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Counter-deception in information fusion ☆

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ABSTRACT

In this article, we develop an entropy-based degree of falsity and combine it with a previously developed conflict-based degree of falsity in order to grade all belief functions. The new entropy-based degree of falsity is based on observing changes in entropy that are not consistent with combining only truthful information. With this measure, we can identify deliberately deceptive information and exclude it from the information fusion process. An experiment is performed comparing conflict and entropy measures and their combination. The effectiveness of the combination of the two measures is suggested.

1. Introduction

Managing false and possibly deliberately deceptive information is, in general, an important issue within an information fusion process. If false and deceptive information is not actively managed, it becomes impossible to trust any conclusions that is based on combining information from several different sources without knowing if one is deceptive. Conclusions that are drawn based on a combination of information from all sources may become degraded or false when truthful information is combined with deceptive information that supports untrue possibilities.

We previously developed methods within the theory of belief functions [1–6] to cluster information about several unrelated problems that should be handled separately when the information about different problems can be mixed up [7–11]. When we know that all information concerns only one problem at hand, this method could be used to identify false pieces of information and allow us to calculate a conflict-based degree of falsity for each piece of evidence [12]. These approaches use a function of the conflict [13,14] in Dempster's rule [2] as criterion function.

Smets [15] developed a methodology for managing a special case of deception where a deceiver may observe a truthful report and send the complement of a truthful belief function as deception instead of the truthful report itself. Pichon et al. [16] later developed a correction scheme that generalizes Shafer's discounting rule [4] by taking into account uncertain meta-knowledge regarding the source relevance and truthfulness. This model now subsumes Smets' model. Furthermore, they recently introduced a contextual correction mechanism [17] for [16].

However, the approach taken by Smets is a special case where the deceiver always sends the complement of what is observed from a truthful source. We think that this is not a realistic strategy by the deceiver, as it is easily countered by

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the counter-deception technique developed in Smets' approach [15]. Instead, we would allow the deceiver to act in any way it chooses and assume it might want to deceive us by supporting some focal elements of the frame of discernment that are wrong but we already somewhat believe. We think that this might be a more realistic approach.

In this article, we develop an entropy-based measure of degree of falsity [18] based on the change in entropy when truthful belief functions are combined with a deceptive belief function. The aim is that this new approach should be able to manage more generic types of deception than Smets' approach. As we have previously developed a conflict-based measure of degree of falsity [12] we will here combine these two approaches into one method for recognizing and managing deceptive information.

In Section 2, we discuss approaches to analyzing belief functions for their likelihood of being false due to deception. In Section 3, we review a previous approach to grading pieces of evidence for their degree of falsity based on their contribution to the conflict [13,14] received from Dempster's rule [2]. We then develop a new complimentary approach for grading pieces of evidence based on such changes in entropy that are not consistent with adding truthful evidence into the combination of all belief functions (Section 4). In Section 5, we combine the previously developed conflict-based degree of falsity with the new entropy-based degree of falsity into a combined degree of falsity. We use this approach to reason about which pieces of evidence might be false and should be either discounted or eliminated from the combination of information from all sources. In Section 6 we conduct an experiment with different numbers of deceptive belief functions and study the performance of the conflict and entropy approaches, and their combinations. Finally, in Section 7, we present the study's conclusions.

2. Analyzing belief functions

A belief function that is constructed to be deliberately false may be discovered in two different ways. Such a belief function is aimed to change the conclusion when analyzing the combination of all belief functions. Thus, it must be different from truthful belief functions.

One way to find this is by observing the conflict when combining a new belief function with all previous belief functions. For each belief function at hand, we may observe the change in conflict if we remove this particular belief function from the entire set of all available belief functions χ [7,19]. This will either lower the conflict or leave it unchanged. From a change in conflict, we can derive a degree of falsity for the belief function in question and, for example, use that to discount this particular belief function [12]. For an alternative approach using discounting rates, see [20].

A second approach is to observe a change in entropy when receiving a new belief function. If we receive a good belief function about the problem at hand we should assume that it will further reduce both the scattering and the nonspecificity of the basic belief by focusing it on small focal sets containing the ground truth. Thus, the belief of the ground truth will gradually become more believed and the entropy of the combined belief function will approach zero. On the other hand, if we receive a false belief function that incrementally changes the belief function a small step towards a uniform mass function, then the entropy of the combined belief function will increase. A very strong false belief function may swap the preferred order of the focal sets and leave the entropy unchanged or increased.

