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Scientific report

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Björn Palmberg

Aeronautics Division, FFA
SE-172 90 Stockholm
Sweden

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Report title

Stress Intensity Factor Formulae for a Through the Thickness Centre Crack in a Sheet of Finite Dimensions

Abstract (not more than 200 words)

Stress intensity factors and stress intensity factor equations for the centre crack in a sheet of finite dimensions have been analyzed. Firstly, a finite width and height correction factor was developed for the sheet loaded in tension along the edge parallel to the crack line. Secondly, the stress intensity factor for the geometry above but with the crack surfaces subjected to a symmetric, partial pressure was obtained using the approximate weight function technique. Thirdly, an equation for this stress intensity factor was developed and verified by finite element analyses. Fourthly, three different stress intensity factor equations for the centre cracked strip subjected to two pairs of symmetrically located splitting forces were studied and compared to the results of finite element computations. It was found that the equation based upon the force-balance method was less accurate compared to the other two equations. Also, in the case of a non symmetric crack surface pressure the equation proposed, based on the force-balance method, showed rather large relative differences compared to the finite element results. Finally, two stress intensity factor equations for the centre cracked strip subjected to a single pair of splitting forces were studied. Compared to finite element results the stress intensity factors obtained by the equation based upon the force-balance method showed large differences. However, more surprisingly, the equation developed by Tada (1973) on basis of the exact mode III stress intensity factor and an asymptotic interpolation showed large differences for the crack tip away from the force pair when compared to the finite element results. The equation is said to have an accuracy better than 1 % but the relative differences amounted to more than 75 % in some cases.

Keywords

Crack, Crack surface pressure, Equation, Finite element, Formula, Method of least squares, Splitting forces, Superposing, Weight function

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Spänningsintensitetsfaktorformler för en genomgående central spricka i en begränsad plåt

Sammanfattning (högst 200 ord)

Spänningsintensitetsfaktorer och spänningsintensitetsfaktorekvationer för en central spricka i en begränsad plåt har studerats. Först skapades en bredd- och höjdkorrektionsfaktor för det fall då plåten är belastad längs kanterna parallella med sprickan. För det andra togs en spänningsintensitetsfaktor fram för samma geometri men med en del av sprickytan belastad med ett symmtriskt yttryck genom att utnyttja den approximativa viktsfunktionstekniken. För det tredje utvecklades en spänningsintensitetsekvation för detta belastningsfall vilken verifierades genom finita elementberäkningar. För det fjärde studerades tre olika spänningsintensitetsfaktorekvationer för den centrala sprickan, i en oändligt lång (hög) plåt, utsatt för två par, symmetriskt belägna, punktkrafter som vill öppna spricka. Den ekvation som var baserad på kraft-balans metoden visade sig vara den minst noggranna. Också för fallet med ett icke symmetrisk yttryck verkande på sprickan så visade den ekvation som baserade sig på kraft-balans metoden relativt stora skillnader jämfört med resultatet av de finita elementberäkningarna. Slutligen studerades två spänningsintensitetsekvationer för den centrala sprickan i en oändligt lång plåt där sprickytan är belastad med ett enda kraftpar. Jämfört med de finita elementberäkningarna så visade sig den ekvation som byggde på kraft-balans metoden ge stora avvikelser. Mer överraskande var att den ekvation som byggde på den exakta lösningen för modus III fallet och en asymptotisk interpolation uppvisade stora skillnader jämfört med finita element resultatet för den sprickspets som är längst ifrån belastningspunkten. Ekvationen sägs ha en noggrannhet bättre än 1 % men de relativa skillnaderna uppgick till mer än 75 % i en del fall.

Nyckelord

Spricka, Sprickuttryck, Ekvation, Finita element, Minsta kvadratmetoden, Punktlaster, Superponering, Viktsfunktion

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Contents

Summary

Stress intensity factors and stress intensity factor equations for the centre crack in a sheet of finite dimensions have been analysed. Firstly, an equation for the case of a uniformly distributed, uniaxial stress along the edges parallel to the crack line was developed by a polynomial fit to data presented in graphical and tabular form. The maximum error in the developed equation is less than 7 % compared to the data that was fitted. The fitted data is said to have an accuracy better than 1 %. In general the error in the developed equation is about 2 %.

Secondly, the stress intensity factor for a centre crack in a rectangular sheet subjected to a symmetrically applied partial crack surface pressure was obtained by using the approximate weight function technique and the newly developed stress intensity factor equation as reference solution.

Thirdly, a stress intensity factor equation for a centre cracked strip subjected to a symmetrically applied partial crack surface pressure was developed and verified by finite element calculations. The equation is called "Modified Newman" since it reduces to an equation proposed by Newman when simplified. The developed equation was compared to an equation, based on the approximate weight function technique, found in the literature. Compared to the finite element results the maximum relative difference in the stress intensity factors obtained using the developed equation is about 5 %.

Fourthly, three different stress intensity factor equations for the centre cracked strip subjected to two pairs of symmetrically located splitting forces were studied and compared to the results of finite element calculations. It was found that the equation based upon the force-balance method was less accurate than the other two equations.

The accuracy of the developed equation is generally much better than 20 % for crack lengths of practical interest and in most cases the accuracy is better than 8 %.

Next the finite element results for a strip with a centre crack subjected to a non symmetric partial crack surface pressure were used to verify an equation developed by Chen et. al. using the force balance method. The partial crack surface pressure was applied from the centre of the crack to a co-ordinate x_U along the crack surface. The stress intensity factor for each one of the two crack tips was studied. The comparison shows rather large relative differences for x_U-values greater than half the crack length for both stress intensity factors. Furthermore, the same equation by Chen,

but for a symmetric crack surface pressure from the co-ordinate $-x_U$ to the co-ordinate x_U, shows relative differences of up to 13 % as compared to the “Modified Newman” equation.

Finally, two stress intensity factor equations for the centre cracked strip subjected to a single pair of splitting forces were studied and compared to finite element results. Again it was found that the stress intensity factor based upon the force-balance method showed large differences compared to the finite element results. More surprisingly, the results of the equation developed through an asymptotic interpolation, by Tada, showed large differences compared to the finite element results. Particularly, for the crack tip away from the splitting forces relative differences of more than 75 % were found. The equation is said to have an accuracy better than 1 %. For the splitting forces acting at the centre of the crack a good correlation was found between the finite element results and the results obtained using the equation.

1 Introduction

Very accurate closed form expressions of the stress intensity factor (SIF) for a centre crack in a sheet of finite width subjected to a remote uniformly distributed, uniaxial stress acting perpendicular to the crack line exist. In the case of both finite width and finite height solutions for the SIF exist, but to the author´s knowledge, only in tabular or graphical form, Ref.[1-2]. Thus, the first objective of this investigation is to develop an equation for the SIF in the case of a centre cracked rectangular sheet.

For a centre crack in a sheet of finite width subjected to a uniformly distributed and symmetrically applied partial crack surface pressure some closed form expressions have been suggested but generally without presenting their accuracy, Ref.[3-6]. Hence, the second objective is to investigate the accuracy of some proposed closed form expressions and to develop an equation for the SIF together with a good estimation of its accuracy.

As an extension of the second objective the same geometry is investigated for a loading consisting of two pairs of splitting forces acting symmetrically with respect to the centre of the crack. Again, some closed form expressions can be found in the literature, Ref.[7-8], but in general without any verification of their accuracy.

For the case of a centre crack in a sheet of finite width subjected to a non symmetric, uniformly distributed, partial crack surface pressure only one closed form expression is known to the author and its accuracy is unknown, Ref.[6]. Thus, the third objective is to investigate the accuracy of this formula.

Finally, a centre cracked strip with the crack subjected to a single pair of splitting forces is investigated. This is an important stress intensity factor solution since it may be used directly as a weight function. Closed form expressions exists and the objective is to verify their accuracy.

2 A SIF-Equation for a Centre Crack in a Rectangular Sheet Subjected to a Uniform Uniaxial Stress

The stress intensity factor for a centre crack in a sheet of finite width and height subjected to a uniformly distributed uniaxial stress, σ, acting on the sheet edges parallel to the crack line can be written,

$$K_I = \sigma\sqrt{\pi a}\beta_W f_H \tag{1}$$

The crack length is 2a from tip to tip, β_W is the boundary correction factor for the finite width and f_H is the correction factor for finite height. In particular, according to Ref.[7], β_W can be expressed as,

$$\beta_W = \frac{1.0 - 0.025\alpha^2 + 0.06\alpha^4}{\sqrt{\cos\left(\frac{\pi}{2}\alpha\right)}} \tag{2}$$

where

$$\alpha = \frac{a}{E} \tag{3}$$

is the normalised crack length and E is the distance from the centre of the crack to the edge of the sheet (2E=W, is the total width of the sheet in this case), see Figure 1. The accuracy of Eq.(2) is better than 0.1% for any α according to the reference.

A finite height correction function, f_H, can be obtained from Ref.[1] or Ref.[2] as the ratio between $\beta(\alpha,\gamma)$ (Figure 1 of Ref.[1] or problem 2.5.1 of Ref.[2]) and β_W according to Eq.(2). That is,

$$f_H(\alpha,\gamma) = \frac{\beta(\alpha,\gamma)}{\beta_W(\alpha)} \tag{4}$$

where the parameter

$$\gamma = \frac{E}{H} \tag{5}$$

is the dimensional ratio E/H, and H is the total height of the sheet. The accuracy of $\beta(\alpha,\gamma)$ is better than 1 % according to Ref.[2], where the function is presented in tabular form for $\alpha \le 0.7$ and $0 \le \gamma \le 1.25$.

Assume that the finite height correction function can be written,

$$f_H = 1.0 + B_1\alpha + B_2\alpha^2 \tag{6}$$

where the coefficients B_i are functions of γ according to,

$$\begin{aligned} B_1 &= \gamma(C_1 + C_2\gamma) \\ B_2 &= \gamma(C_3 + C_4\gamma) \end{aligned} \tag{7}$$

The method of least squares fit then yields the coefficients C_i according to,

$C_1 = 0.170398,\ C_2 = 0.43604,$

$$C_3 = \text{-}0.55270,\ C_4 = 1.68076 \tag{8}$$

The maximum absolute relative difference in the product $\beta_W f_H$ with respect to $\beta(\alpha,\gamma)$ is less than 7 % for the ranges $\alpha \leq 0.7$ and $\gamma \leq 1.25$. The average of the absolute relative differences, based on 88 compared values, is 2.05 %. A comparison between the product $\beta_W f_H$ and $\beta(\alpha,\gamma)$ is shown in Figure 2.

3 The SIF for a Centre Crack in a Rectangular Sheet Subjected to a Symmetric Partial Crack Surface Pressure

The stress intensity factor for a centre crack subjected to a symmetrically applied crack surface pressure acting over a part of the crack surfaces can be obtained from the solution above using the approximate weight function technique.

A Cartesian co-ordinate system with its origin at the centre of the crack and its x-direction parallel to the crack is introduced. Furthermore, it is assumed that the crack surfaces are subjected to a uniform pressure, p, acting over the range $[-x_U , x_U]$, see Figure 3. Then the stress intensity factor can be written,

$$K_I = p\sqrt{\pi a}\beta_{EH} \tag{9}$$

where $\beta_{EH} = \beta_{EH}(\alpha, \gamma, \kappa)$ is the load and boundary correction function and,

$$\kappa = x_U/E \tag{10}$$

is the normalised boundary of the crack surface pressure. γ is given by Eq.(5).

The particular weight function technique used herein is the approximate method according to Fett, Ref.[8], in which the crack surface displacement, for the reference solution, is expressed as,

$$u(\rho, \alpha) = \sqrt{8}\frac{\sigma E}{E'}\alpha\beta_W f_H \sum_{v=0}^{\infty} D_v (1-\rho)^{v+1/2} \tag{11}$$

where

$$\rho = x/a \tag{12}$$

and E' is the generalised modulus of elasticity. β_W and f_H are given by Eqs.(2) and (6), respectively. The coefficients D_v are determined from the requirement of self consistency and from crack surface boundary conditions. The approximate weight function, $h(\rho,\alpha)$ is obtained as,

$$h(\rho,\alpha)=\frac{E'}{EK_I}\frac{\partial u}{\partial \alpha}=$$

$$\sqrt{\frac{2}{\pi\alpha E}}\sum_{v=0}^{\infty}\left\{D_v\left(1+\frac{2\alpha}{\beta_W f_H}\frac{\partial(\beta_W f_H)}{\partial\alpha}-2v\right)+2\alpha\frac{\partial D_v}{\partial\alpha}\right\}(1-\rho)^{v+1/2}+ \quad (13)$$

$$\sqrt{\frac{2}{\pi\alpha E}}\sum_{v=0}^{\infty}D_v(2v+1)(1-\rho)^{v-1/2}$$

The stress intensity factor for a symmetric partial crack surface loading with a uniform pressure, p, acting over the interval $[-x_U , x_U]$ is given by the integral,

$$K_I = p\int_{-x_U}^{x_U} h(x,a)dx \quad (14)$$

Combining Eqs.(9), (13) and (14) gives the load and boundary correction function β_{EH} according to,

$$\beta_{EH}(\alpha,\gamma,\kappa)=\frac{1}{\sqrt{\pi a}}\int_{-x_U}^{x_U} h(x,a)dx \quad (15)$$

The function β_{EH} has been computed as function of α for a few values of λ and κ and the result is shown in Figure 4 and Table 1. The accuracy of β_{EH} is assessed for λ=0 in the next section.

4 SIF-Equation for a Centre Cracked Strip Subjected to a Symmetric Partial Crack Surface Pressure

In order to evaluate the accuracy of the stress intensity factor derived in previous section a slightly alternative approach is presented below. Also, the objective is to obtain a closed form expression for the stress intensity factor in the case of γ=0.

For a centre crack in a sheet of finite width a closed form expression for the weight function, developed by Tada, is given in Ref.[7] in terms of the stress intensity factor for the centre crack loaded by a single pair of splitting forces acting at the location x_U along the crack surfaces.

$$K_I^{\substack{A\\B}}(\alpha,\rho)=\frac{P}{\sqrt{\pi a}}\sqrt{\frac{\pi\alpha}{2}\tan\left(\frac{\pi\alpha}{2}\right)}\cdot \left[1+\frac{\left(\pi-\sqrt{\pi^2-4}\right)\sqrt{1-\rho^2}\left(1-\cos\left(\frac{\pi\alpha}{2}\right)\right)}{\sqrt{\pi^2-4}}\right]f^{\substack{A\\B}} \tag{16}$$

where α is given by Eq.(3), ρ is given by Eq.(12) and

$$f^{\substack{A\\B}}=\frac{\cos\left(\frac{\pi\kappa}{2}\right)\left(\sin\left(\frac{\pi\alpha}{2}\right)\pm\sin\left(\frac{\pi\kappa}{2}\right)\right)}{\sin\left(\frac{\pi\alpha}{2}\right)\sqrt{\sin^2\left(\frac{\pi\alpha}{2}\right)-\sin^2\left(\frac{\pi\kappa}{2}\right)}} \tag{17}$$

The plus and minus signs refer to crack tips A and B, respectively. κ is given by Eq.(10). In the case of a symmetric loading (two pairs of splitting forces) with respect to x=0 the factor f becomes,

$$f=\frac{2\cos\left(\frac{\pi\kappa}{2}\right)}{\sqrt{\sin^2\left(\frac{\pi\alpha}{2}\right)-\sin^2\left(\frac{\pi\kappa}{2}\right)}} \tag{18}$$

by superposition. The error in the stress intensity factor is less than 1% for any crack length, α, and location ρ of the splitting forces P according to Ref.[7].

Based on Eq.(16) the weight function can be written directly as,

$$h(\alpha,\rho)=\frac{1}{\sqrt{2E}}\sqrt{\tan\left(\frac{\pi\alpha}{2}\right)}\cdot$$

$$\left[1+\frac{\left(\pi-\sqrt{\pi^2-4}\right)\sqrt{1-\rho^2}\left(1-\cos\left(\frac{\pi\alpha}{2}\right)\right)}{\sqrt{\pi^2-4}}\right]^{A} f^{B} \qquad (19)$$

The integration in Eq.(15) has been carried out using both the approximate weight function (Eq.(13)) with $\gamma = 0$ and the closed form weight function according to Eq.(19) for a few selected ranges $[-x_U , x_U]$ of the partial uniform crack surface pressure. The results of the integrations have been compared and the relative difference between the two solutions is shown in Figure 5. For α less than 0.9 the largest absolute relative difference found is less than 4.5%.

The first term in Eq.(19), for symmetric loading, can be written,

$$h_1(x,a)=\sqrt{\frac{\pi\alpha}{E}}\sqrt{\frac{\tan\left(\frac{\pi\alpha}{2}\right)}{\frac{\pi\alpha}{2}}}\frac{\cos\left(\frac{\pi\kappa}{2}\right)}{\sqrt{\sin^2\left(\frac{\pi\alpha}{2}\right)-\sin^2\left(\frac{\pi\kappa}{2}\right)}} \qquad (20)$$

The second square root, in Eq.(20), can be identified as a finite width correction factor for a strip of infinite height having a centre crack subjected to a remote uniform, uniaxial stress. According to Ref.[7] the accuracy of this finite width correction is better than 5% for any α less than 0.5.

Integrating Eq.(20) with respect to x for a crack surface pressure, p, acting over the range $[-x_U , x_U]$ yields,

$$K_I=\frac{2}{\pi}p\sqrt{\pi a}\sqrt{\frac{\tan\left(\frac{\pi\alpha}{2}\right)}{\frac{\pi\alpha}{2}}}\sin^{-1}\left(\frac{\sin\left(\frac{\pi\kappa}{2}\right)}{\sin\left(\frac{\pi\alpha}{2}\right)}\right) \qquad (21)$$

The contribution from the second term in the weight function, Eq.(19), is in general relatively small. Only for large α-values combined with small ρ-values the contribution from the second term may reach 30 % of the first term in the weight function. Eqs.(1), (2) and (21) suggest that the stress intensity factor for a partially loaded (symmetric) crack in a strip of infinite height can be expressed as,

$$K_I = \frac{2}{\pi} p \sqrt{\pi a} \frac{1.0 + A_1\alpha^2 + A_2\alpha^4}{\sqrt{\cos\left(\frac{\pi}{2}\alpha\right)}} \sin^{-1}\left(\frac{\sin\left(\frac{\pi\kappa}{2}\right)}{\sin\left(\frac{\pi\alpha}{2}\right)}\right) \tag{22}$$

where

$$A_1 = -0.025 + B_1\left(1 - \frac{\kappa}{\alpha}\right)$$
$$A_2 = 0.06 + B_2\left(1 - \frac{\kappa}{\alpha}\right) \tag{23}$$

A least squares fit of Eq.(22) with respect to the solution obtained using the approximate weight function (Eqs.(13-15)) gives the coefficients B_1 and B_2. The numerical values obtained are,

$$B_1 = 0.321549 \; ; \; B_2 = -0.324864 \tag{24}$$

The maximum absolute error in the fit is less than 3.3 % for α-values less than 0.9 as compared to the weight function solution. Eq.(22) is reduced to Eq.(1) for $\gamma=0$ when the crack surface pressure extends over the entire crack length, $\kappa=\alpha$.

