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A Stress Intensity Factor Equation for Cracks at an Open Hole in a Sheet of Finite Dimensions Subjected to a Uniform, Uniaxial Stress

Abstract (not more than 200 words)

In many applications it is more convenient to have an equation for computing the stress intensity factor instead of interpolating in tables or reading values from graphs. This is particulatly true if there are many parameters involved as in the case of radial cracks at a circular hole in a rectangular sheet.

The aim of this investigation is to develop a stress intensity factor equation for cracks at a hole in a limited sheet. It is assumed that the sheet is loaded along the edges parallel to the cracks with a uniform, unaxial stress. The aim is also to assess the accuracy of the equation through comparisons to different numerical solutions.

An equation consisting of five parts has been developed. The first part is the stress intensity factor for cracks at a hole in an infinite sheet. The second part is a finite width correction. The third part makes a correction for eccentricity. The fourth part represents a finite height correction. Finally, the fifth part includes a correction for interaction between the hole and the finite height.

The accuracy of the stress intensity factor equation is generally better than 20% and in most cases better than 8%.

Keywords

Finite element analysis, Cracks, Stress intensity factor, Two dimensional

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Rapportens titel (i översättning)

En ekvation för spänningsintensitetsfaktorn för sprickor vid ett öppet hål i en plåt med begränsade dimensioner utsatt för en jämn enaxlig belastning

Sammanfattning (högst 200 ord)

I många tillämpningar är det bekvämt med en ekvation för beräkning av spänningsintensitetsfaktorn i stället för att interpolera i tabeller eller läsa värden från ett diagram. Detta gäller i synnerhet om antalet parametrar är stort, vilket är fallet för en eller två radiella sprickor vid ett cirkulärt hål i en begränsad rektangulär plåt.

Syftet med föreliggande undersökning är att utveckla en spänningsintensitetsfaktorekvation för sprickor vid ett hål i en begränsad plåt. Plåten antas vara belastad med en jämnt fördelad spänning längs kanterna parallella med sprickan. Syften är också att uppskatta noggrannheten i spänningsintensitetsfaktorekvationen genom jämförelser mot olika numeriska lösningar.

En spänningsintensitetsekvation bestående av 5 delar har utvecklats. Den första delen avser sprickor vid hålen i en obegränsad plåt. Den andra delen är en breddkorrektion. Den tredje delen ger en korrektion för excentricitet. Den fjärde delen avser korrektion för begränsad höjd. Slutligen den femte delen utgör korrektion för interaktion mellan höjd och hål. Noggrannheten hos spänningsintensitetsfaktorekvtionen är generellt bättre än 20% och i de flesta fall bättre än 8%.

Nyckelord

Finit elementanalys, Sprickor, Spänningsintensitetsfaktorer, Tvådimensionell

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Contents

Summary

In many applications it is more convenient to have an equation for computing the stress intensity factor instead of interpolating in different tables or reading the values from a graph. This is particularly true when there is a relatively large number of parameters involved as in the case of a single or two radial cracks emanating from an open hole in a sheet of finite dimensions.

A few suggestions to an equation for the stress intensity factor, in the case of infinite height, exist in the literature. These equations are generally based upon a basic equation for an open hole geometry multiplied by a correction factor for the finite width. Unfortunately, the accuracy of these equations has not been investigated particularly well.

The aim of the present investigation is to develop an equation for the stress intensity factor in the case of a single or two cracks at the hole in a sheet of finite dimensions. It is assumed that the sheet is subjected to a uniformly distributed, uniaxial stress acting on the edges of the sheet parallel to the prospective crack path. Also, the aim is to obtain an estimation of the accuracy of the developed equation by comparisons to different numerical solutions.

An equation consisting of five parts has been developed. The first part is the stress intensity factor for a single or two radial cracks at an open hole in an infinite sheet. The second part is a standard finite width correction. The third part is a correction for an eccentric location of the equivalent line crack centre with respect to the width. The fourth part represents a finite height correction for a sheet without a hole. Finally, the fifth part is a correction for the interaction between the hole and the finite height.

The accuracy of the developed equation is generally much better than 20 % for crack lengths of practical interest and in most cases the accuracy is better than 8 %.

1 Introduction

In many applications it is more convenient to have an equation to compute the stress intensity factor instead of interpolating in tables or reading values from a graph. This is particularly true when there is a relatively large number of parameters (ratios between dimensions) involved.

Consider a sheet or a plate of finite dimensions having a circular hole with radius R. The total width and height (length) of the sheet are denoted W and H, respectively. The sheet is loaded by uniform tensile stresses along the edges parallel to the width direction. In the cases of a single or two radial cracks emanating from the hole there may be four parameters involved, E/R, E/W, E/H and a_2/a_1, see Figure 1. E is the distance from the centre of the hole to the nearest edge parallel to the height direction of the sheet. The crack lengths, a_1 and a_2, are measured from the hole edge. Thus, the stress intensity factor for through the thickness cracks can be written,

$$K_I = K_I\left(\frac{a_1}{E-R}, \frac{a_2}{a_1}, \frac{E}{W}, \frac{E}{R}, \frac{E}{H}\right) \qquad (1)$$

A few suggestions to an equation for the stress intensity factor exist in the literature, in the case of infinite height. The accuracy of these equations has not been investigated particularly well, mainly because there has not been that many numerical solutions available for a comparison where all the parameters have been varied. Furthermore, most of the numerical solutions that exist, for the finite width sheet, have treated the problem with two diametrically located symmetrical cracks.

In the present investigation different numerical solutions to the problem are compared. Some additional finite element analyses are performed to complement the literature data. An approximate equation for the stress intensity factor, for a single crack, two symmetrical cracks or two cracks of unequal lengths, is suggested. The accuracy of the proposed equation is assessed by comparisons to the literature solutions and the finite element solutions.

2 SIF for Cracks at an Open Hole in an Infinite Sheet

The stress intensity factor for radial cracks at an open hole in a sheet of infinite width and height, subjected to a remote uniform, uniaxial stress perpendicular to the crack line, has been derived by various methods. Bowie, Ref.[1], solved this problem already in 1956 for both a single crack and for two symmetrical cracks. Tweed and Rooke, Ref.[2-4], used the Mellin transform technique combined with a Gauss-Chebyshev integration formula to obtained the solution for the stress intensity factors of a single crack, two symmetrical cracks and two cracks of unequal lengths, respectively. Lai et. al. combined a complex variable method with a least square method in a boundary integral form, Ref.[5], to obtain the same stress intensity factors as Tweed and Rooke. Newman used the boundary collocation method to obtain the solution for two symmetrical cracks, Ref.[6]. Finally, Rubinstein and Sadegh obtained the solution for a single crack on basis of dislocation pile-up, Ref.[7]. The results of the different investigators are all presented in Table 1 as the normalized stress intensity factor.

The normalized stress intensity factor is defined according to,

$$F_{OH} = \frac{K_I}{\sigma_\infty \sqrt{\pi a}} \tag{2}$$

where K_I is the mode I stress intensity factor, σ_∞ is the remote uniform stress perpendicular to the crack line and a is the crack length measured from the hole edge.

It should be noticed that the results presented in table 1 for Tweed and Rooke are computed using their solution method. The only difference is that the singular integrals resulting from the Mellin transform were reduced to 40 linear algebraic equations instead of 24, which should give a higher accuracy. The accuracy of the Tweed and Rooke solution for a single crack is better than 1 % according to Ref.[6] and according to themselves, Ref.[4], the results should be accurate to about 0.1 %. The relative difference between their solution and the numerical result obtained for a single crack using their solution technique is less than 0.07 %. Compared to the solutions by Lai et. al. and by Rubinstein and Sadegh the relative differences are almost negligible, as can be seen in table 1. The accuracy of the Newman solution for two symmetrical cracks is better than 0.1 % according to Ref.[6] and the difference compared to the solution by Tweed and Rooke is less than 0.7 %.

Grandt, Ref.[8], has suggested least squares fittings to the results of Bowie according to,

$$F_{OH}^{1} = 0.6762 + \frac{0.4367}{0.1623 + \dfrac{a}{2R}} \tag{3}$$

for a single crack and,

$$F_{OH}^{2} = 0.9439 + \frac{0.3433}{0.1386 + \dfrac{a}{2R}} \tag{4}$$

in the case of two symmetrical cracks.

Newman, Ref.[9], has suggested polynomial fittings to the results of Bowie according to,

$$F_{OH}^{1} = 0.707 - 0.18\lambda + 6.55\lambda^2 - 10.54\lambda^3 + 6.85\lambda^4 \tag{5}$$

for the single crack and in the case of two symmetrical cracks,

$$F_{OH}^{2} = 1.0 - 0.15\lambda + 3.46\lambda^2 - 4.47\lambda^3 + 3.52\lambda^4 \tag{6}$$

where

$$\lambda = \frac{1}{1 + (a/R)} \tag{7}$$

A third equation for the single crack is given by Rooke, Baratta and Cartwright, Ref.[10], on basis of the solution by Tweed and Rooke, Ref.[2],

$$F_{OH}^{1} = \left[1.12 - 0.119\left(\arctan\left(\frac{a}{R}\right)\right)^{2.748}\right]\left(1 + 0.5\lambda^2 + 1.5\lambda^4\right) \tag{8}$$

Finally, Schijve, Ref.[11], has suggested equations based on the numerical result of Newman for two symmetrical cracks and a modified conversion factor based on a suggestion by Shah, Ref.[12]. For the single crack Schijve obtains the following equation,

$$F_{OH}^{1} = \left[\frac{1}{0.539 + 1.93\dfrac{a}{R} + 2\left(\dfrac{a}{R}\right)^2} + \frac{\lambda + 2}{2}\right]\sqrt{\frac{\lambda + 1}{2}}\left[1 + \frac{a}{R}\frac{\lambda^3}{5}\right] \tag{9}$$

and for the case with two symmetrical cracks the equation becomes,

$$F_{OH}^{2}=\frac{1}{0.539+1.93\frac{a}{R}+2\left(\frac{a}{R}\right)^{2}}+\frac{\lambda+2}{2} \qquad (10)$$

The equation for two symmetrical cracks is within 0.2 % of the Newman data except for a/R=0.01 where the Newman data is supposed to be too low.

Using the numerical results by Tweed and Rooke, presented in table 1, as reference, the relative errors of the different equations for a single crack and for two symmetrical cracks have been calculated. The results are shown in Figures 2 and 3. In general the equations by Schijve, Eq.(9-10), give the smallest relative errors, and although, it is an approximate analysis the accuracy is said to be better than 0.4 %. Furthermore, the equation by Schijve is the only one that gives the exact asymptotic value as the single crack length tends to infinity, $F_{OH}^{1}(a/R \to \infty)=1/\sqrt{2}$. The equations by Newman Eq.(5), and Rooke et.al., Eq.(8), give approximately the correct asymptotic value for the single crack. The asymtotic value as a/R tends to infinity is correct for both Newman´s and Schijve´s equations in the case of two symmetrical cracks..

In the case of two cracks of unequal lengths some SIF solutions are presented as graphs and tables in the literature, Refs.[3, 5, 13]. The numerical results, based on the solution technique by Tweed and Rooke, Ref.[3], have been compared to the tabulated solution by Lai et. al., Ref.[5]. For $a_2/a_1 \le 1$ the maximum relative difference found, 1.7 %, occurs for $a_1/R = 0.02$. All other relative differences are less than 1 %.

An equation for the stress intensity factor can be obtained by further modifying the already modified conversion factor. For the crack, a_1 with crack tip A, the stress intensity factor can be written,

$$K_{I}^{A}=\sigma_{\infty}\sqrt{\pi a_1}F_{OH}^{A} \qquad (11)$$

where the normalized stress intensity factor is expressed as,

$$F_{OH}^{A}=\left(\frac{1}{0.539+1.93\frac{a_1}{R}+2\left(\frac{a_1}{R}\right)^{2}}+\frac{\lambda_1+2}{2}\right)f_{c}^{A} \qquad (12)$$

with the two-times modified conversion factor equal to,

$$f_{c}^{A}=\sqrt{\frac{1}{2}\left(1+\frac{\lambda_1}{\lambda_2}\right)}\left[1+\frac{a_1}{R}\frac{\lambda_1^{3}}{5}\left(1-\frac{4}{\pi}\tan^{-1}\left(\frac{a_2}{a_1}\right)\right)\right] \qquad (13)$$

where

$$\lambda_1 = \frac{1}{1+a_1/R} \quad ; \quad \lambda_2 = \frac{1}{1+a_2/R} \tag{14}$$

A relative comparison, for $\lambda_1/\lambda_2 \leq 1$, between the results of Eq.(11) and the results obtained using the solution technique by Tweed and Rooke is shown in Figure 4. The maximum relative difference in the figure is less than 1.6 %. It should be observed that Eq.(12) reduces to Eq.(9) when a_2 is zero and to Eq.(10) when a_2 equals a_1.

For the crack a_2, with tip B, the stress intensity factor may be written,

$$K_I^B = \sigma_\infty \sqrt{\pi a_2} F_{OH}^B \tag{15}$$

where the normalized stress intensity factor is,

$$F_{OH}^B = \left(1 + \frac{1}{0.539 + 1.93\frac{a_2}{R} + 2\left(\frac{a_2}{R}\right)^2} + \frac{\lambda_2}{2} \right) f_c^B \tag{16}$$

with f_c^B given by,

$$f_c^B = \sqrt{\frac{1}{2}\left(1+\frac{\lambda_2}{\lambda_1}\right)}\left[1+\frac{a_2}{R}\frac{\lambda_2^3}{5}\left(1-\frac{4}{\pi}\tan^{-1}\left(\frac{a_1}{a_2}\right)\right)\right] \tag{17}$$

Obviously, Eq.(15) is identical to Eq.(11) but with the crack length indices shifted. A comparison is made between the results of Eq.(15) and the results obtained using the solution technique by Tweed and Rooke in Figure 5. The comparison is made for $\lambda_1/\lambda_2 \leq 1$ and the maximum relative difference, 7.8 %, occur for $a_2/R = 0.02$. In general the figure shows that large relative differences occur for small crack lengths, a_2, and that they increase with increasing crack length a_1.

3 Finite Width Correction

The most commonly used correction for cracks at an open hole in a sheet of finite width is a modification of the secant correction suggested by Newman, Ref.[9]. The secant correction for a centre line crack in a strip of finite width (without a hole) was proposed by Isida on basis of the results of Feddersen, Ref.[14]. The secant correction is written,

$$F_W = \frac{1}{\sqrt{\cos\left(\frac{\pi}{2}\frac{a}{E}\right)}} \tag{18}$$

where E is the distance from the centre of the line crack to the edge of the sheet and the crack length is 2a from tip to tip.

The modification for an open hole consists of a factor, f_{sc}, that takes into account the increase in stress concentration at the edge of the hole due to the finite width. The factor f_{sc} is given by,

$$f_{sc}^{N} = \left[\cos\left(\frac{\pi}{2}\frac{1}{E/R}\right)\right]^{-1/2} \approx \sqrt{\frac{K_{tg}^{N}}{3}} \tag{19a}$$

for the side of the hole having its edge closest to the edge of the sheet, and

$$f_{sc}^{F} = \left[\cos\left(\frac{\pi}{2}\frac{E/W}{E/R\,(1-E/W)}\right)\right]^{-1/2} \approx \sqrt{\frac{K_{tg}^{F}}{3}} \tag{19b}$$

at the side of the hole with its edge farthest away from the sheet edge. The distance from the centre of the hole to the nearest edge of the sheet is E ($E \le W/2$). K_{tg} is the gross stress concentration factor.

The second modification is the introduction of an equivalent normalized crack length. The cracks and the open hole itself are considered as a line crack of length $2a_{eq}$ from tip to tip. The centre of this line crack has a distance to the edge of the sheet, at crack tip A, equal to E_{eq}. The equivalent normalized crack length, for the crack, a_1 (with tip A), may then be written,

$$\alpha_{eq}^{A} = \frac{a_{eq}}{E_{eq}} = \frac{2 + \frac{a_1 + a_2}{R}}{2\frac{E}{R} - \frac{a_1 - a_2}{R}} \tag{20a}$$

where,

$$E_{eq} = E + R + a_2 - \frac{a_{eq}}{2} = E - \frac{a_1 - a_2}{2} \qquad (21)$$

Correspondingly for crack, a_2 (with tip B), the equivalent normalized crack length is,

$$\alpha_{eq}^{B} = \frac{a_{eq}}{W - E_{eq}} = \frac{2 + \frac{a_1 + a_2}{R}}{2\frac{E}{R}\left(\frac{1 - E/W}{E/W}\right) + \frac{a_1 - a_2}{R}} \qquad (20b)$$

Replacing the normalized crack length, a/E, in Eq.(18) with the equivalent normalized crack length and introducing the factor f_{sc} yields the following equation for the finite width correction, at crack tip A,

$$F_W^A = \left[\cos\left(\frac{\pi}{2}\frac{1}{E/R}\right)\cos\left(\frac{\pi}{2}\alpha_{eq}^{A}\right)\right]^{-1/2} \qquad (22a)$$

The finite width correction at crack tip B is obtained analogously,

$$F_W^B = \left[\cos\left(\frac{\pi}{2}\frac{R/W}{(1 - E/W)}\right)\cos\left(\frac{\pi}{2}\alpha_{eq}^{B}\right)\right]^{-1/2} \qquad (22b)$$

Equation (22a) is identical to the finite width correction factor suggested by Newman if $a_1 = a_2$ or $a_2 = 0$.

For the centre cracked sheet the secant correction has an accuracy better than 0.3 % for $2a/W \leq 0.7$ and the accuracy is about 1 % for $2a/W = 0.8$. For the open hole sheet the accuracy of the secant correction has not been verified.

A more accurate finite width correction for the centre cracked sheet has been developed by Tada, Ref.[14],

$$F_W = \frac{1 - 0.025\alpha^2 + 0.06\alpha^4}{\sqrt{\cos\left(\frac{\pi}{2}\alpha\right)}} \qquad (23)$$

This correction factor has an accuracy better than 0.1 % for any $\alpha = a/E$.

4 Eccentricity Correction

As mentioned in the previous section the equivalent crack length, a_{eq}, is obtained by viewing the cracks and the hole as an internal line crack of length a_1+a_2+2R from tip to tip. The distance from the centre of this internal line crack to the edge of the sheet (at the crack tip A side) has been denoted E_{eq}, which in general is different from W/2. Thus a correction for the eccentric location of the crack centre is made in the finite width correction if the cracks, a_1 and a_2, are of different lengths and/or the hole is displaced from the sheet centre line. Furthermore, the finite width correction implies that the total width of the sheet becomes $2E_{eq}$ at the crack tip A side and $2(W-E_{eq})$ at the crack tip B side, which is not correct. To adjust for this error an additional correction for eccentricity is introduced. According to Ref.[15], the eccentricity correction is given by,

$$F_E^A = \sin\left(\pi\frac{E_{eq}}{W}\right) + \left[\frac{1+\sqrt[4]{\cos\left(\frac{\pi}{2}\alpha_{eq}^A\right)}}{2}\right]^2 \left[1-\sin\left(\pi\frac{E_{eq}}{W}\right)\right] \qquad (24a)$$

for the crack with tip A if $E_{eq}/W \leq 0.5$ and,

$$F_E^A = \left(\frac{\dfrac{1}{\sqrt{\cos\left(\frac{\pi}{14}\alpha_{eq}^A\left(\frac{3+E_{eq}/W}{1-E_{eq}/W}\right)\right)}}-1}{1+0.21\sin\left[8\tan^{-1}\left(\left(2\frac{E_{eq}}{W}-1\right)^{0.9}\right)\right]}+1\right)\sqrt{\cos\left(\frac{\pi}{2}\alpha_{eq}^A\right)} \qquad (24b)$$

if $E_{eq}/W \geq 0.5$.

Above, α and E in the original formulae, for a line cracked sheet, have been substituted for α_{eq}^A and E_{eq}, respectively.

Compared to the solution by Isida, given in Ref.[14], Eq.(24a) is within 1.5 % for $\alpha_{eq}^A \leq 0.7$ and any $E_{eq}/W \leq 0.5$. For $0.7 < \alpha_{eq}^A \leq 0.9$ differences of up to 6.5 % have been found in a few cases. The maximum relative difference for Eq.(24b) is less than 1.1 % for $\alpha_{eq}^B \leq 0.7$ and any

$E_{eq}/W \geq 0.5$. For $0.7 < \alpha_{eq}^{B} < 0.9$ the maximum relative difference found, for any E_{eq}/W, is 8.1 %. The accuracy of Isida´s solution is better than 1 %.

For the crack with tip B a corresponding closed form solution is presented in Ref.[15],

$$F_E^B = \left(\frac{\dfrac{1}{\sqrt{\cos\left(\dfrac{\pi}{14}\alpha_{eq}^{B}\left(\dfrac{4-E_{eq}/W}{E_{eq}/W}\right)\right)}} - 1}{1+0.21\sin\left[8\tan^{-1}\left(\left(1-2\dfrac{E_{eq}}{W}\right)^{0.9}\right)\right]} + 1 \right)\sqrt{\cos\left(\frac{\pi}{2}\alpha_{eq}^{B}\right)} \qquad (24c)$$

if $E_{eq}/W \leq 0.5$ and,

$$F_E^B = \sin\left(\pi\left(1-\frac{E_{eq}}{W}\right)\right) + \left[\frac{1+\sqrt[4]{\cos\left(\dfrac{\pi}{2}\alpha_{eq}^{B}\right)}}{2}\right]^2 \left[1-\sin\left(\pi\left(1-\frac{E_{eq}}{W}\right)\right)\right] \qquad (24d)$$

if $E_{eq}/W \geq 0.5$.

Compared to the solution by Isida the relative difference of Eq.(24c) is less than 1.1 % for $\alpha_{eq}^{A} \leq 0.7$ and any $E_{eq}/W \leq 0.5$. The maximum relative difference found is 8.1 %. Eq.(24d) have relative differences less than 1.5 % for $\alpha_{eq}^{B} \leq 0.7$ and any $E_{eq}/W \geq 0.5$. The maximum relative difference found in this case is 6.5 %.

5 Finite Height Correction

For a centre line crack of length 2a from tip to tip in a sheet of finite width and height the correction due to the finite height can be obtained from the complete stress intensity factor divided by the stress intensity factor corrected only for the finite width. Thus,

$$F_H = \frac{K_I}{\sigma\sqrt{\pi a}F_W} \tag{25}$$

F_W, for the centre line crack, is given by Eq.(23). K_I, with an accuracy better than 1 % for $\alpha \leq 0.7$, can be found in Ref.[6] and in Ref.[16]. A polynomial fit of Eq.(25) according to the method of least squares yields, Ref.[17],

$$F_H = 1 + B_1\alpha + B_2\alpha^2 \tag{26}$$

where

$$\begin{cases} B_1 = 0.170218\gamma + 0.43604\gamma^2 \\ B_2 = -0.55270\gamma + 1.68076\gamma^2 \end{cases} \tag{27}$$

and

$$\gamma = \frac{E}{H} \tag{28}$$

H is the total height of the sheet or the plate. The accuracy in the polynomial fit is better than 7 % for $\alpha \leq 0.7$ and $\gamma \leq 1.25$. The average of the absolute relative errors in the fit is less than 2.1 %.

Introduction of α_{eq}^A, according to Eq.(20a), and γ_{eq}^A according to,

$$\gamma_{eq}^A = \frac{E}{H}\left(1 - \frac{1}{2}\frac{a_1 - a_2}{E}\right) \tag{29a}$$

into Eq.(26) is assumed to give an approximate finite height correction factor for crack tip A in a sheet with an open hole. For crack tip B the corresponding equivalent parameters are α_{eq}^B and

$$\gamma_{eq}^B = \frac{E}{H}\left(\frac{1 - E/W}{E/W} - \frac{1}{2}\frac{a_2 - a_1}{E}\right) \tag{29b}$$

6 A Stress Intensity Factor Equation

From above the complete stress intensity factor for one or two cracks at an open hole in a sheet or a plate of finite dimensions subjected to a uniformly distributed, uniaxial stress perpendicular to the crack line can be written,

$$K_I^A = \sigma\sqrt{\pi a_1}F^A$$
$$K_I^B = \sigma\sqrt{\pi a_2}F^B \tag{30}$$

where

$$F^A = F_{OH}^A F_W^A F_E^A F_H^A$$
$$F^B = F_{OH}^B F_W^B F_E^B F_H^B \tag{31}$$

In the following, the open hole corrections, F_{OH}^A and F_{OH}^B, according to Schijve (Eqs.(12, 16)), the modified finite width corrections, F_W^A and F_W^B (Eqs.(22a, 22b)), the eccentricity corrections, F_E^A and F_E^B (Eqs.(24a, 24b, 24c and 24d)) and the finite height corrections, F_H^A and F_H^B (Eq.(26)), have been used to calculate the normalized stress intensity factors F^A and F^B.

7 Finite Element Analysis

Finite element analyses of cracks at an open hole in a sheet of finite width and height have been made using the finite element code STRIPE, Ref.[18]. In the case of a single radial crack at the hole an element mesh for one half of the plate was developed using 78 solid elements and 564 nodes, see Figure 6. In the case of two symmetrical radial cracks at the hole an element mesh for one quarter of the plate having 57 solid elements and 382 nodes was developed. The radius of the open hole and the thickness of the sheet were chosen to 3 mm and 2 mm, respectively. STRIPE uses the hp-version of adaptive solution technique which explains the rather few number of elements used for the basic meshes. The maximum polynomial order, p, was limited to 6 in the analyses, except in a few cases where higher p-values were used to check the convergence of the solution.