We will use both of these approaches to identify which belief functions may be deceptive in order to manage or eliminate them completely from the combination. It is important to note that combining truthful information with deceptive information leads to high conflict and entropy, while the reverse is not true. High conflict and entropy can arise through misrepresentation, mixed up belief function from failed clustering, or measurement errors, etc. A prerequisite for this approach is that the number of deceptive belief functions is less than the number of true belief function, otherwise we will eliminate the truth. If so, we can observe an initial upturn followed by a fall in the conflict as more deceptive information is included.

3. Conflict-based degree of falsity

We interpret the conflict received when combining a set of basic belief assignments (bbas) χ, as if there is at least one bba in χ that violates the representation of the frame of discernment Ω. Such a bba is interpreted as if it does *not* belong to the evidence that refer to the problem at hand [19] described by Ω. Instead, it should be removed from χ.

A conflict when combining all bbas in χ may thus be interpreted as a piece of evidence on a metalevel stating that at least one bba that is placed in χ does not actually belong to χ. On the metalevel, we reason only about the inclusion of bbas in χ, the frame of discernment is $\Theta = \{\text{Adp}, \neg\text{Adp}\}$ where AdP is short for χ being an *adequate partition* [19] of all bbas (i.e., with all bbas in χ), which means we can have metalevel evidence that the partition is either adequate or not. This can be reformulated to $\Theta = \{\forall j.e_j \in \chi,\ \exists j.e_j \notin \chi\}$, where AdP is refined to the first element of Θ, and $\neg$Adp is refined to the second element. This indicates that we can have evidence that all bbas belong to χ or at least one bba currently in χ does not. In addition, it is possible to refine the frame on the metalevel as $\Theta = \left\{\forall j.e_j \in \chi, \{e_q \notin \chi\}_q\right\}$, but we will use the first formulation.

We represent the conflict as,

$$
\begin{aligned}
&m_\chi\left(\exists j.e_j \notin \chi\right) = c_0, \\
&\qquad m_\chi(\Theta) = 1 - c_0,
\end{aligned}
\tag{1}
$$

where χ is the set of all bbas, c_0 is the conflict when combining all bbas, e_j is bba number j, and Θ is the frame of discernment on the metalevel.

Let us study one particular piece of evidence e_q in χ. If e_q is removed from χ, the conflict when combining all remaining bbas in χ decreases from c_0 to c_q. This decrease is interpreted as if there exists some evidence on the metalevel indicating that e_q does not belong to χ [12],

$$\begin{aligned} & m_{\Delta\chi}\left(e_q \notin \chi\right), \\ & m_{\Delta\chi}\left(\Theta\right), \end{aligned} \tag{2}$$

where $\Delta\chi$ is a label for this piece of evidence.

The conflict that remains c_q after e_q has been removed from χ is interpreted as evidence on the metalevel that there is at least one other bba e_j, $j \neq q$, that does not belong to $\chi - \{e_q\}$.

We have,

$$\begin{aligned} m_{\chi-\{e_q\}}\left(\exists j \neq q . e_j \notin \left(\chi - \{e_q\}\right)\right) &= c_q, \\ m_{\chi-\{e_q\}}\left(\Theta\right) &= 1 - c_q. \end{aligned} \tag{3}$$

Using eqs. (1) and (3), we can derive eq. (2) by stating that the belief in the proposition that there is at least one bba that does not belong to χ, $\exists j . e_j \notin \chi$, must be equal, regardless of whether we base that belief on (1) before e_q is taken out from χ, or on the combination of (2) and (3) after e_q is taken out from χ.

