_i\}_{i=1}^{n}}(\Theta) = 1 . \quad (22)$$

Thus, Eq. (21) becomes the subjective assessment of SE_j as calculated using the subjective input assessments of all activities A_i that impact upon SE_j.

What is calculated for supporting effects from subjective assessment of activities can in a second phase be calculated for decisive conditions using the newly calculated assessments of supporting effects. In the same way we can calculate the subjective assessment of the military end state from the assessment of decisive conditions.

3.2 Combining assessments regarding plan elements using the CIM

At the activities level we have a frame of discernment

$$\Theta_A = \{\mathrm{AdP}, \neg\mathrm{AdP}\} \tag{23}$$

In order to map this onto the problem of combining assessments, Sec. 3.1, we must first generalize the discounting operation.

The discounting operation was introduced to handle the case when the source of some piece of evidence is lacking in credibility [11]. The credibility of the source, $0 < \alpha < 1$, also became the credibility of the piece of evidence. The situation was handled by discounting each supported proposition other than Θ with the credibility α and by adding the discounted mass to Θ;

$$m^{\%}(A) = \begin{cases} \alpha m(A), & A \neq \Theta \\ 1 - \alpha + \alpha m(\Theta), & A = \Theta \end{cases} \tag{24}$$

We generalize the discounting operation by allowing the credibility to take values in the interval $-1 \le \alpha < 1$.

Definition 1. Let $m: 2^{\Theta} \to [0, 1]$ be a bba where $-1 \le \alpha < 1$. Then

$$m^{\%}(A) = \begin{cases} \alpha m(A), & A \neq \Theta \\ 1 - \alpha + \alpha m(\Theta), & A = \Theta \end{cases} \tag{25}$$

is a generalized discounting of *m* where $m^{\%}(A)$ is an inverse simple support function (ISSF) [16] whenever $\alpha < 0$.

Before combining the mass functions we discount them using the impact values of the CIM. This ensures that each activity influences the supporting effect to its proper degree.

For SE_j and A_i we have

$$m_{A_i}^{\alpha_{kj}}(A) = \begin{cases} \alpha_{A_i SE_j} m_{A_i}(\mathrm{AdP}), & A = \mathrm{AdP} \\ \alpha_{A_i SE_j} m_{A_i}(\neg\mathrm{AdP}), & A = \neg\mathrm{AdP} \\ 1 - \alpha_{A_i SE_j} m_{A_i}(\mathrm{AdP}) - \alpha_{A_i SE_j} m_{A_i}(\neg\mathrm{AdP}), & A = \Theta \end{cases} \tag{26}$$

where the discounting factor is defined as

$$\alpha_{A_i SE_j} \triangleq \frac{\text{impact}(A_i, SE_j)}{10} \quad . \tag{27}$$

This is a generalization of the discounting operator where discounting factors may assume values less than 0, i.e., $\alpha_{kj} = \{-0{,}9, -0{,}8, -0{,}7, \ldots, 0{,}9\}$.

We combine all bbas on the activities level and bring the result to the supporting effects level. At the supporting effects level we have a similar frame of discernment

$$\Theta_{SE} = \{\text{AdP}, \neg\text{AdP}\} \quad . \tag{28}$$

Using Eq. (21), Eq. (26) and Eq. (27), we define

$$\begin{aligned}
m_{SE_j}(\text{AdP}) &\triangleq m^{\alpha_{ji}}_{\oplus\{m_{A_i}\}_{i=1}^{n}}(\text{AdP}) \\
&= K\left\{\prod_i [1-\alpha_{A_i SE_j} m_{A_i}(\neg\text{AdP})] - \prod_i [1-\alpha_{A_i SE_j} m_{A_i}(\text{AdP}) - \alpha_{A_i SE_j} m_{A_i}(\neg\text{AdP})]\right\} \\
&= K\left\{\prod_i \left[1-\frac{\text{impact}(A_i, SE_j)}{10} m_{A_i}(\neg\text{AdP})\right] - \prod_i \left[1-\frac{\text{impact}(A_i, SE_j)}{10} m_{A_i}(\text{AdP}) - \frac{\text{impact}(A_i, SE_j)}{10} m_{A_i}(\neg\text{AdP})\right]\right\}
\end{aligned} \tag{29}$$