The STRIPE analysis gives the stress intensity factor for local points along the crack front. Thus, a symmetric variation in the stress intensity factor with respect to the sheet mid-plane is obtained in this case. The maximum value occurs in the centre of the sheet. In the following an average value of the stress intensity factor according to,

$$K_I = \sqrt{\frac{K_{I,1}^2(z_2 - z_1) + \sum_{i=2}^{m-1} K_{I,i}^2(z_{i+1} - z_{i-1}) + K_{I,m}^2(z_m - z_{m-1})}{2T}} \qquad (32)$$

is used, where m is the number of equally spaced data points along the crack front, T is the total thickness and z is a co-ordinate in the thickness direction representing the local points.

Figures 7 and 8 show two typical examples of the convergence of the stress intensity factor as function of p for two selected combinations of the parameters E/R, E/H and a/R. Similar results were obtained for other combinations of the parameters. The solutions converge rather rapidly and the results presented in the following should be accurate to within ±2 %.

7.1 Two Symmetrical Cracks

Figures 9 and 10 show the normalized stress intensity factor as function of the normalized crack length in the case of two symmetrical cracks at the open hole. The parameter E/R was varied in the range 1.7 to 10, as can be seen in the figures, while the parameter E/H was either equal to 0.25 or 0.5. A few computations were made with smaller values of E/H (0.1) in order to check the effect of increasing the finite height. It was found that increasing the finite height to values greater than 4 times E had a very small effect on the stress intensity factor (less than 1 %).

Also shown in Figures 9 and 10 are curves of the normalized stress intensity factor, F ($F=F^A=F^B$), calculated, using Eq.(30), for the same values of E/R and E/H as used in the FE-analyses. The correlation between the calculated values and those obtained in the FE analyses is rather good for E/H=0.25 whereas the correlation is rather poor for E/H=0.5, particularly for small values of E/R. The average of the absolute relative differences, the maximum and the minimum relative differences between the FE-solutions and the corresponding values obtained using Eq.(30) are presented in Table 2.

7.2 A Single Crack

Figure 11 shows the normalized stress intensity factor as function of the normalized crack length obtained using STRIPE with E/R=2 and four different values on E/H. The effect of the finite height is almost negligible for E/H≤0.25 as can be seen in the figure. Also shown in the figure are curves obtained using Eq.(30) and the same parameter values. The agreement between the FE-solution and the equation is rather good for E/H≤0.3333. However, for E/H=0.5 rather large discrepancy between the STRIPE solution and the results of the equation can be observed in figure.

A finite element analysis was also made for the case of E/R=4 and E/H=0.25. The result of this analysis is shown in Figure 12 as a comparison to the result obtained using Eq.(30). A very good agreement between the STRIPE solution and Eq.(30) is obtained in this case as can be seen in figure.

Table 3 gives the average of the absolute relative differences as well as the maximum and the minimum relative differences between the STRIPE solutions and the corresponding values obtained using Eq.(30).

8 Comparisons to Literature data

The results obtained using the finite element code STRIPE and the stress intensity factor according to Eq.(30) have been compared to different stress intensity factor solutions found in the literature, Refs.[5-6, 14-15, 19-24].

8.1 Two Symmetrical Cracks

Firstly, the case of E/R=2 and E/H=0.25 is studied. Newman has obtained a solution to this case using the boundary collocation technique, Refs.[6, 14]. The solution is accurate according to Ref.[6] but no value on the accuracy is given. Lai et. al. used complex variable technique and the method of least squares in a boundary integral form to obtain the solution, Ref.[5]. Cartwright and Parker used a similar technique with boundary point matching and the method of least squares, Ref.[19]. Figure 13 shows the different solutions in terms of the normalized stress intensity factor as function of the normalized crack length. Included in the figure are the results obtained using STRIPE and a curve representing Eq.(31).

The three different solutions from the literature agree rather well but they all under-estimates the stress intensity factor compared to the STRIPE solution. The equation, on the other hand, slightly over-estimates the stress intensity factor as compared to the STRIPE solution.

Secondly, the case of E/R=4 and E/H=0.25 is studied. Again a solution by Newman can be found in Refs.[6, 14]. The solution is based on the boundary collocation method and the accuracy is given as better than 0.1 %. Furthermore, a solution by Kitagawa and Yuuki based on conformal mapping is also available in Ref.[6]. The accuracy is said to be better than 1 %. Lai et. al. give a third solution, Ref.[5], and finally, Cartwright and Parker, Ref.[19], suggest a fourth solution.

Again, the four solutions from the literature agree rather favourably as can be seen in Figure 14. Furthermore, both the equation and the STRIPE solution give slightly higher values on the stress intensity factor in general than the solutions from the literature.

The third case studied is a comparison for E/R=10 and E/H=0.25. This case has been investigated by Kitagawa and Yuuki, Ref.[6], Lai et. al., Ref.[5], and Cartwright and Parker, Ref.[19]. Figure 15 shows the comparison between the different solutions for the normalized stress intensity factor. Included in the figure is the result obtained using Eq.(31). A rather good agreement between the different solutions is found. Compared to the solutions from the literature a slight over-estimation of the stress intensity factor is obtained by using the equation.

Finally, a comparison between the results of Eq.(31), the solutions by Cartwright and Parker, Ref.[19], and STRIPE for E/H=0.5 and E/R=2, 2.5, 4 and 10 is made in Figure 16. The best correlation is obtained for E/R=10. Furthermore, the correlation between the STRIPE solution and the solution by Cartwright and Parker is rather good for all E/R-values in common. The equation, on the other hand, shows a rather poor correlation, particularly for E/R=2, indicating that the finite height correction in the equation may be poor in combination with small values of E/R.

The averages of the absolute relative differences between the values of the numerical solutions and the values calculated using the equation for different parameter values are summarised in Table 2. Also shown in the table are the maximum and minimum relative differences, respectively, for each comparison.

8.2 Single Crack

Firstly, the results of Eq.(31) and STRIPE are compared to the results by Lai et. al., Ref.[5, 20], where the ratio E/H is constant and equal to 0.25. According to Schijve and Lai, Ref.[21], a ratio E/H $\leq$ 0.25 should be sufficiently small to yield representative results even for larger plates. The comparison is shown in Figures 17-19 for different values of the ratio E/R. It should be noted that there exists a difference of about 2.5 % between the solutions of Ref.[5] and Ref.[20] for the case of E/R=4.0, see Figure 18. According to the authors of these references the results of the former reference, published 1991, should be the most accurate.

In general the equation yields slightly greater values of the normalized stress intensity factor for all three E/R-ratios. Also, the correlation between the values calculated using the equation and those obtained using STRIPE is generally better than the correlation to the literature data. The finite height correction factor, in the equation for this case (E/H=0.25), is between 1.0021 and 1.0266 depending on E/R and a/(E-R). As can be seen in the figures the largest relative differences occur for E/R equal to 2.

Secondly, a comparison of the values calculated using the equation to the results by Lai and Schijve, Ref.[20], is made. The ratio E/R is constant and equal to 4.0. The comparison is presented in Figure 20 for different values on E/H. It can be noted in figure 20 that the difference in stress intensity factor for different values of E/H, smaller than 0.25, is small as already pointed. The equation yields stress intensity factors which in general are slightly greater than those presented in Ref.[20].

A third comparison of the results of the equation is made with respect to the results by Wiklander, presented in Ref.[22]. In this case the hole is eccentrically located in the sheet such that E/W=0.09091. The comparison is shown in Figure 21 for different values on E/R and E/H. For E/H=0.2 and E/R=2.0 the average of the absolute relative differences between Eq.(31) and the numerical result is 4.3 %. Similarly, for the following three combinations of E/H and E/R, 0.25 and 2.5, 0.3 and 3.0 and 0.4 and 4.0 the averages of the absolute relative differences are 1.9 %, 2.3 % and 1.8 %, respectively. The maximum relative differences in the four cases are 8.5 %, 5.2 %, 9.2 % and 15.6 %, respectively.

Comparisons have also been made with respect to the results obtained by Wang et. al., Ref.[23]. Wang et. al. used the boundary collocation method to obtain the stress intensity factor solutions. Unfortunately, many of their results are for rather extreme values of E/R (1.11<E/R<42.0), using a square plate (E/H=0.5). There is a rather good correlation between the solutions by Wang et. al. and Lai et. al. for E/R=4 and E/H=0.5, see Figure 22. It should be pointed out, though, that interpolations have been made with respect to the Wang data in order to obtain figure 22. Also shown in the figure is the result obtained using Eq.(31). The correlation between the result of Eq.(31) and the two solutions from the literature is rather good.

Finally, a comparison is made between the solutions of Wang et. al., STRIPE and the equation for the parameter combination E/R=2 and E/H=0.5, see Figure 23. There is a rather good correlation between the solutions by Wang et. al. and STRIPE for normalized crack lengths in the range $0.05 \leq a/(E-R) \leq 0.5$. The values obtained using the equation show a very poor correlation to the numerical data for all crack lengths as already observed for two symmetrical cracks with the same parameters.

In Table 3 the averages of the absolute relative differences between the values of the numerical solutions and the values obtained using the equation are summarised. Furthermore, the maximum and the minimum relative difference, respectively, for each comparison are presented in the table.

8.3 Two Cracks of Unequal Length

Two cracks of unequal lengths at a open hole in a sheet of finite dimensions have been investigated by Lai et. al., Ref.[5], using complex variable technique and a least squares method. Their results are presented in both tabular and graphical form. The results are limited to E/H equal to 0.25 and E/R equal to 2, 4 and 10.

For crack tip A, the stress intensity factors, according to the tables in Ref.[5], have been compared to the stress intensity factors according to

Eq.(30) in Figures 24-26. The relative differences between the stress intensity factors are shown as function of the normalized crack length, $a_1/(E-R)$, for different values of the dimensionless crack length, a_2/R. Relative differences greater than 10 % are found for $a_1/(E-R)$-values greater than 0.55 when E/R is equal to 2. For E/R-values greater than 2 the largest relative differences (> 8 %) are found for $a_1/(E-R)$-values less than 0.2. The relative differences increase with increasing ratio a_2/a_1, except for E/R=2, as can be seen in Figure 27.

Identical figures are obtained for the comparisons between the stress intensity factors according to the tables in Ref.[5] and those computed using Eq.(30) for crack tip B provided that the crack length indices are shifted.

9 Discussion

According to Schijve and Lai, Ref.[21], and the STRIPE results of the present investigation the influence of a finite height is very small for heights resulting in $E/H \leq 0.25$. Hence, the differences observed between the stress intensity factor calculated using Eq.(30) and the values obtained by the numerical methods, for $E/H \leq 0.25$, should be due to inaccuracy in the numerical solutions and shortcomings of Eq.(30). Figure 28 shows the relative differences between the stress intensity factor calculated using Eq.(30) and corresponding values obtained using either STRIPE, the solution by Lai et. al., Ref.[5], or the solution by Lai and Schijve, Ref.[20]. The relative differences are shown as function of the normalized crack length, a/(E-R) for E/H less than or equal to 0.25 and different E/R values. The relative differences are less than 5 % for normalized crack lengths less than 0.3. In general the relative differences increase with increasing crack length which indicates that the finite width correction may be too large for large values of a/(E-R).

Figure 29 shows a similar comparison but for the ratio E/H=0.5. In this case the relative differences are very large, particularly for small crack lengths combined with narrow sheets.

Figures 16 and 23 clearly show that the finite height correction for a centre cracked sheet is insufficient for the open hole sheet when the sheet becomes small compared to the hole. The finite height correction factor according to Eq.(26) gives a correction as function of crack length for the interaction between the finite height and the finite width but it does not consider the effect of the open hole.

A more appropriate finite height correction factor may be based upon both the dimensional ratios H/R and E/H instead of just E/H. Figure 30 shows the ligament stress distribution for a plate with an open hole, as obtained using STRIPE, for E/R=2 and various values on H/R. There are rather small differences in the stress distribution for $H/R \geq 7$ whereas rather dramatic changes occur for $H/R<6$. This is consistent with previous statements of small influence of the height for $E/H \leq 0.25$.

Based on the ligament stress distributions for the 7 different values on H/R and the approximate weight function technique, Ref.[24], the stress intensity factors for a single crack and two symmetrical cracks at the open hole in a sheet having E/R=2 were calculated. The reference stress intensity factor, in the approximate weight function method, was taken as Eq.(30) and the reference stress distribution was the stress distribution for H/R=20.

The quotient between the stress intensity factors computed by the weight function technique, for the different values on H/R, and the corresponding reference stress intensity factors yield ratios that should be equal to the effect of varying the H/R-ratio. Figures 31 and 32 show these ratios as function of the normalized crack length for different values of H/R. For H/R-values greater than 7 the ratios are almost independent of the crack length and close to unity. The data points, in figures 31 and 32, have been fitted by straight lines assuming an equation according to,

$$F_{HR}\left(\frac{a}{E-R},\frac{H}{R}\right)=C_1\frac{a}{E-R}+C_2 \tag{33}$$

where C_1 and C_2 are functions of the ratio H/R, see Figure 33. A study of C_1 and C_2 shows that the logarithm of $-C_1$ as function of H/R is a straight line and the double logarithm of C_2 as function of H/R is a straight line. Also, the difference in C_1 and C_2 due to the number of cracks is rather small. Thus an additional correction factor for the finite height due to the influence of H/R can be written,

$$F_{HR}=-e^{B_{11}(H/R)+B_{12}}\left(\frac{a}{E-R}\right)+e^{e^{B_{21}(H/R)+B_{22}}} \tag{34}$$

where the coefficients B_{ij} can be obtained by a least squares fit giving,

$$\begin{matrix} B_{11}=-0.733 \\ B_{12}=2.096 \end{matrix} \; ; \; \begin{matrix} B_{21}=-0.817 \\ B_{22}=2.374 \end{matrix} \tag{35}$$

F_{HR} according to the Eq.(34) yields correction factors which in general are slightly smaller than those obtained by the weight function technique and shown in figures 31 and 32. The relative differences vary between -3.4 % and 6.2 % with the largest absolute errors occurring for the smallest H/R ratio.

The effect of F_{HR} is negligible for H/R≥8 but increases rapidly as H/R decreases. Only four of the investigated combinations of E/R and E/H yield ratios of H/R less than eight. In Figure 34 the normalized stress intensity factor as function of the normalized crack length is shown for these four combinations as obtained by STRIPE and by Eq.(31) using the additional finite height correction.

Taking into account the correction factor for the interaction between the finite height and the hole radius the complete stress intensity factor is written,

$$\begin{aligned} K_I^A &= \sigma\sqrt{\pi a_1}F_{New}^A \\ K_I^B &= \sigma\sqrt{\pi a_2}F_{New}^B \end{aligned} \tag{36}$$

where

$$F^A_{New} = F^A F^A_{HR}$$
$$F^B_{New} = F^B F^B_{HR} \tag{37}$$

and

$$F^A_{HR} = -e^{B_{11}(H/R)+B_{12}}\left(\frac{a_1}{E-R}\right) + e^{e^{B_{21}(H/R)+B_{22}}}$$
$$F^B_{HR} = -e^{B_{11}(H/R)+B_{12}}\left(\frac{a_2}{W-E-R}\right) + e^{e^{B_{21}(H/R)+B_{22}}} \tag{38}$$

10 Relative Differences

The relative differences between the stress intensity factors calculated using Eq.(36) and those obtained using either the STRIPE solutions, the data presented by Lai et. al., Ref.[5], or the data by Lai and Schijve, Ref.[20], as reference have been computed. The results are shown as function of the normalized crack length in Figures 35 to 36 for a single crack and in Figures 37 to 38 for two symmetrical cracks. For two cracks of unequal lengths the results are shown in Figures 39 to 40, with respect to crack tip A, as function of the normalized crack length at tip A. Provided that the crack lengths are interchanged the crack tip B results are identical to those of crack tip A, since the hole is located to the centre of the sheet.

For a single or two symmetrical cracks with $a/(E-R) \leq 0.8$ in a sheet having $E/H \leq 0.25$ the accuracy of Eq.(36) is generally between –4 % and 7 %. In a sheet having $E/H > 0.25$ the accuracy of Eq.(36) is not so good but compared to Eq.(30) the range of relative differences has shifted from (–25 %, 11 %) to (-11 %, 22 %) for cracks with $a/(E-R) \leq 0.8$. Thus, Eq.(36) is more likely to give a conservative stress intensity factor than Eq.(30).

For two cracks of unequal lengths the accuracy of Eq.(36) is generally better than 8 % if $E/R \geq 4$. However, for small cracks ($a_1/(E-R) < 0.1$) at tip A combined with relatively large cracks at tip B ($a_2/a_1 > 10$) the accuracy decreases with decreasing crack length at tip A and increasing ratio a_2/a_1. This implies that the stress intensity factor, at tip B, for the case of continuing damage ($a_1 >> a_2$) will be over estimated by the equation. The accuracy for $E/R = 2$ is generally better than 11 % for $a_2/a_1 > 1$ whereas the accuracy decreases with increasing crack length at tip A for $a_2/a_1 \leq 1$.

11 Conclusions

A stress intensity factor equation for a single or two radial cracks at an open hole in a sheet, or plate, of finite dimensions has been developed and verified. The sheet is subjected to a uniformly distributed, uniaxial stress acting on the sheet edges parallel to the crack line.

In order to verify the equation over large ranges of parameter values several numerical solutions found in the literature have been used together with finite element analyses. The finite element analyses have been performed using the finite element code STRIPE.

It has been found that the standard finite height correction factor based on a centre cracked sheet, without a hole, is insufficient in cases where the height is small relative the hole radius ($H/R \leq 8$). In such cases an additional finite height correction factor has been proposed. This additional correction factor is based upon weight function solutions for the case of E/R=2 and different H/R-ratios.

Compared to the stress intensity factors by Lai et. al., Lai and Schijve and those computed using the adaptive finite element method the developed equation for the stress intensity factor has an accuracy which generally is much better than 20 % if $E/H \leq 0.25$. In most of the cases investigated the accuracy is better than 8 %. A poorer accuracy is obtained for cases where $E/R \leq 2$ combined with relatively large crack lengths ($a/(E-R) > 0.6$) and in cases of continuing damage.

For the ratio $E/H > 0.25$ the correlation between the stress intensity factors due to the equation and those by STRIPE or Lai and Schijve is less good as compared to the correlation for $E/H \leq 0.25$. For $a/(E-R) < 0.8$ the maximum relative difference is less than 18 %. Again the poorest accuracy is obtained for E/R=2.

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a/R	F_{OH} (Bowie)		F_{OH} (Tweed& Rooke)		F_{OH} (Lai et. al.)		F_{OH} (New-man)	F_{OH} (Rubin-stein& Sadegh)
	Single Crack	Two Cracks	Single Crack	Two Cracks	Single Crack	Two Cracks	Two Cracks	Single Crack
0.01			3.293	3.293		.	3.272	
0.02			3.225	3.225	3.268	3.267	3.224	
0.04			3.097	3.100		3.073	3.101	
0.06			2.980	2.985	2.976	2.975	2.986	
0.08			2.872	2.881		2.880	2.882	
0.10	2.73	2.73	2.772	2.786	2.775	2.785	2.786	2.771
0.12			2.681	2.699				
0.14			2.595	2.619				
0.15				2.581			2.581	
0.16			2.516	2.545				
0.18			2.443	2.476				
0.20	2.30	2.41	2.374	2.413	2.375	2.414	2.413	2.373
0.25			2.222	2.273			2.274	
0.30	2.04	2.15	2.093	2.156			2.156	
0.40	1.86	1.96	1.885	1.971	1.886	1.971	1.971	
0.50	1.73	1.83	1.728	1.833			1.833	1.727
0.60	1.64	1.71	1.605	1.726	1.605	1.726	1.726	
0.75			1.465					
1.00	1.37	1.45	1.306	1.472	1.306	1.472	1.472	1.305
1.50	1.18	1.29	1.127	1.323	1.127	1.323	1.324	
2.00	1.06	1.21	1.031	1.244	1.031	1.245	1.244	1.030
3.00	0.94	1.14	0.930	1.163	0.930	1.164	1.164	
4.00			0.878	1.123				
5.00	0.81	1.07	0.846	1.098	0.846			
6.00			0.824	1.082				0.823
7.00			0.808	1.070	0.808			
9.00			0.787	1.055	0.787			
10.00			0.779	1.049				0.779

Table 1. Normalized stress intensity factor for cracks at an open hole in a sheet of infinite size subjected to a remote uniform stress perpendicular to the crack line.

		RELATIVE DIFFERENCES BETWEEN EQUATIONS AND NUMERICAL SOLUTIONS IN %													
		STRIPE				Newman		Lai et.al. (1991)		Cartwright & Parker				Kitagawa et. al	
E/R	E/H	0.25		0.5		0.25		0.25		0.25		0.5		0.25	
	RELA-TIVE ERROR		With F_{HR}		With F_{HR}		With F_{HR}		With F_{HR}		With F_{HR}		With F_{HR}		With F_{HR}
1.7	Average	5.60	7.00												
	Max.	19.38	18.39												
	Min.	0.92	4.05												
2.0	Average	2.71	3.29	20.66	8.24	4.62	5.56	4.75	5.69	5.67	6.43	19.88	7.71		
	Max.	12.04	11.44	14.60	27.24	12.49	12.15	11.03	10.95	8.91	9.01	-15.44	17.95		
	Min.	0.05	1.06	-24.96	-2.74	1.11	2.62	0.89	2.41	3.81	4.74	-21.98	-0.25		
2.5	Average	1.06	1.11	14.45	5.93										
	Max.	5.84	5.65	15.52	16.50										
	Min.	-0.14	-0.05	-18.50	-9.95										
3.0	Average	0.83	0.82												
	Max.	2.69	2.63												
	Min.	-0.77	-0.76												
4.0	Average	1.05	1.05	8.36	7.67	2.89	2.89	3.03	3.03	4.57	4.57	4.79	4.05	3.02	3.02
	Max.	-0.04	-0.04	15.36	14.74	5.11	5.11	4.63	4.63	24.49	24.48	24.73	24.08	4.10	4.09
	Min.	-2.18	-2.18	-11.12	-10.57	1.58	1.58	2.17	2.17	2.54	2.54	-6.49	-5.91	2.30	2.30
10.0	Average			5.20	5.20			2.31	2.31	2.45	2.45	3.58	3.58	1.86	1.86
	Max.			14.61	14.61			4.85	4.85	13.56	13.56	23.71	23.71	3.13	3.13
	Min.			-6.51	-6.51			0.72	0.72	-2.18	-2.18	-7.27	-7.27	0.98	0.98

Table 2. Results, in terms of relative differences, of comparison between stress intensity factors for two symmetrical cracks obtained using equations and numerical solutions.

		RELATIVE DIFFERENCE BETWEEN EQUATIONS AND NUMERICAL SOLUTIONS %															
		STRIPE				Lai & Schijve (1990)		Lai et. al. (1991)						Wang et. al.			
E/H	E/R	2.0		4.0		4.0		2.0		4.0		10.0		2.0		4.0	
	RELATIVE ERROR		With F_{HR}		With F_{HR}		With F_{HR}		With F_{HR}		With F_{HR}		With F_{HR}		With F_{HR}		With F_{HR}
0.16667	Average	1.37	1.34			4.32	4.32										
	Max.	5.72	9.04			8.43	8.43										
	Min.	-1.28	-1.24			-3.71	-3.71										
0.20000	Average					4.34	4.34										
	Max.					7.32	7.32										
	Min.					-3.42	-3.42										
0.25000	Average	1.55	2.21	1.82	1.82	4.55	4.55	4.33	5.38	2.69	2.69	1.07	1.07				
	Max.	7.45	10.67	0.55	0.55	6.21	6.21	12.62	12.54	3.61	3.62	2.00	2.00				
	Min.	-0.44	1.07	-5.26	-5.26	-3.29	-3.28	0.79	2.30	1.57	1.57	0.08	0.08				
0.33333	Average	2.93	5.06														
	Max.	10.40	14.11														
	Min.	-3.06	4.02														
0.50000	Average	19.18	11.44			4.35	3.73							22.87	15.05	7.60	6.71
	Max.	12.00	34.72			12.75	12.23							15.37	35.60	5.57	5.57
	Min.	-23.86	6.03			-9.73	-8.40							-39.96	-10.17	-25.62	-24.47

Table 3. Results, in terms of relative differences, of comparisons between stress intensity factors for a single crack obtained using equations and numerical solutions.