That is,

$$\mathrm{Bel}_{\chi}\left(\exists j . e_j \notin \chi\right) = \mathrm{Bel}_{\Delta\chi \oplus (\chi - \{e_q\})}\left(\exists j . e_j \notin \chi\right). \tag{4}$$

On the left hand side (LHS) of eq. (4) we have,

$$\mathrm{Bel}_{\chi}\left(\exists j . e_j \notin \chi\right) = m_{\chi}\left(\exists j . e_j \notin \chi\right) = c_0 \tag{5}$$

and, on the right hand side (RHS) eq. (4) we have,

$$\mathrm{Bel}_{\Delta\chi \oplus (\chi - \{e_q\})}\left(\exists j . e_j \notin \chi\right) = c_q + m_{\Delta\chi}\left(e_q \notin \chi\right)\left(1 - c_q\right). \tag{6}$$

By stating that LHS = RHS, we derive the basic belief number (bbn) of eq. (2) as,

$$\begin{aligned} m_{\Delta\chi}\left(e_q \notin \chi\right) &= \tfrac{c_0 - c_q}{1 - c_q}, \\ m_{\Delta\chi}\left(\Theta\right) &= \tfrac{1 - c_0}{1 - c_q}. \end{aligned} \tag{7}$$

We call this the conflict-based degree of falsity of e_q. For additional details, see [12]. An extensive example that goes through all derivations of eqs. (1)–(7) can be found in [19].

4. Entropy-based degree of falsity

Let us measure the change in entropy by observing the change in the aggregated uncertainty functional (AU) of the combination of all belief functions, both with and without the particular belief function in question e_q.

4.1. Aggregated uncertainty functional

The aggregated uncertainty functional AU was discovered by several authors around the same time [21–23]. AU is defined as

$$AU\left(\mathrm{Bel}\right) = \max_{\{p_x\}_{x \in \Omega}} \left\{ -\sum_{x \in \Omega} p\left(x\right) \log_2 p\left(x\right) \right\} \tag{8}$$

where $\{p_x\}_{x \in \Omega}$ is the set of all probability distributions such that $p_x \in [0, 1]$ for all $x \in \Omega$,

$$\sum_{x \in \Omega} p\left(x\right) = 1 \tag{9}$$

and

$$\mathrm{Bel}\left(A\right) \leq \sum_{x \in A} p\left(x\right) \tag{10}$$

for all $A \subseteq \Theta$. For an overview, see [24]. The AU measure corresponds to measures of nonspecificity and scattering that generalize Hartley information [25] and Shannon entropy [26].

An algorithm for numeric computation of AU was found by Meyerowitz et al. [27]. See [28] for implementation. For additional discussion on entropy, and a new definition of entropy of a bba see a recent paper by Jiroušek and Shenoy [29].

We define the entropy as a normalization of AU [30,31],

$$Ent\left(\{m_j\}\right) = \frac{AU\left(\oplus\{m_j\}\right)}{\log_2|\Omega|} \tag{11}$$

where m_j is the set of all bbas under combination, $AU \in [0, \log_2|\Omega|]$ and $Ent \in [0, 1]$.

Using Ent and AU, we may define an entropy-based degree of falsity for a deceptive piece of evidence as

$$\begin{aligned} m_{\Delta Ent}\left(e_q \notin \chi\right) &= Ent_q\left(\left\{m_j \mid j \neq q\right\}_j\right) - Ent_0\left(\left\{m_j\right\}_j\right), \\ m_{\Delta Ent}\left(\Theta\right) &= 1 - m_{\Delta Ent}\left(e_q \notin \chi\right), \end{aligned} \tag{12}$$

where Ent_0 is the entropy with e_q included in the combination, and Ent_q is the entropy without e_q, under the assumption that $m_{\Delta Ent}\left(e_q \notin \chi\right) \geq 0$. In the same way as in dealing with the change of conflict in removing a single belief function, we interpret an increase in entropy here as an indication to some extent that the belief function in question is false. Provided that the difference in eq. (12) is positive and that there is no change in the bbn in the order of focal element by degree of support, this may serve as an adequate measure of falsity for a deceptive piece of evidence based on a change in entropy. For a deceptive piece of evidence that changes the order of focal elements we may have a negative difference. For truthful evidence we expect a negative difference and would like to define the degree of falsity as zero. For a general and incremental approach that takes these situations into account see section 4.2.

4.2. Incremental steps of entropy change

Let us focus on e_q, which we want to evaluate by changes in entropy Ent. Because the entropy might increase when we remove e_q we will study a series of incremental changes. We will discount the mass function m_q at different rates and observe the incremental changes in entropy. We have [4],

$$m_q^{\%}(A) = \begin{cases} \alpha m_q(A), & A \subset \Omega \\ 1 - \alpha + \alpha m_q(A), & A = \Omega \end{cases} \tag{13}$$

where $0 \leq \alpha \leq 1$. Let α be defined as

$$\alpha = \tfrac{i}{n}, \tag{14}$$

where n is a parameter of choice with $0 \leq i \leq n$.