Furthermore, Eq.(22), with $A_1=A_2=0$, is identical to the equation suggested by Newman, Jr, Ref.[3]. For that reason the equation is called the “Modified Newman” equation herein.

The same approach, fitting an equation to the results obtained using the approximate weight function technique, was taken by Jiam-Zhong and Wu, Ref.[5]. They derived the following expression for the stress intensity factor,

$$K_I = \frac{2}{\pi} p \sqrt{\pi a} \left[\frac{\pi}{2} - \frac{1}{r_1}\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{\sin\left(\frac{\pi\kappa}{2}\right)}{\sin\left(\frac{\pi\alpha}{2}\right)}\right)\right)\right]\beta_W \tag{25}$$

where

$$r_1 = 1 + 0.24\kappa^2 \tag{26}$$

and β_W is identical to Eq.(2).

Eq.(25) is said to have a high accuracy because of the high accuracy in the weight function used (better than 2 % for $\alpha \leq 0.85$). Compared to the results obtained with the approximate weight function used herein (Eqs.(13-15) the maximum relative difference found for $\alpha \leq 0.85$ is 3.6 %.

4.1 Finite Element Analysis

The finite element (FE) code STRIPE, which uses the hp-version of adaptive technique, has been used to assess the accuracy of some stress intensity factor expressions. Only volume elements are included in the STRIPE code but they may have rather extreme ratios between the side lengths without any practical loss in accuracy. Thus, the elements can be used to model a relatively thin sheet. However, as a result of using volume elements a complete 3-dimensional analysis is obtained. This means that the stress intensity factor varies across the sheet thickness. In the STRIPE model the stress intensity factor is computed for a number of locations along the crack front. Even in the case of a through the thickness crack in a model using only one layer of elements for the thickness direction several stress intensity factor values are obtained for the thickness direction. This causes some difficulties when comparing the results to 2-dimensional analytical solutions. Also, the comparison between the results of the 3-dimensional model and the results of the 2-dimensional analytical solutions becomes less relevant when the length of the crack is close to the thickness of the sheet or the crack surface load is close to the crack front.

Firstly, the accuracy of the FE-model was assessed by computing the stress intensity factor for a centre cracked rectangular plate loaded with a uniformly distributed stress acting on the edges parallel to the crack surfaces. The width to height ratio of the FE-model was 0.25 and the thickness to width ratio was 0.01. The width itself was 100 mm. Totally, the model included 140 20-noded volume elements representing the upper half of the rectangular sheet. The number of elements in the crack plane was 40. It was assumed that Eq.(1) would be a good reference for comparisons since the height correction is almost negligible for γ=0.25 (less than 1.5 %).

Figure 6 shows the result of the STRIPE computation as a relative comparison to Eq.(1). For each crack length investigated, three different values of the stress intensity factor, from the STRIPE computation, were considered. The comparison shows that the value obtained for the plate surfaces gives the best correlation with the equation. The largest relative difference, 0.93 %, occurs for the normalised crack length, a/E, of 0.95. The average values (root mean squared) and the centre values obtained in the STRIPE computation are consistently higher than the corresponding values from the equation, as can be seen in the figure.

Secondly, to further assess the accuracy of the FE model the same centre cracked plate was considered but now subjected to a symmetrically applied partial crack surface pressure extending from $x = -x_U$ to $x = x_U$. The normalised stress intensity factor in this case is obtained as,

$$\beta = \frac{K_I}{K_0} \tag{27}$$

where K_0 is the exact solution for the infinite plate given by,

$$K_0 = \frac{2p\sqrt{\pi a}}{\pi}\sin^{-1}\left(\frac{x_U}{a}\right) \tag{28}$$

K_I is given by Eq.(22) or Eq.(25). For the plate of finite width the combination of Eqs.(22), (27), and (28) yields,

$$\beta = \frac{\left(\beta_W + \frac{\left(1-\frac{\kappa}{\alpha}\right)\left(B_1 + B_2\alpha^2\right)\alpha^2}{\sqrt{\cos\left(\frac{\pi}{2}\alpha\right)}}\right)\sin^{-1}\left(\frac{\sin\left(\frac{\pi\kappa}{2}\right)}{\sin\left(\frac{\pi}{2}\alpha\right)}\right)}{\sin^{-1}\left(\frac{\kappa}{\alpha}\right)} \tag{29}$$

where, as before, α and κ are given by Eqs.(3) and (10), respectively.

β_W is the finite width correction for the remotely loaded centre cracked plate according to Eq.(2). The coefficients B_1 and B_2 are given by Eq.(24).

Similarly, the combination of Eqs.(25), (27) and (28) yields,

$$\beta = \left[\frac{\pi}{2} - \frac{1}{r_1}\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{\sin\left(\frac{\pi\kappa}{2}\right)}{\sin\left(\frac{\pi\alpha}{2}\right)}\right)\right)\right]\frac{\beta_W}{\sin^{-1}\left(\frac{\kappa}{\alpha}\right)} \tag{30}$$

where r_1 is given by Eq.(26).

Figure 7 shows the normalised stress intensity factor as function of the normalised loading position, κ/α, as obtained using the STRIPE model and Eq.(28) for four different crack lengths. Again, it is the values on the plate surface obtained by the STRIPE model that have been used. The normalised loading position, in the figure, represents the boundaries of the partial crack surface pressure. In Figures 8 and 9 comparisons are made between the normalised stress intensity factors computed using STRIPE and those calculated using Eqs.(29) and (30), respectively.

The maximum absolute relative difference in the STRIPE solution is about 5 % as compared to the equations. The curves of relative differences, in figures 8 and 9, show some unevenness. This is due to several factors. Firstly, the STRIPE solution was obtained by applying the crack surface pressure over the range from x=0 to $x=x_U$ and computing K_I^A and K_I^B separately. These two stress intensity factors were then superposed to obtain the total solution. Secondly, in order to obtain a sufficiently large number of loading positions for each crack length with the rather coarse mesh used, the computations had to be repeated with small changes of the finite element mesh in between. Changing the mesh also changes the results of the computations to some extent. In some cases this resulted in several different stress intensity factor values for the same loading position, as can be seen in the figures. However, this gives an indication of the relative errors in the STRIPE solution due to mesh variations.

The comparison above was made to verify the STRIPE model but it also confirms the accuracy of the approximate weight function technique. It seems as the approximate weight function technique gives stress intensity factors with an accuracy better than 5 % for $\alpha \leq 0.9$ and $\gamma < 0.25$.

4.2 Finite Height Correction

In section 3 a load and boundary correction factor, β_{EH}, for the centre cracked rectangular sheet with the crack surfaces partially subjected to a symmetrically applied uniform pressure was obtained in tabular form. In the previous section a closed form expression for the case of the height being greater than two times the width of the sheet was developed. Using this closed form expression as a base a correction factor for the case where the height is smaller than two times the width can be obtained.

A study of β_{EH}, presented in Table 1, suggests a correction factor of the following form,

$$f_H = 1 + \gamma\left(D_1 + D_2\gamma + \frac{(\alpha-\kappa)}{1-\kappa}(D_3 + D_4\gamma)\right) \tag{31}$$

to be applied to the normalised stress intensity factor according to Eq.(29). α, κ and γ are given by Eqs.(3), (9) and (5), respectively. The D_i coefficients are functions of κ according to,

$$\begin{aligned} D_1 &= 0.0894194\left(1-4.330e^{-0.458/\kappa}\right)\\ D_2 &= -0.111202\left(1-21.01e^{-0.518/\kappa}\right)\\ D_3 &= -0.499953\left(1-0.352e^{-0.633/\kappa}\right)\\ D_4 &= 3.024540\left(1-1.262e^{-0.756/\kappa}\right)\end{aligned} \tag{32}$$

where the numerical values have been obtained by using the method of least squares. The complete stress intensity factor may now be written,

$$K_I = \frac{2p\sqrt{\pi a}}{\pi} \sin^{-1}(\kappa/\alpha)\beta f_H \tag{33}$$

where β is given by Eq.(29).

Relative comparisons are made in Figures 10 to 13 between the stress intensity factor according to Eq.(33) above and the stress intensity factor obtained by the approximate weight function method, Eq.(14). The comparison is made for five selected values of γ and four selected values of κ as shown by the figure captions. In general the relative differences are small, a few percent, but increases rapidly for normalised crack lengths above 0.9. The latter is due to numerical difficulties associated with the approximate weight function technique for large crack lengths. Furthermore, for the plate having γ=1 and loaded with κ=0.1 relative differences of up to 8.6 % are obtained for short crack lengths, as can be seen in figure 10.

5 SIF-Equations for a Centre Cracked Strip Subjected to Two Pairs of Splitting

The same centre cracked plate as a in previous sections was subjected to two pairs of splitting forces, symmetrically located with respect to the centre of the crack, see Figure 14. In this case Eq.(16) by Tada applies, which, in the symmetric case, may be written,

$$K_I = \frac{2P}{\sqrt{\pi a}} \sqrt{\frac{\pi\alpha}{2}\tan\left(\frac{\pi\alpha}{2}\right)} \left[\frac{1 + 0.297\sqrt{1-\rho^2}\left(1-\cos\left(\frac{\pi\alpha}{2}\right)\right)}{\sqrt{1 - \frac{\cos^2\left(\frac{\pi\alpha}{2}\right)}{\cos^2\left(\frac{\pi\kappa}{2}\right)}}} \right] \qquad (34)$$

where $\rho = \kappa/\alpha$ has been introduced and the approximate value 0.297 is used instead of $\pi/\sqrt{\pi^2 - 4} - 1$.

A second solution, based on the Force-Balance Method, is suggested by Chen et.al., Ref.[9], which reads,

$$K_I = \frac{2P}{\sqrt{\pi a}} \frac{\pi}{\sqrt{1-\rho^2}\cos^{-1}\left(\frac{2\alpha^2 - \kappa^2 - 1}{1-\kappa^2}\right)} \qquad (35)$$

The stress intensity factor, K_0, for an infinite plate where the crack is subjected to two pairs of splitting forces is exactly,

$$K_0 = \frac{2P}{\sqrt{\pi a}} \frac{1}{\sqrt{1-\rho^2}} \qquad (36)$$

The normalised stress intensity factor, K_I/K_0, as function of the normalised loading position, x_U/a, for four different normalised crack lengths has been compared for the two equations above in Figure 15. The maximum relative difference is 11.6 %. The formula by Chen et. al. results in larger values than the formula by Tada except for the largest crack length where the two solutions intersect each other at a normalised loading position of $\rho \cong 0.4$.

5.1 Finite Element comparison

In order to find out which one is the most accurate of the two equations, studied in the previous section, a finite element analysis was carried out. The same STRIPE model as used in section 4.1 was modified for this task.

In the STRIPE model it is not possible to apply the four point forces representing the two pairs of splitting forces. Instead a concentrated uniform crack surface pressure, acting over 1/2000 of the specimen width and the complete thickness, was applied, see Figure 16. Two approaches, a direct one and one based upon a polynomial fit followed by a superposition were used to obtain the stress intensity factor.

In the direct approach the concentrated crack surface pressure was actually applied in the STRIPE model. This approach has a drawback in difficulties to obtain reliable values for loading positions close to the centre of the crack and close to the crack tip because of the sizes of the finite elements.

The superposition approach made use of the already computed solutions for the uniform crack surface pressure acting over the entire range [0, x_U]. These solutions were fitted very accurately with polynomials, the maximum absolute relative errors being less than 2.7 %, in the range 0.1 $<x_U/a<0.9$. In particular, the average of the absolute relative errors for crack tip A was less than 0.65 %. The maximum relative error for crack tip B was less than 0.53 % in the range $0.025< x_U/a <0.95$.

Using the polynomials the stress intensity factor for a concentrated crack surface pressure was obtained as the difference in polynomial value for $\kappa = \kappa^+$ and $\kappa = \kappa^-$, where,

$$\kappa^- = \frac{x_U}{E} - \frac{1}{2000}; \quad \kappa^+ = \frac{x_U}{E} + \frac{1}{2000} \tag{37}$$

In addition to the two equations in section 4.1, Eqs.(29) and (30), and in order to obtain a direct comparison to the STRIPE model results the modified Newman equation, Eq.(22), was used. Superposition of Eq.(22) with itself for $\kappa = \kappa^+$ and $\kappa = \kappa^-$ yields,

$$K_I = \frac{2p\sqrt{\pi a}}{\pi}\left[\sin^{-1}\left(\frac{\sin\left(\frac{\pi\kappa^+}{2}\right)}{\sin\left(\frac{\pi}{2}\alpha\right)}\right)f_W^+ - \sin^{-1}\left(\frac{\sin\left(\frac{\pi\kappa^-}{2}\right)}{\sin\left(\frac{\pi}{2}\alpha\right)}\right)f_W^-\right] \tag{38}$$

$$f_W^s = \beta_W + \frac{\left(1 - \frac{\kappa^s}{\alpha}\right)\left(B_1 + B_2\alpha^2\right)\alpha^2}{\sqrt{\cos\left(\frac{\pi}{2}\alpha\right)}} \quad ; \quad s = +, - \tag{39}$$

where β_W according to Eq.(2) has been introduced.

The exact stress intensity factor for an infinite plate with two pairs of symmetrically located concentrated crack surface pressures is obtained as,

$$K_0 = \frac{2p\sqrt{\pi a}}{\pi}\left[\sin^{-1}\left(\frac{\kappa^+}{\alpha}\right) - \sin^{-1}\left(\frac{\kappa^-}{\alpha}\right)\right] \tag{40}$$

by superposing Eq.(28) with itself for $\kappa = \kappa^+$ and $\kappa = \kappa^-$.

A study was made regarding the effect of applying the load as a concentrated crack surface pressure instead of as point forces. Substituting the point force P in Eq.(36) by the force created of the concentrated crack surface pressures gives,

$$K_0 = \frac{2p\left(\kappa^+ - \kappa^-\right)\sqrt{\pi a}}{\pi\alpha}\frac{1}{\sqrt{1-\rho^2}} \tag{41}$$

The relative difference between Eq.(40) and Eq.(41), as shown in Figure 17 for some specific crack lengths, indicates that the error made by using concentrated crack surface pressures instead of point forces is very small for loading positions $x_U/a < 0.9$.

Figure 18 shows the normalised stress intensity factor as function of normalised loading position for four different crack lengths as obtained using STRIPE and Eq.(40). The results of both approaches to obtain the stress intensity factor are included in the figure. The results obtained using the polynomial fit and superposition approach have been fitted with new polynomials with coefficients according to the figure caption. Due to the waviness of the curves obtained by the initial polynomial fit and superposition approach the relative errors for the new polynomials are as high as 14.5 %, but the average of the absolute relative errors is less than 4 %. Also, compared to the direct approach the new polynomials have relative errors less than 4.5 %.

Figures 19 to 22 show the relative difference between the results of the three equations (Eqs. 34, 35 and 38) above and the STRIPE results. The results from the formula by Tada and the "Modified Newman" equation are all within 5 % of the STRIPE results. The results obtained using the formula by Chen et. al. show relative differences of up to 14.7 %. Obviously, the equation by Chen et. al. is less accurate than the other two equations.

6 The SIF and a SIF-Equation for a Centre Cracked Strip Subjected to a Non Symmetric Crack Surface Pressure

In section 4.1 the stress intensity factor for a centre cracked strip subjected to a crack surface pressure extending from x=0 to $x=x_U$ was computed using STRIPE. The solutions for crack tips A and B were fitted very accurately with polynomials in section 5.1 before they were superposed. In this section the results before superposing the polynomials are compared to a closed form solution by Chen et. al., Ref.[6], obtained using the force balance method.

The polynomials for the STRIPE solution are written,

$$P_i^{\overset{A}{B}}(x_U/a)=\sum_{j=0} q_{i,j}^{\overset{A}{B}}(x_U/a)^{j-1} \quad ; \qquad i=1,2,3,4 \tag{42}$$

where the coefficients q are presented in Table 2. The upper limit of the summation index j was variable depending on the accuracy of the fit. It should be observed that the polynomials give the stress intensity factor based upon a crack surface pressure of 100 MPa and that they have the same unit as the stress intensity factor ($\mathrm{MPa}\sqrt{\mathrm{mm}}$).