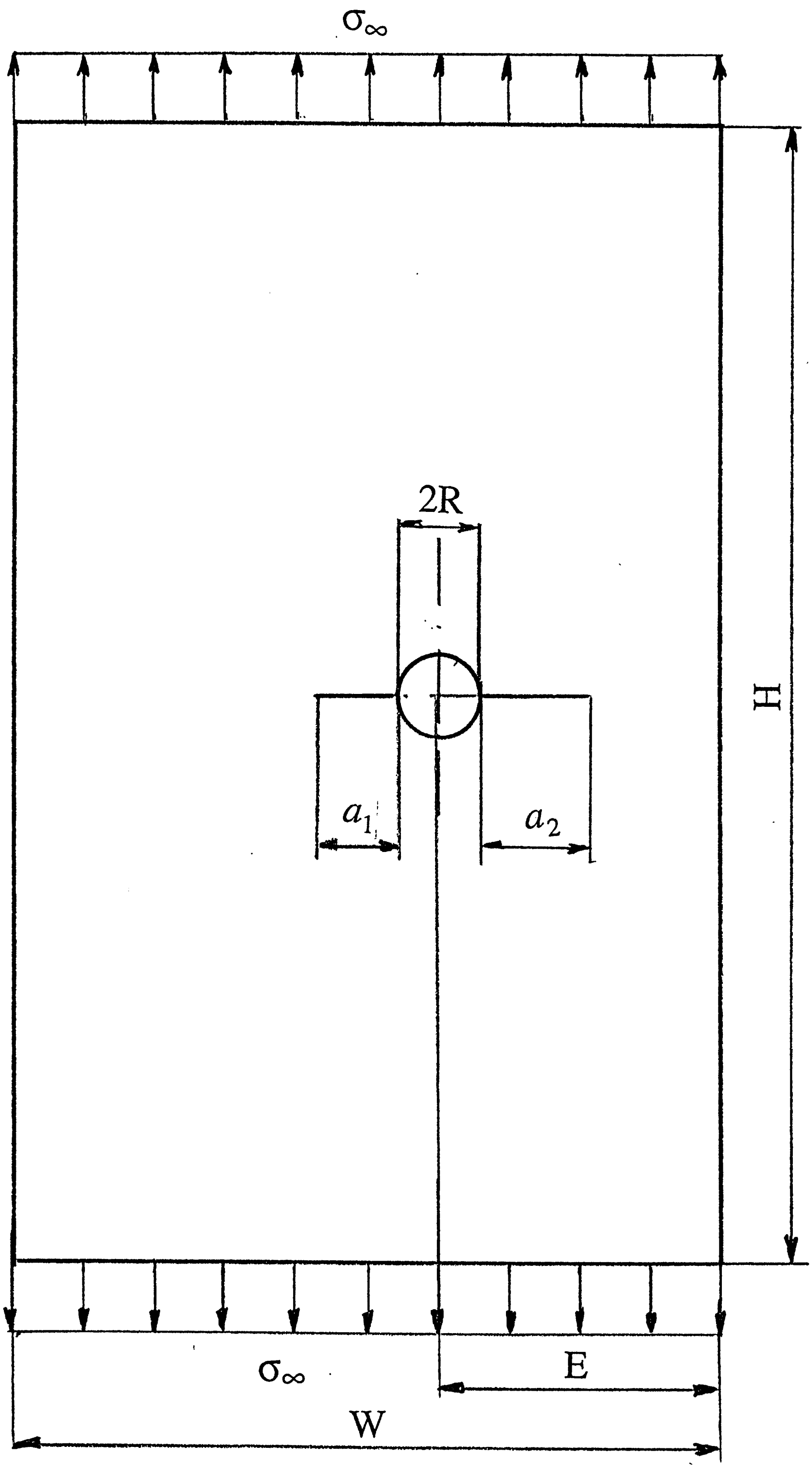

Figure 1. Definition of geometric parameters.

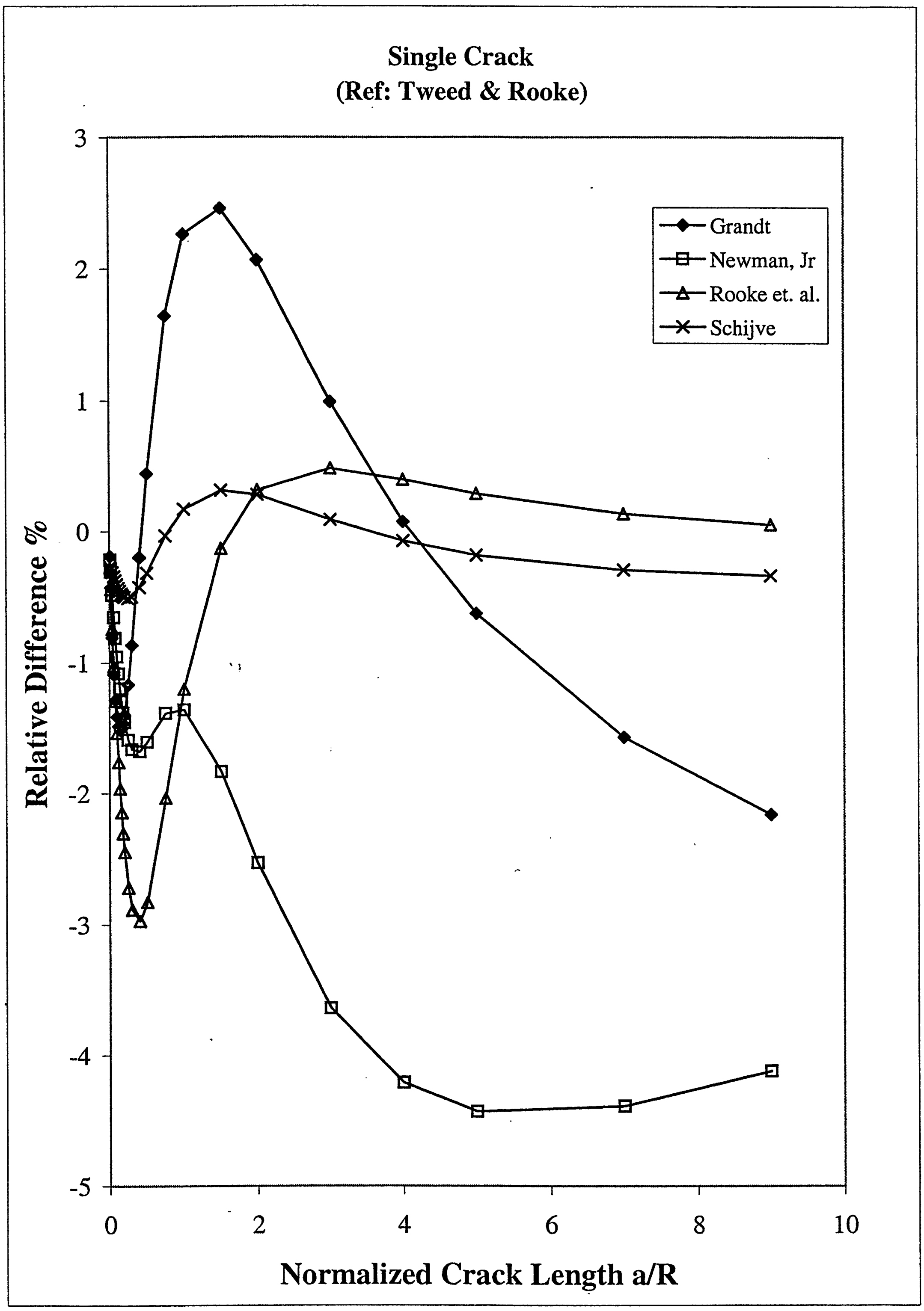

Figure 2. Relative comparison of different solutions for the stress intensity factor of a single crack at an open hole in a sheet of infinite dimensions.

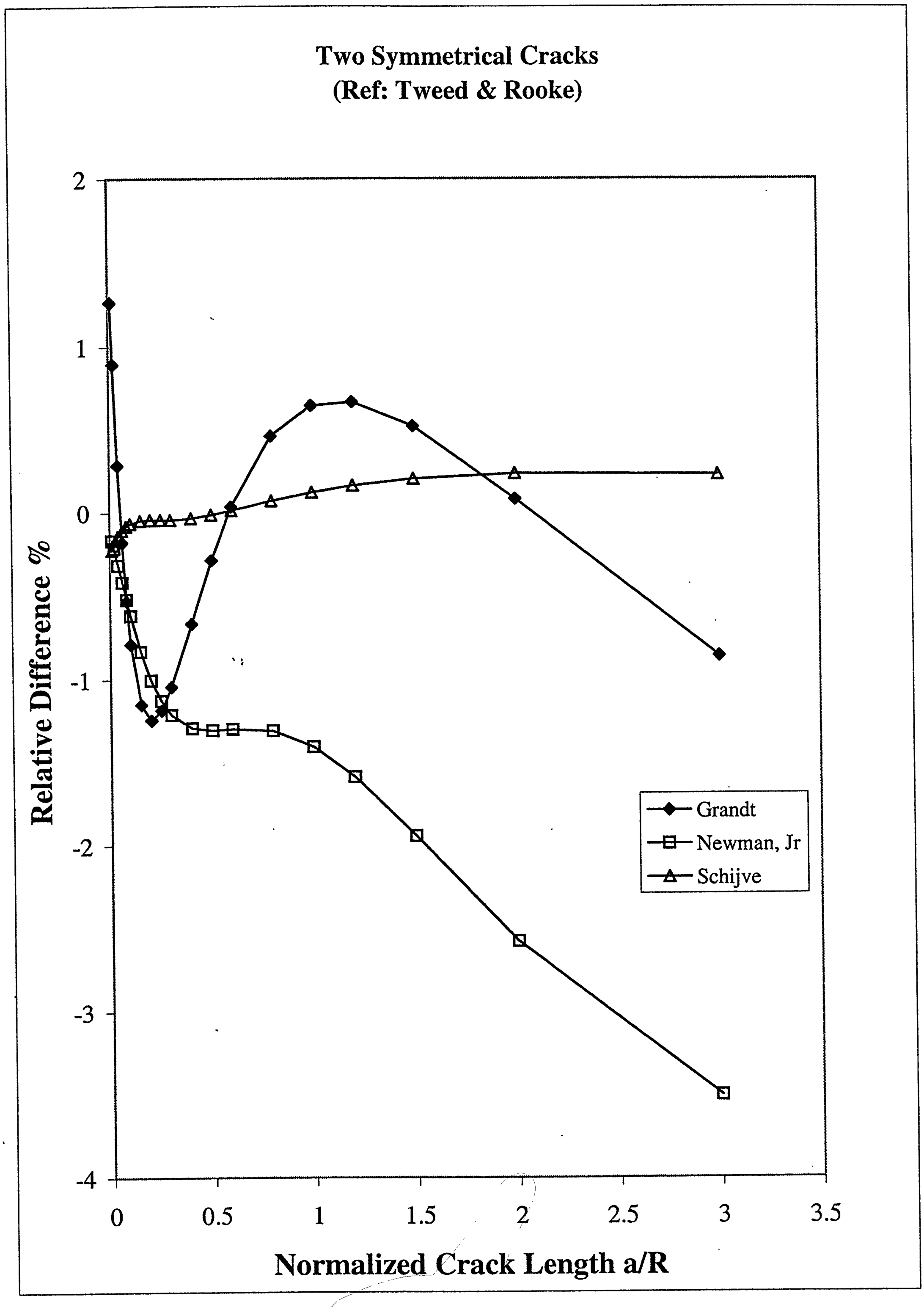

Figure 3. Relative comparison of different solutions for the stress intensity factor of two symmetric cracks at an open hole in a sheet of infinite dimensions.

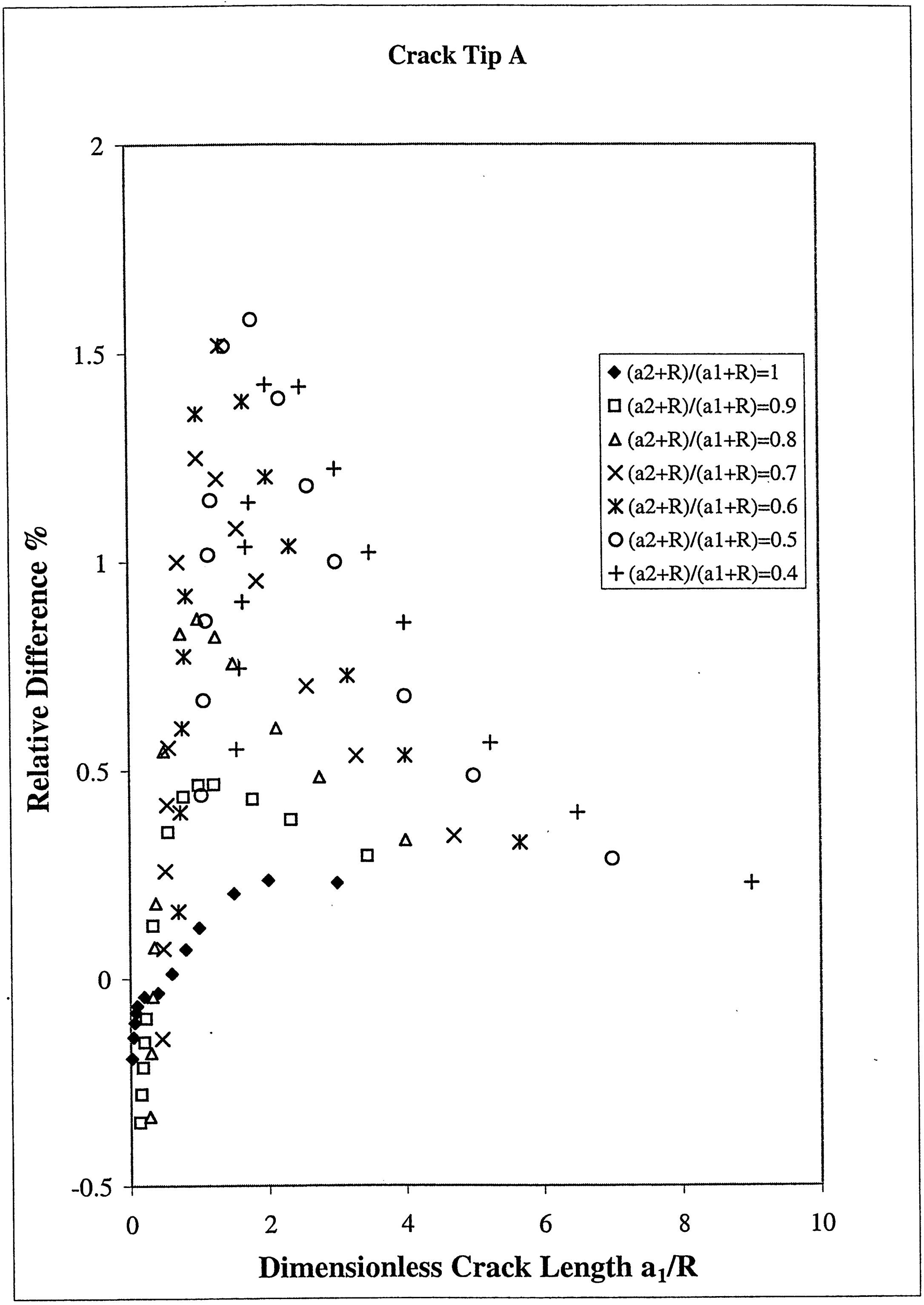

Figure 4. Relative comparison between a closed form solution and a numerical solution according to Tweed and Rooke for the stress intensity factor at crack tip A of two cracks having unequal lengths at an open hole in a sheet of infinite dimensions.

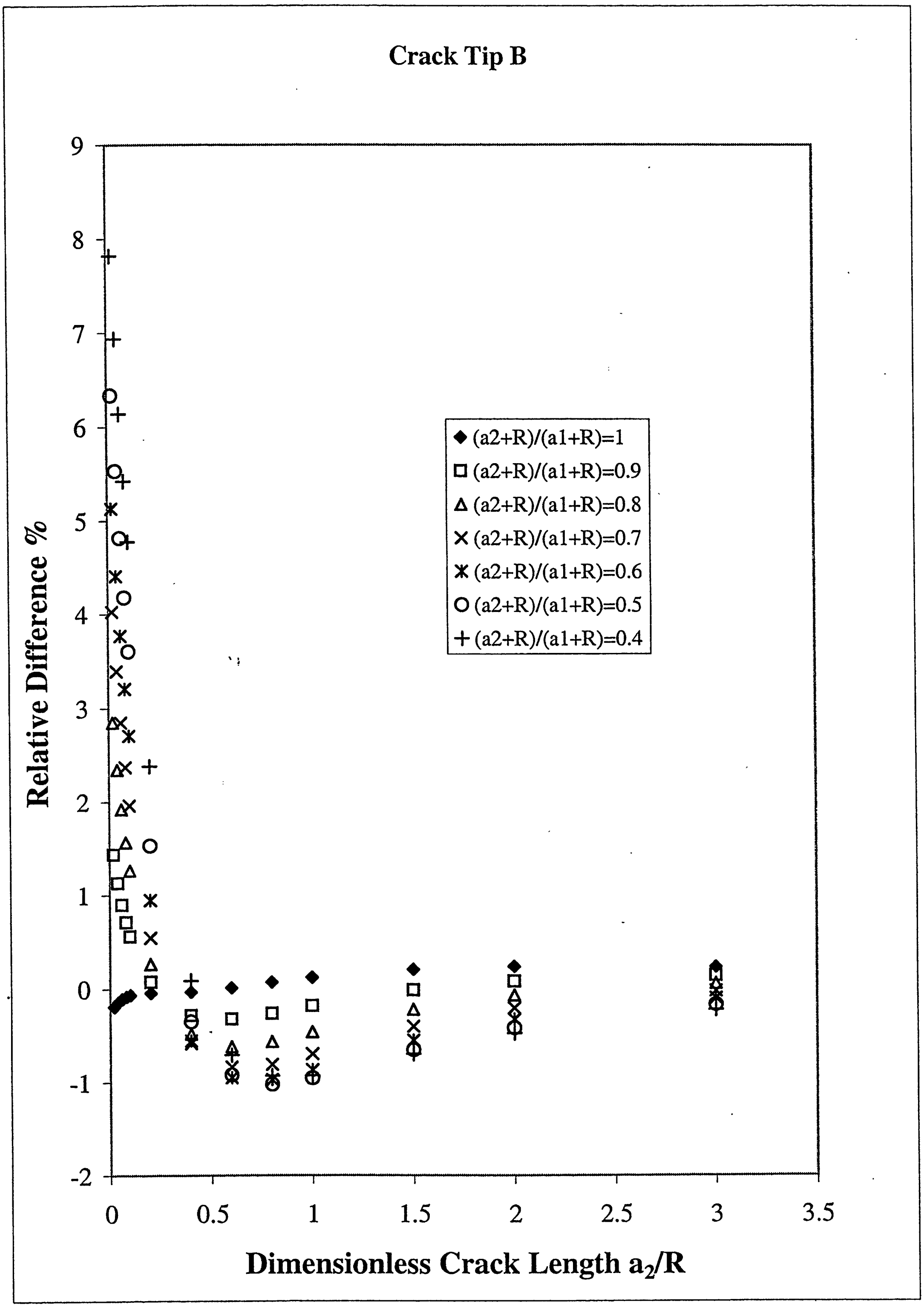

Figure 5. Relative comparison between a closed form solution and a solution according to Tweed and Rooke for the stress intensity factor at crack tip B of two cracks having unequal lengths at an open hole in a sheet of infinite dimensions.

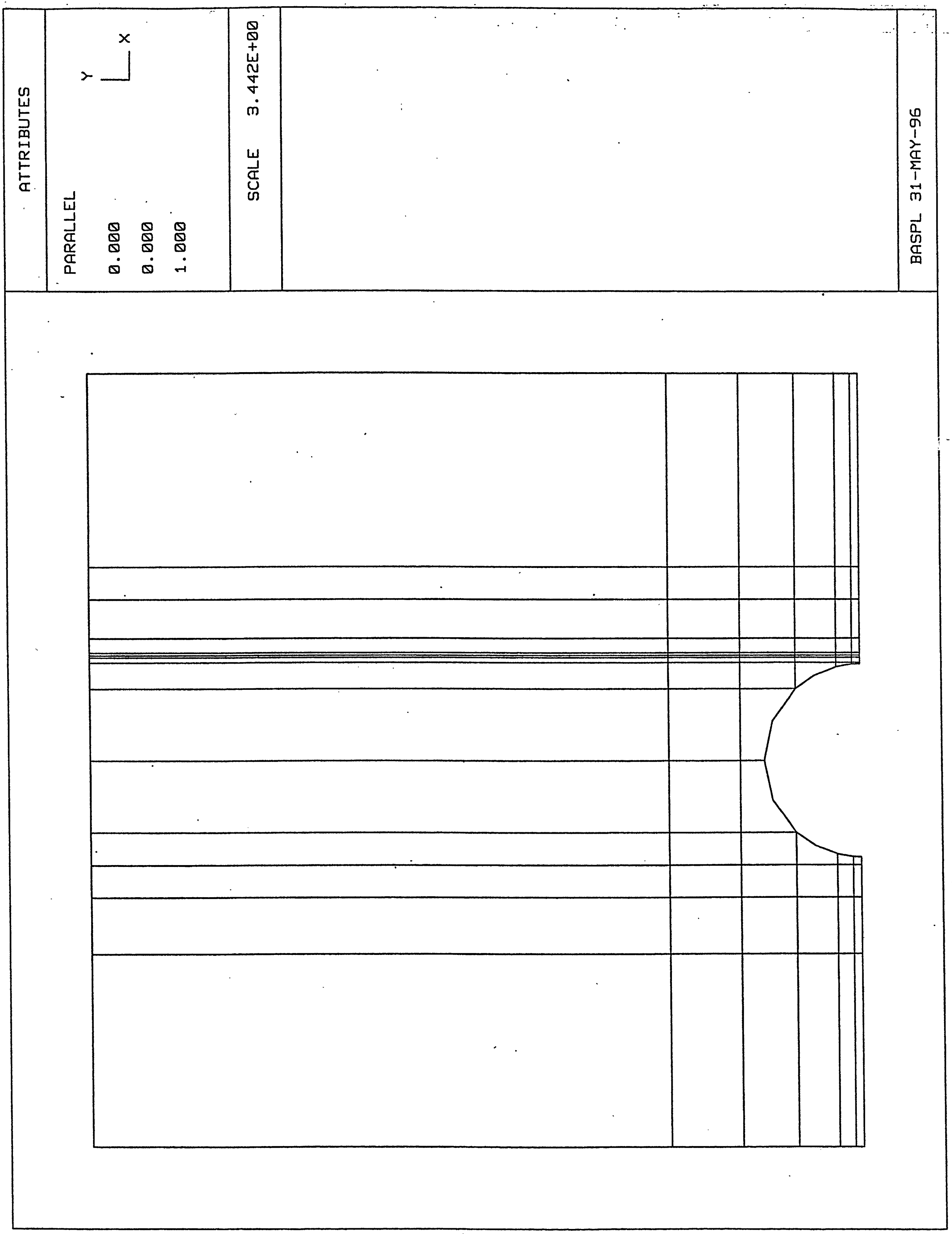

Figure 6. Finite element mesh for computation of the stress intensity factor for a single crack at an open hole in a sheet of finite dimensions.

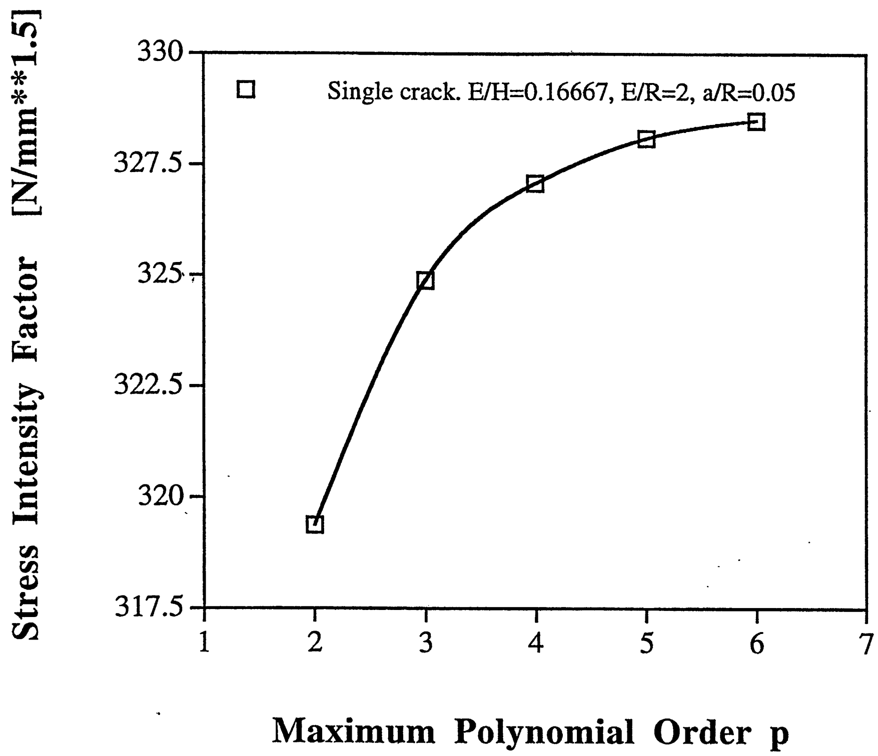

Figure 7. Example of the conversion rate for the p-version finite element computation.

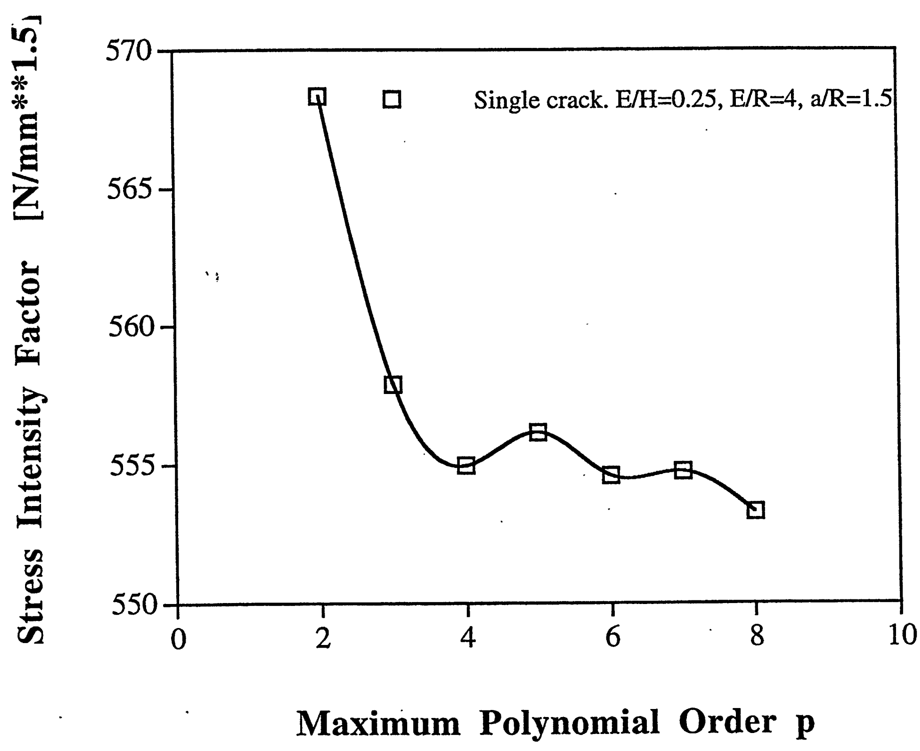

Figure 8. Example of the conversion rate for the p-version finite element computation.