We have,

$$m_q^i(A) = \begin{cases} \tfrac{i}{n} m_q(A), & A \subset \Omega \\ 1 - \tfrac{i}{n} + \tfrac{i}{n} m_q(A), & A = \Omega \end{cases}. \tag{15}$$

Let $\Delta Ent_q^{k+1,k}$ be the incremental change in entropy between two situations using m_q^{k+1} and m_q^k, respectively, in the calculation of $\Delta Ent_q^{k+1,k}$.

We have,

$$\Delta Ent_q^{k+1,k} = Ent_q\left(\left\{m_q^{k+1}, m_j \mid j \neq q\right\}_j\right) - Ent_q\left(\left\{m_q^{k}, m_j \mid j \neq q\right\}_j\right). \tag{16}$$

We may extend eq. (12) using eq. (16) to define an incremental entropy-based degree of falsity as

$$\begin{aligned} m_{\Delta Ent}\left(e_q \notin \chi\right) &= \frac{1}{2}\sum_{k=0}^{n-1} \begin{cases} 0, & \forall 0 \leq l \leq k.\ \Delta Ent_q^{l+1,l} \leq 0 \\ \left|\Delta Ent_q^{k+1,k}\right|, & \text{otherwise} \end{cases}, \\ m_{\Delta Ent}\left(\Theta\right) &= 1 - m_{\Delta Ent}\left(e_q \notin \chi\right), \end{aligned} \tag{17}$$

using eq. (16). Here we sum up all incremental absolute differences to ensure a positive mass assignment.

As long as we receive a sequence of negative incremental changes, we consider m_q to be true. However, if there is a positive incremental change this is interpreted (to a degree) that this piece of evidence is false. The sequential inclusion of m_q may eventually cause a flip in the preferred focal element, followed by a series of negative incremental changes that must be counted towards the degree of falsity when the distribution becomes more and more focused around false focal elements.

This information, $m_{\Delta Ent}\left(e_q \notin \chi\right)$, can serve as an indication that m_q might be deliberately false, and may function as an indication even if the direct conflict with the main body of truthful evidence is low.

5. Combine degree of falsity with change of entropy

To be able to draw conclusions that are as sharp as possible regarding which belief functions are deceptive we prefer to eliminate any measuring noise before proceeding. We do this by assuming that at least one belief function is true and set $m_{\Delta\chi}\left(e_q \notin \chi\right) = 0$ and $m_{\Delta Ent}\left(e_q \notin \chi\right) = 0$ by subtraction from the belief function e_q with the lowest values. Note, that this may be two different belief functions. We have,

$$min\Delta\chi = \min_q m_{\Delta\chi}\left(e_q \notin \chi\right) \tag{18}$$

and

$$min\Delta Ent = \min_q m_{\Delta Ent}\left(e_q \notin \chi\right), \tag{19}$$

and get

$$\forall q.\, m^*_{\Delta\chi}\left(e_q \notin \chi\right) = m_{\Delta\chi}\left(e_q \notin \chi\right) - min\Delta\chi \tag{20}$$

and

$$\forall q.\, m^*_{\Delta Ent}\left(e_q \notin \chi\right) = m_{\Delta Ent}\left(e_q \notin \chi\right) - min\Delta Ent. \tag{21}$$

Furthermore, as the change in entropy is much smaller than the degree of falsity, and we prefer both approaches to have the same influence when deciding if a belief function is deceptive, we scale all $m^*_{\Delta Ent}$ by an influence quotient (Iq) that is the quotient in maximum values of and $m^*_{\Delta\chi}$ and $m^*_{\Delta Ent}$, in eqs. (20) and (21). We have,

$$Iq = \frac{\max_q m^*_{\Delta\chi}\left(e_q \notin \chi\right)}{\max_t m^*_{\Delta Ent}\left(e_t \notin \chi\right)} \tag{22}$$

and get,

$$\forall q.\, m^{**}_{\Delta Ent}\left(e_q \notin \chi\right) = Iq \cdot m^*_{\Delta Ent}\left(e_q \notin \chi\right) \tag{23}$$