The closed form solution by Chen et. al. is rather lengthy as can be seen in the following. The stress intensity factor is written,

$$\begin{aligned} K_I^A &= p\sqrt{\frac{a}{\pi}}(F_1+F_2)\beta_{Wp}^A \\ K_I^B &= p\sqrt{\frac{a}{\pi}}(F_1-F_2)\beta_{Wp}^B \end{aligned} \tag{43}$$

where the parts in front of the β-factors are the exact stress intensity factors for an infinitely large plate. The functions F_1 and F_2 are given by,

$$\begin{aligned} F_1 &= \sin^{-1}(x_U/a)-\sin^{-1}(x_L/a) \\ F_2 &= \sqrt{1-(x_L/a)^2}-\sqrt{1-(x_U/a)^2} \end{aligned} \tag{44}$$

where x_L and x_U represents the lower and upper boundaries of the crack surface pressure, respectively. In the present case x_L is equal to zero. The boundary correction factors β are given by,

$$\beta_{Wp}^{\overset{A}{B}} = \frac{\left(F_6 - Q_1\right)\left(F_1F_5 \mp 2\alpha F_2F_3\right) \pm 2\left(F_7 - 2Q_2\right)\left(F_1F_3 \mp \alpha F_2F_4\right)}{F_3\left(F_1^2F_5 - 2\alpha^2F_2^2F_4\right)} \quad (45)$$

where

$$F_3 = \sqrt{1-\alpha^2}$$
$$F_4 = \ln\left(\frac{1+F_3}{\alpha}\right) \quad (46a)$$

$$F_5 = F_3 + \alpha^2 F_4$$
$$F_6 = \frac{\pi}{2}\frac{\left(x_U - x_L\right)}{E} \quad (46b)$$
$$F_7 = \frac{\left(x_U + x_L\right)}{E}F_6$$

and

$$Q_1 = \begin{cases} \frac{x_U}{E}\left(\eta_{x_U} + \omega_{x_U}\right) - \frac{x_L}{E}\left(\eta_{x_L} + \omega_{x_L}\right) - \\ \vartheta_{x_U} + \vartheta_{x_L} - \psi_{x_U} + \psi_{x_L} \end{cases}$$
$$Q_2 = \begin{cases} \left(\frac{x_U}{E}\right)^2\left(\eta_{x_U} + \omega_{x_U}\right) - \left(\frac{x_L}{E}\right)^2\left(\eta_{x_L} + \omega_{x_L}\right) - \\ \alpha\frac{F_2F_3}{4} - \frac{1}{4}\left(\vartheta_{x_U} - \vartheta_{x_L} - \psi_{x_U} + \psi_{x_L}\right) \end{cases} \quad (46c)$$

with

$$\eta_{x_U} = \tan^{-1}\left(\frac{F_3}{1+\alpha}\sqrt{\frac{a+x_U}{a-x_U}}\right)$$
$$\eta_{x_L} = \tan^{-1}\left(\frac{F_3}{1+\alpha}\sqrt{\frac{a+x_L}{a-x_L}}\right) \quad (46d)$$

$$\omega_{x_U} = \tan^{-1}\left(\frac{F_3}{1+\alpha}\sqrt{\frac{a-x_U}{a+x_U}}\right)$$
$$\omega_{x_L} = \tan^{-1}\left(\frac{F_3}{1+\alpha}\sqrt{\frac{a-x_L}{a+x_L}}\right) \quad (46e)$$

$$\vartheta_{x_U} = \tan^{-1}\left(\frac{1-\sqrt{1-(x_U/a)^2}-(x_U/E)}{(x_U/a)F_3}\right)$$
$$\vartheta_{x_L} = \tan^{-1}\left(\frac{1-\sqrt{1-(x_L/a)^2}-(x_L/E)}{(x_L/a)F_3}\right) \quad (46f)$$

$$\psi_{x_U} = \tan^{-1}\left(\frac{1-\sqrt{1-(x_U/a)^2}+(x_U/E)}{(x_U/a)F_3}\right)$$
$$\psi_{x_L} = \tan^{-1}\left(\frac{1-\sqrt{1-(x_L/a)^2}+(x_L/E)}{(x_L/a)F_3}\right) \quad (46g)$$

Figures 23 and 24 show the relative difference between the polynomial solution, Eq.(42), and the solution by Chen et. al., Eq.(43), for crack tips A and B, respectively. The relative differences are rather large for both crack tips in combination with normalized loading positions greater than 0.5.

If $x_L = -x_U$ is introduced in Eq.(43) then an equation corresponding to the "Modified Newman" equation is obtained. In Figure 25 a comparison is shown for this case. The relative difference between the two equations increases with increasing crack length. Also, the equation by Chen et. al. results in stress intensity factors smaller than those obtained with the Modified Newman equation.

7 The SIF for a Centre Cracked Strip Subjected to a Single Pair of Splitting Forces

Finally, the centre cracked strip was subjected to a single pair of splitting forces acting at the position x_U along the crack surfaces. Again the equation by Tada, Eq.(16), is directly applicable.

In addition to Eq.(16) a second solution, based upon the force-balance method, by Chen et. al., Ref.[10], is included in the comparison. The solution by Chen et. al. can be written,

$$K_I^A = \frac{P}{2\sqrt{\pi a}} \frac{\pi(1+\rho)}{\sqrt{1-\rho^2}\tan^{-1}\left(\frac{1}{\alpha}\sqrt{\frac{1-\alpha^2}{1-\rho^2}}\right)}$$
$$K_I^B = \frac{P}{2\sqrt{\pi a}} \frac{\pi(1-\rho)}{\sqrt{1-\rho^2}\tan^{-1}\left(\frac{1}{\alpha}\sqrt{\frac{1-\alpha^2}{1-\rho^2}}\right)} \tag{47}$$

The exact stress intensity factor for an infinite plate loaded by a single pair of point forces acting at the position x_U along the crack surfaces is,

$$\begin{cases} K_0^A = \dfrac{P}{\sqrt{\pi a}}\sqrt{\dfrac{a+x_U}{a-x_U}} \\ K_0^B = \dfrac{P}{\sqrt{\pi a}}\sqrt{\dfrac{a-x_U}{a+x_U}} \end{cases} \tag{48}$$

It is easily verified that the normalised stress intensity factor, K_I/K_0 based upon Eqs.(47) and (48) becomes identical for crack tip A and crack tip B if E in α becomes infinite.

The normalised stress intensity factor as function of the normalised loading position for four different crack lengths as obtained using Eqs.(16), (47) and (48) is shown in Figures 26 and 27. The relative differences between the results of the two equations are within 11.5 % for crack tip A but goes up to 157 % for crack tip B.

7.1 STRIPE Solution for the Single Force Pair

The STRIPE model results, already obtained in section 4.1, were used to evaluate the stress intensity factors K_I^A and K_I^B for a concentrated crack surface pressure applied over the range $\kappa = \kappa^-$ to $\kappa = \kappa^+$. The evaluation made use of the polynomial fit described in sections 5.1 and 6 and the principle of superposition.

The exact stress intensity factor for an infinitely large sheet subjected to a concentrated crack surface pressure applied between κ^- and κ^+ becomes,

$$K_0^A = \frac{p\sqrt{\pi a}}{\pi}\left[\sin^{-1}\left(\frac{\kappa^+}{\alpha}\right) - \sin^{-1}\left(\frac{\kappa^-}{\alpha}\right) - \sqrt{1-\left(\frac{\kappa^+}{\alpha}\right)^2} + \sqrt{1-\left(\frac{\kappa^-}{\alpha}\right)^2}\right]$$

$$K_0^B = \frac{p\sqrt{\pi a}}{\pi}\left[\sin^{-1}\left(\frac{\kappa^+}{\alpha}\right) - \sin^{-1}\left(\frac{\kappa^-}{\alpha}\right) + \sqrt{1-\left(\frac{\kappa^+}{\alpha}\right)^2} - \sqrt{1-\left(\frac{\kappa^-}{\alpha}\right)^2}\right] \qquad (49)$$

The STRIPE model results from sections 4.1, 5.1 and 6 can be written

$$K_{I,U}^{\overset{A}{B}}(\alpha_j,\kappa) = P_j^{\overset{A}{B}}(\alpha_j,\kappa) \qquad j = 1,2,3,4 \qquad (50)$$

Subscript U has been added to the stress intensity factors to indicate that they correspond to a uniform crack surface pressure applied over the range x=0 to $x=x_U$. P_j are the fitted polynomials with coefficients according to Table 2.. For the concentrated crack surface pressure the superposition of Eq.(50) with itself results in,

$$K_I^{\overset{A}{B}}(\alpha_j,\kappa) = K_{I,U}^{\overset{A}{B}}(\alpha_j,\kappa^+) - K_{I,U}^{\overset{A}{B}}(\alpha_j,\kappa^-) \qquad j = 1,2,3,4 \qquad (51)$$

The normalised stress intensity factor, K_I/K_0, as function of the normalised loading position as obtained using Eq.(51) and Eq.(49) is shown in Figures 28 and 29 for four different crack lengths. Also shown in the figures are the results for a pair of splitting forces acting at the centre of the crack as obtained by Tada, Ref.[7]. Furthermore, some results obtained directly using the STRIPE-model with the concentrated crack surface pressure actually applied are shown (thus not involving the polynomial fit). Finally, the figures show solid lines representing new polynomials for the normalised stress intensity factors K_I^i/K_0^i. These polynomials were obtained by the method of least squares. Also, the polynomial orders and their coefficients are given in the figures.

The maximum relative difference in the new polynomial fits with respect to Eq.(51) is considerably different for crack tip A and crack tip B. For crack tip A the maximum relative difference is 27.2 % whereas for crack tip B the maximum relative difference is 4.8 %. The rather large relative error for crack tip A is due to the waviness of the curves obtained using Eq.(51). This waviness is partly artificial and due to the evaluation procedure. Comparing the new polynomials to the results obtained using the direct approach gives a maximum relative difference, for crack tip A, of 4.4 %.

From figures 28 and 29 it is clear that the finite width correction (the normalised stress intensity factor) is different for the two crack tips. Thus, the proposed equation by Chen et. al. cannot be correct.

Figures 30 and 31 show the relative difference between the stress intensity factors according to Tada and those obtained using the STRIPE results combined with polynomial fits for crack tips A and B, respectively. For crack tip A the relative difference is less than 15 %, whereas, for crack tip B the relative difference becomes greater than 75 % for long cracks and loading positions close to the crack tip.

Figures 32 and 33 show the comparison between the results of Chen et. al. and STRIPE. In this case the relative differences are within 23 % for crack tip A and exceed 40 % for crack tip B.

Asymptotically, the equation by Tada gives the exact stress intensity factor as the width of the plate becomes infinite. Also, for the splitting forces acting in the centre of the crack the equation results in the formula proposed for this case in Ref.[7]. The first term in Eq.(16) represents the exact solution to a periodic array of collinear cracks, each one subjected to a single pair of splitting forces. Thus, Eq.(16) is obtained as an interpolation between the exact solution for collinear cracks and the solution for a splitting force in the centre of the crack. This interpolation does not seem to consider crack tip B correctly.

The normalised stress intensity factors obtained using STRIPE show a good correlation with the results obtained using the formula for the splitting forces acting in the centre of the crack.

8 Conclusion

A stress intensity factor equation has been developed for the centre cracked rectangular sheet subjected to a uniformly distributed stress acting on the edges parallel to the crack line. In general, the equation has an accuracy better than 4 %.

A stress intensity factor equation has been developed for the centre cracked strip having a crack subjected to a symmetrically applied and uniformly distributed partial crack surface pressure. The equation represents a least squares fit with respect to results obtained using the approximate weight function technique. Furthermore, the results of the equation correlated very well with some finite element results obtained using the STRIPE code. The accuracy of the formula is estimated to be better than 5 %. Since, the formula has many similarities with an equation proposed by Newman it was called the "Modified Newman" equation.

A finite height correction factor for the centre cracked strip subjected to a partial crack surface pressure has been developed. Stress intensity factors obtained using the approximate weight function technique were in general fitted very well using the equation for the centre cracked strip multiplied with the finite height correction factor. The accuracy is assumed to be better than 5 % for a very large range of width to height ratios. However, for ratios greater than 1.5 the accuracy is not so good in combination with small cracks.

Two stress intensity factor equations, found in the literature, for a centre cracked strip subjected to two pairs of splitting forces applied symmetrically with respect to the centre of the strip were verified using the finite element code STRIPE. Because of point forces not being possible to apply in the finite element model two pairs of concentrated crack surface pressures were used instead. The technique of using concentrated crack surface pressures was verified by comparing the exact solutions for an infinite plate with a crack subjected to the two loading alternatives. Also, the STRIPE result was compared to the result of the Modified Newman equation. The latter was achieved by using the principle of superposition. It was found that the equation proposed by Tada, which is said to have an accuracy better than 1 %, yielded results within 5 % of the finite element results. Also, the Modified Newman equation gave results within 5 % of the finite element results. The third equation, proposed by Chen et. al., based on the force balance method was less accurate and showed relative differences of up to 14.7 % as compared to the finite element results.

Non symmetric loading with respect to the centre of the strip has been studied in two cases. Firstly, a uniformly distributed crack surface pressure acting from the centre of the crack to a co-ordinate x_U on the crack surface was investigated. A rather lengthy formula, based upon the force balance method, was compared to some results obtained using the finite element model developed. Rather large relative differences were found for both crack tips. Using the principle of superposition stress intensity factors calculated from the formula were compared to corresponding factors obtained by the Modified Newman equation. Also in this case rather large relative differences were found.

The second non symmetric loading condition studied was a single pair of splitting forces acting at the position x_U on the crack surfaces. Firstly, two equations from the literature were compared for this loading case. For the crack tip nearest the point forces the relative difference in stress intensity factors for the two equations was less than 11.5 %. However, for the crack tip away from the point forces relative differences of more than 150 % were found.

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a/E	E/H=0	E/H=0.25	E/H=0.5	E/H=0.75	E/H=1.0
0.11	0.733	0.7401	0.7554	0.7791	0.8111
0.128	0.5799	0.5872	0.6039	0.6297	0.6648
0.146	0.4911	0.4987	0.5162	0.5437	0.5813
0.163	0.4299	0.4375	0.4558	0.4848	0.5245
0.181	0.3843	0.392	0.411	0.4413	0.4831
0.199	0.3489	0.3566	0.3762	0.4078	0.4517
0.217	0.3205	0.3282	0.3485	0.3814	0.4272
0.235	0.2973	0.305	0.3258	0.3601	0.4079
0.252	0.2779	0.2856	0.3071	0.3426	0.3925
0.27	0.2616	0.2693	0.2914	0.3283	0.3802
0.288	0.2477	0.2554	0.2782	0.3164	0.3704
0.306	0.2358	0.2434	0.2669	0.3065	0.3626
0.324	0.2254	0.2331	0.2573	0.2982	0.3566
0.341	0.2165	0.2242	0.249	0.2915	0.3521
0.359	0.2087	0.2164	0.242	0.2859	0.3489
0.377	0.2019	0.2096	0.2359	0.2815	0.3469
0.395	0.196	0.2037	0.2308	0.278	0.3459
0.413	0.1908	0.1986	0.2265	0.2754	0.3459
0.43	0.1863	0.1941	0.2229	0.2735	0.3468
0.448	0.1825	0.1903	0.22	0.2724	0.3485
0.466	0.1792	0.187	0.2177	0.272	0.3511
0.484	0.1764	0.1843	0.2159	0.2723	0.3545
0.502	0.1741	0.182	0.2147	0.2732	0.3586
0.519	0.1722	0.1802	0.214	0.2747	0.3636
0.537	0.1708	0.1788	0.2138	0.2769	0.3693
0.555	0.1698	0.1779	0.2141	0.2797	0.3759
0.573	0.1691	0.1774	0.2149	0.2831	0.3833
0.591	0.1689	0.1772	0.2161	0.2872	0.3917
0.608	0.1691	0.1775	0.2179	0.2919	0.401
0.626	0.1696	0.1782	0.2202	0.2974	0.4114
0.644	0.1706	0.1792	0.223	0.3037	0.4228
0.662	0.172	0.1808	0.2264	0.3108	0.4355
0.68	0.1738	0.1828	0.2304	0.3189	0.4496
0.697	0.1761	0.1853	0.2351	0.3279	0.4653
0.715	0.1789	0.1883	0.2406	0.3381	0.4826
0.733	0.1823	0.1919	0.2469	0.3497	0.502
0.751	0.1863	0.1962	0.2541	0.3627	0.5237
0.769	0.191	0.2012	0.2624	0.3774	0.5481
0.786	0.1965	0.2071	0.272	0.3942	0.5757
0.804	0.2029	0.2139	0.2831	0.4135	0.6071
0.822	0.2104	0.2219	0.2959	0.4356	0.6432
0.84	0.2192	0.2313	0.3109	0.4615	0.6852
0.858	0.2294	0.2422	0.3284	0.4918	0.7345
0.875	0.2414	0.2551	0.3493	0.528	0.7934
0.893	0.2553	0.27	0.374	0.5715	0.8646
0.911	0.2705	0.2866	0.4028	0.6235	0.951
0.929	0.2898	0.3078	0.4408	0.6936	1.0682

Table 1. Normalized stress intensity factor K_I/K_0 for a centre crack in a rectangular sheet subjected to a partial crack surface pressure applied over $-0.1 \leq x/E \leq 0.1$. $K_0 = p\sqrt{\pi a}$.

a/E	E/H=0.0	E/H=0.25	E/H=0.5	E/H=0.75	E/H=1.00
0.26	0.8645	0.881	0.9267	1.0018	1.1062
0.275	0.7731	0.79	0.8381	0.9176	1.0285
0.29	0.7127	0.7299	0.7802	0.8637	0.9806
0.304	0.6668	0.6843	0.7366	0.8239	0.9465
0.319	0.63	0.6477	0.7019	0.7929	0.9211
0.334	0.5995	0.6173	0.6734	0.7681	0.9018
0.349	0.5738	0.5917	0.6496	0.7479	0.8871
0.364	0.5517	0.5698	0.6295	0.7314	0.8761
0.378	0.5326	0.5508	0.6124	0.7179	0.8682
0.393	0.516	0.5344	0.5977	0.707	0.8629
0.408	0.5015	0.52	0.5852	0.6982	0.8599
0.423	0.4889	0.5074	0.5746	0.6914	0.8589
0.438	0.4778	0.4964	0.5655	0.6862	0.8598
0.452	0.4681	0.4868	0.5579	0.6827	0.8625
0.467	0.4596	0.4785	0.5516	0.6806	0.8667
0.482	0.4523	0.4713	0.5466	0.6798	0.8725
0.497	0.446	0.4651	0.5426	0.6803	0.8799
0.512	0.4407	0.4599	0.5397	0.682	0.8887
0.526	0.4363	0.4556	0.5379	0.685	0.899
0.541	0.4327	0.4522	0.5369	0.6891	0.9107
0.556	0.4299	0.4496	0.5369	0.6943	0.9239
0.571	0.4279	0.4478	0.5379	0.7007	0.9387
0.586	0.4267	0.4468	0.5397	0.7083	0.9551
0.6	0.4262	0.4465	0.5425	0.7172	0.9731
0.615	0.4265	0.447	0.5462	0.7272	0.9929
0.63	0.4275	0.4483	0.5508	0.7386	1.0145
0.645	0.4292	0.4503	0.5565	0.7514	1.0381
0.66	0.4317	0.4531	0.5631	0.7657	1.0638
0.674	0.435	0.4568	0.5709	0.7815	1.0919
0.689	0.4392	0.4613	0.5798	0.7991	1.1226
0.704	0.4443	0.4668	0.59	0.8186	1.156
0.719	0.4502	0.4732	0.6015	0.8401	1.1925
0.734	0.4573	0.4808	0.6146	0.864	1.2326
0.748	0.4654	0.4895	0.6293	0.8904	1.2766
0.763	0.4747	0.4994	0.6458	0.9197	1.3251
0.778	0.4854	0.5108	0.6644	0.9524	1.3787
0.793	0.4977	0.5239	0.6855	0.9889	1.4384
0.808	0.5117	0.5388	0.7093	1.03	1.5052
0.822	0.5276	0.5557	0.7362	1.0762	1.5801
0.837	0.5458	0.5751	0.7668	1.1286	1.665
0.852	0.5668	0.5975	0.8022	1.189	1.7625
0.867	0.5912	0.6235	0.8433	1.2592	1.8758
0.882	0.6182	0.6524	0.8897	1.3392	2.0056
0.896	0.6481	0.6846	0.9427	1.4319	2.1572
0.911	0.6816	0.7208	1.0043	1.5417	2.3382
0.926	0.7187	0.7613	1.0768	1.675	2.561