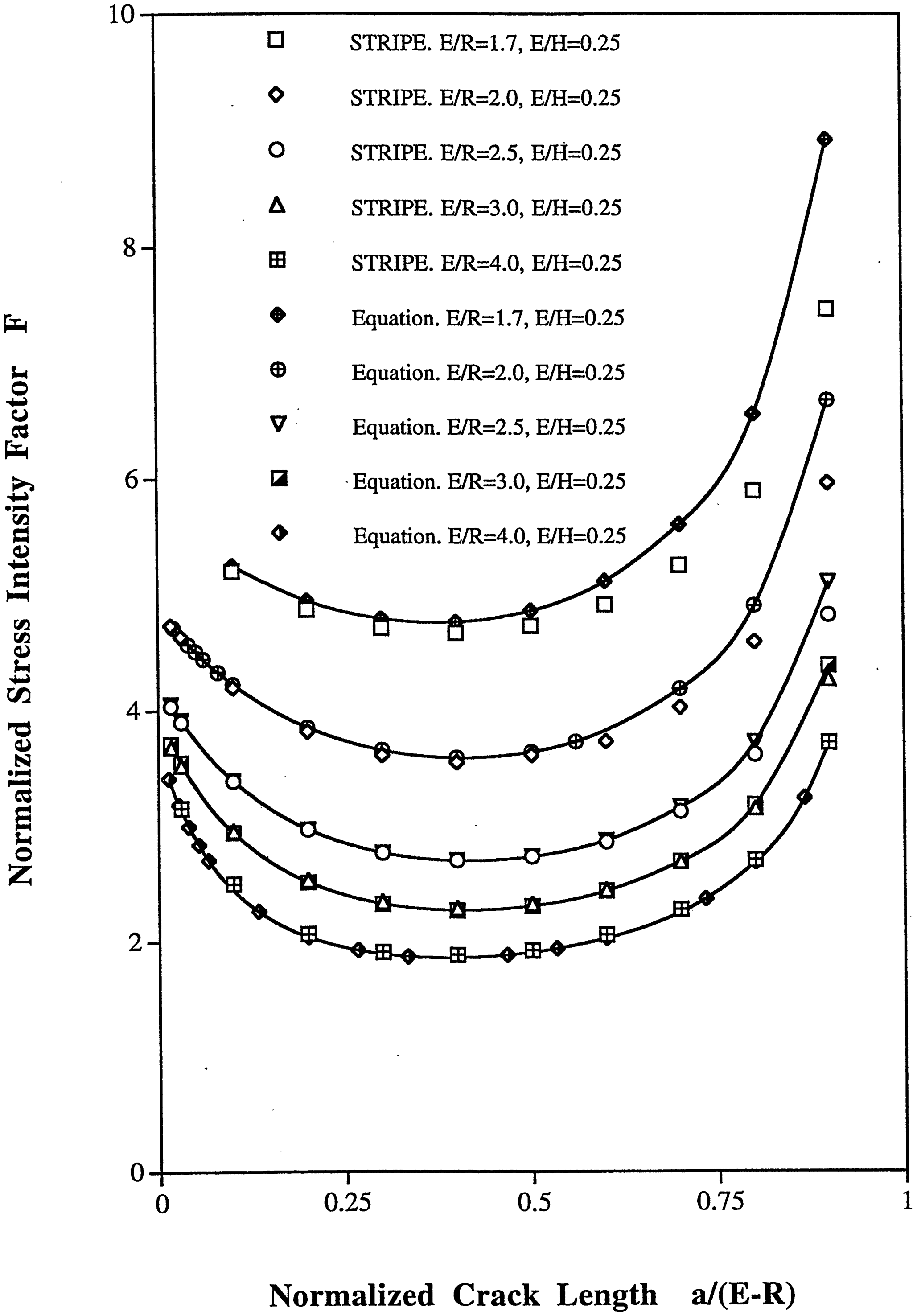

Figure 9. Comparison of normalized stress intensity factors computed using the finite element code STRIPE and the proposed equation (Eq.31) for the case of two symmetric cracks at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress. E/H=0.25, various E/R ratios.

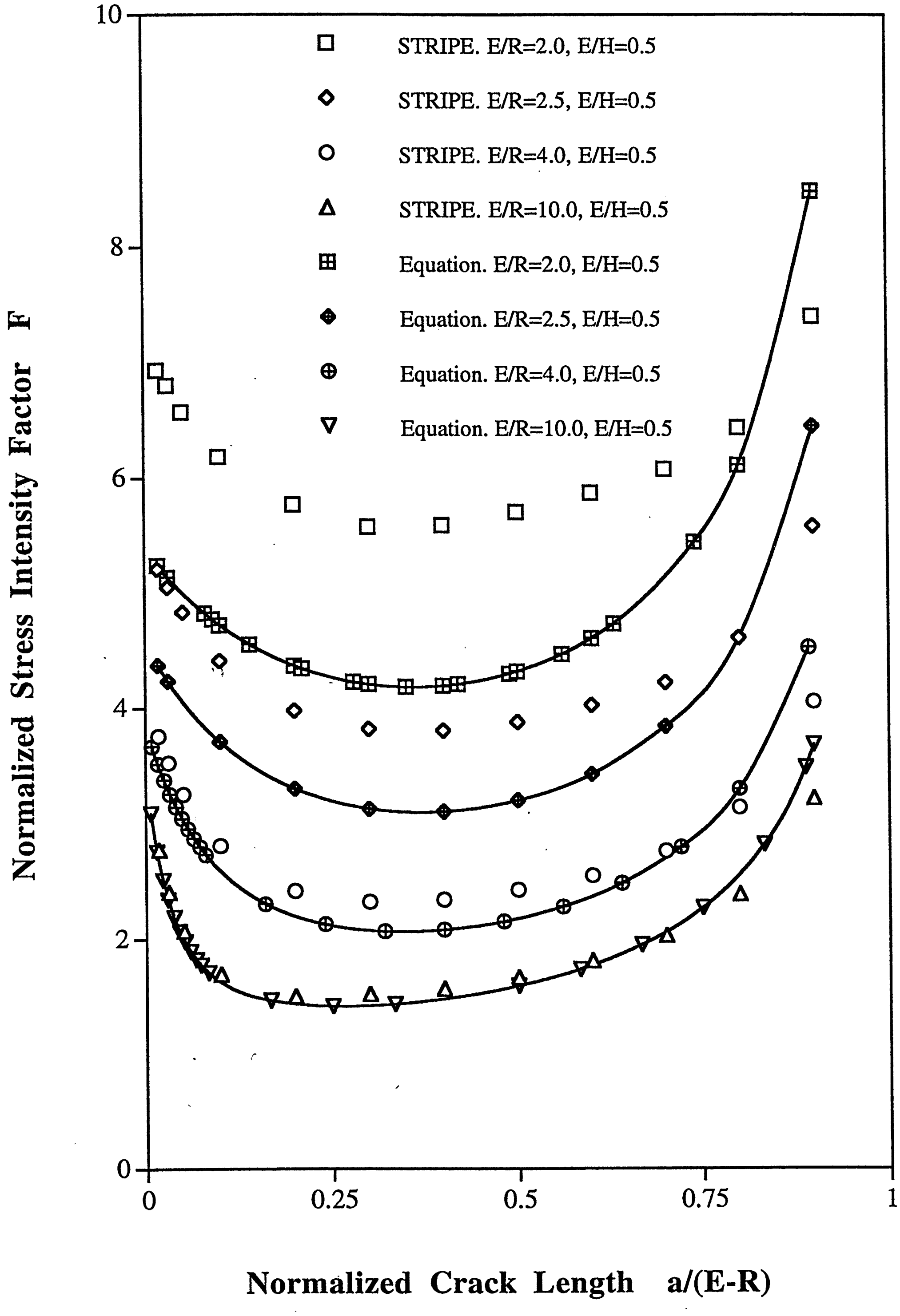

Figure 10. Comparison of normalized stress intensity factors computed using the finite element code STRIPE and the proposed equation (Eq.31) for the case of two symmetric cracks at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress. E/H=0.50, various E/R ratios.

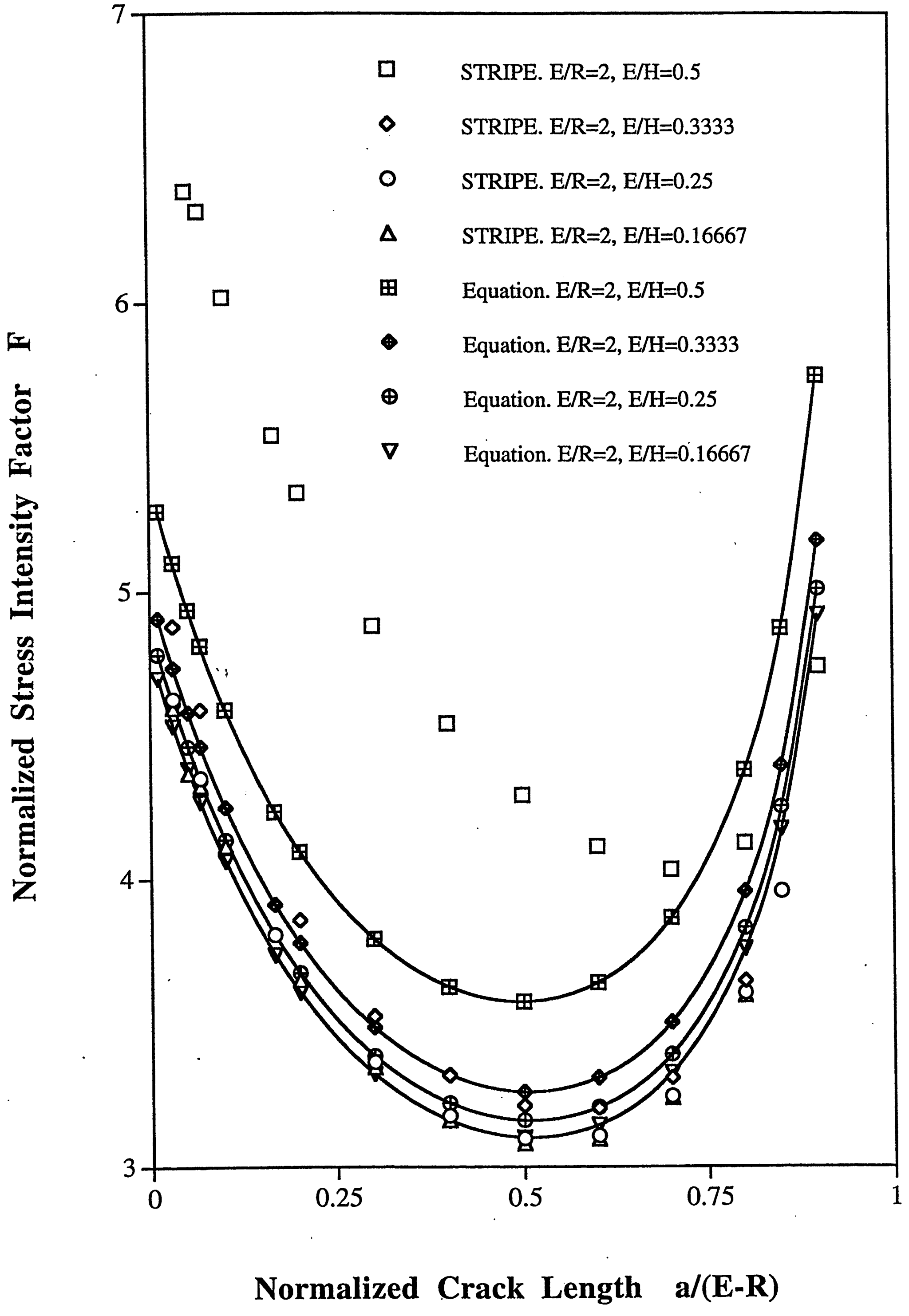

Figure 11. Comparison of normalized stress intensity factors computed using the finite element code STRIPE and the proposed equation (Eq.31) for the case of a single crack at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress. E/R=2, various E/H ratios.

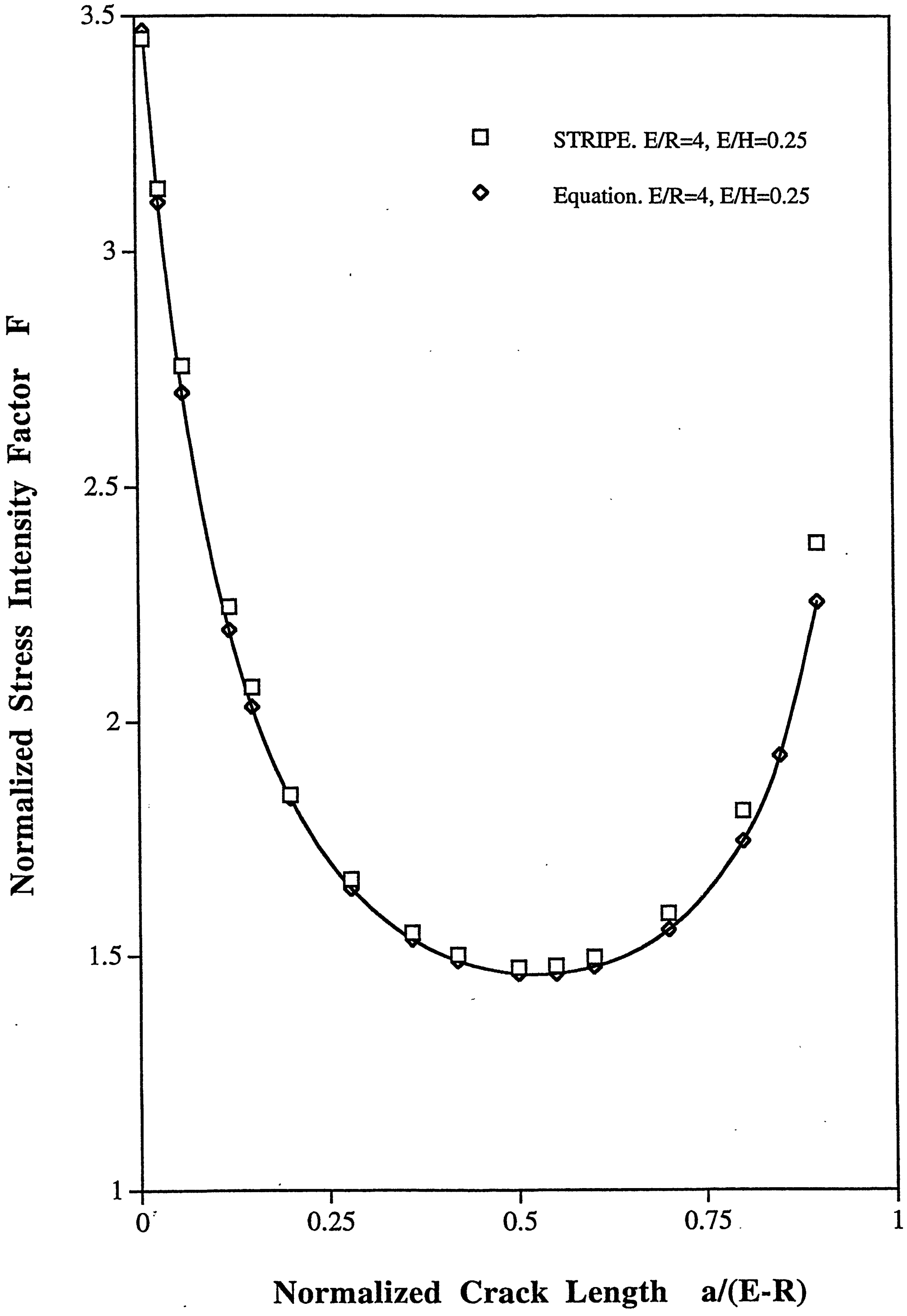

Figure 12. Comparison of normalized stress intensity factors computed using the finite element code STRIPE and the proposed equation (Eq.31) for the case of a single crack at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress. E/R=4 and E/H=0.25.

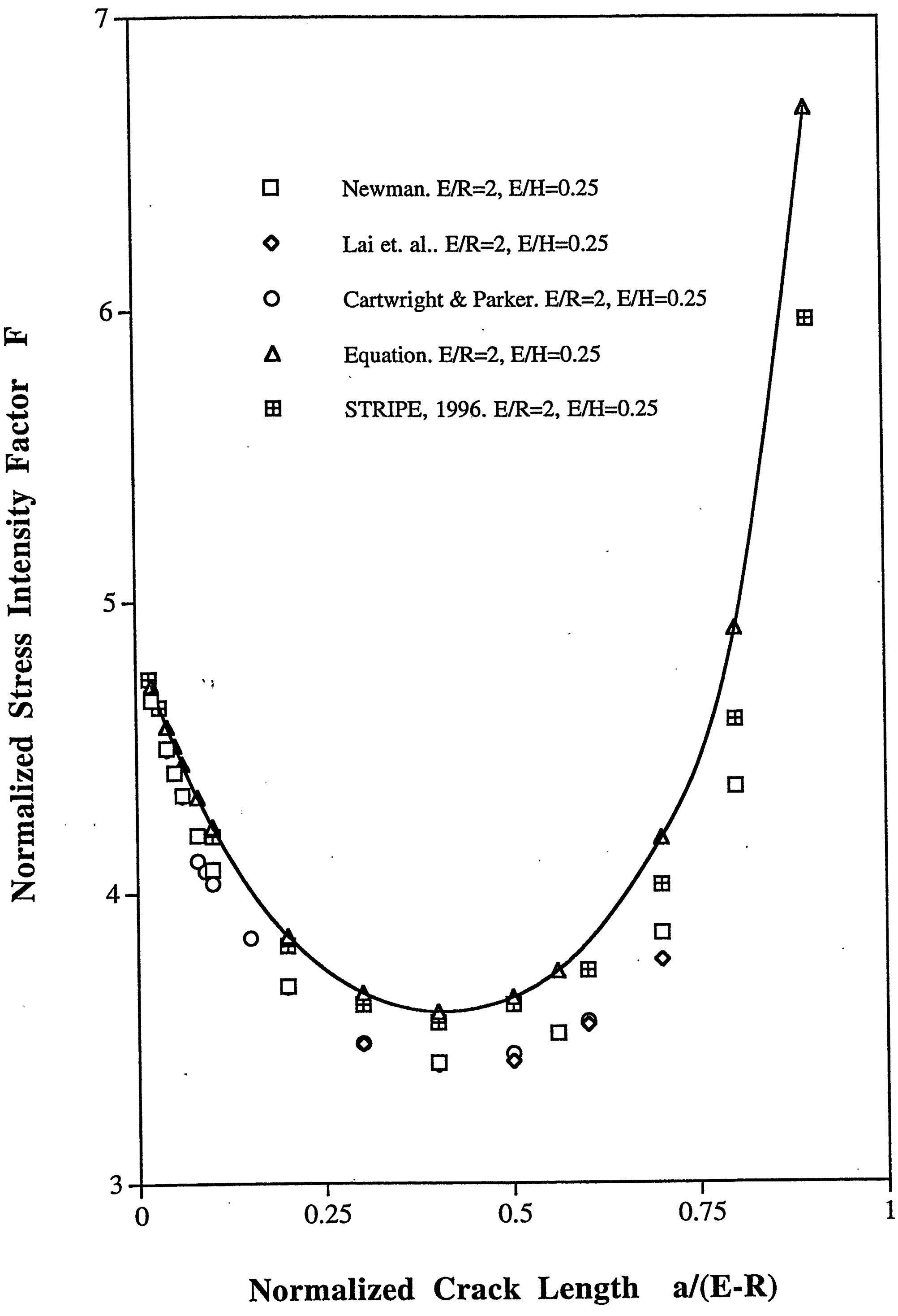

Figure 13. Comparison of normalized stress intensity factors obtained from the literature, STRIPE computations and by using the proposed equation (Eq.31) for the case of two symmetrical cracks at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress, E/R=2 and E/H=0.25.

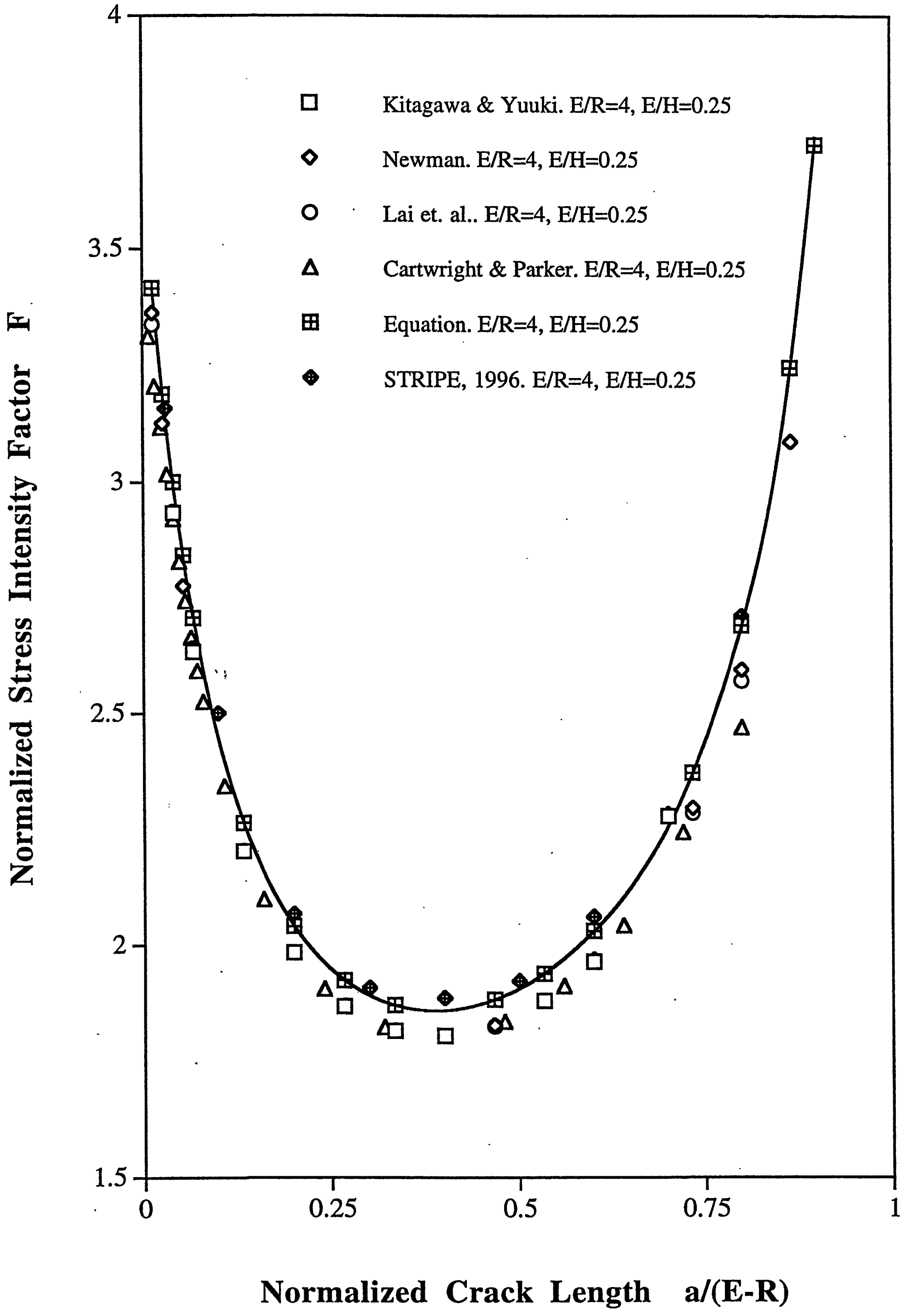

Figure 14. Comparison of normalized stress intensity factors obtained from the literature, STRIPE computations and by using the proposed equation (Eq.31) for the case of two symmetric cracks at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress. E/R=4 and E/H=0.25.