In order to find which pieces of evidence might be false, we combine $m^*_{\Delta\chi}\left(e_q \notin \chi\right)$ with $m^{**}_{\Delta Ent}\left(e_q \notin \chi\right)$ in two different ways. First, we combine them using Dempster's disjunctive rule, where the product's mass $m_{\Delta\chi}\left(e_q \notin \chi\right) \cdot m_{\Delta Ent}\left(e_q \notin \chi\right)$ assigned to $e_q \notin \chi$ is the degree to which both measures simultaneously claim that the belief function is false. Secondly, we combine them by Dempster's rule, i.e., $m_{\Delta\chi}\left(e_q \notin \chi\right) \oplus m_{\Delta Ent}\left(e_q \notin \chi\right)$. This is a conflict-free combination as both mass functions have the same foci. Because Dempster's rule assigns mass to $e_q \notin \chi$ in an orthogonal combination when at least one of the two measures supports $e_q \notin \chi$, the mass assigned to $e_q \notin \chi$ corresponds to the statement that at least one of the belief functions is false. It is called probabilistic sum.

We get the product,

$$\begin{aligned} m_{\Delta\chi\cdot\Delta Ent}\left(e_q \notin \chi\right) &= m^*_{\Delta\chi}\left(e_q \notin \chi\right) \cdot m^{**}_{\Delta Ent}\left(e_q \notin \chi\right), \\ m_{\Delta\chi\cdot\Delta Ent}\left(\Theta\right) &= 1 - m_{\Delta\chi\oplus\Delta Ent}\left(e_q \notin \chi\right), \end{aligned} \tag{24}$$

and the probabilistic sum

$$\begin{aligned} m_{\Delta\chi\oplus\Delta Ent}\left(e_q \notin \chi\right) = m^*_{\Delta\chi}\left(e_q \notin \chi\right) + m^{**}_{\Delta Ent}\left(e_q \notin \chi\right) \\ - m^*_{\Delta\chi}\left(e_q \notin \chi\right) \cdot m^{**}_{\Delta Ent}\left(e_q \notin \chi\right), \\ m_{\Delta\chi\oplus\Delta Ent}\left(\Theta\right) = 1 - m_{\Delta\chi\oplus\Delta Ent}\left(e_q \notin \chi\right), \end{aligned} \tag{25}$$

respectively, by using eqs. (20), (18), (7) in calculation of $m^*_{\Delta\chi}$, and eqs. (23), (22), (21), (17), (16), (15), (11) and the algorithm in [28] to compute eq. (8) in calculation of $m^{**}_{\Delta Ent}$.

Based on this results (of eq. (24) and (25)) we can manage all m_q $(\forall q)$ in one of several different ways:

1. We may discount all m_q based on $m_{\Delta\chi\cdot\Delta Ent}\left(e_q \notin \chi\right)$ or $m_{\Delta\chi\oplus\Delta Ent}\left(e_q \notin \chi\right)$ using eq. (13) with $\alpha = 1 - m_{\Delta\chi\cdot\Delta Ent}\left(e_q \notin \chi\right)$ or $\alpha = 1 - m_{\Delta\chi\oplus\Delta Ent}\left(e_q \notin \chi\right)$. Evidence with a high degree of combined conflict-based and entropy-based falsity will be discounted to its degree with a low α. Subsequently, we handle all evidence with whatever mass remains after discounting as if it is true. This approach is somewhat crude and may not be the most preferable way to manage all evidence.

Table 1
Performance over 100 experiments with ten true and one false belief function.

	Conflict mean	Standard deviation	Entropy mean	Standard deviation	Product mean	Standard deviation	Probabilistic sum mean	Standard deviation
True bf	0.029	0.051	0.237	0.240	0.016	0.035	0.250	0.252
False bf	0.787	0.018	0.656	0.224	0.519	0.181	0.924	0.056
Diff	0.758		0.419		0.503		0.674	