and

$$\begin{aligned}
m_{SE_j}(\neg\text{AdP}) &\triangleq m^{\alpha_{ji}}_{\oplus\{m_{A_i}\}_{i=1}^{n}}(\neg\text{AdP}) \\
&= K\left\{\prod_i [1-\alpha_{A_i SE_j} m_{A_i}(\text{AdP})] - \prod_i [1-\alpha_{A_i SE_j} m_{A_i}(\text{AdP}) - \alpha_{A_i SE_j} m_{A_i}(\neg\text{AdP})]\right\} \\
&= K\left\{\prod_i \left[1-\frac{\text{impact}(A_i, SE_j)}{10} m_{A_i}(\text{AdP})\right] - \prod_i \left[1-\frac{\text{impact}(A_i, SE_j)}{10} m_{A_i}(\text{AdP}) - \frac{\text{impact}(A_i, SE_j)}{10} m_{A_i}(\neg\text{AdP})\right]\right\}
\end{aligned} \tag{30}$$

with

$$\begin{aligned}
m_{SE_j}(\Theta) &\triangleq m^{\alpha_{ji}}_{\oplus\{m_{A_i}\}_{i=1}^{n}}(\Theta) \\
&= K\prod_i [1-\alpha_{A_i SE_j} m_{A_i}(\text{AdP}) - \alpha_{A_i SE_j} m_{A_i}(\neg\text{AdP})] \\
&= K\prod_i \left[1-\frac{\text{impact}(A_i, SE_j)}{10} m_{A_i}(\text{AdP}) - \frac{\text{impact}(A_i, SE_j)}{10} m_{A_i}(\neg\text{AdP})\right]
\end{aligned} \tag{31}$$

where $m_{DC_j}(\text{AdP}) + m_{DC_j}(\neg\text{AdP}) + m_{DC_j}(\Theta) = 1$.

In the same way we may calculate the support for decisive conditions, and the military end state. For decisive conditions, we have

$$m_{DC_j}(\text{AdP}) = K\left\{\prod_i\left[1 - \frac{\text{impact}(SE_i, DC_j)}{10} m_{SE_i}(\neg\text{AdP})\right] - \prod_i\left[1 - \frac{\text{impact}(SE_i, DC_j)}{10} m_{SE_i}(\text{AdP}) - \frac{\text{impact}(SE_i, DC_j)}{10} m_{SE_i}(\neg\text{AdP})\right]\right\} \quad (32)$$

and

$$m_{DC_j}(\neg\text{AdP}) = K\left\{\prod_i\left[1 - \frac{\text{impact}(SE_i, DC_j)}{10} m_{SE_i}(\text{AdP})\right] - \prod_i\left[1 - \frac{\text{impact}(SE_i, DC_j)}{10} m_{SE_i}(\text{AdP}) - \frac{\text{impact}(SE_i, DC_j)}{10} m_{SE_i}(\neg\text{AdP})\right]\right\} \quad (33)$$

with

$$m_{DC_j}(\Theta) = K\prod_i\left[1 - \frac{\text{impact}(SE_i, DC_j)}{10} m_{SE_i}(\text{AdP}) - \frac{\text{impact}(SE_i, DC_j)}{10} m_{SE_i}(\neg\text{AdP})\right] \quad (34)$$

where $m_{DC_j}(\text{AdP}) + m_{DC_j}(\neg\text{AdP}) + m_{DC_j}(\Theta) = 1$.

Similarly, for the military end state, we have

$$m_{MES}(\text{AdP}) = K\left\{\prod_i\left[1 - \frac{\text{impact}(DC_i, MES)}{10} m_{DC_i}(\neg\text{AdP})\right] - \prod_i\left[1 - \frac{\text{impact}(DC_i, MES)}{10} m_{DC_i}(\text{AdP}) - \frac{\text{impact}(DC_i, MES)}{10} m_{DC_i}(\neg\text{AdP})\right]\right\} \quad (35)$$

and

$$m_{MES}(\neg\text{AdP}) = K\left\{\prod_i\left[1 - \frac{\text{impact}(DC_i, MES)}{10} m_{DC_i}(\text{AdP})\right] - \prod_i\left[1 - \frac{\text{impact}(DC_i, MES)}{10} m_{DC_i}(\text{AdP}) - \frac{\text{impact}(DC_i, MES)}{10} m_{DC_i}(\neg\text{AdP})\right]\right\} \quad (36)$$

with

$$m_{MES}(\Theta) = K\prod_i\left[1 - \frac{\text{impact}(DC_i, MES)}{10} m_{DC_i}(\text{AdP}) - \frac{\text{impact}(DC_i, MES)}{10} m_{DC_i}(\neg\text{AdP})\right] \quad (37)$$

where $m_{MES}(\text{AdP}) + m_{MES}(\neg\text{AdP}) + m_{MES}(\Theta) = 1$.