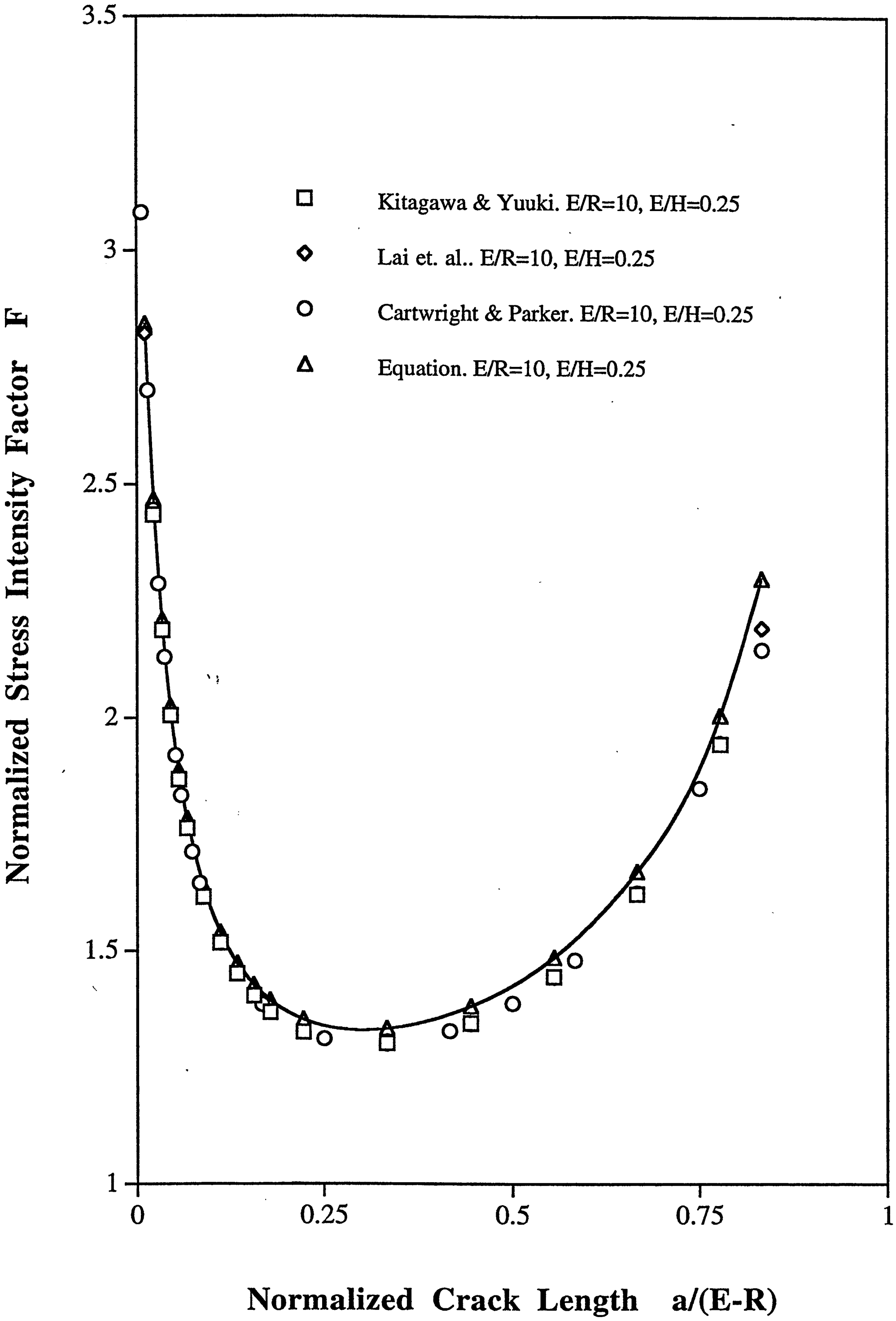

Figure 15. Comparison of normalized stress intensity factors obtained from the literature, STRIPE computations and by using the proposed equation (Eq.31) for the case of two symmetric cracks at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress. E/R=10 and E/H=0.25.

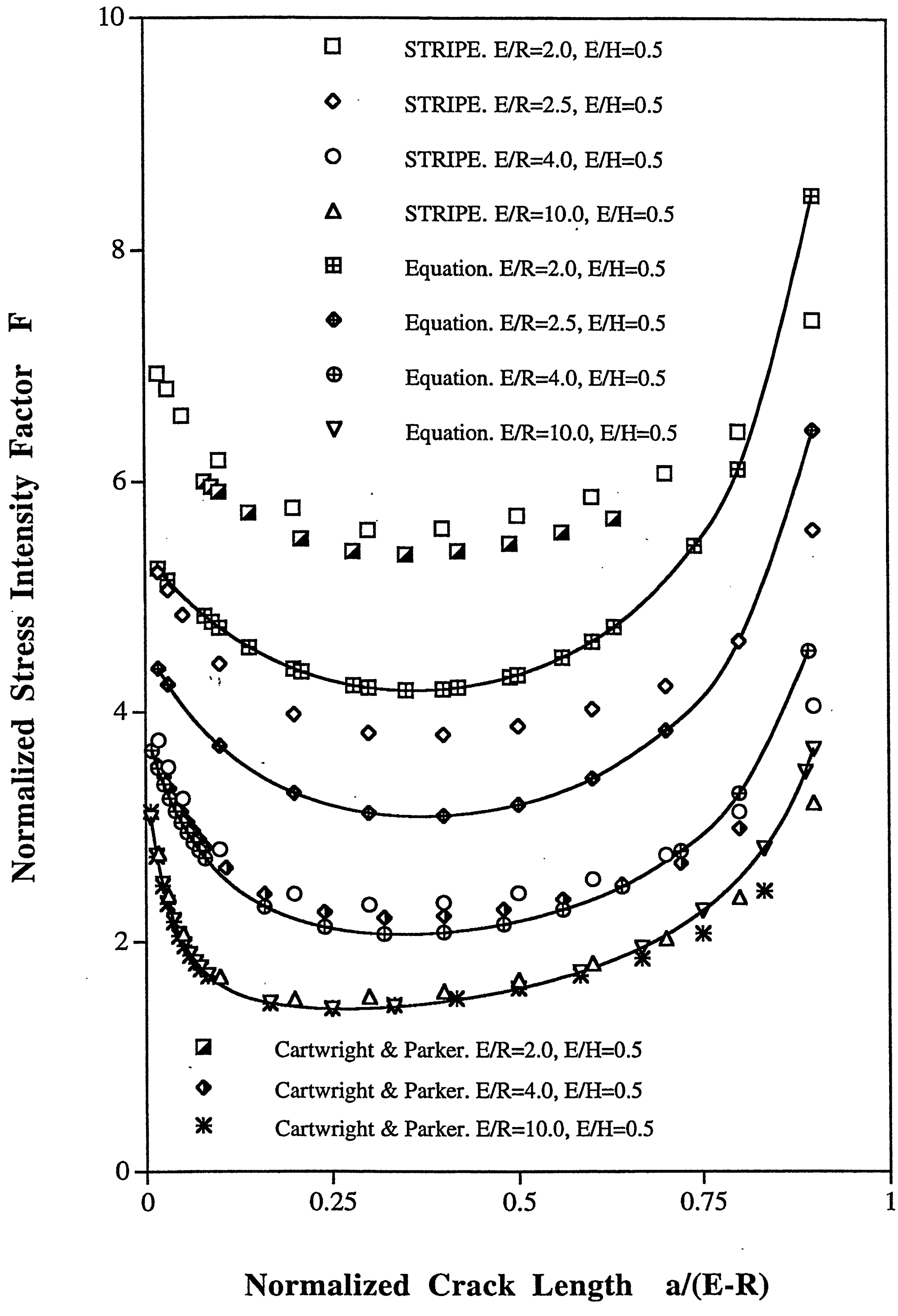

Figure 16. Comparison of normalized stress intensity factors obtained from the literature, STRIPE computations and by using the proposed equation (Eq.31) for the case of two symmetric cracks at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress. E/H=0.5, various E/R ratios.

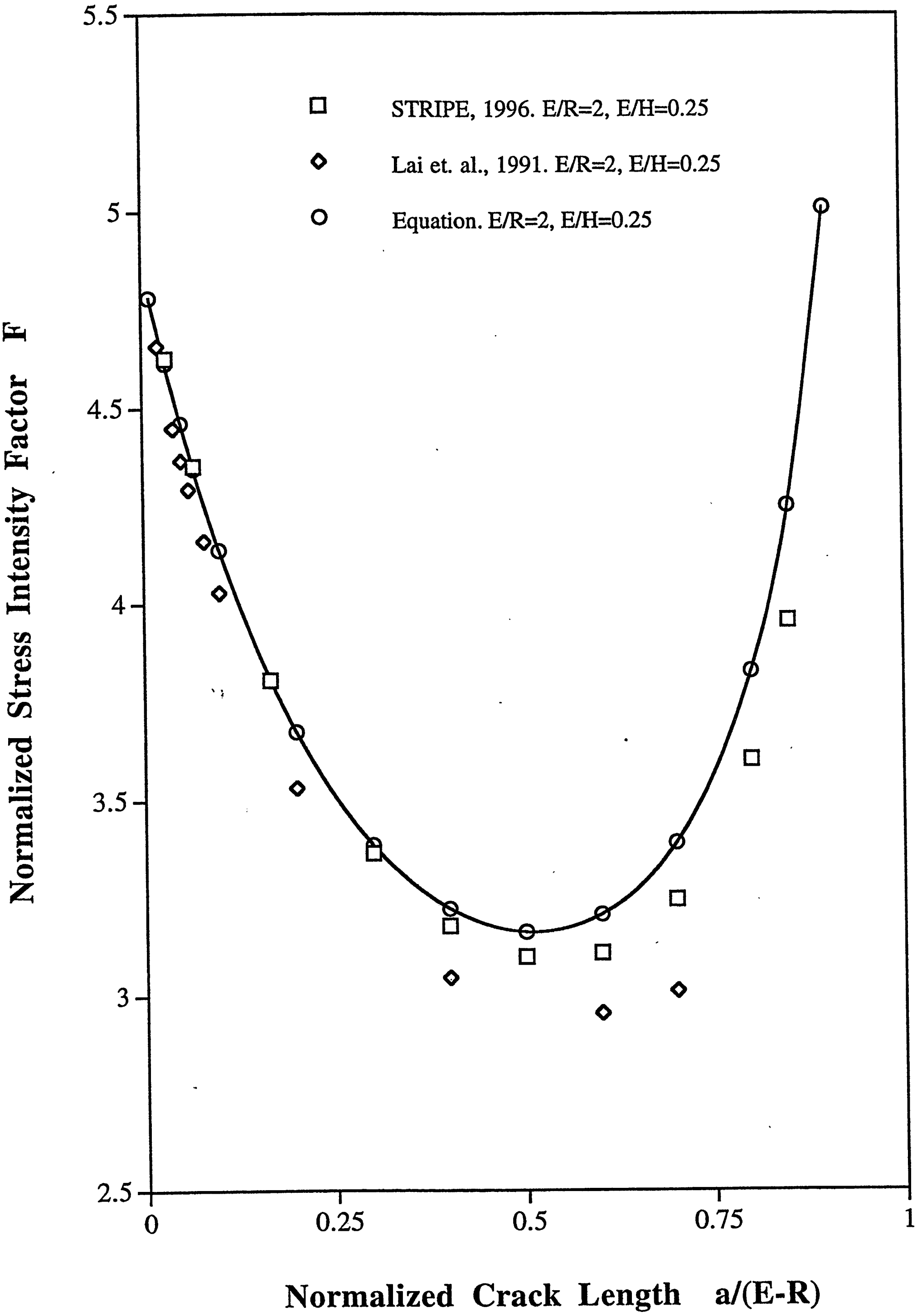

Figure 17. Comparison of normalized stress intensity factors obtained from the literature, STRIPE computations and by using the proposed equation (Eq.31) for the case of a single crack at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress. E/R=2 and E/H=0.25.

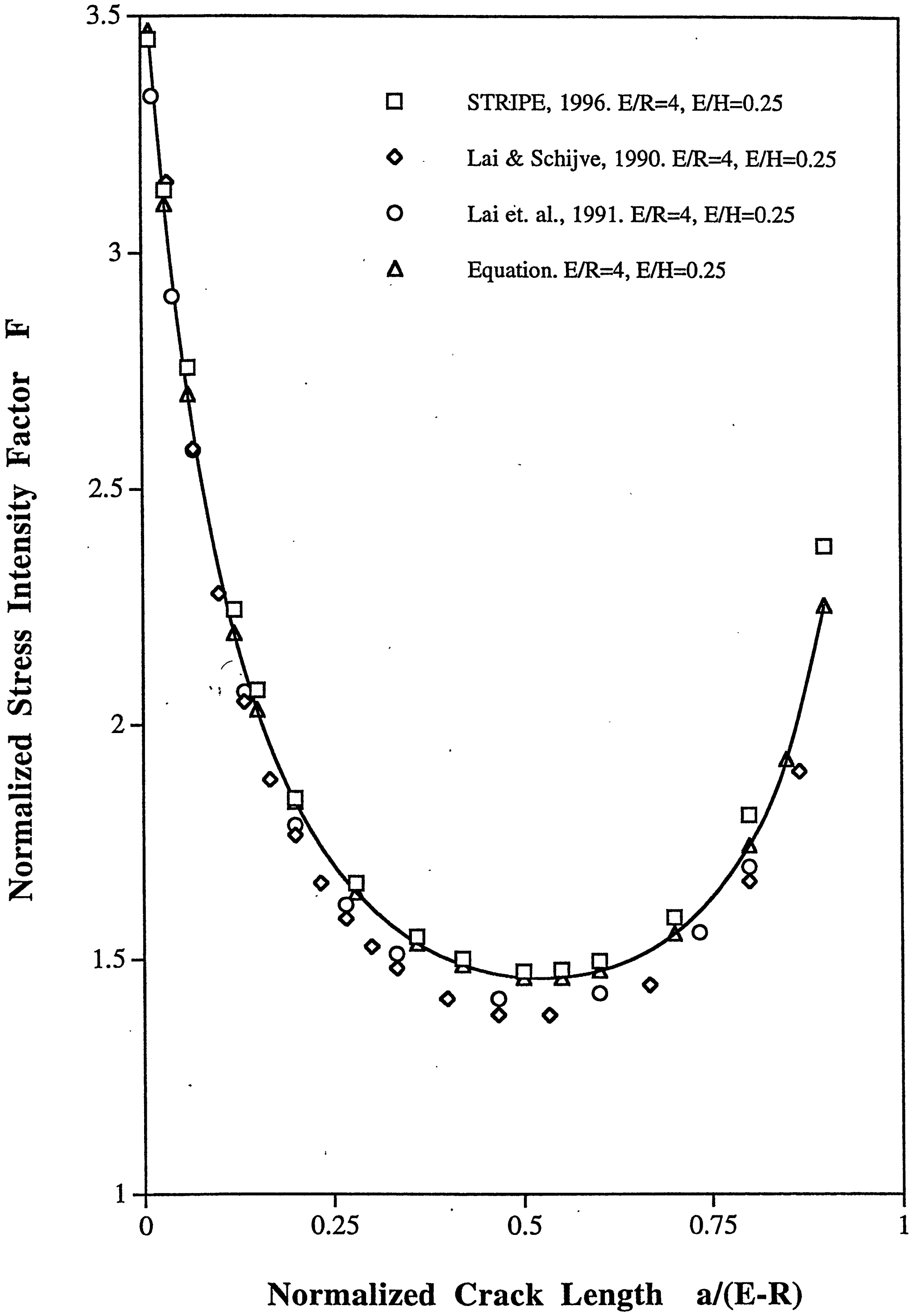

Figure 18. Comparison of normalized stress intensity factors obtained from the literature, STRIPE computations and by using the proposed equation (Eq.31) for the case of a single crack at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress. E/R=4 and E/H=0.25.

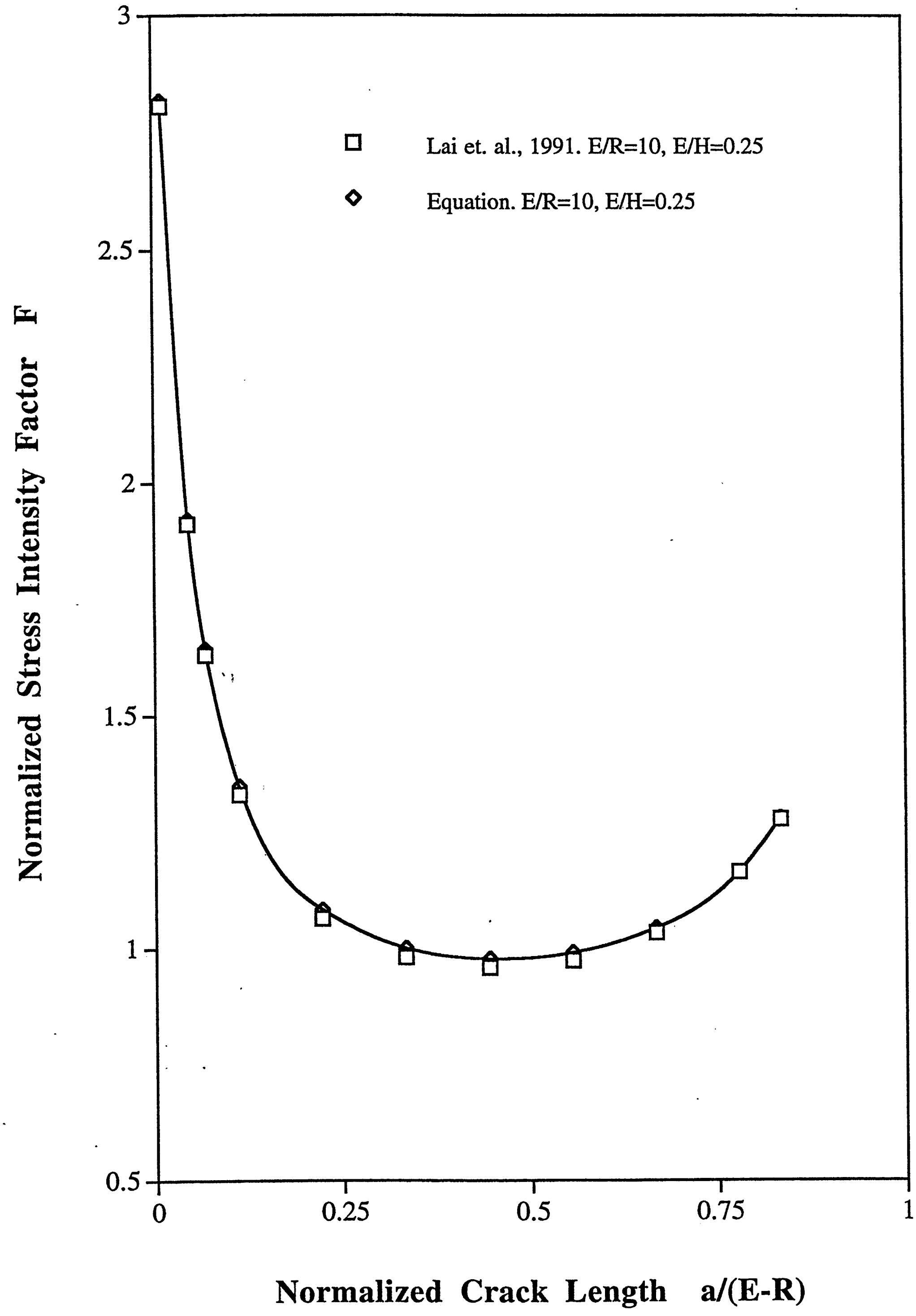

Figure 19. Comparison of normalized stress intensity factors obtained from the literature and by using the proposed equation (Eq.31) for the case of a single crack at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress. E/R=10 and E/H=0.25.

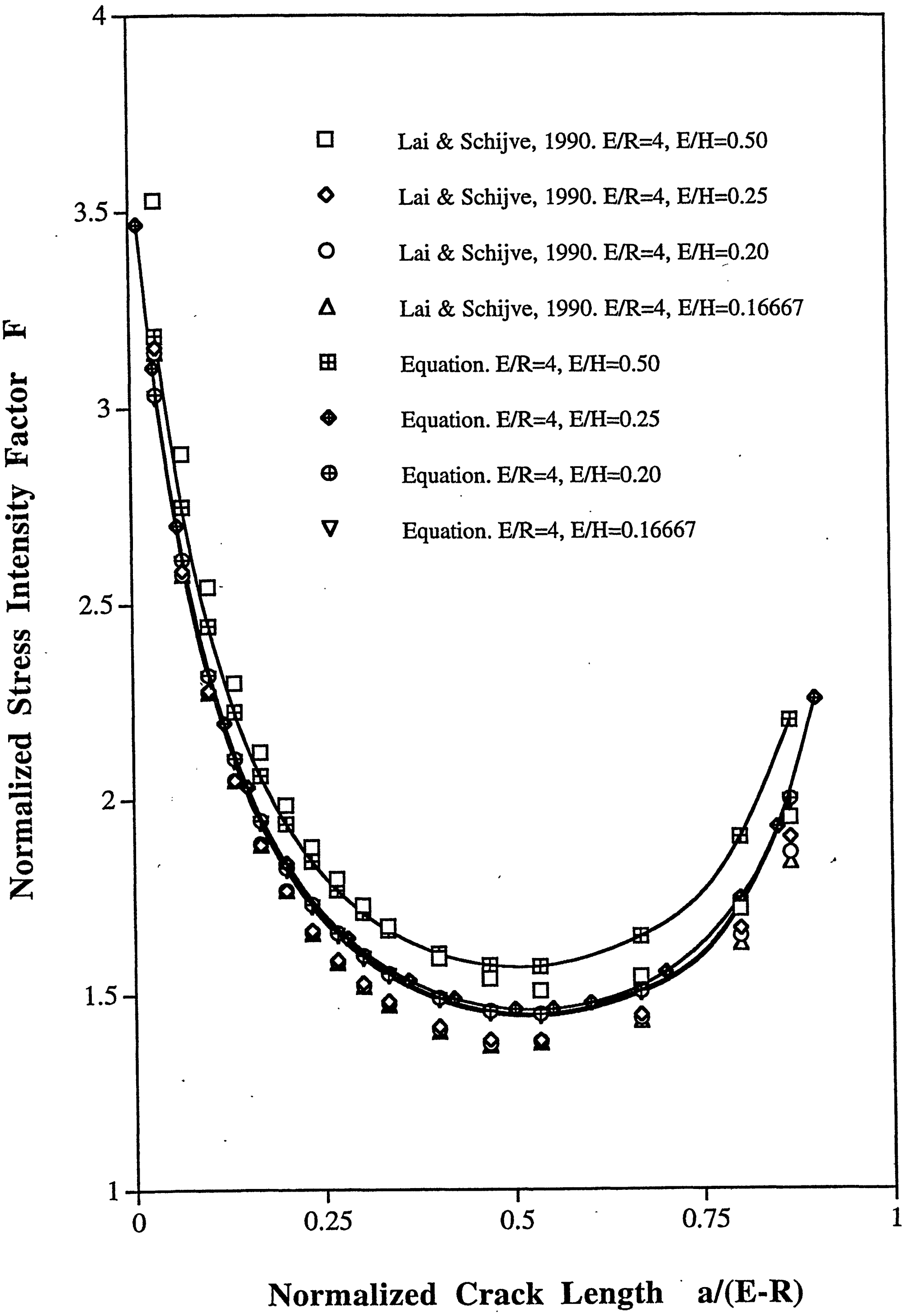

Figure 20. Comparison of normalized stress intensity factors obtained from the literature and by using the proposed equation (Eq.31) for the case of a single crack at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress, E/R=4, various E/H ratios.

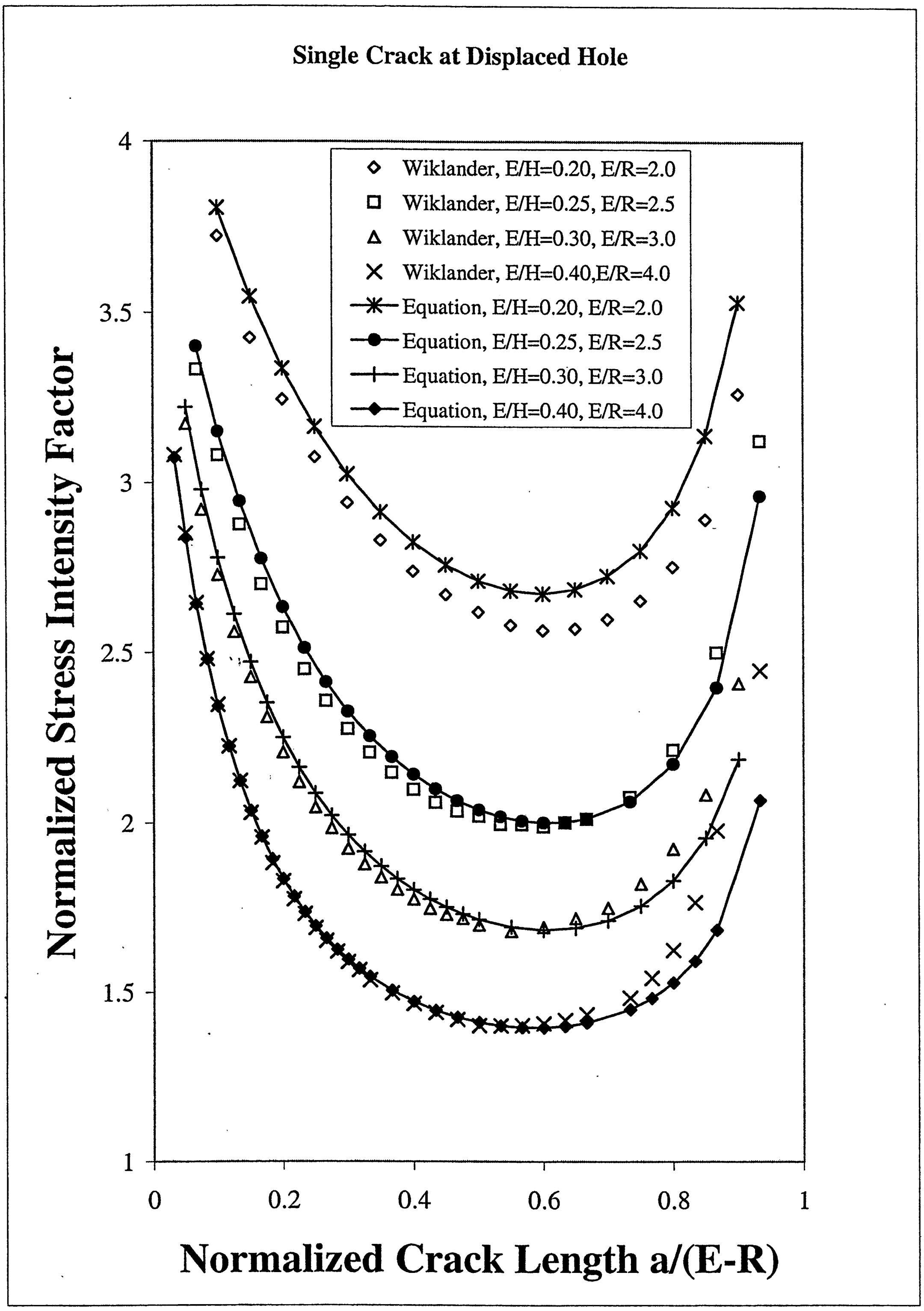

Figure 21. Comparison of normalized stress intensity factors obtained from the literature and by using the proposed equation (Eq.31) for the case of a single crack at an eccentrically located open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress.