2. A more refined approach is to perform sequential incremental discounts using increments of $\alpha = 1 - m_{\Delta\chi \cdot \Delta Ent}(e_q \notin \chi)$ or $\alpha = 1 - m_{\Delta\chi \oplus \Delta Ent}(e_q \notin \chi)$ as was suggested in [12]. Thus, instead of performing a direct discount of each piece of evidence by its degree of falsity we begin with a smaller incremental discount made individually for each belief function in proportion to its degree of falsity. After these initial discounts we recalculate conflict and update the degree of falsity. The process is performed sequentially in several small steps. With this approach it is possible to manage the conflict by appropriate discounts to obtain a smooth discounting process (compared to if we would have fully discounted each belief function to its degree of falsity) that bring the conflict down to an acceptable level.
3. A third approach is to evaluate and rank all m_q based on $m_{\Delta\chi \cdot \Delta Ent}(e_q \notin \chi)$ or $m_{\Delta\chi \oplus \Delta Ent}(e_q \notin \chi)$, and if there is a natural partition of all m_q into two groups (corresponding to true and false belief functions) we eliminate the false group from the combination. A natural partition of all m_q ranked by $m_{\Delta\chi \cdot \Delta Ent}(e_q \notin \chi)$ or $m_{\Delta\chi \oplus \Delta Ent}(e_q \notin \chi)$ can be said to exist if there is one gap in the evaluation significantly larger than the second largest difference within the ranking. If no such natural partition exists, we use one of the discount methods.

We think that managing all evidence in an interactive and incremental way using eq. (24), (25) and the third approach whenever possible, is a good way to find and manage deceptive information in an information fusion process. Although the first approach is acceptable in situations with many belief functions, the second method is more robust with fewer belief functions. The second method has a higher computational complexity, but the result is never worse than the first method, see [12] for details. However, the third approach, whenever possible, should be preferred because it eliminates the problem of false information rather than downgrading it.

6. Experiment with deception

In this section we study five experiments each with a frame of discernment of five elements $\Omega = \{A, B, C, D, E\}$ where A is the ground truth in each experiment. In every experiment we have ten true belief functions. They support a focal element that is a superset of A, or the set A itself, and Θ.

In addition we assign between one and five deceptive belief function, respectively, in each of these five experiments that does not support any focal element that contains A.

Each of the five experiments is repeated 100 times and averages are calculated for $m^*_{\Delta\chi}$, $m^{**}_{\Delta Ent}$, $m_{\Delta\chi \cdot \Delta Ent}(e_q \notin \chi)$, and $m_{\Delta\chi \oplus \Delta Ent}(e_q \notin \chi)$ using eqs. (20), (23), (24) and (25). These numbers are available in Tables 1–5 in columns 2, 4, 6, and 8 with column titles Conflict mean, Entropy mean, Product mean, and Probabilistic sum[1] mean, together with their standard deviations in columns 3, 5, 7, and 9 with columns titles Standard deviation. The difference between the averages for all false belief functions (in row 3) and all true belief functions (in row 2), respectively, is tabulated in row 4 for comparison regarding the effectiveness between the two measures, i.e., conflict (20) and entropy (23), and their product (24) and probabilistic sum (25). Note, that we use our knowledge about the experiment set-up to calculate these measures for false and true belief functions, respectively. We are not focused in this experiment in deriving the status of unknown belief functions.

In Table 1 we observe the superiority of the conflict measure $m^*_{\Delta\chi}$ over the entropy measure $m^{**}_{\Delta Ent}$ in situations with only one deceptive belief function and ten true belief functions, with an average difference over all 100 examples between false and true belief functions of 0.758 for the conflict measure and 0.419 for the entropy measure. The conflict measure is also superior to both the product and probabilistic sum of the two measures in this situation. The same conclusion also stands in the case of two deceptive belief functions (see Table 2).

For the case with three false belief functions the performance of the conflict measure is decreasing. The entropy measure continues to show stable performance. In this situation the best performance is provided by the probabilistic sum of the two measures (see Table 3).

For the case of four and five false belief functions in Tables 4 and 5, respectively, we notice the sharp deterioration in the performance of the conflict measure. In the case of five false belief functions this measure is not able to differentiate between true and false belief functions in this experiment. The entropy measure, however, continues to perform well, and the probabilistic sum of the two measure continue to differentiate well between the true and false belief functions.

[1] Here, *Product* refers to the product in the first row of eq. (24), and *Probabilistic sum* is a reference to the right hand side of the first row of eq. (25).

Table 2
Performance over 100 experiments with ten true and two false belief functions.