Table 1. Normalized stress intensity factor K_I/K_0 for a centre crack in a rectangular sheet subjected to a partial crack surface pressure applied over –0.25=<x/E=<0.25. $K_0 = p\sqrt{\pi a}$.

a/E	E/H=0.0	E/H=0.25	E/H=0.5	E/H=0.75	E/H=1.00
0.343	0.9225	0.944	1.012	1.1266	1.2879
0.356	0.8513	0.8733	0.9443	1.0644	1.2337
0.37	0.8028	0.8252	0.8989	1.0242	1.2013
0.383	0.7655	0.7881	0.8645	0.9948	1.1794
0.396	0.7351	0.758	0.837	0.9723	1.1643
0.409	0.7097	0.7329	0.8144	0.9546	1.1541
0.422	0.6881	0.7115	0.7955	0.9407	1.1477
0.435	0.6695	0.6931	0.7797	0.9298	1.1444
0.448	0.6534	0.6773	0.7663	0.9215	1.1437
0.462	0.6394	0.6635	0.7551	0.9154	1.1454
0.475	0.6273	0.6515	0.7458	0.9113	1.1493
0.488	0.6167	0.6411	0.7381	0.909	1.155
0.501	0.6075	0.6322	0.7319	0.9082	1.1627
0.514	0.5997	0.6245	0.7271	0.909	1.1721
0.527	0.593	0.618	0.7235	0.9113	1.1832
0.54	0.5874	0.6126	0.7211	0.9149	1.196
0.553	0.5828	0.6083	0.7199	0.9199	1.2104
0.567	0.5792	0.6049	0.7198	0.9263	1.2266
0.58	0.5766	0.6025	0.7207	0.9339	1.2444
0.593	0.5748	0.601	0.7227	0.9429	1.2641
0.606	0.5738	0.6003	0.7257	0.9533	1.2855
0.619	0.5738	0.6006	0.7298	0.965	1.3089
0.632	0.5746	0.6017	0.735	0.9782	1.3343
0.645	0.5762	0.6037	0.7413	0.9929	1.3617
0.659	0.5787	0.6065	0.7486	1.0092	1.3915
0.672	0.5821	0.6103	0.7572	1.0272	1.4236
0.685	0.5865	0.6151	0.767	1.0469	1.4584
0.698	0.5917	0.6208	0.7781	1.0686	1.496
0.711	0.598	0.6276	0.7907	1.0924	1.5367
0.724	0.6053	0.6355	0.8047	1.1185	1.5809
0.737	0.6138	0.6446	0.8204	1.147	1.6288
0.75	0.6236	0.655	0.8379	1.1785	1.681
0.764	0.6346	0.6668	0.8573	1.2129	1.7379
0.777	0.6471	0.6801	0.879	1.2508	1.8002
0.79	0.6613	0.6953	0.9033	1.2929	1.8688
0.803	0.6772	0.7121	0.9302	1.3392	1.9441
0.816	0.6952	0.7313	0.9604	1.3909	2.0277
0.829	0.7158	0.7532	0.9947	1.4491	2.1215
0.842	0.7386	0.7774	1.0327	1.5137	2.2256
0.856	0.7645	0.805	1.0759	1.5869	2.3434
0.869	0.7952	0.8376	1.1266	1.6724	2.4805
0.882	0.8276	0.8722	1.1817	1.7666	2.6328
0.895	0.8639	0.9111	1.2444	1.8749	2.8085
0.908	0.9119	0.9624	1.3257	2.0135	3.0317
0.921	0.94	0.9936	1.3877	2.1341	3.2384

Table 1. Normalized stress intensity factor K_I/K_0 for a centre crack in a rectangular sheet subjected to a partial crack surface pressure applied over –0.333=<x/E=<0.333 $K_0 = p\sqrt{\pi a}$.

a/E	E/H=0.00	E/H=0.25	E/H=0.50	E/H=0.75	E/H=1.00
0.51	1.0686	1.1008	1.2312	1.46	1.7872
0.52	1.0271	1.0598	1.1942	1.4305	1.7688
0.53	0.998	1.0311	1.1695	1.4133	1.7627
0.539	0.9756	1.0092	1.1514	1.4026	1.7631
0.549	0.9576	0.9916	1.1377	1.3964	1.7682
0.559	0.9428	0.9772	1.1272	1.3936	1.7767
0.569	0.9305	0.9653	1.1194	1.3934	1.7881
0.579	0.9204	0.9556	1.1136	1.3955	1.802
0.588	0.912	0.9476	1.1098	1.3997	1.8183
0.598	0.9052	0.9412	1.1076	1.4057	1.8366
0.608	0.8998	0.9361	1.1069	1.4134	1.857
0.618	0.8956	0.9324	1.1075	1.4227	1.8793
0.628	0.8927	0.9298	1.1096	1.4336	1.9036
0.637	0.8908	0.9284	1.1128	1.4461	1.9299
0.647	0.89	0.9281	1.1173	1.4601	1.9582
0.657	0.8902	0.9287	1.123	1.4756	1.9885
0.667	0.8915	0.9304	1.1299	1.4927	2.021
0.677	0.8937	0.9331	1.138	1.5114	2.0556
0.686	0.8969	0.9368	1.1474	1.5318	2.0925
0.696	0.901	0.9415	1.158	1.5538	2.1319
0.706	0.9062	0.9473	1.1698	1.5778	2.1738
0.716	0.9124	0.9541	1.183	1.6035	2.2185
0.726	0.9196	0.9619	1.1977	1.6313	2.2661
0.735	0.9279	0.9709	1.2137	1.6613	2.3168
0.745	0.9374	0.9811	1.2314	1.6935	2.3709
0.755	0.948	0.9925	1.2507	1.7283	2.4287
0.765	0.96	1.0052	1.2719	1.7657	2.4905
0.775	0.9732	1.0193	1.2949	1.806	2.5567
0.784	0.9879	1.035	1.3201	1.8496	2.6278
0.794	1.0042	1.0523	1.3475	1.8967	2.7042
0.804	1.0222	1.0714	1.3774	1.9477	2.7866
0.814	1.0421	1.0924	1.4102	2.003	2.8756
0.824	1.064	1.1156	1.446	2.0633	2.9722
0.833	1.0881	1.1412	1.4853	2.1289	3.0772
0.843	1.1149	1.1695	1.5285	2.2009	3.1919
0.853	1.1443	1.2006	1.576	2.2798	3.3175
0.863	1.177	1.2353	1.6287	2.3672	3.4563
0.873	1.213	1.2735	1.6868	2.4634	3.6091
0.882	1.2532	1.316	1.7516	2.5709	3.7799
0.892	1.2967	1.3622	1.8226	2.6895	3.9687
0.902	1.3448	1.4133	1.9019	2.8226	4.1815
0.912	1.3918	1.4636	1.9832	2.9628	4.4087
0.922	1.4379	1.5134	2.0681	3.1146	4.6588

Table 1. Normalized stress intensity factor K_I/K_0 for a centre crack in a rectangular sheet subjected to a partial crack surface pressure applied over $-0.5 =< x/E =< 0.5$. $K_0 = p\sqrt{\pi a}$.

Crack tip A	$q_{i,j}$			
j=	**i=1**	**i=2**	**i=3**	**i=4**
1	-0.1753	-1.6049	0.6798	-0.6096
	244.8888	523.9402	211.1279	1006.5613
3	-992.4958	-4322.8127	7243.6957	-2502.2250
4	7192.9019	30157.8244	-62689.9077	21476.5445
5	-23463.1345	-94590.7155	265855.5007	-73311.0860
6	40570.4335	151252.2765	-605033.3511	125922.8715
7	-35158.9224	-119446.0956	759579.7803	-105866.5806
8	12028.8495	37064.5296	-495885.9895	34706.1028
9			131619.5600	
Crack tip B				
j=	**i=1**	**i=2**	**i=3**	**i=4**
1	0.0250	0.0482	0.0885	0.1103
2	181.3532	297.3493	477.8228	874.5063
3	-76.8574	-93.4088	-141.7923	-270.5775
4	-7.7603	-93.8718	-135.7170	-258.1570
5	33.2575	151.0280	199.5988	335.0025
6	-24.1625	-81.6333	-109.0848	-168.1017

Table 2. Polynomial coefficients for the stress intensity factor at crack tips A and B of a strip subjected to a uniform crack surface pressure between $0<x<x_U$.

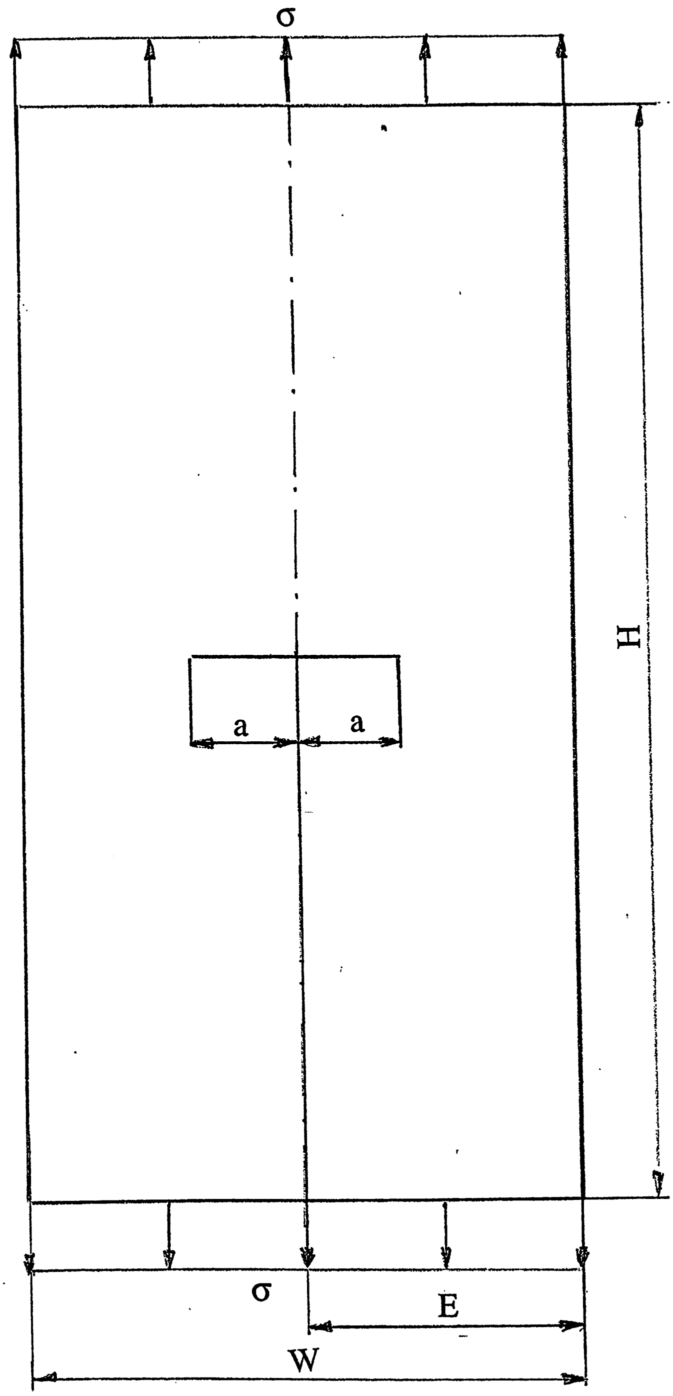

Figure 1. Dimensions and loading of centre cracked rectangular plate.

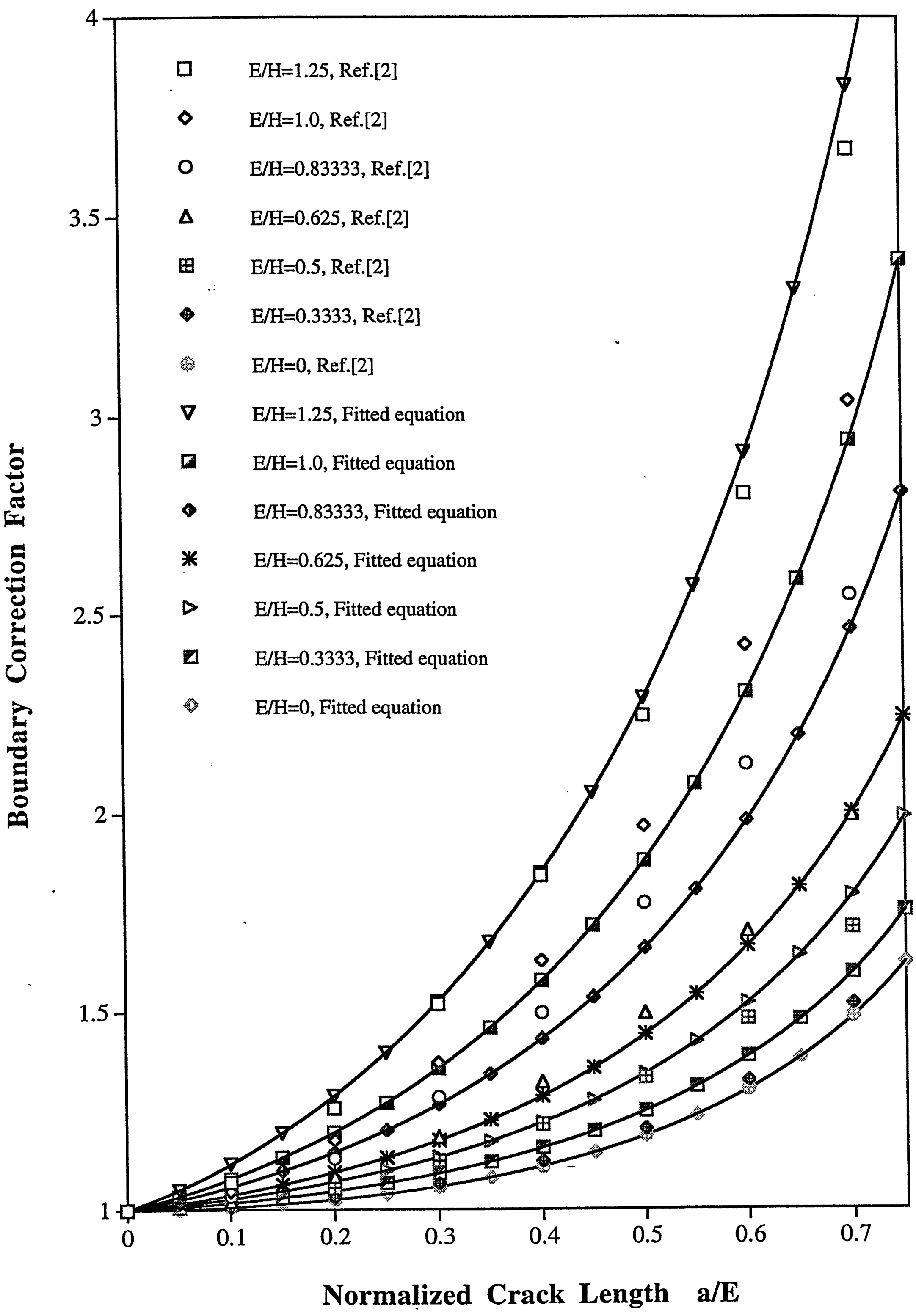

Figure 2. Comparison of boundary correction factors for a centre cracked plate of finite width and height subjected to a uniformly distributed stress acting on the edges parallel to the crack line.

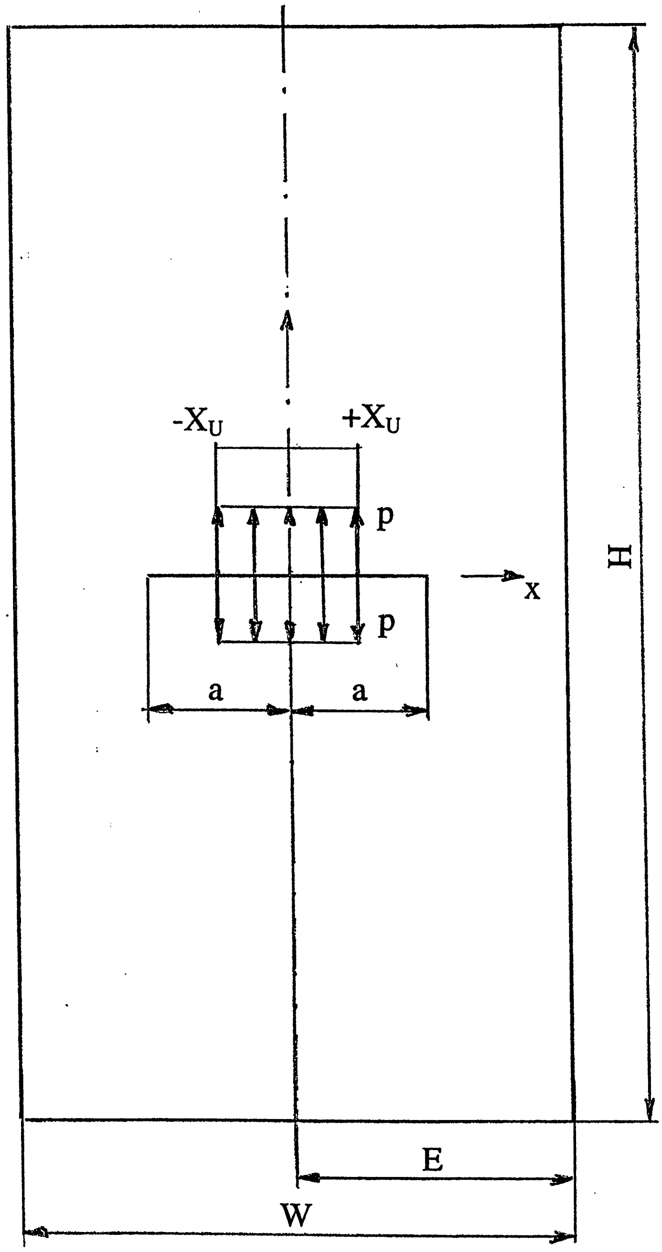

Figure 3. Centre cracked rectangular plate subjected to a symmetrically applied uniform partial crack surface pressure.