With these calculations we have all pieces of a subjective EBA algorithm (Algorithm 1).

Algorithm 1: Subjective EBA

- For all SE_j calculate: $m_{SE_j}(\text{AdP})$, $m_{SE_j}(\neg\text{AdP})$, $m_{SE_j}(\Theta)$ using Eq. (29), Eq. (30), Eq. (31).
- For all DC_j calculate: $m_{DC_j}(\text{AdP})$, $m_{DC_j}(\neg\text{AdP})$, $m_{DC_j}(\Theta)$ using Eq. (32), Eq. (33), Eq. (34).
- Calculate: $m_{MES}(\text{AdP})$, $m_{MES}(\neg\text{AdP})$, $m_{MES}(\Theta)$ using Eq. (35), Eq. (36), Eq. (37).
- Return all calculated values.

In Figure 9 the calculated values of Algorithm 1 are presented in the upper part labelled "Impact", together with the initial subjective assessments $m_{A_j}(\text{AdP})$, $m_{A_j}(\neg\text{AdP})$, and $m_{A_j}(\Theta)$ in the lower part labelled "Activities" within the CSMT. Obviously, $m(\text{AdP})$ is indicated by green, $m(\neg\text{AdP})$ by red and the uncommitted $m(\Theta)$ by gray.

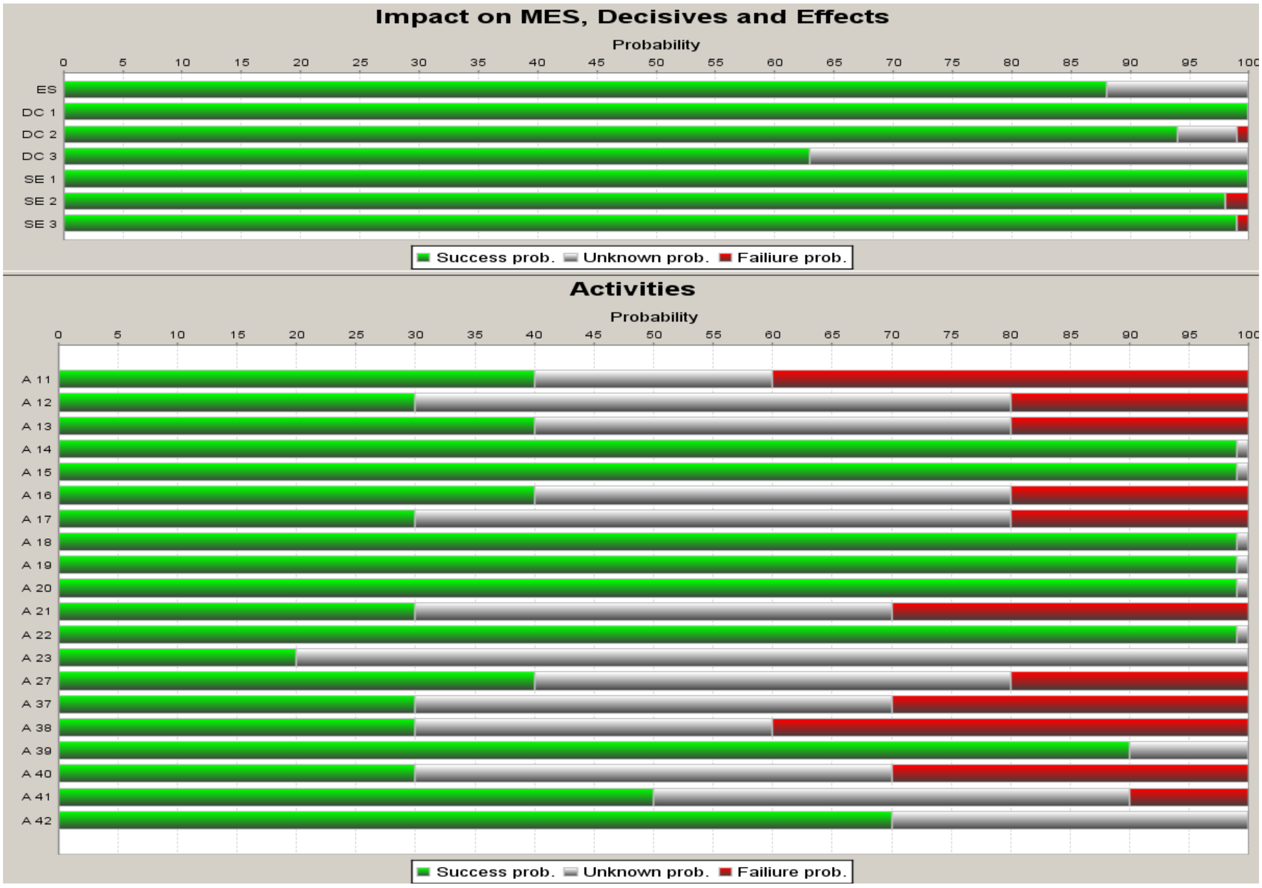

Figure 9: Subjective effects-based assessment (EBA) in the collaborative synchronization management tool (CSMT).

In order to further enhance the usability it may be of value to include a diagram of the change over time for these assessments. In Figure 10 this is exemplified for the Military End State as calculated by Eq. (35), Eq. (36) and Eq. (37) at different times.