	Conflict mean	Standard deviation	Entropy mean	Standard deviation	Product mean	Standard deviation	Probabilistic sum mean	Standard deviation
True bf	0.086	0.123	0.287	0.283	0.046	0.074	0.327	0.309
False bf	0.748	0.082	0.554	0.242	0.418	0.198	0.884	0.073
Diff	0.662		0.266		0.371		0.557	

Table 3
Performance over 100 experiments with ten true and three false belief functions.

	Conflict mean	Standard deviation	Entropy mean	Standard deviation	Product mean	Standard deviation	Probabilistic sum mean	Standard deviation
True bf	0.188	0.204	0.190	0.286	0.046	0.081	0.333	0.313
False bf	0.633	0.146	0.547	0.173	0.344	0.145	0.836	0.075
Diff	0.445		0.357		0.298		0.504	

Table 4
Performance over 100 experiments with ten true and four false belief functions.

	Conflict mean	Standard deviation	Entropy mean	Standard deviation	Product mean	Standard deviation	Probabilistic sum mean	Standard deviation
True bf	0.264	0.255	0.068	0.198	0.020	0.058	0.313	0.294
False bf	0.415	0.218	0.552	0.114	0.221	0.128	0.746	0.086
Diff	0.151		0.484		0.202		0.434	

Table 5
Performance over 100 experiments with ten true and five false belief functions.

	Conflict mean	Standard deviation	Entropy mean	Standard deviation	Product mean	Standard deviation	Probabilistic sum mean	Standard deviation
True bf	0.314	0.2848	0.002	0.005	0.001	0.002	0.315	0.284
False bf	0.208	0.169	0.583	0.015	0.119	0.094	0.672	0.061
Diff	−0.105		0.581		0.119		0.357	

As the probabilistic sum of the two measures, i.e., combination by Dempster's rule $m_{\Delta\chi\oplus\Delta Ent}\left(e_q \notin \chi\right)$ eq. (25) always outperforms the product of the two measure $m_{\Delta\chi\cdot\Delta Ent}\left(e_q \notin \chi\right)$ eq. (24), we will never recommend using the product.

While it is obvious from the performance values in Tables 1–5 that the entropy measure alone works well enough, using the probabilistic sum of the two measure for increased robustness in problems with few deceptive belief functions seems to be a good approach in this experiment.

How to use conflict, entropy and the probabilistic sum of both measure in another situation is likely domain depended and needs to be studied. It seems safe to conclude that entropy alone (17) or the probabilistic sum of both measures (25) are the preferred candidates in order to differentiate between true information and deception.

7. Conclusions

We have developed an approach for counter-deception in information fusion. This method combines the study of conflict in Dempster's rule with observation of changes in entropy to determine which belief functions are deceptive. We conclude from the experiment performed that the entropy measure and the probabilistic sum of both measures perform well in differentiating between true information and deception. With this methodology, we can prevent deceptive information from being included in the information fusion process.

References

[1] A.P. Dempster, Upper and lower probabilities induced by a multivalued mapping, Ann. Math. Stat. 38 (1967) 325–339.
[2] A.P. Dempster, A generalization of Bayesian inference, J. R. Stat. Soc., Ser. B 30 (1968) 205–247.
[3] A.P. Dempster, The Dempster–Shafer calculus for statisticians, Int. J. Approx. Reason. 48 (2008) 365–377.
[4] G. Shafer, A Mathematical Theory of Evidence, Princeton University Press, Princeton, 1976.
[5] R.Y. Yager, M. Fedrizzi, J. Kacprzyk (Eds.), Advances in the Dempster–Shafer Theory of Evidence, Wiley, New York, 1994.
[6] P. Smets, R. Kennes, The transferable belief model, Artif. Intell. 66 (1994) 191–234.
[7] J. Schubert, On nonspecific evidence, Int. J. Intell. Syst. 8 (1993) 711–725.
[8] J. Schubert, Managing inconsistent intelligence, in: Proceedings of the 3rd International Conference on Information Fusion, 2000, pp. TuB4/10–16.
[9] J. Schubert, Clustering belief functions based on attracting and conflicting metalevel evidence using Potts spin mean field theory, Inf. Fusion 5 (2004) 309–318.
[10] J. Schubert, Clustering decomposed belief functions using generalized weights of conflicts, Int. J. Approx. Reason. 48 (2008) 466–480.