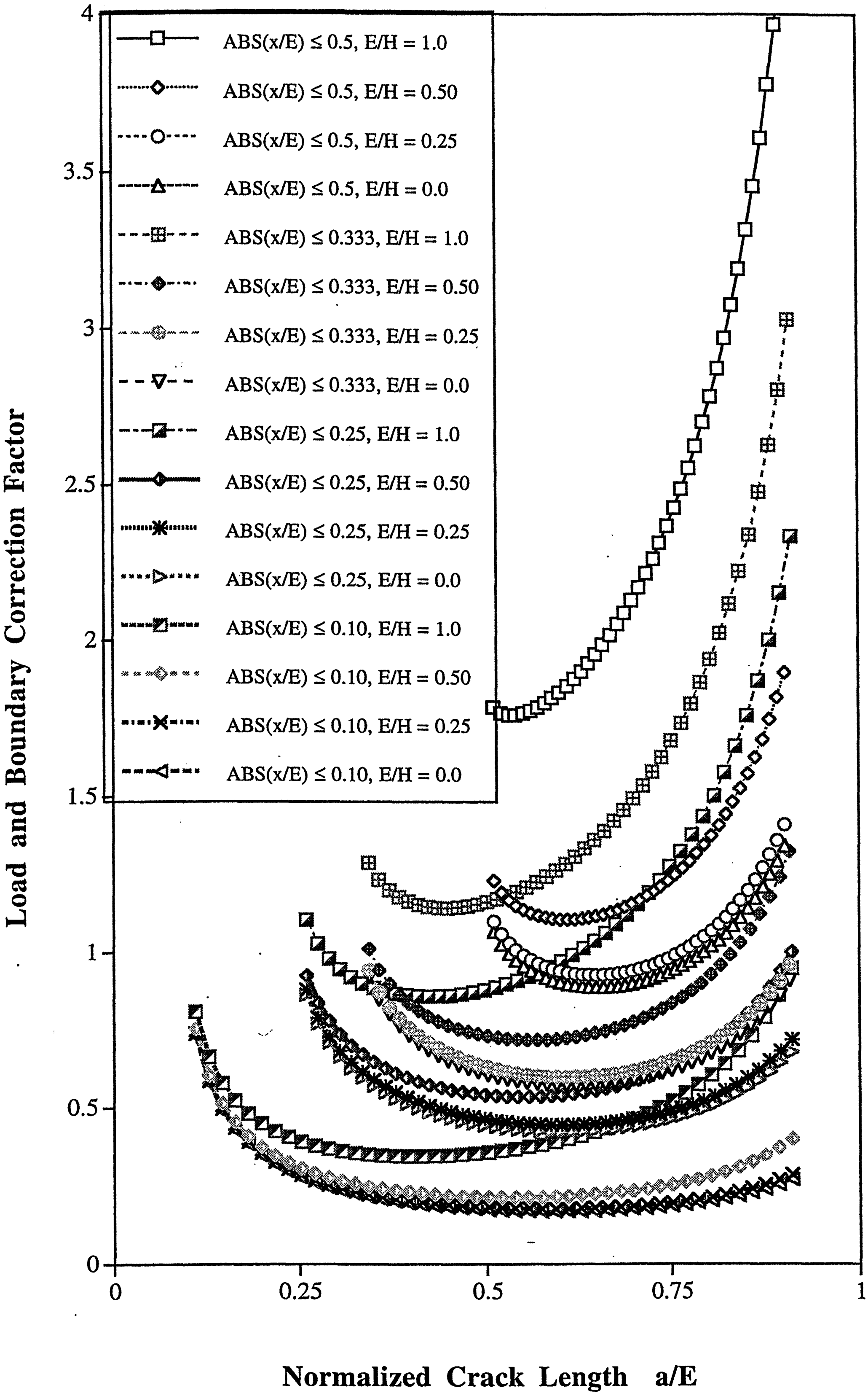

Figure 4. The load and boundary correction factor for a centre cracked rectangular plate subjected to a symmetrically applied uniformly distributed partial crack surface pressure.

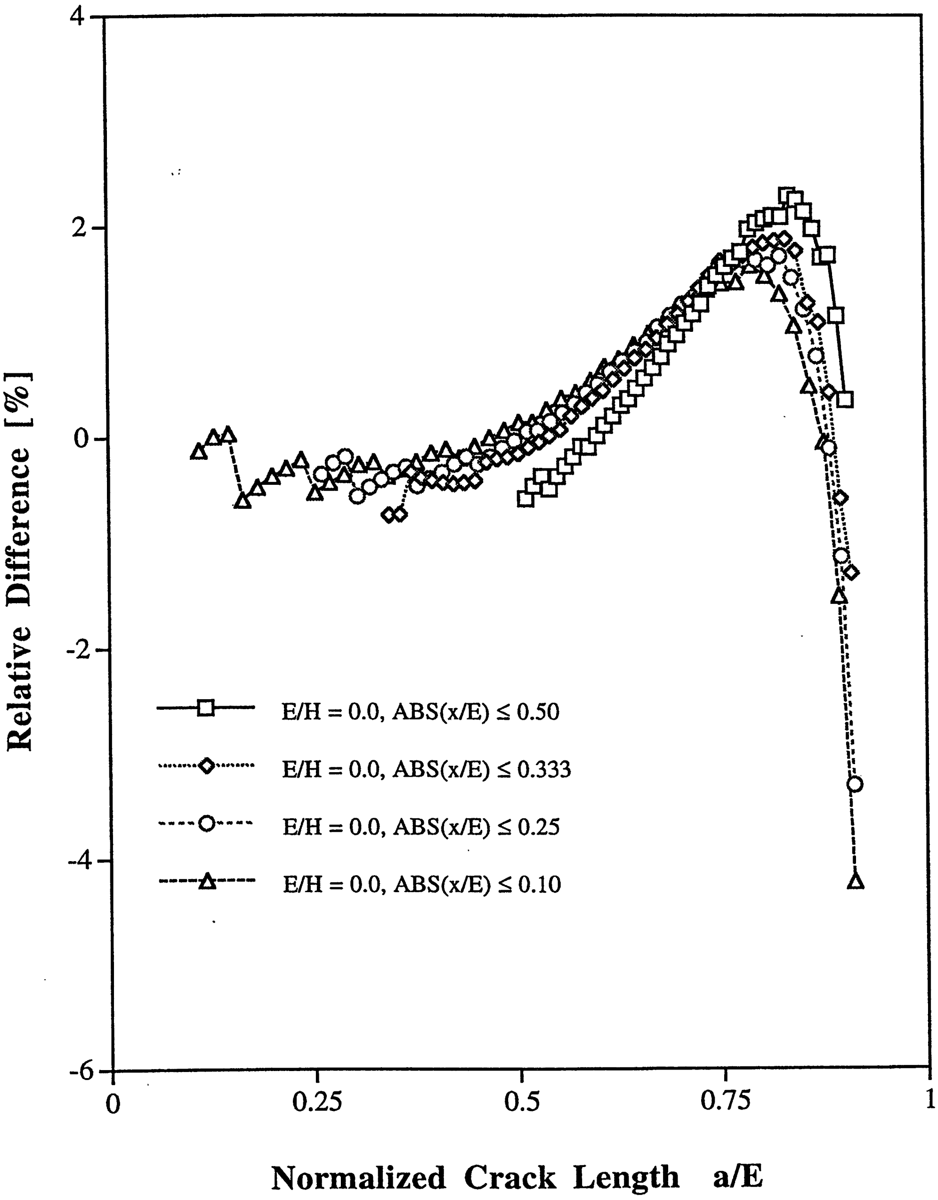

Figure 5. Relative difference between the stress intensity factors obtained using the approximate weight function method and the integration of the weight function proposed by Tada.

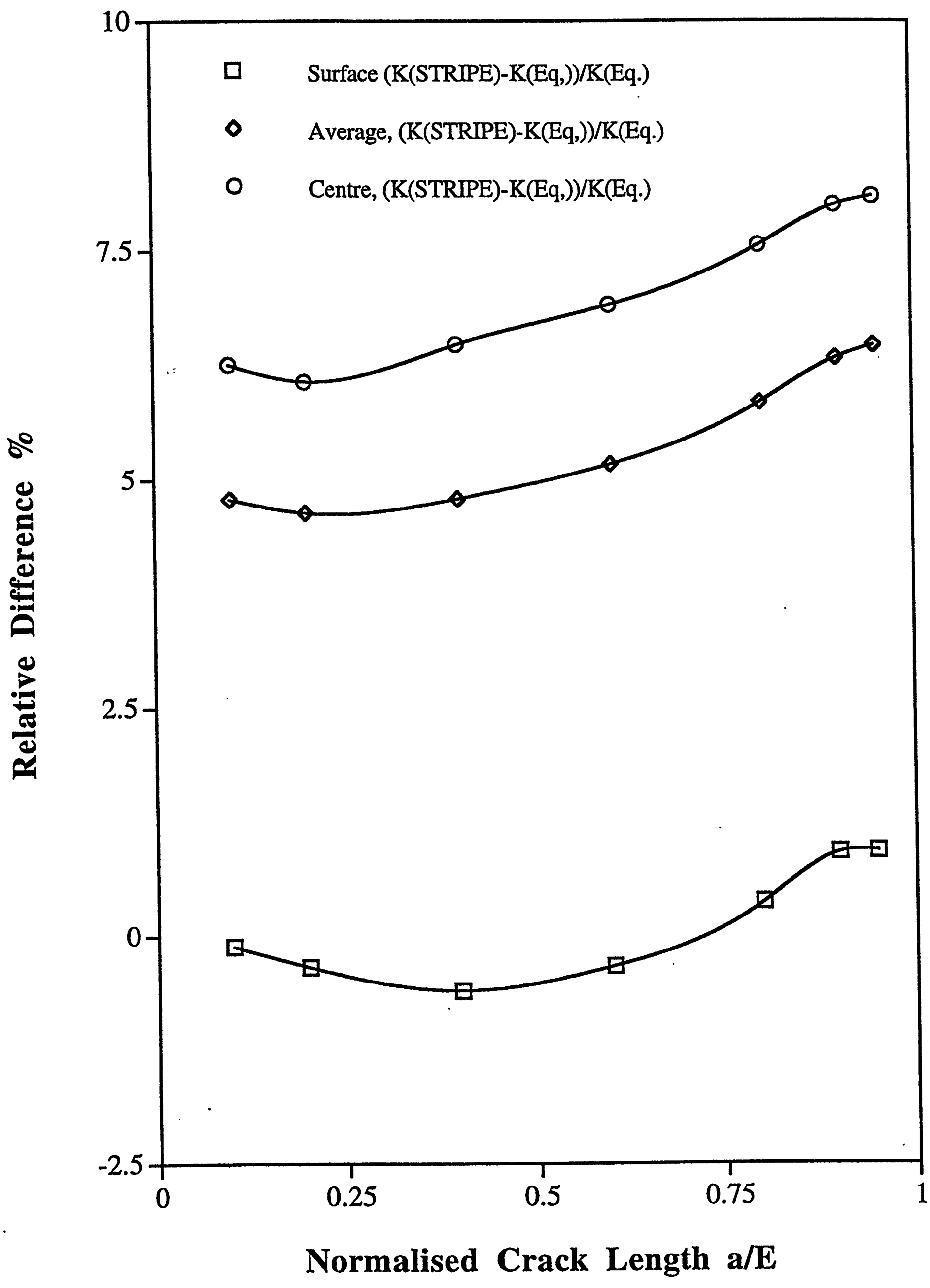

Figure 6. Comparison of the fully 3D solution by STRIPE to the proposed SIF-equation for a centre cracked rectangular plate subjected to a uniformly distributed stress on the edges parallel to the crack.

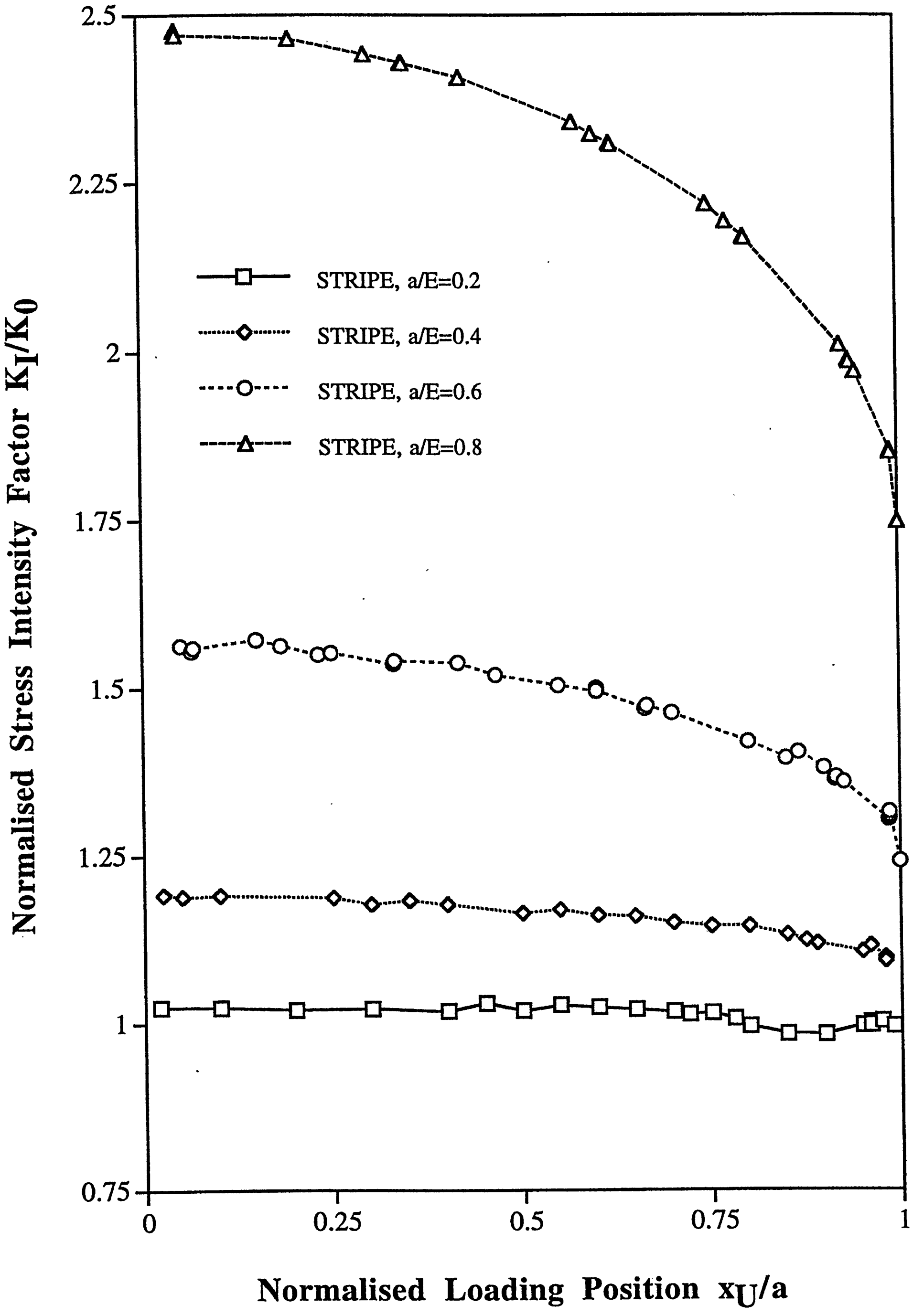

Figure 7. Normalised stress intensity factor, for a centre cracked strip, as function of the limits, $[-x_U, x_U]$, of a uniformly distributed partial crack surface pressure and the crack length.

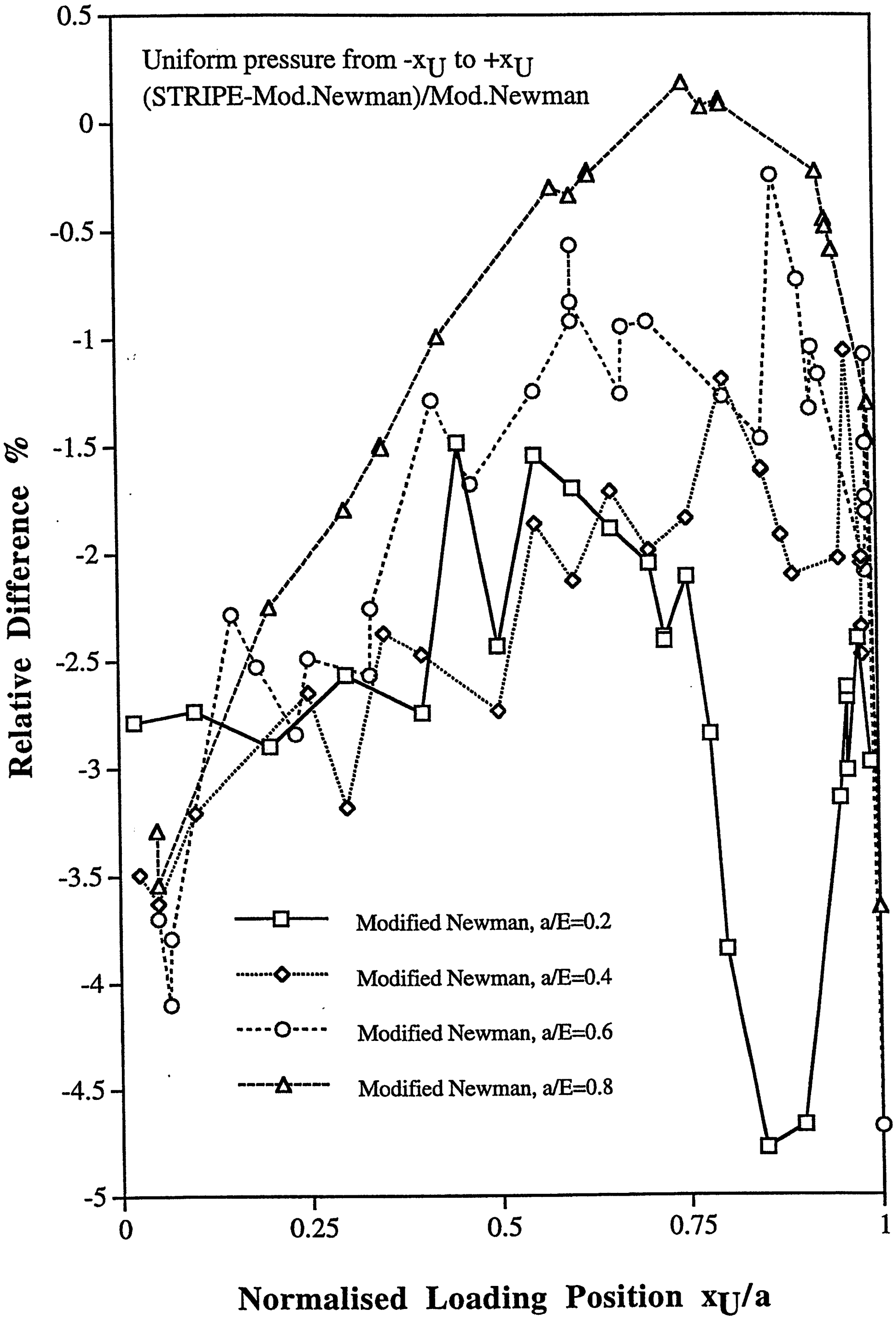

Figure 8. Relative comparison of finite element results to results of the proposed "Modified Newman" equation for a centre cracked strip subjected to a uniformly distributed, symmetrically applied , partial crack surface pressure $[-x_U, x_U]$.