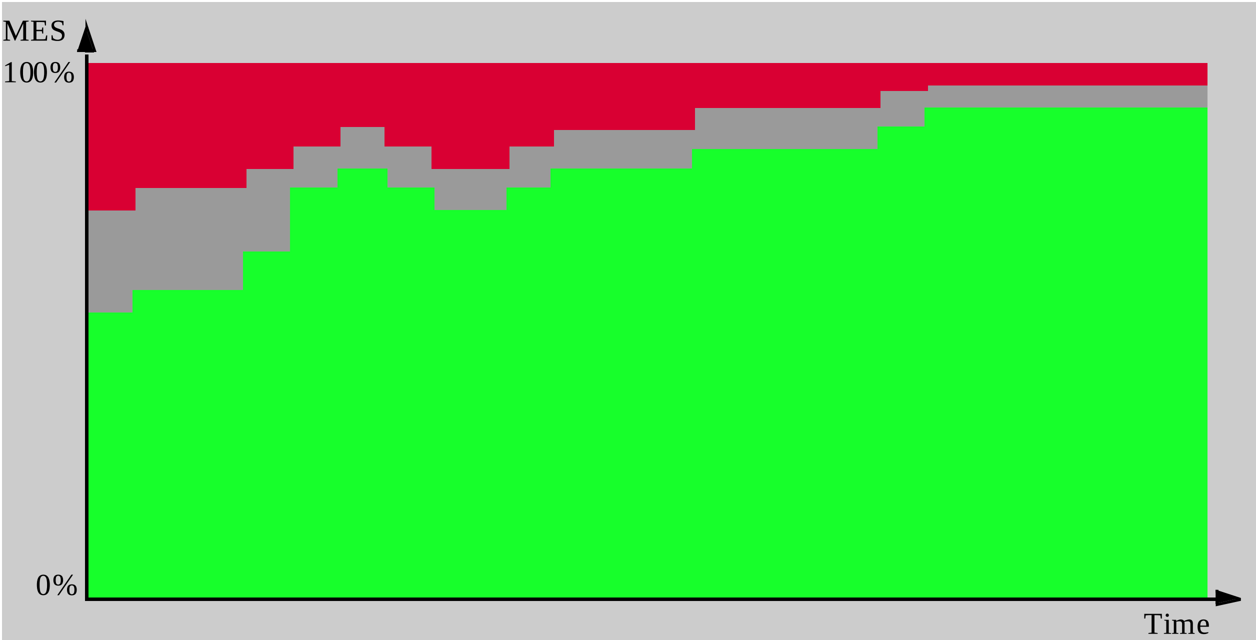

Figure 10: Subjective assessments over time of Military End State (MES).

4 CONCLUSIONS

We have demonstrated that it is possible to evaluate and refine a plan within effects-based planning using morphological analysis of the cross impact matrix. Furthermore, we show that we can find the decisive influences from activities by using Dempster-Shafer theory and sensitivity analysis. By doing both we can find any weaknesses and all strengths of the plan as described by the cross impact matrix before the effects-based execution phase.

We have developed a subjective effects-based assessment method for making subjective assessment of plans and plan elements within the effects-based approach to operations. We have shown that such subjective assessments can be performed of all supporting effects, decisive conditions and the military end state by taking human subjective assessments about activities as input and extending those assessments to all other plan elements using a cross impact matrix.

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