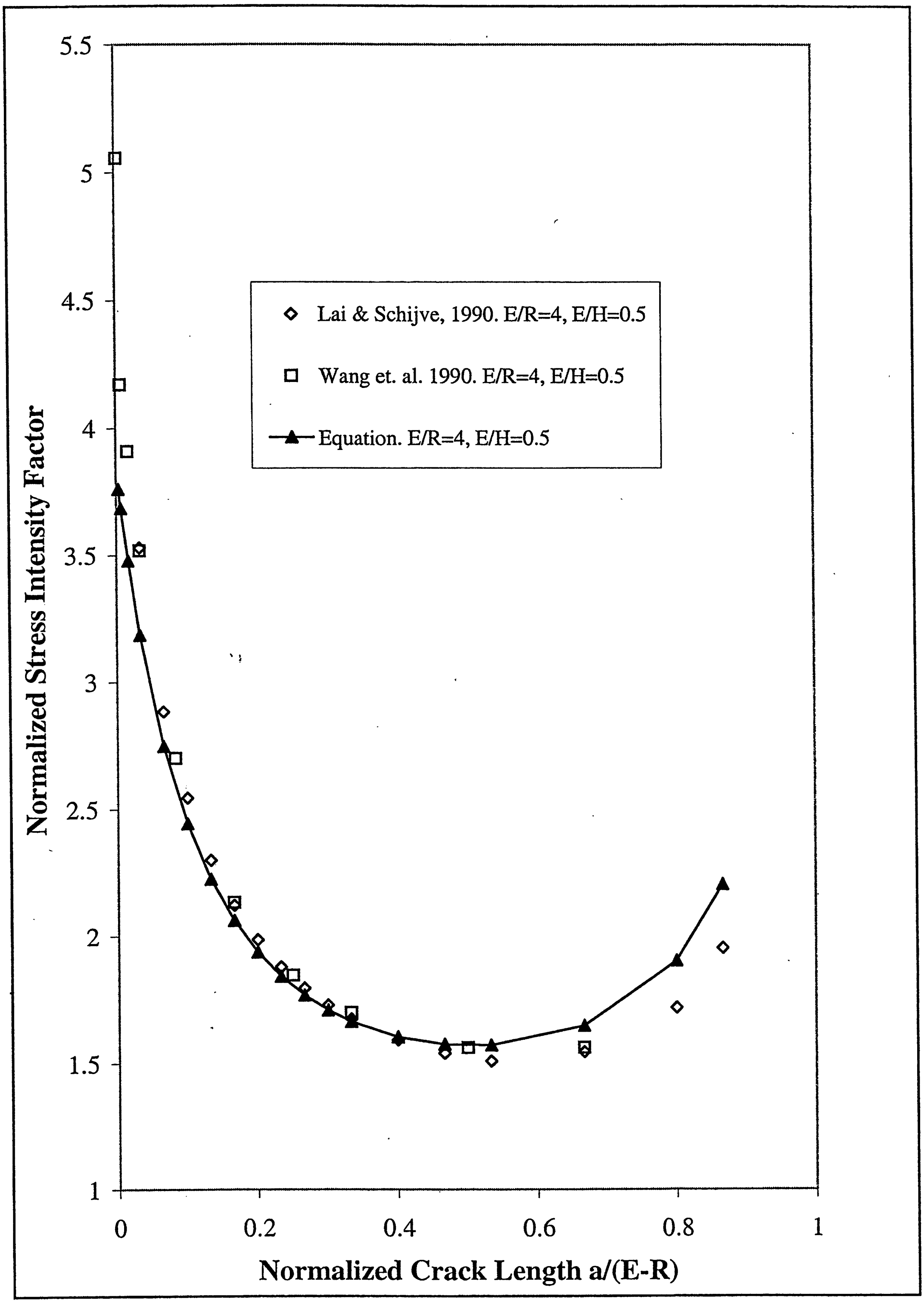

Figure 22. Comparison of normalized stress intensity factors obtained from the literature and by using the proposed equation (Eq.31) for the case of a single crack at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress. E/R=4 and E/H=0.5.

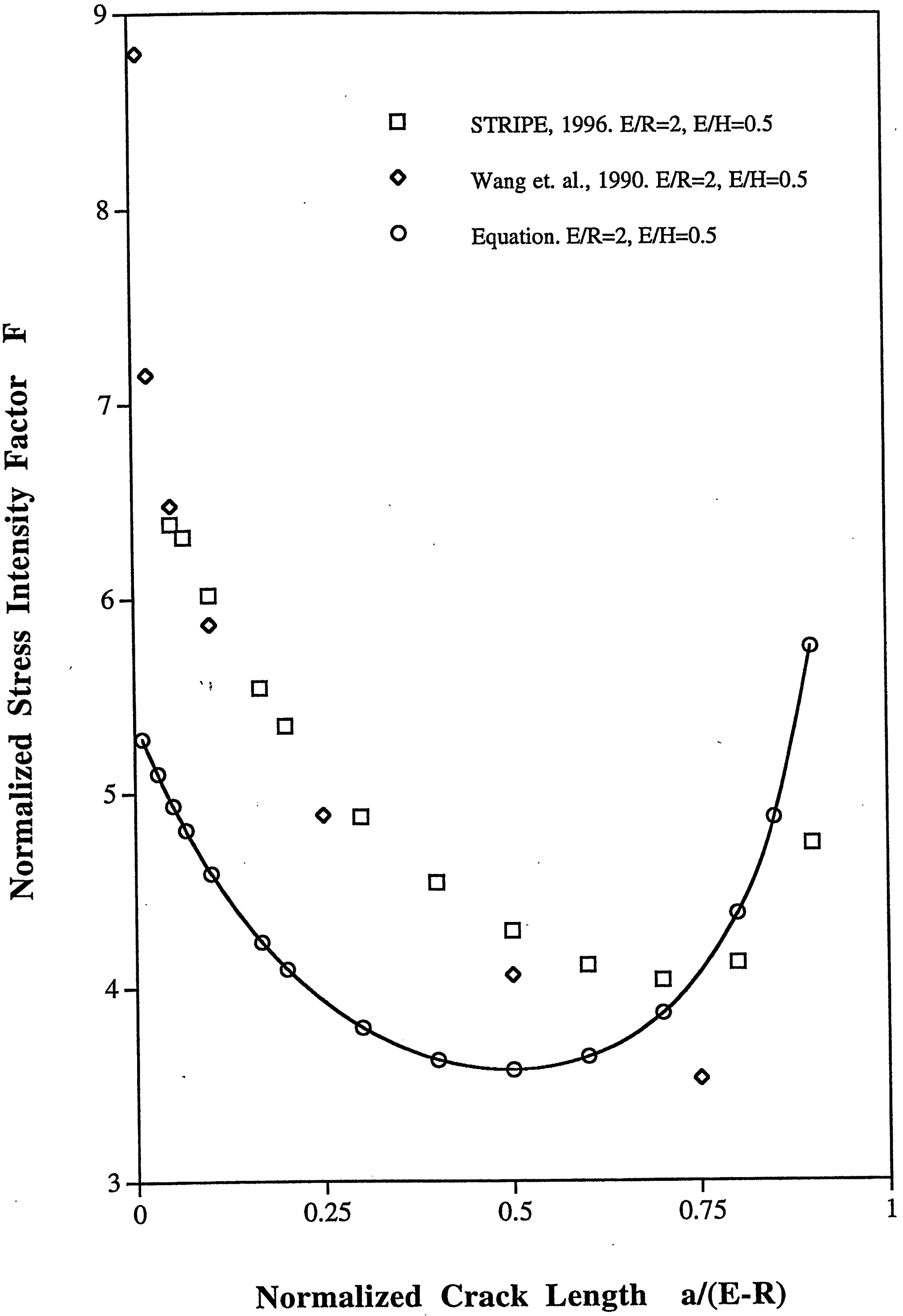

Figure 23. Comparison of normalized stress intensity factors obtained from the literature, STRIPE computations and by using the proposed equation (Eq.31) for the case of a single crack at an open hole in a sheet of finite dimensions, subjected to a uniform, uniaxial stress. E/R=2 and E/H=0.5.

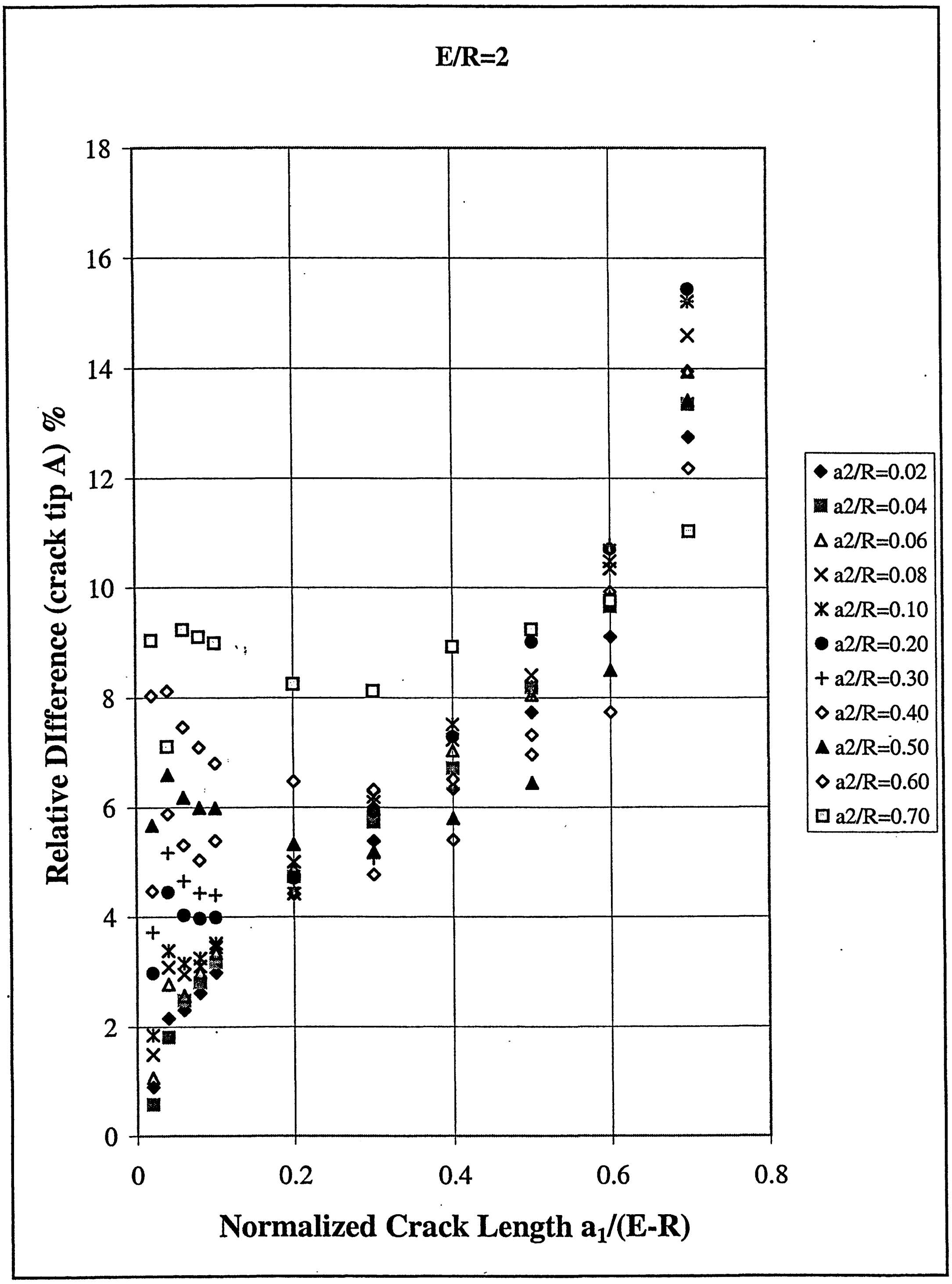

Figure 24. Relative comparison between proposed equation (Eq. 30) and tabulated literature data for the stress intensity factor at crack tip A for two cracks of unequal lengths at an open hole in a sheet of finite dimensions. E/R=2 and E/H=0.25.

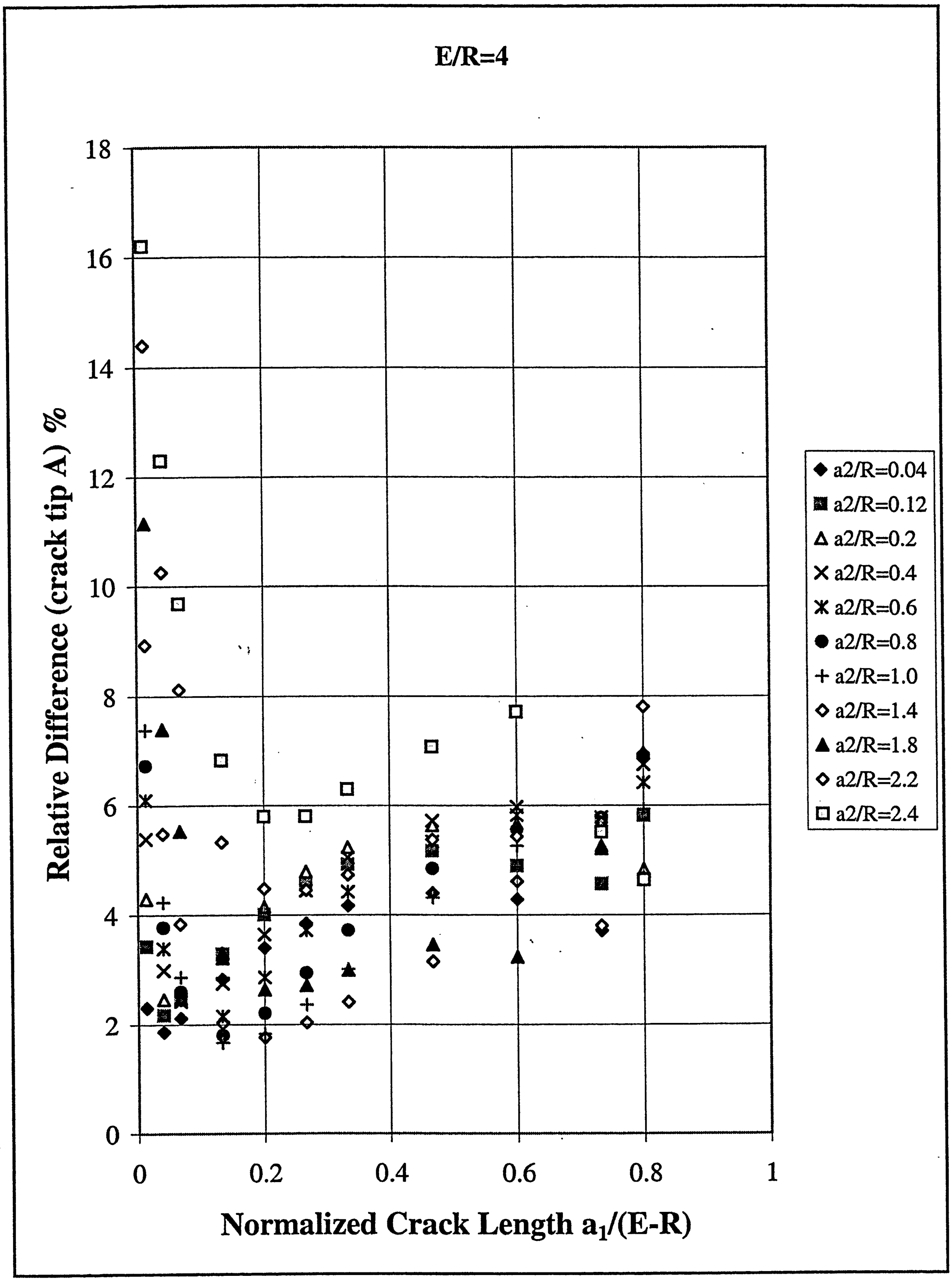

Figure 25. Relative comparison between proposed equation (Eq. 30) and tabulated literature data for the stress intensity factor at crack tip A for two cracks of unequal lengths at an open hole in a sheet of finite dimensions. E/R=4 and E/H=0.25.

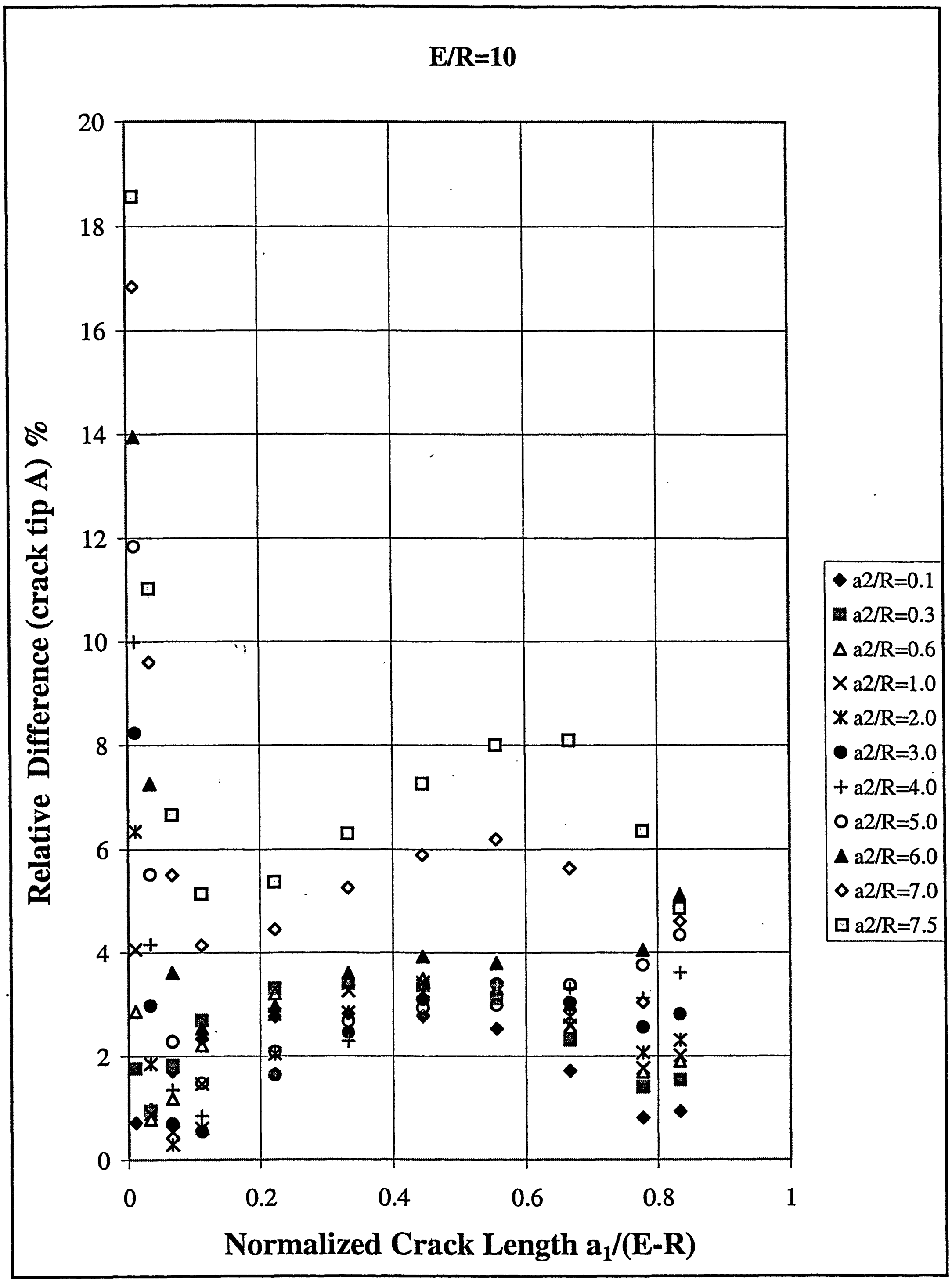

Figure 26. Relative comparison between proposed equation (Eq. 30) and tabulated literature data for the stress intensity factor at crack tip A for two cracks of unequal lengths at an open hole in a sheet of finite dimensions. E/R=10 and E/H=0.25.

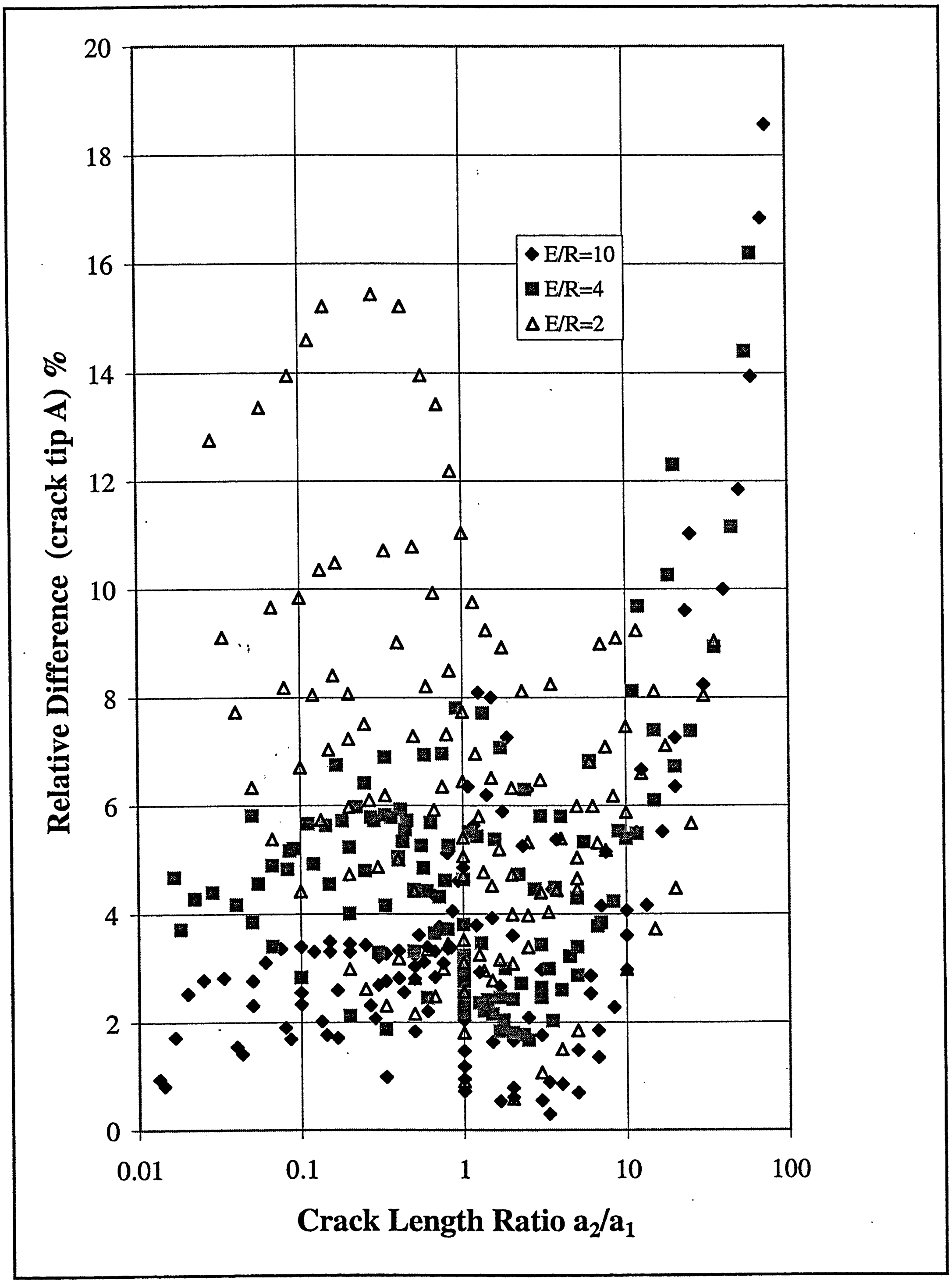

Figure 27. Relative differences, at crack tip A, as function of the ratio a_2/a_1 for different values of E/R.