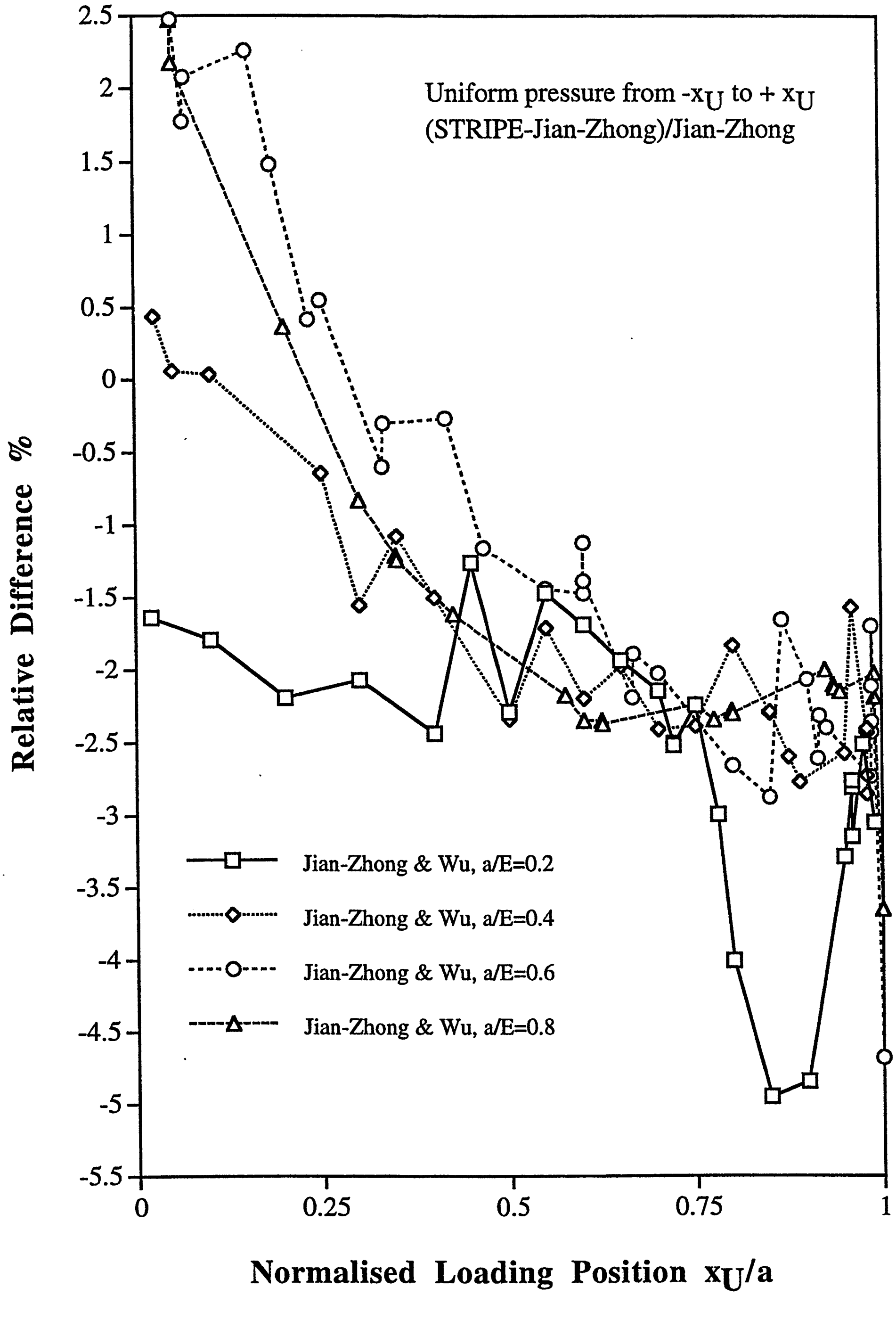

Figure 9. Relative comparison of finite element results to results of the equation proposed by Jian-Zhong and Wu for a centre cracked strip subjected to a uniformly distributed, symmetrically applied , partial crack surface pressure $[-x_U, x_U]$.

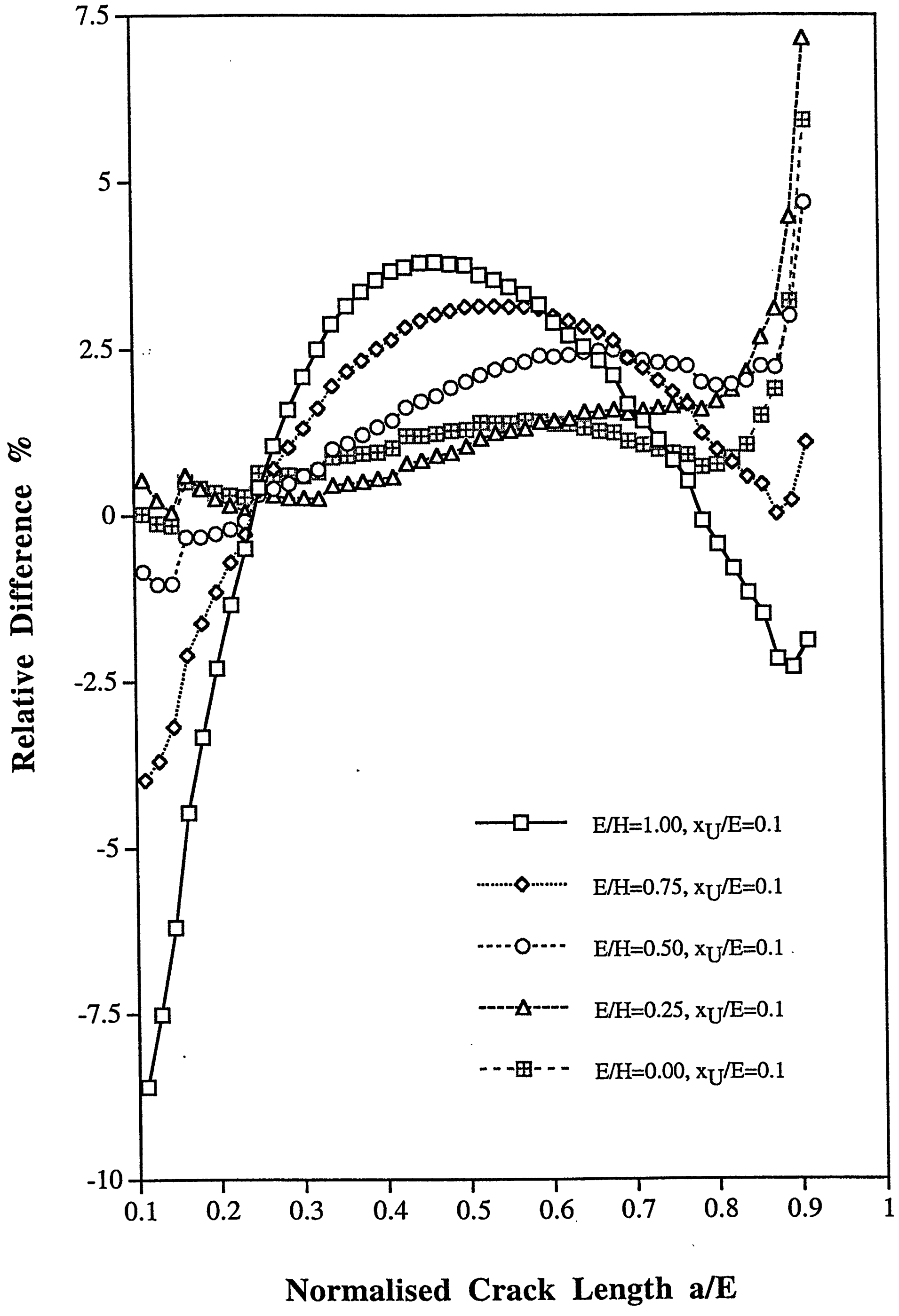

Figure 10. Relative differences between the stress intensity factors according to the proposed equation and the results of the weight function solution for a rectangular sheet subjected to a partial crack surface pressure. $x_U/E = 0.1$.

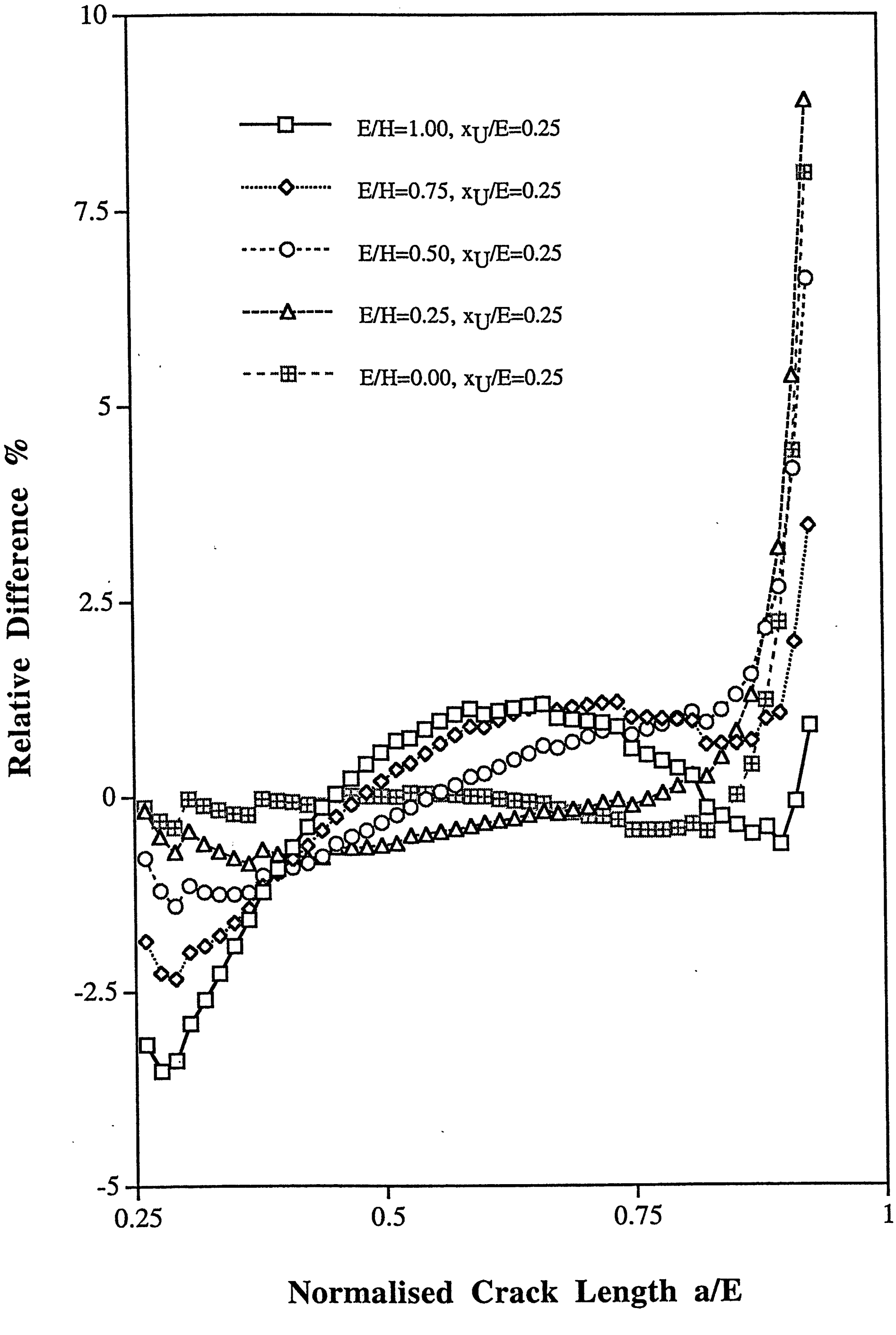

Figure 11. Relative differences between the stress intensity factors according to the proposed equation and the results of the weight function solution for a rectangular sheet subjected to a partial crack surface pressure. $x_U/E = 0.25$.

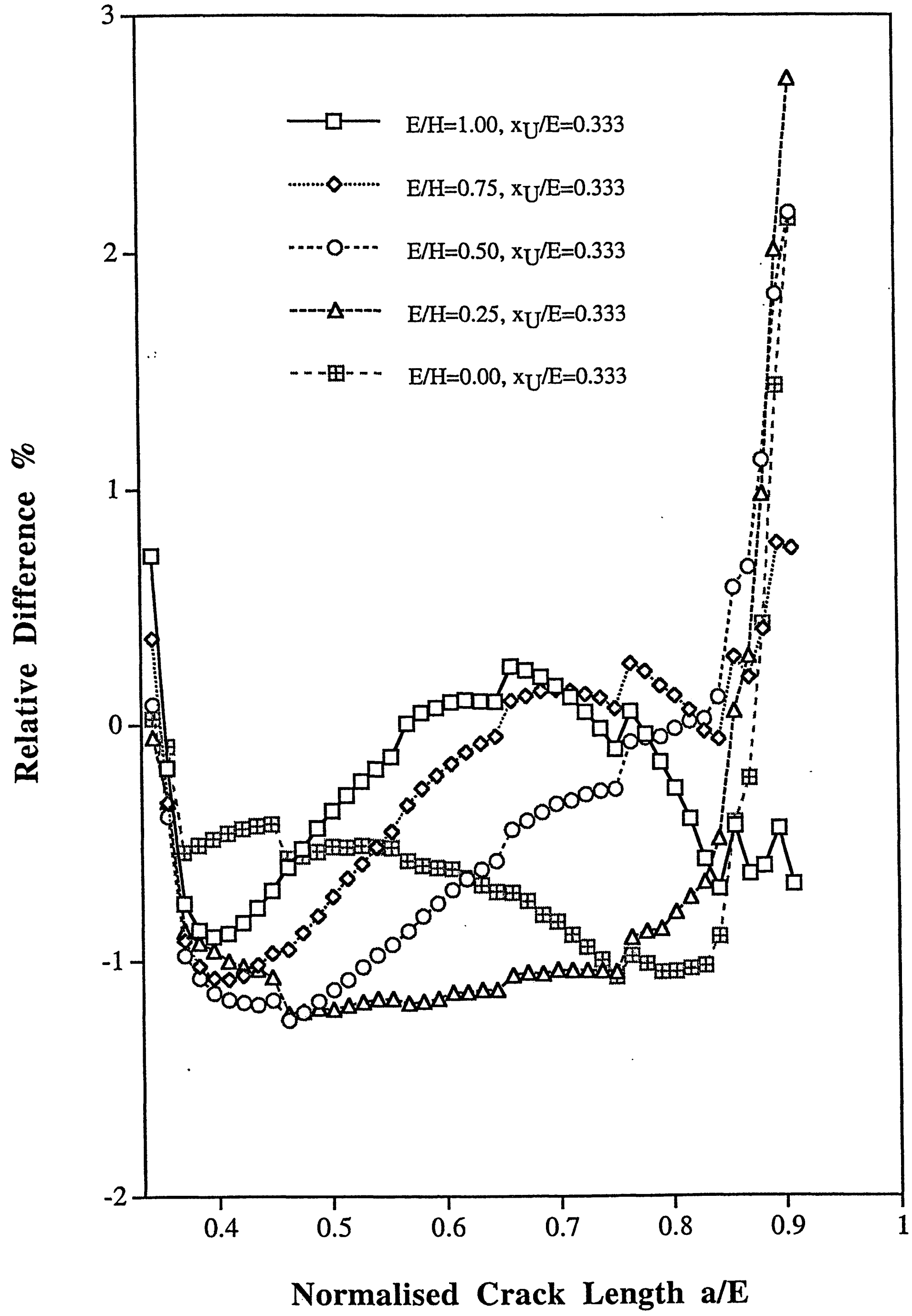

Figure 12. Relative differences between the stress intensity factors according to the proposed equation and the results of the weight function solution for a rectangular sheet subjected to a partial crack surface pressure. $x_U/E = 0.333$.

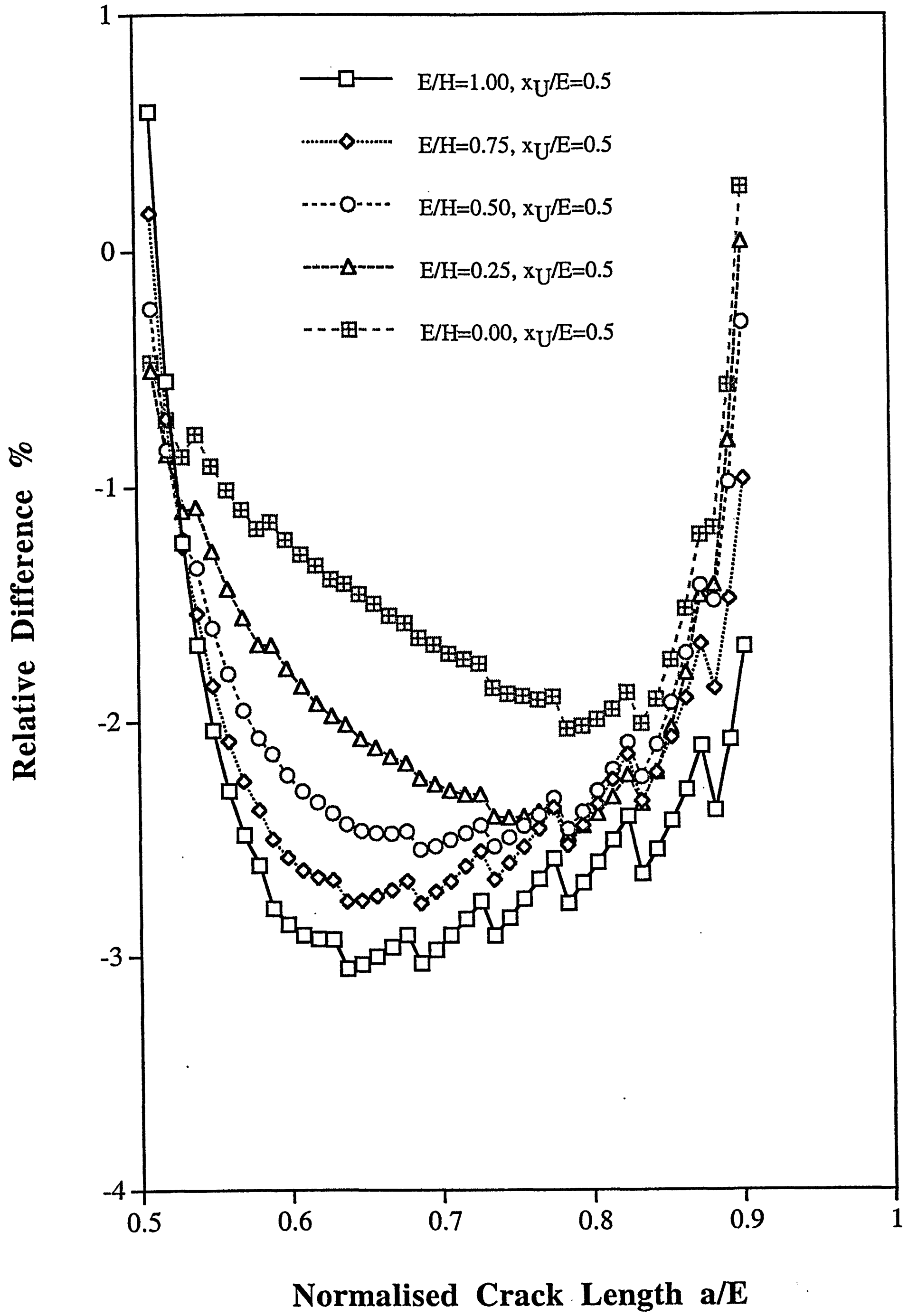

Figure 13. Relative differences between the stress intensity factors according to the proposed equation and the results of the weight function solution for a rectangular sheet subjected to a partial crack surface pressure. $x_U/E = 0.5$.