[11] J. Schubert, H. Sidenbladh, Sequential clustering with particle filtering – estimating the number of clusters from data, in: Proceedings of the 8th International Conference on Information Fusion, 2005, paper A4-3.
[12] J. Schubert, Conflict management in Dempster–Shafer theory using the degree of falsity, Int. J. Approx. Reason. 52 (2011) 449–460.
[13] P. Smets, Analyzing the combination of conflicting belief functions, Inf. Fusion 8 (2007) 387–412.
[14] J. Schubert, The internal conflict of a belief function, in: T. Denœux, M.-H. Masson (Eds.), Belief Function: Theory and Applications, in: AISC, vol. 164, Springer, Heidelberg, 2012, pp. 169–177.
[15] P. Smets, Managing deceitful reports with the transferable belief model, in: Proceedings of the 8th International Conference on Information Fusion, 2005, pp. 893–899.
[16] F. Pichon, D. Dubois, T. Denœux, Relevance and truthfulness in information correction and fusion, Int. J. Approx. Reason. 53 (2012) 159–175.
[17] F. Pichon, D. Mercier, É. Lefèvre, F. Delmotte, Proposition and learning of some belief function contextual correction mechanisms, Int. J. Approx. Reason. 72 (2016) 4–42.
[18] J. Schubert, Entropy-based counter-deception in information fusion, in: J. Vejnarova, V. Kratochvil (Eds.), Belief Functions: Theory and Applications, in: Lecture Notes in Artif. Intell., vol. 9861, Springer, Cham, 2016, pp. 174–181.
[19] J. Schubert, Specifying nonspecific evidence, Int. J. Intell. Syst. 11 (1996) 525–563.
[20] J. Klein, O. Colot, Singular sources mining using evidential conflict analysis, Int. J. Approx. Reason. 52 (2011) 1433–1451.
[21] C.W.R. Chau, P. Lingras, S.K.M. Wong, Upper and lower entropies of belief functions using compatible probability functions, in: J. Komorowski, Z.W. Raś (Eds.), Proceedings of the 7th International Symposium on Methodologies for Intelligent Systems, in: Lecture Notes in Artif. Intell., vol. 689, Springer, Heidelberg, 1993, pp. 306–315.
[22] Y. Maeda, H. Ichihashi, An uncertainty measure with monotonicity under the random set inclusion, Int. J. Gen. Syst. 21 (1993) 379–392.
[23] D. Harmanec, G.J. Klir, Measuring total uncertainty in Dempster–Shafer theory: a novel approach, Int. J. Gen. Syst. 22 (1994) 405–419.
[24] G.J. Klir, Uncertainty and Information: Foundation of Generalized Information Theory, Wiley, Hoboken, 2006.
[25] R.V.L. Hartley, Transmission of information, Bell Syst. Tech. J. 7 (1928) 535–563.
[26] C.E. Shannon, A mathematical theory of communication, Bell Syst. Tech. J. 27 (1948) 379–423, 623–656.
[27] A. Meyerowitz, F. Richman, E. Walker, Calculating maximum-entropy probability for belief functions, Int. J. Uncertain. Fuzziness Knowl.-Based Syst. 2 (1994) 377–389.
[28] D. Harmanec, G. Resconi, G.J. Klir, Y. Pan, On the computation of uncertainty measure in Dempster–Shafer theory, Int. J. Gen. Syst. 25 (1996) 153–163.
[29] R. Jiroušek, P.P. Shenoy, Entropy of belief functions in the Dempster–Shafer theory: a new perspective, in: J. Vejnarova, V. Kratochvil (Eds.), Belief Functions: Theory and Applications, in: Lecture Notes in Artif. Intell., vol. 9861, Springer, Cham, 2016, pp. 3–13.
[30] J. Schubert, Constructing multiple frames of discernment for multiple subproblems, in: E. Hüllermeier, R. Kruse, F. Hoffmann (Eds.), Proceedings of the 13th International Conference on Information Processing and Management of Uncertainty, in: CCIS, vol. 80, Springer, Heidelberg, 2010, pp. 189–198.
[31] J. Schubert, Constructing and evaluating alternative frames of discernment, Int. J. Approx. Reason. 53 (2012) 176–189.