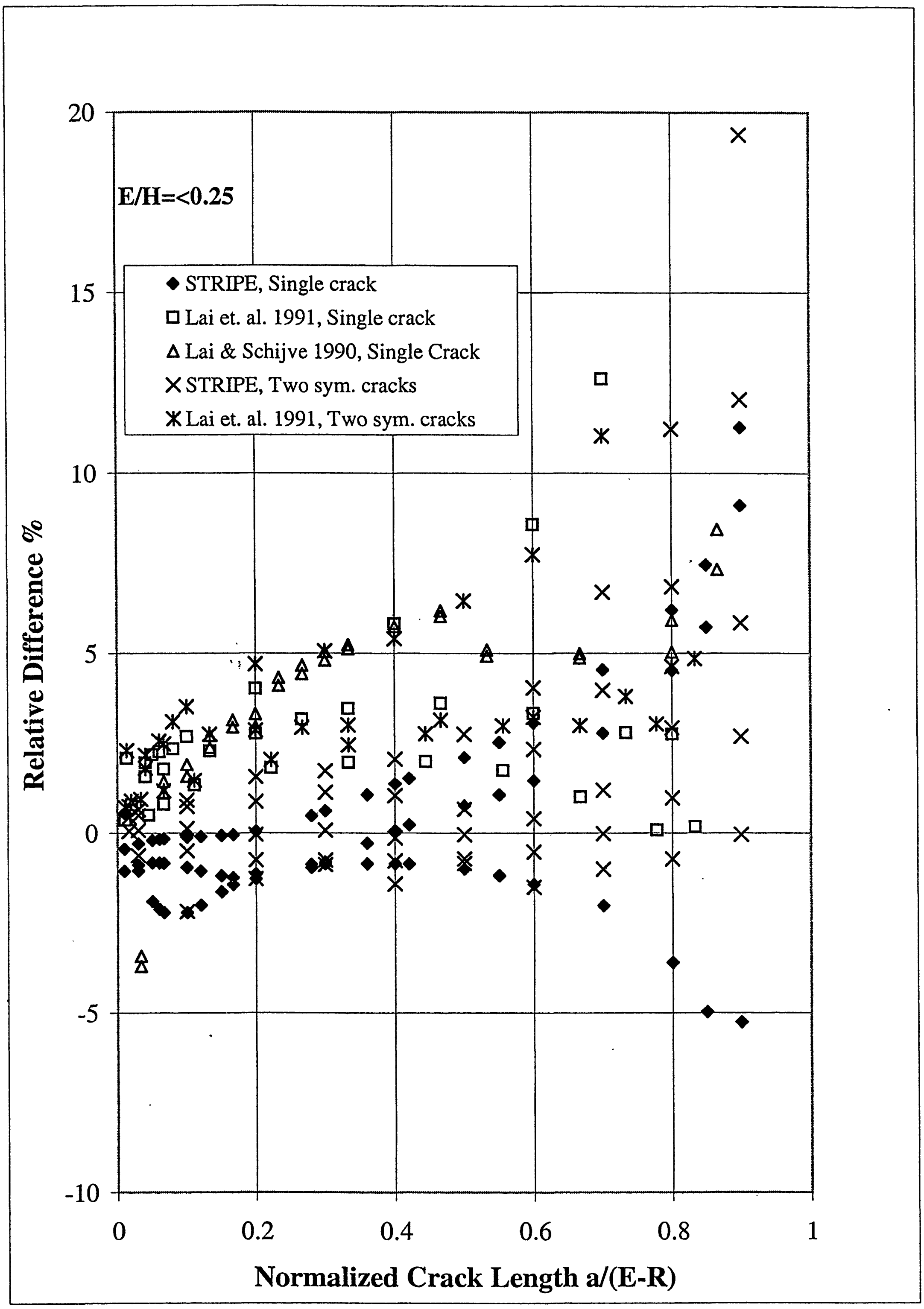

Figure 28. Relative differences between stress intensity factors computed using the proposed equation (Eq. 30), STRIPE and the paper by Lai et. al... E/H=0.25

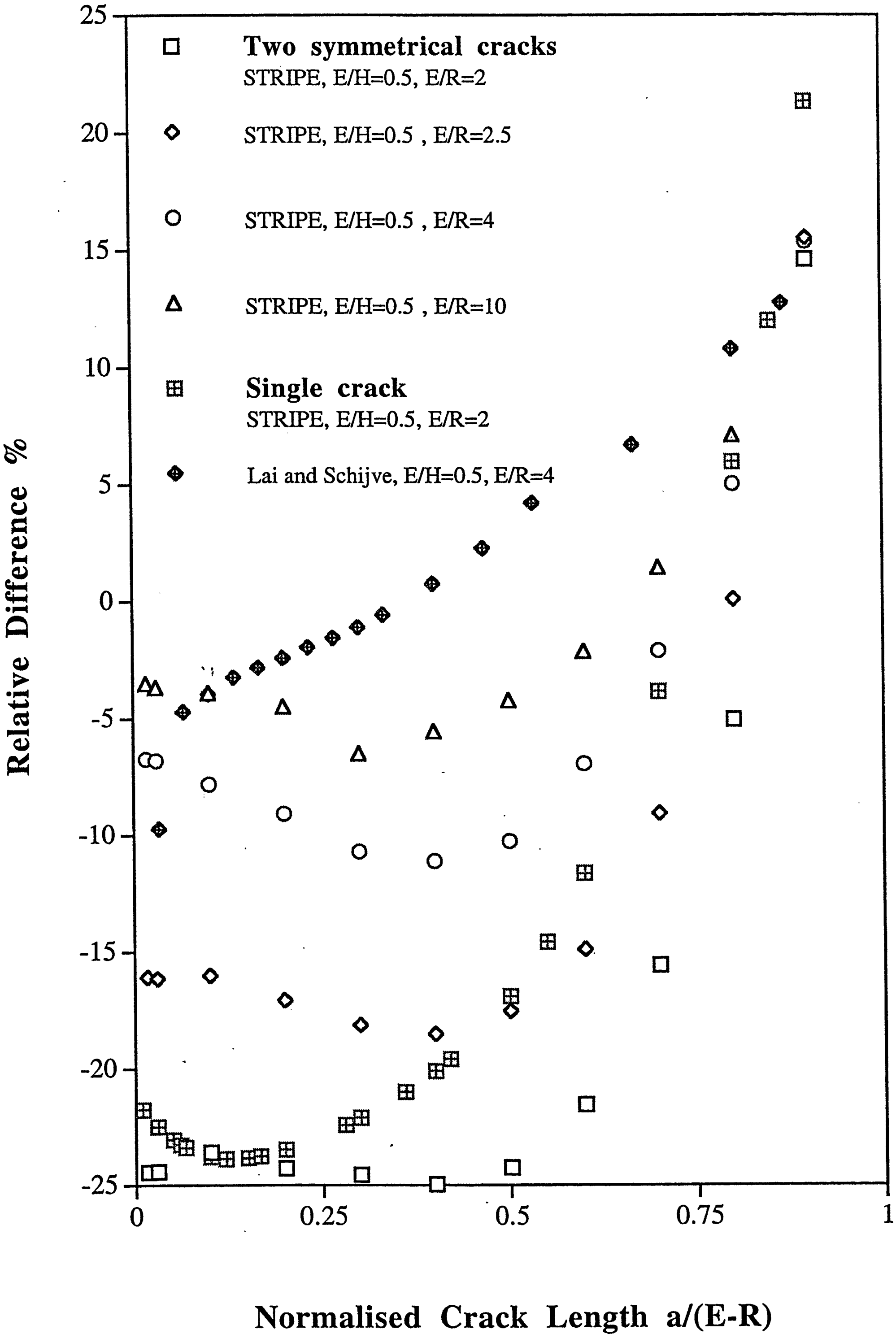

Figure 29. Relative differences between stress intensity factors computed using the proposed equation (Eq. 30), STRIPE and the paper by Lai and Schijve. E/H=0.5

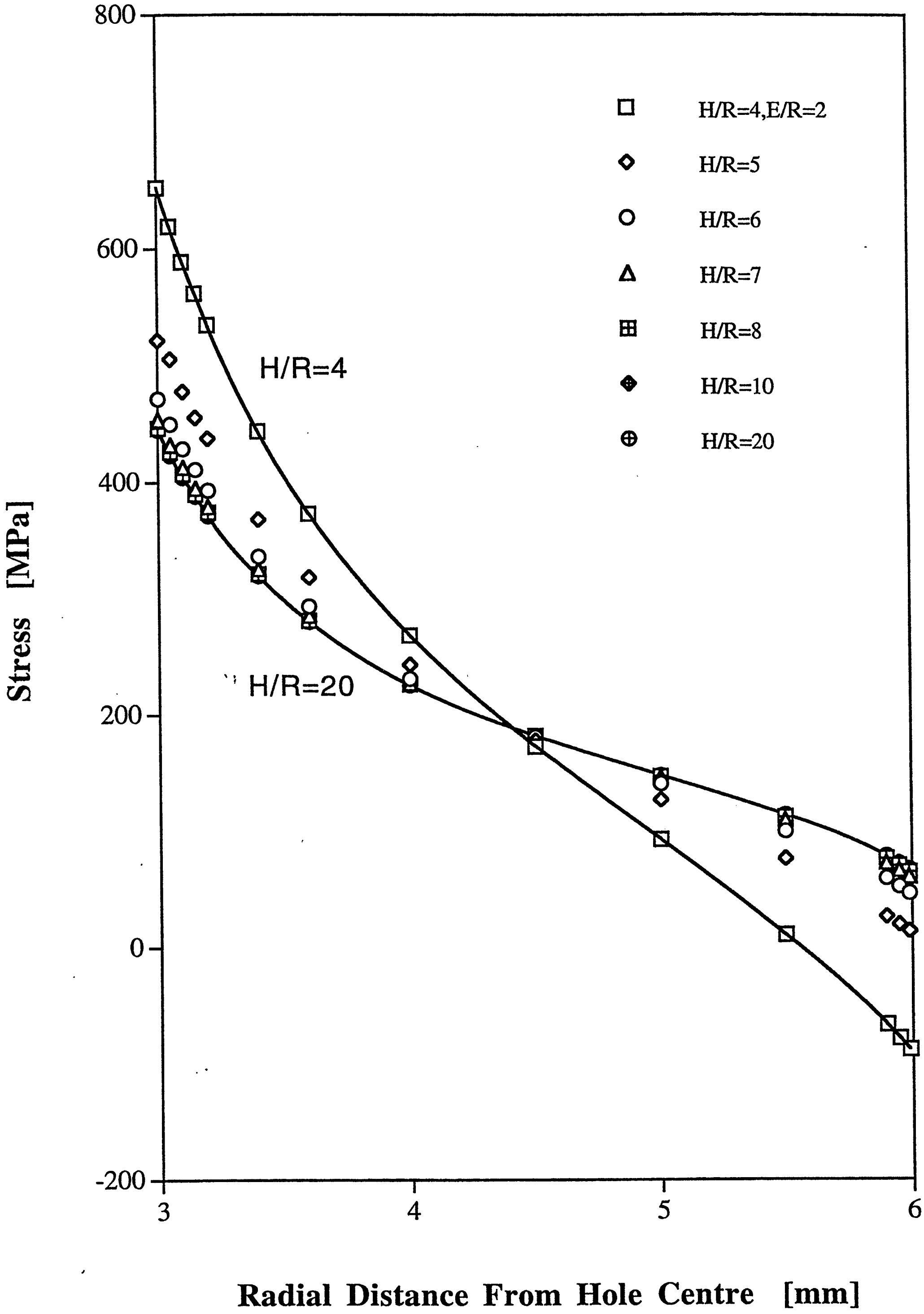

Figure 30. Ligament stress distribution, as obtained using STRIPE, for E/R = 2 and various H/R ratios.

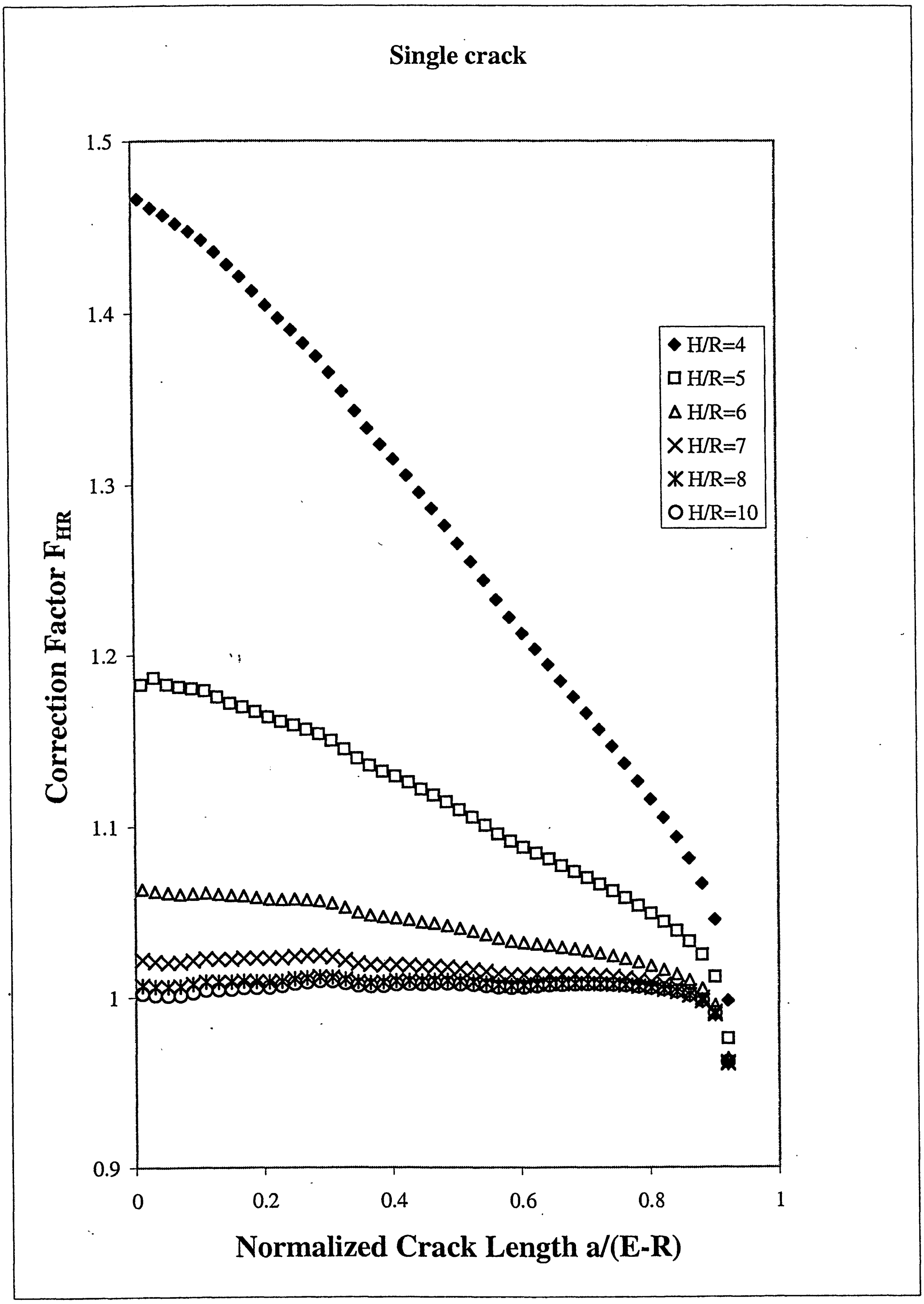

Figure 31. Additional finite height correction factor based upon the quotient between the stress intensity factor computed using the weight function technique and SIF calculated using the proposed equation (Eq.30). Single crack at open hole.

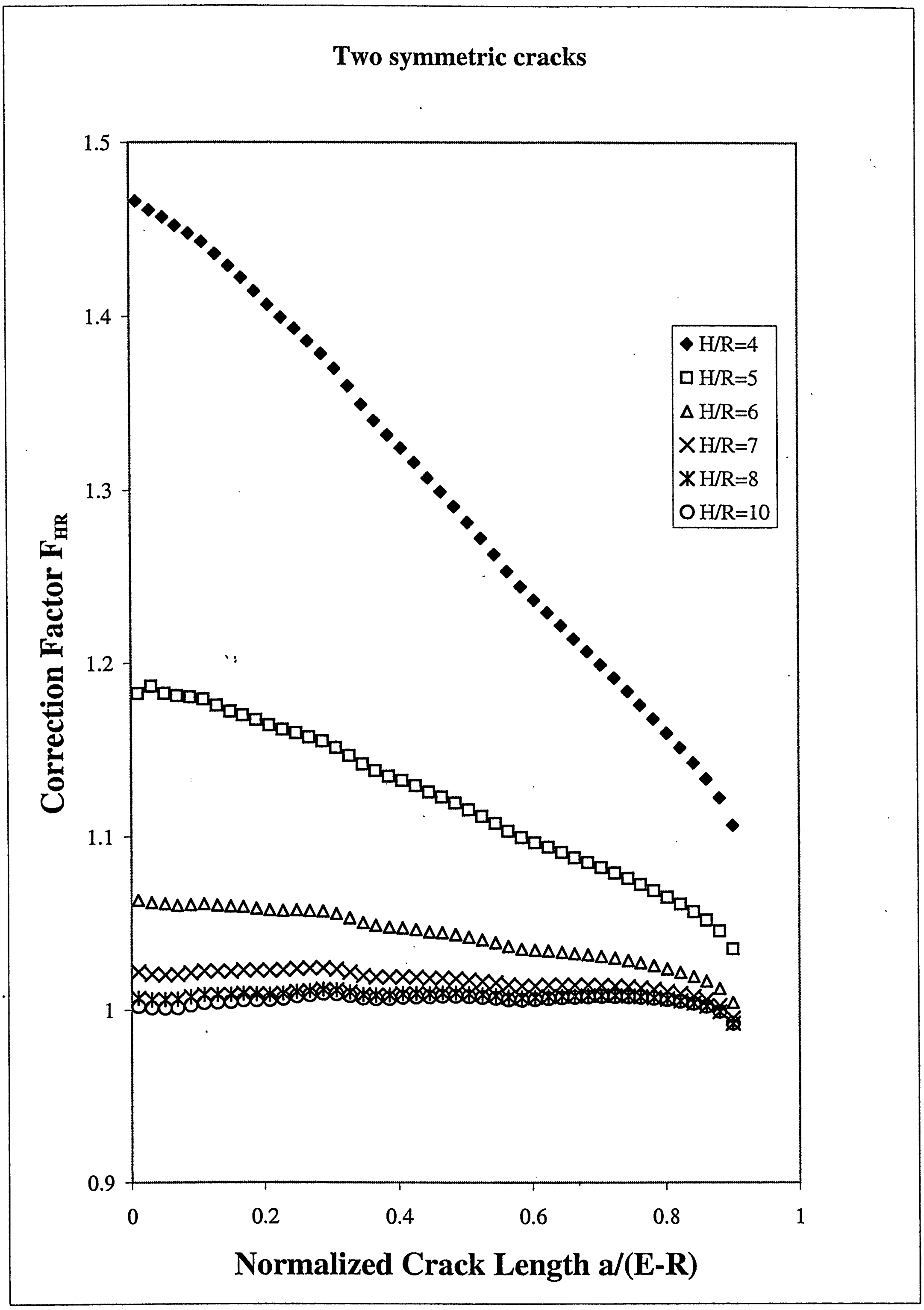

Figure 32. Additional finite height correction factor based upon the quotient between the stress intensity factor computed using the weight function technique and SIF calculated using the proposed equation (Eq.30). Two symmetric cracks at open hole.

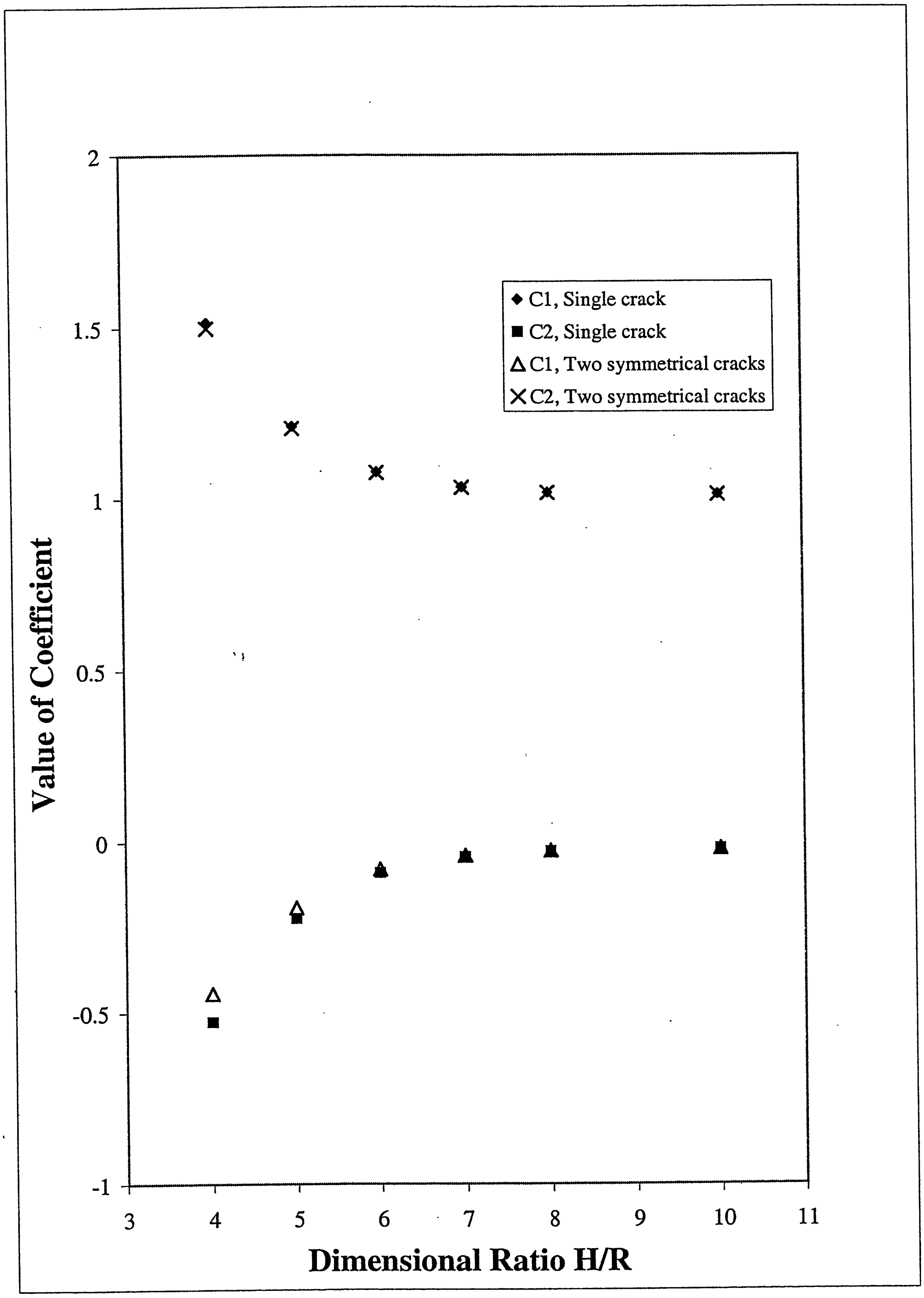

Figure 33. Coefficients C_1 and C_2 as function of the dimensional ratio H/R.

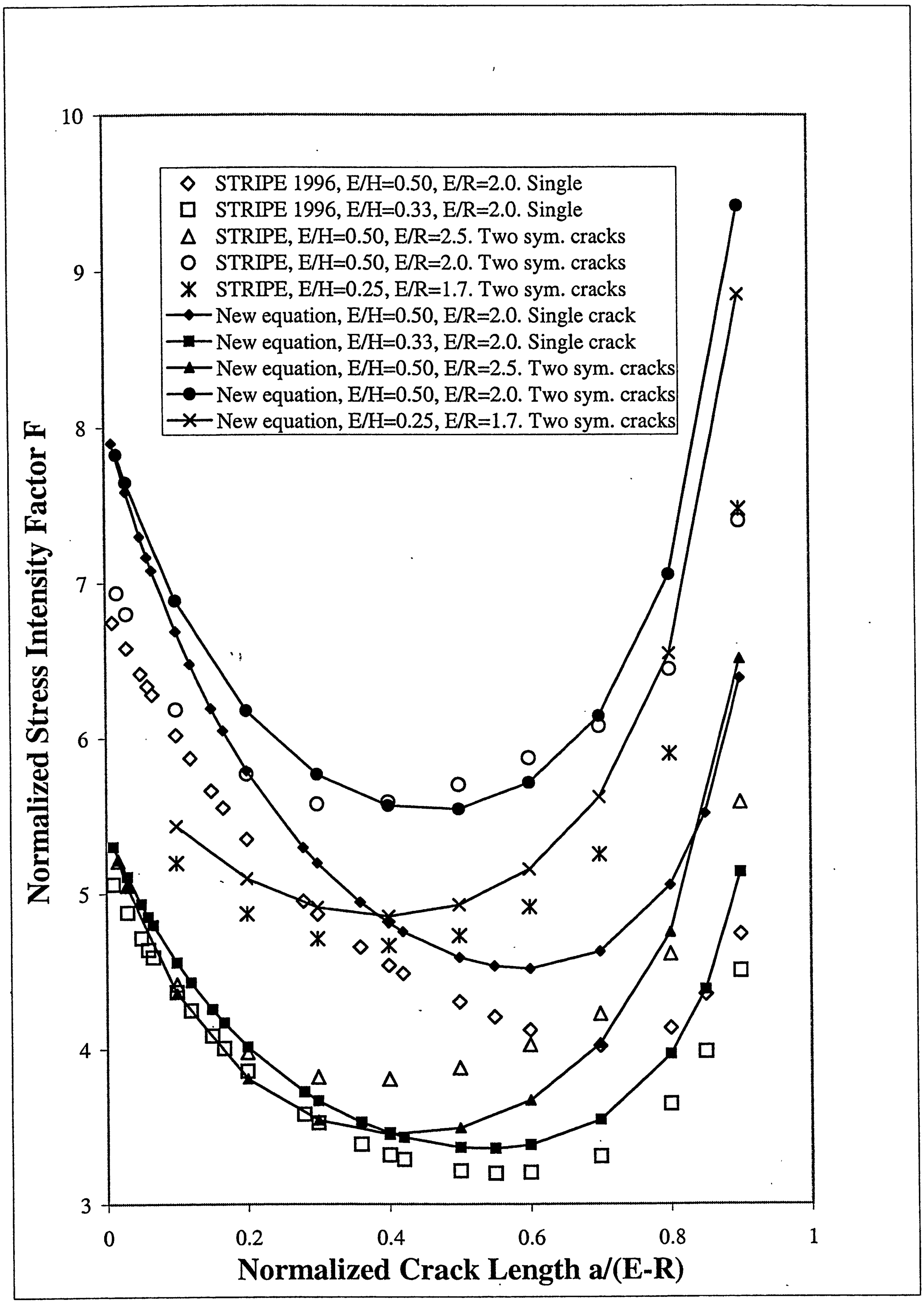

Figure 34. Comparison between normalized stress intensity factors obtained using STRIPE and corresponding factors obtained using the proposed equation with an additional correction factor for the finite height.