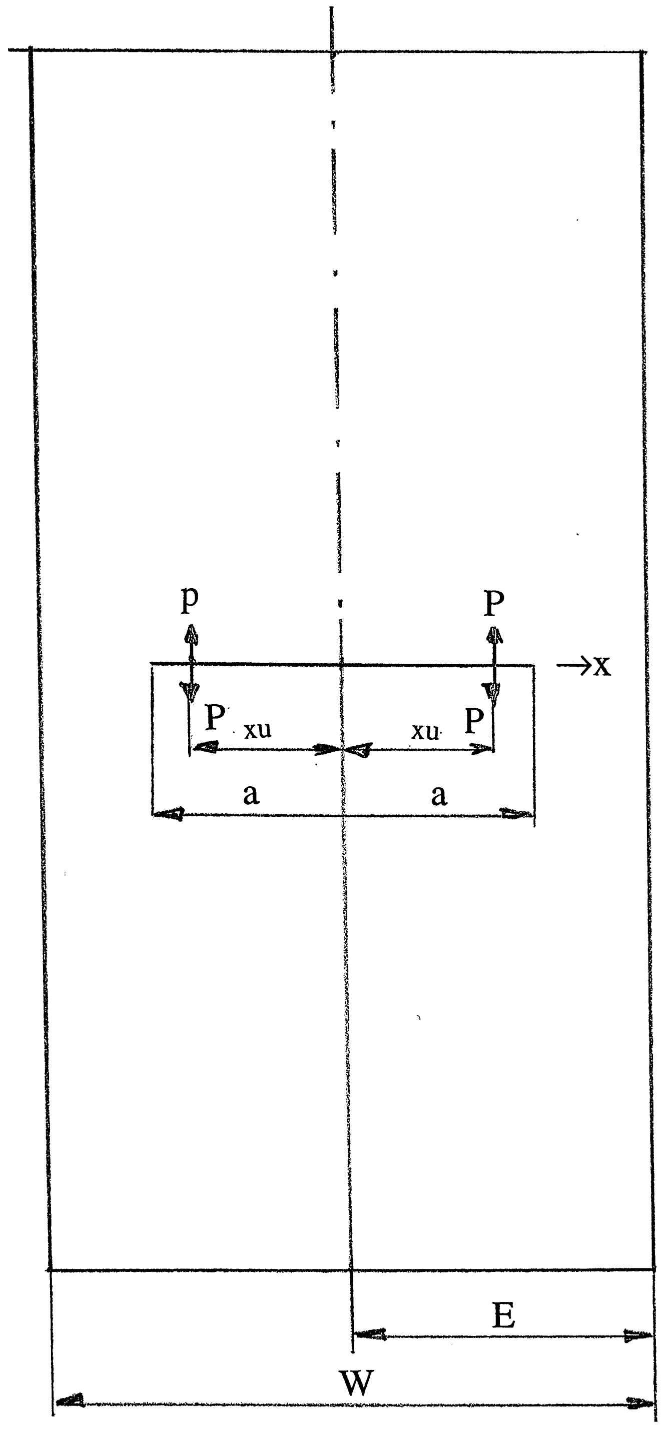

Figure 14. Geometry and loading for a centre cracked strip subjected to two pairs of splitting forces.

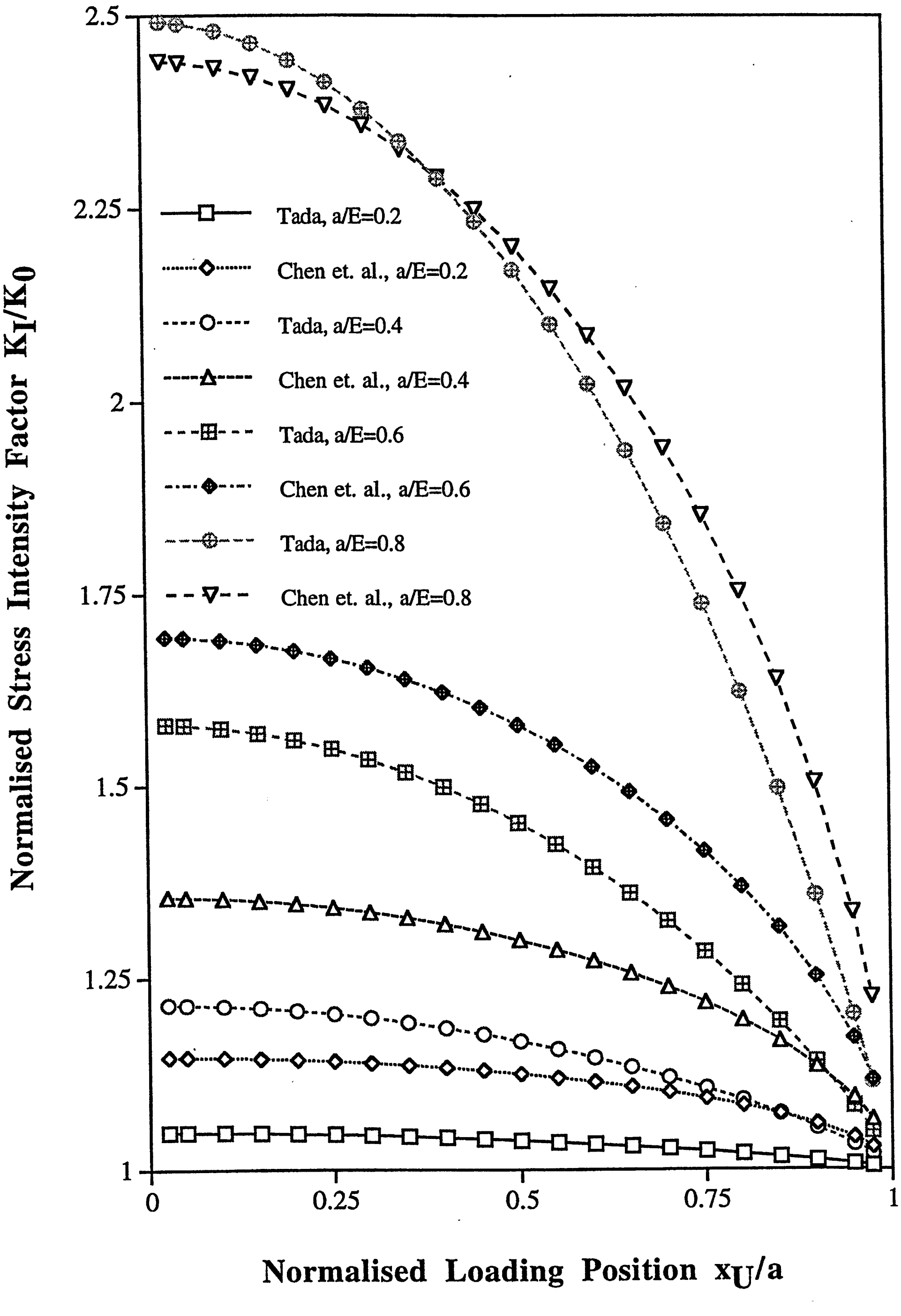

Figure 15. Comparison of the normalised stress intensity factors as obtained by the equations of Tada and Chen et. al., respectively for a centre cracked plate subjected to two pairs of splitting forces.

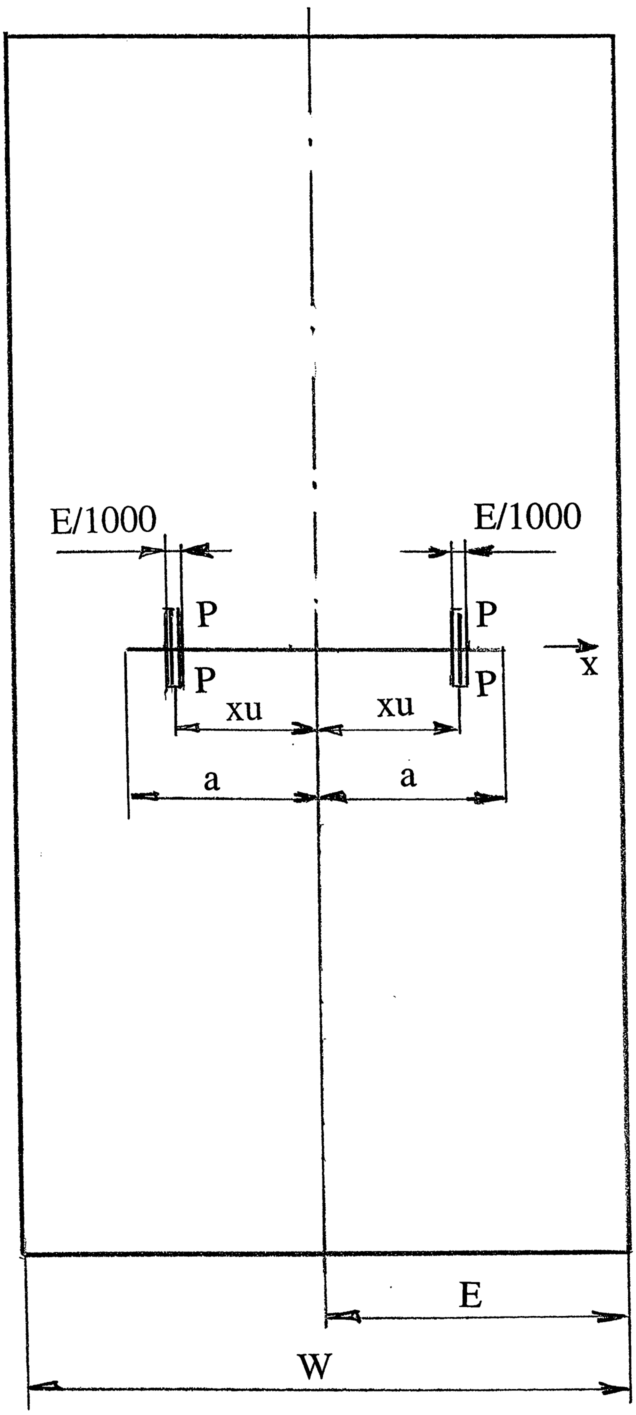

Figure 16. Geometry and loading for a centre cracked strip subjected to two concentrated crack surface pressures.

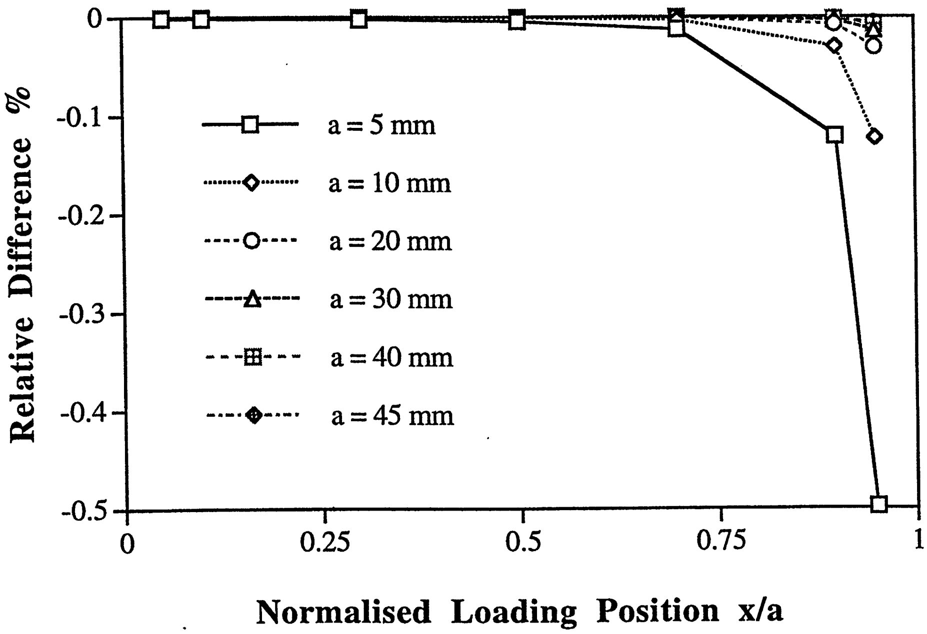

Figure 17. Relative comparison of the effect of using two concentrated crack surface pressures instead of two pairs of splitting forces.

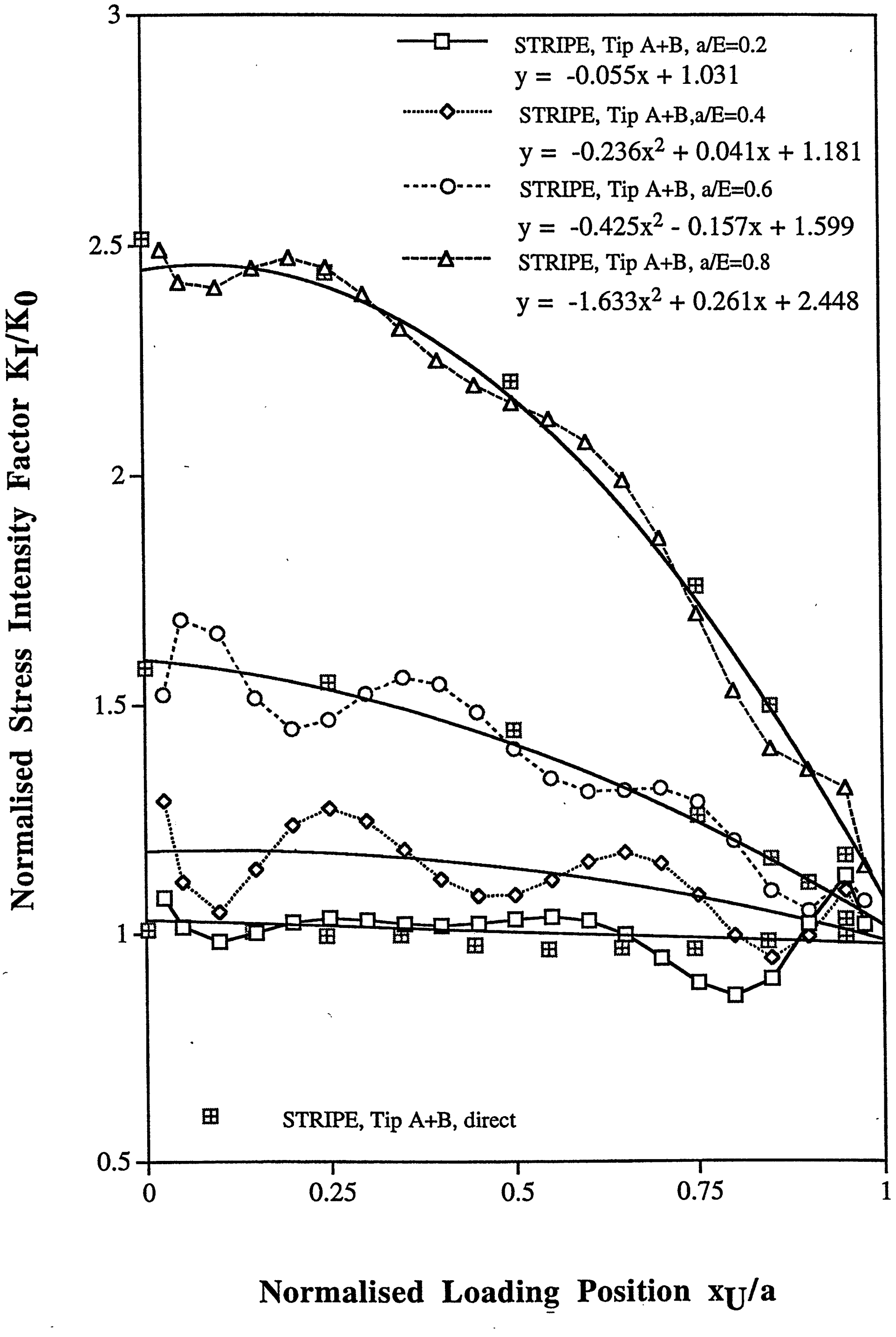

Figure 18. The normalised stress intensity factor for a centre cracked strip subjected to two concentrated crack surface pressures, as obtained using the FE method.