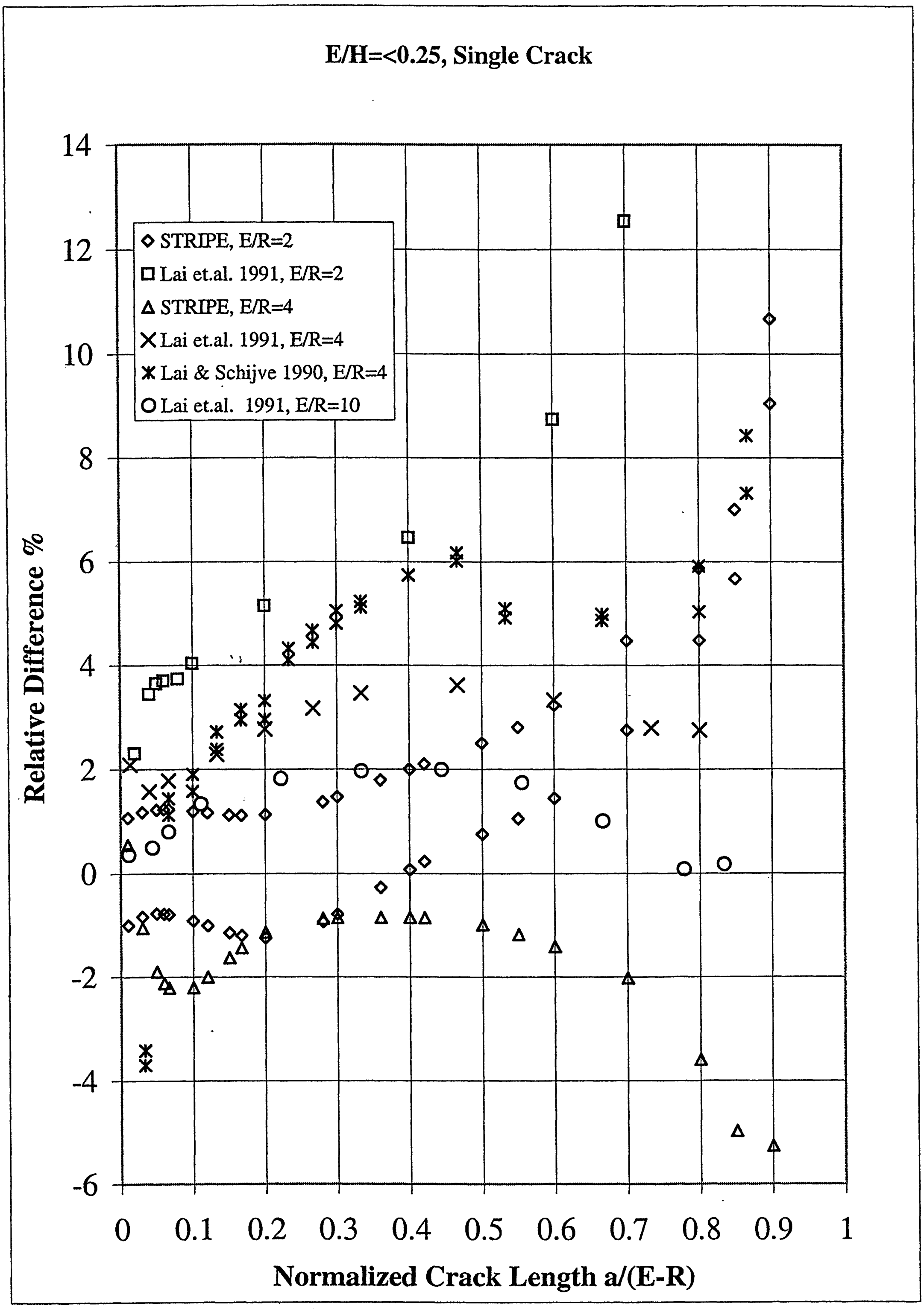

Figure 35. Relative differences obtained by comparisons between normalized stress intensity factors obtained using STRIPE or the literature and corresponding factors obtained using the proposed equation with an additional correction factor for the finite height. Single crack in sheet having $E/H \leq 0.25$.

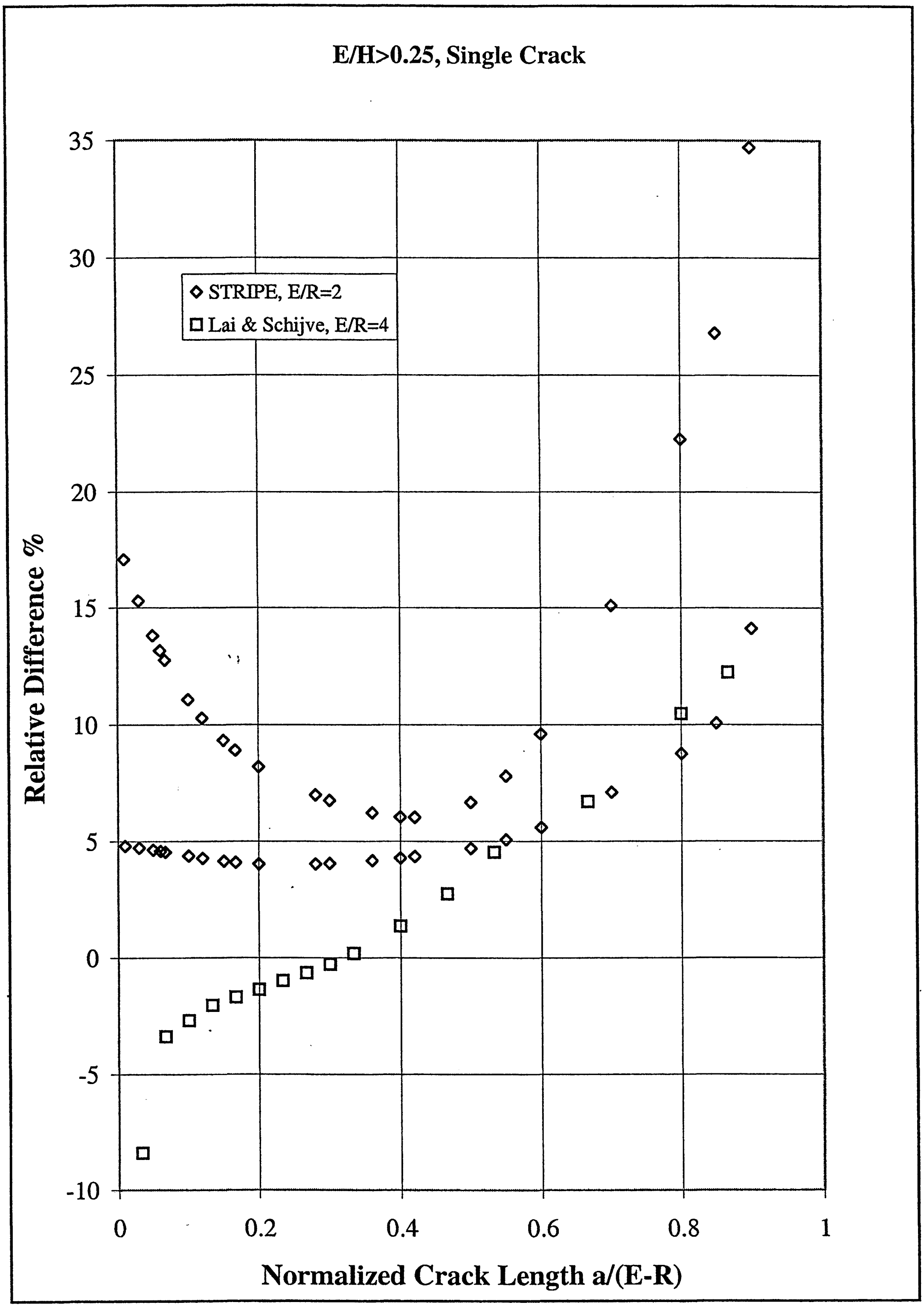

Figure 36. Relative differences obtained by comparisons between normalized stress intensity factors obtained using STRIPE or the literature and corresponding factors obtained using the proposed equation with an additional correction factor for the finite height. Single crack in sheet having E/H>0.25.

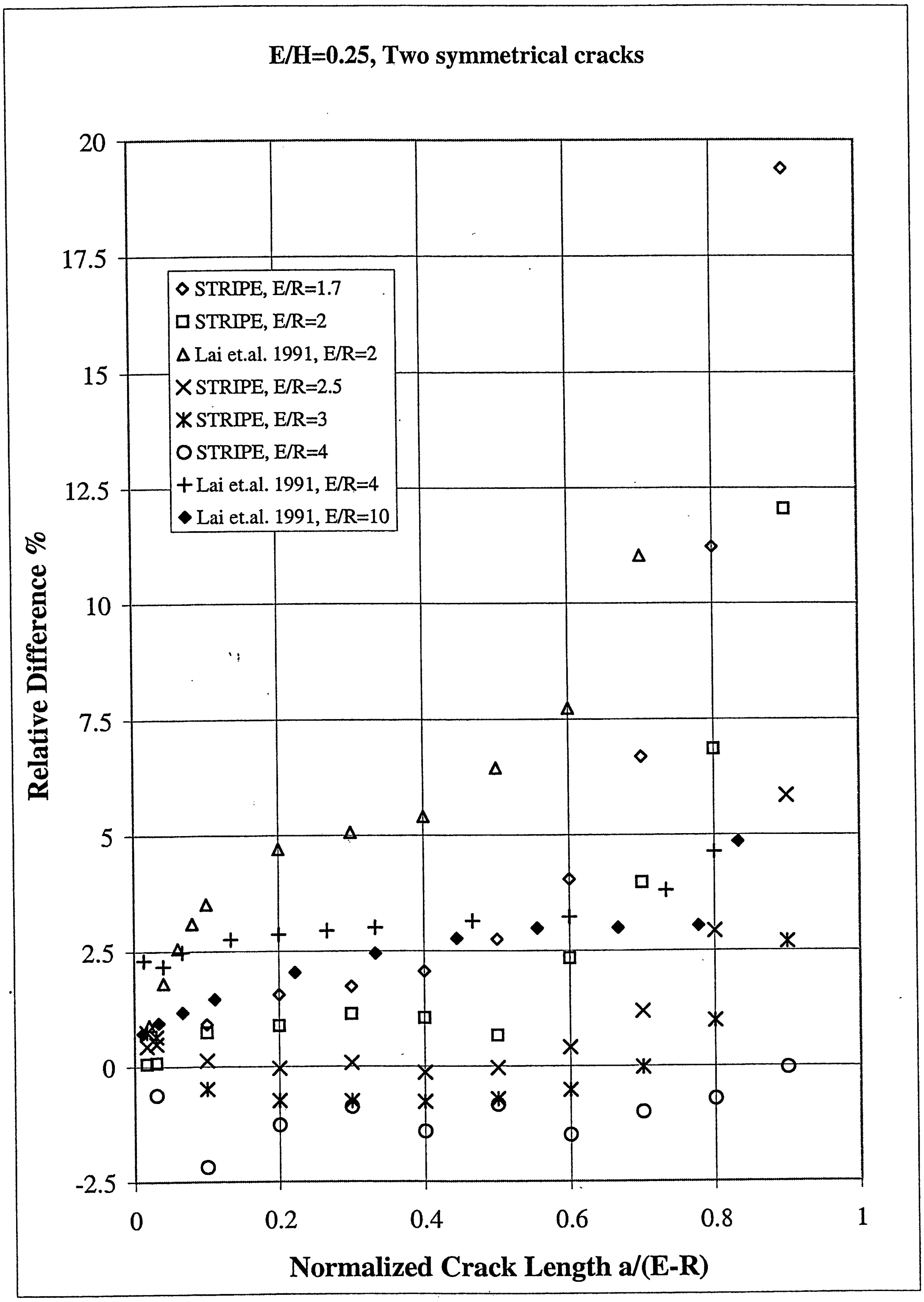

Figure 37. Relative differences obtained by comparisons between normalized stress intensity factors obtained using STRIPE or the literature and corresponding factors obtained using the proposed equation with an additional correction factor for the finite height. Two symmetrical cracks in a sheet having E/H≤0.25.

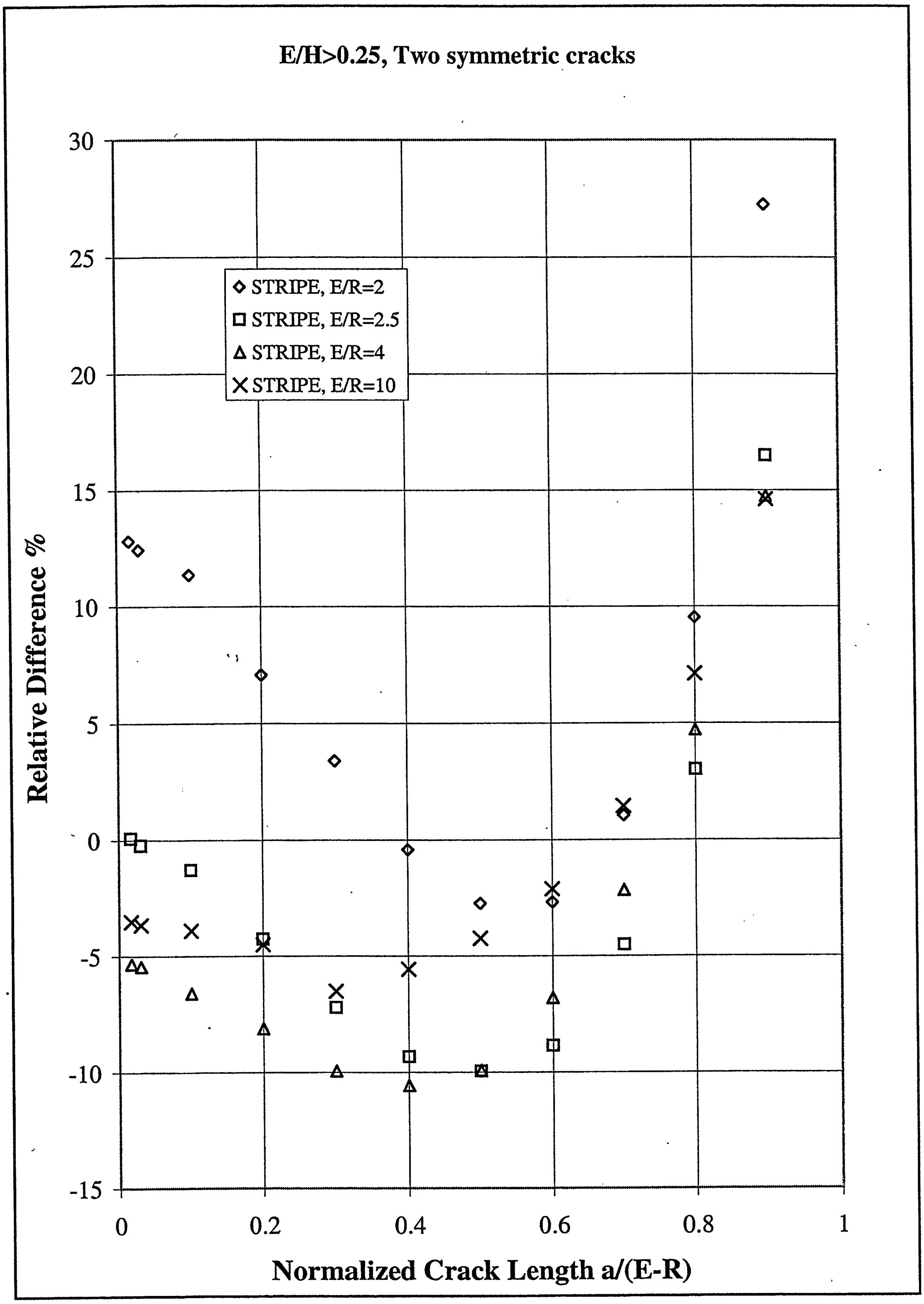

Figure 38. Relative differences obtained by comparisons between normalized stress intensity factors according to STRIPE and corresponding factors obtained using the proposed equation with an additional correction factor for the finite height. Two symmetrical cracks in a sheet having E/H>0.25.

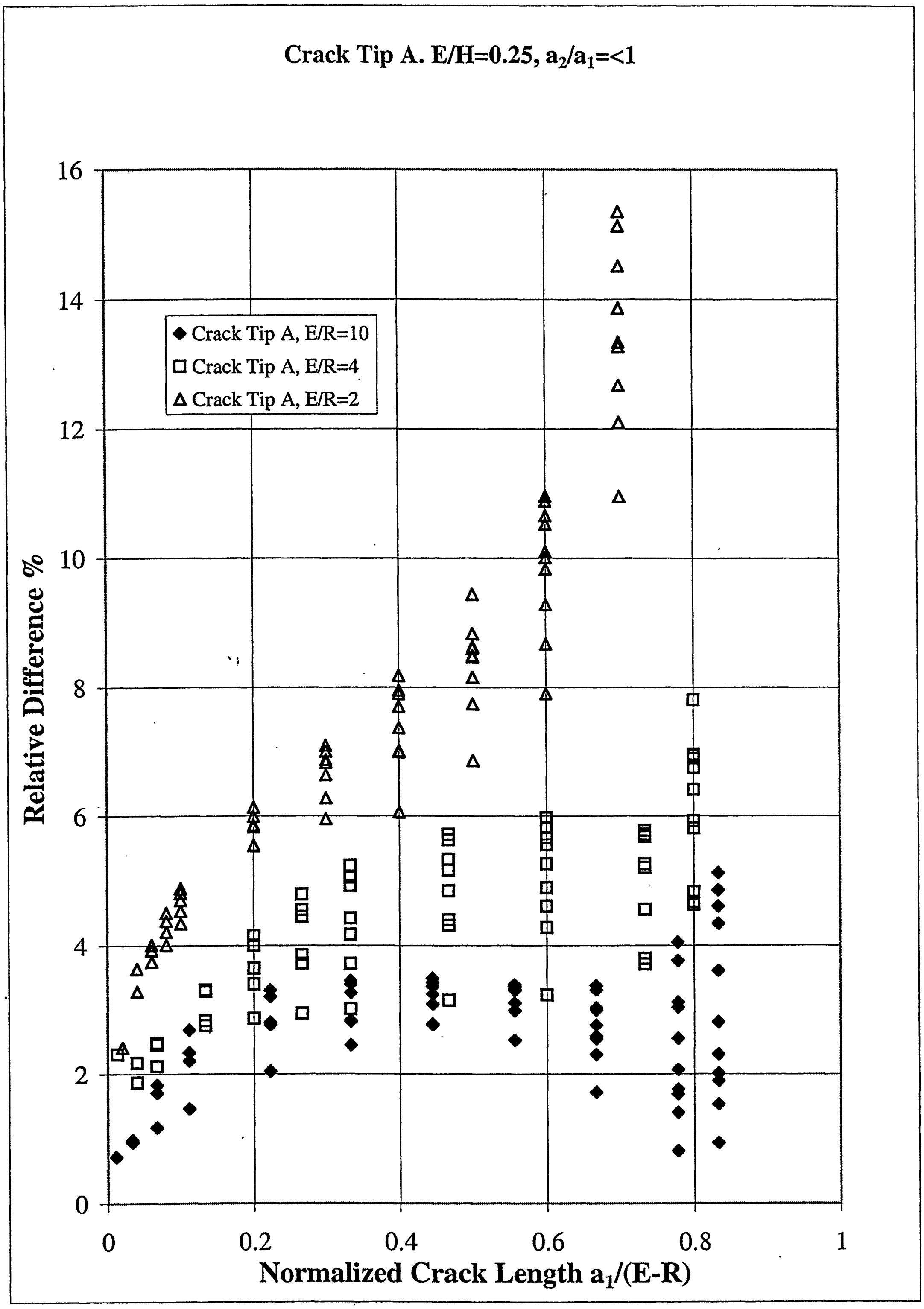

Figure 39. Relative differences obtained by comparisons between normalized stress intensity factors according to Lai et. al. and corresponding factors obtained by the proposed equation. Two cracks of unequal lengths. $a_2/a_1 \leq 1$.

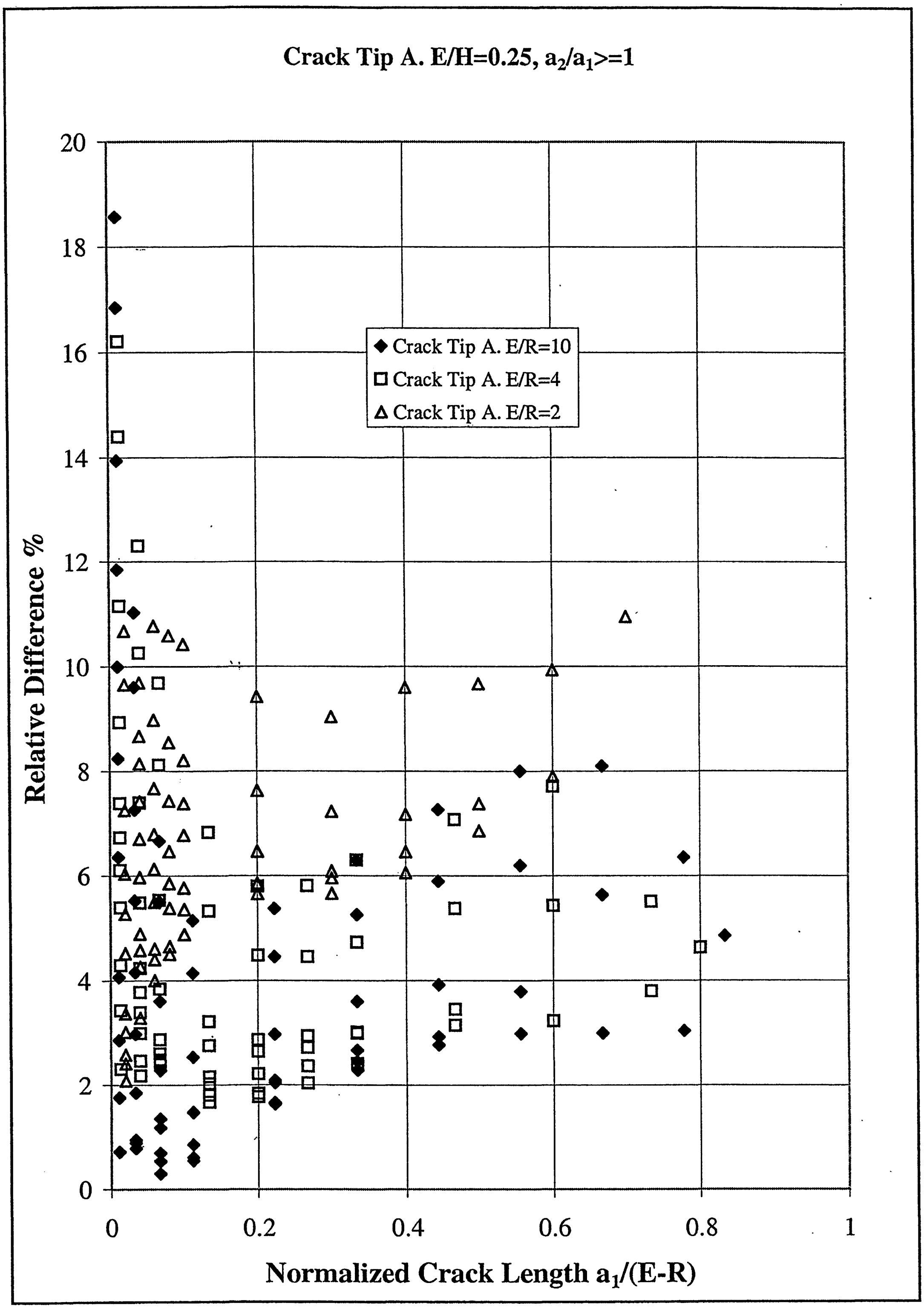

Figure 40. Relative differences obtained by comparisons between normalized stress intensity factors according to Lai et. al. and corresponding factors obtained by the proposed equation. Two cracks of unequal lengths. $a_2/a_1 \geq 1$.