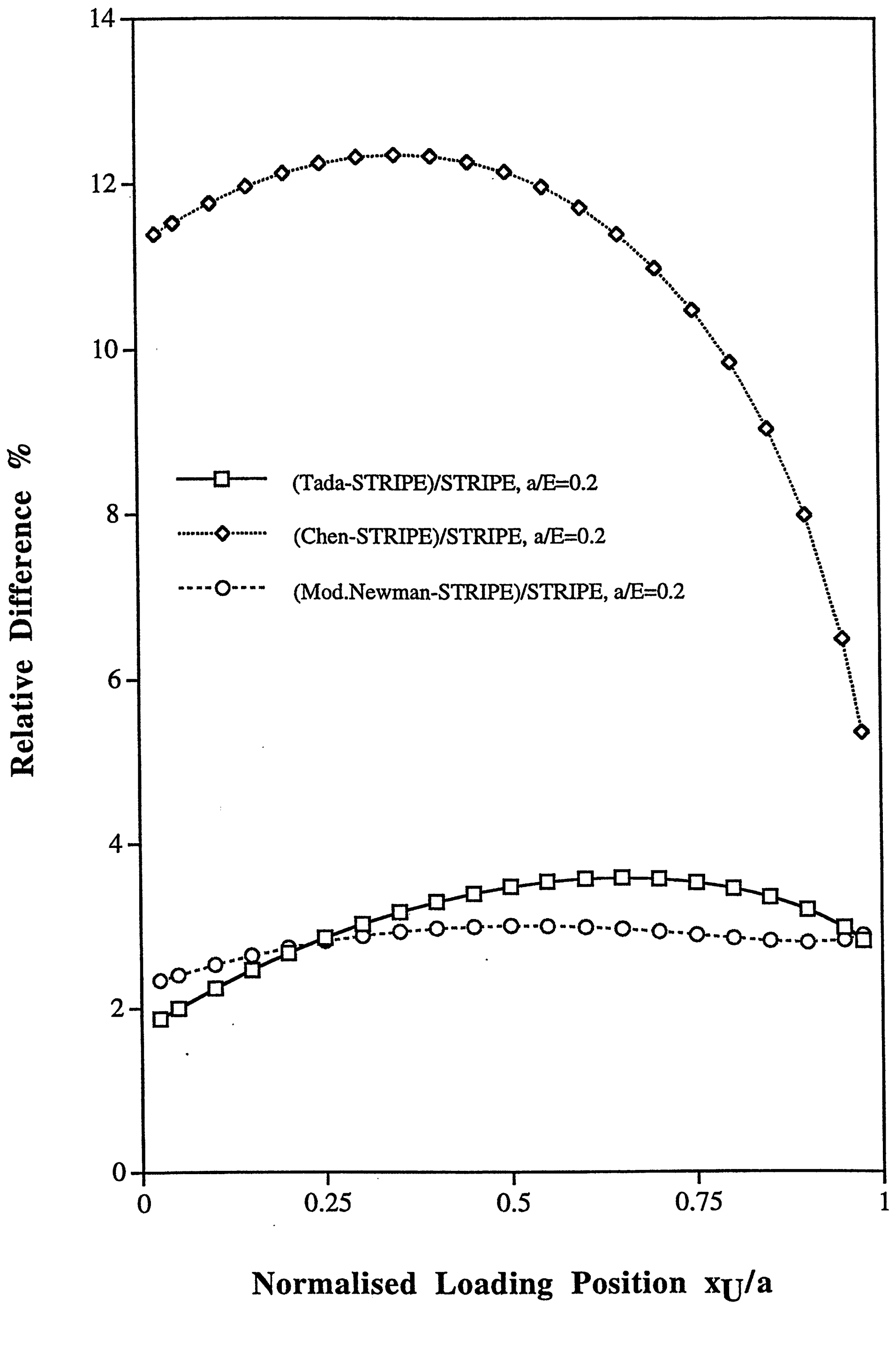

Figure 19. The results of three equations with respect to the results of a finite element analysis for a centre cracked strip subjected to two pairs of splitting forces. Normalized crack length a/E = 0.2.

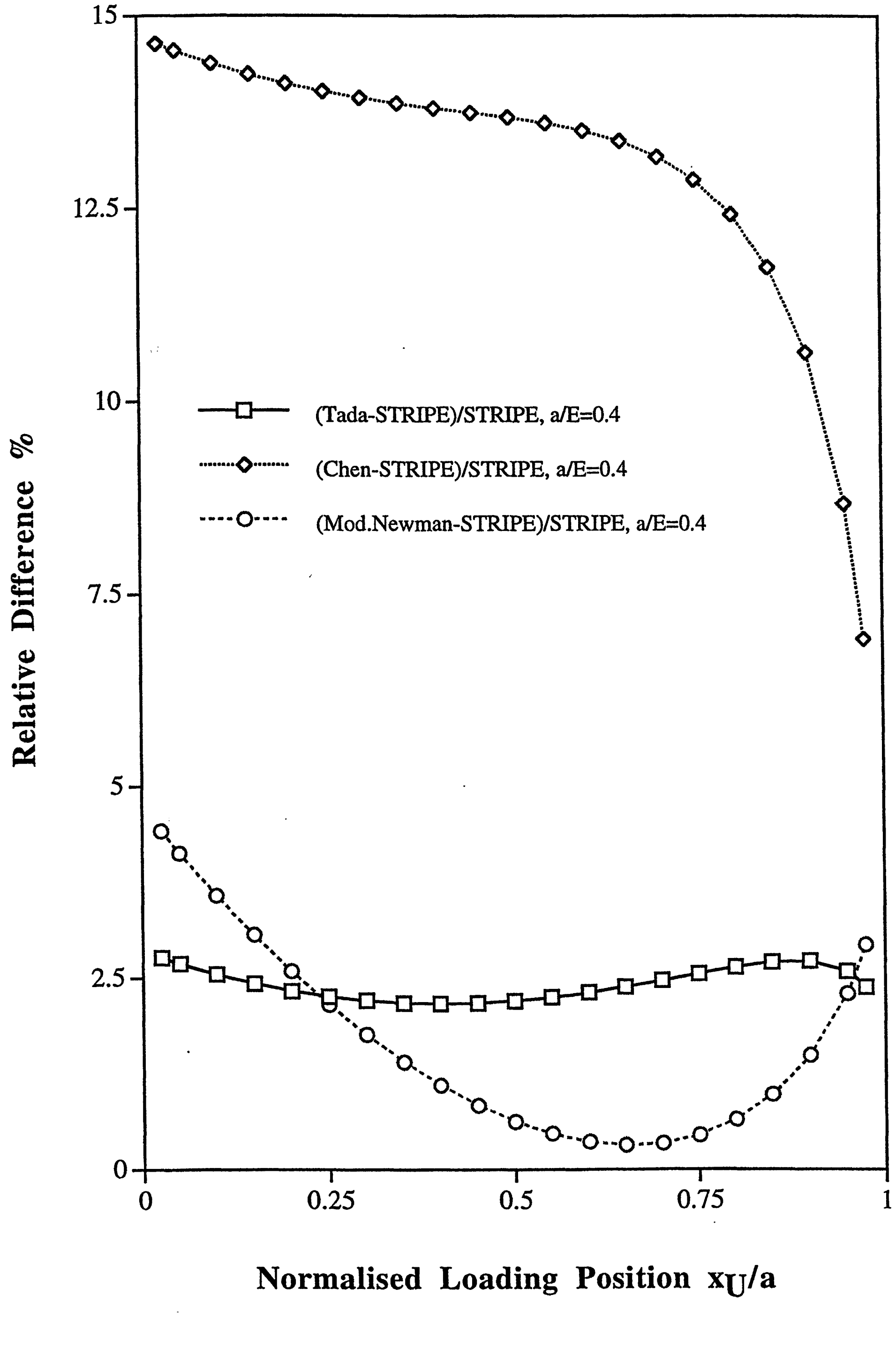

Figure 20. The results of three equations with respect to the results of a finite element analysis for a centre cracked strip subjected to two pairs of splitting forces. Normalized crack length a/E = 0.4.

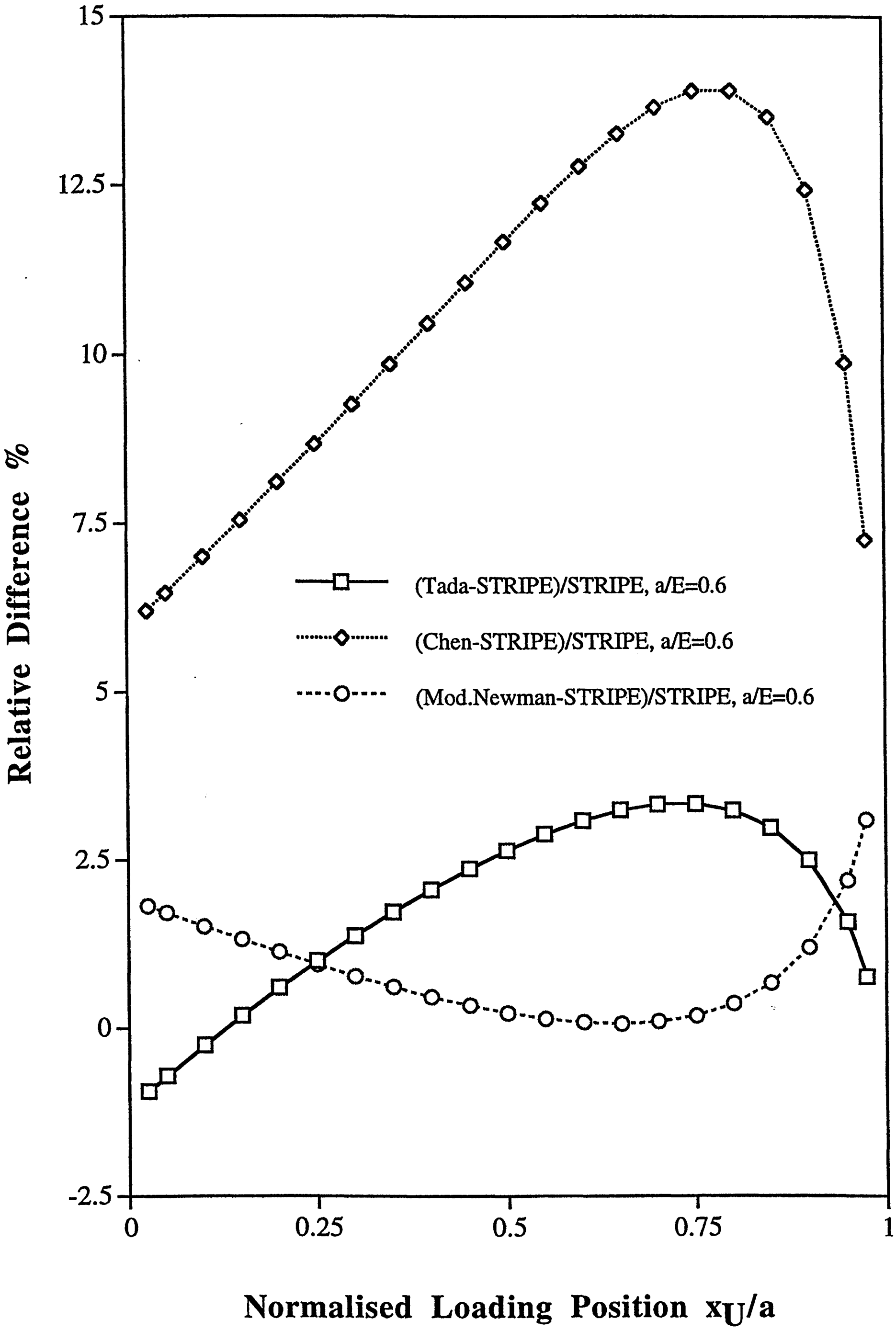

Figure 21. The results of three equations with respect to the results of a finite element analysis for a centre cracked strip subjected to two pairs of splitting forces. Normalized crack length a/E = 0.6.

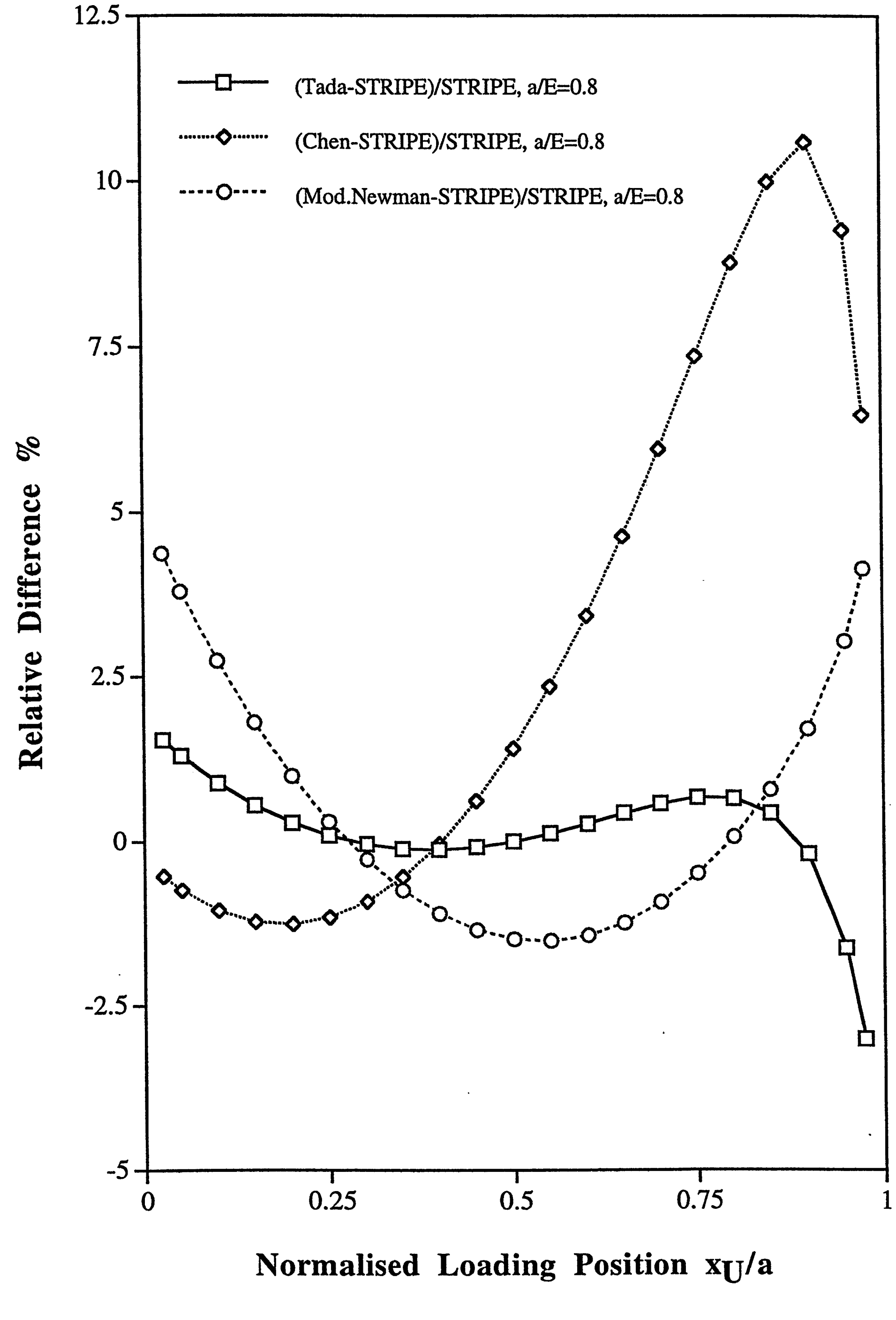

Figure 22. The results of three equations with respect to the results of a finite element analysis for a centre cracked strip subjected to two pairs of splitting forces. Normalized crack length a/E = 0.8.

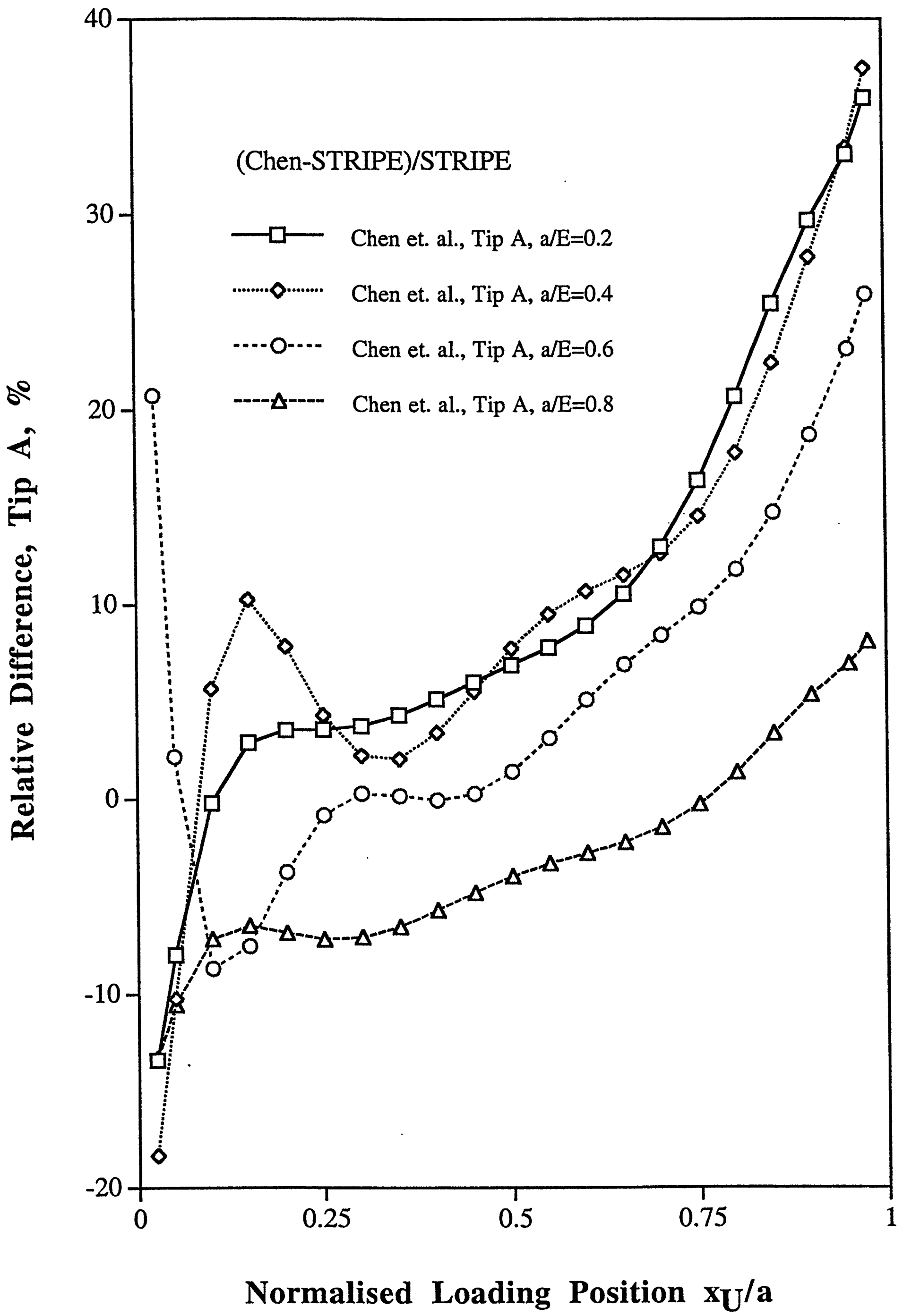

Figure 23. Relative difference between the results, for crack tip A, calculated using the equation by Chen et. al. and the results obtained using the finite element model for a centre cracked strip subjected to a crack surface pressure from 0 to x_U.

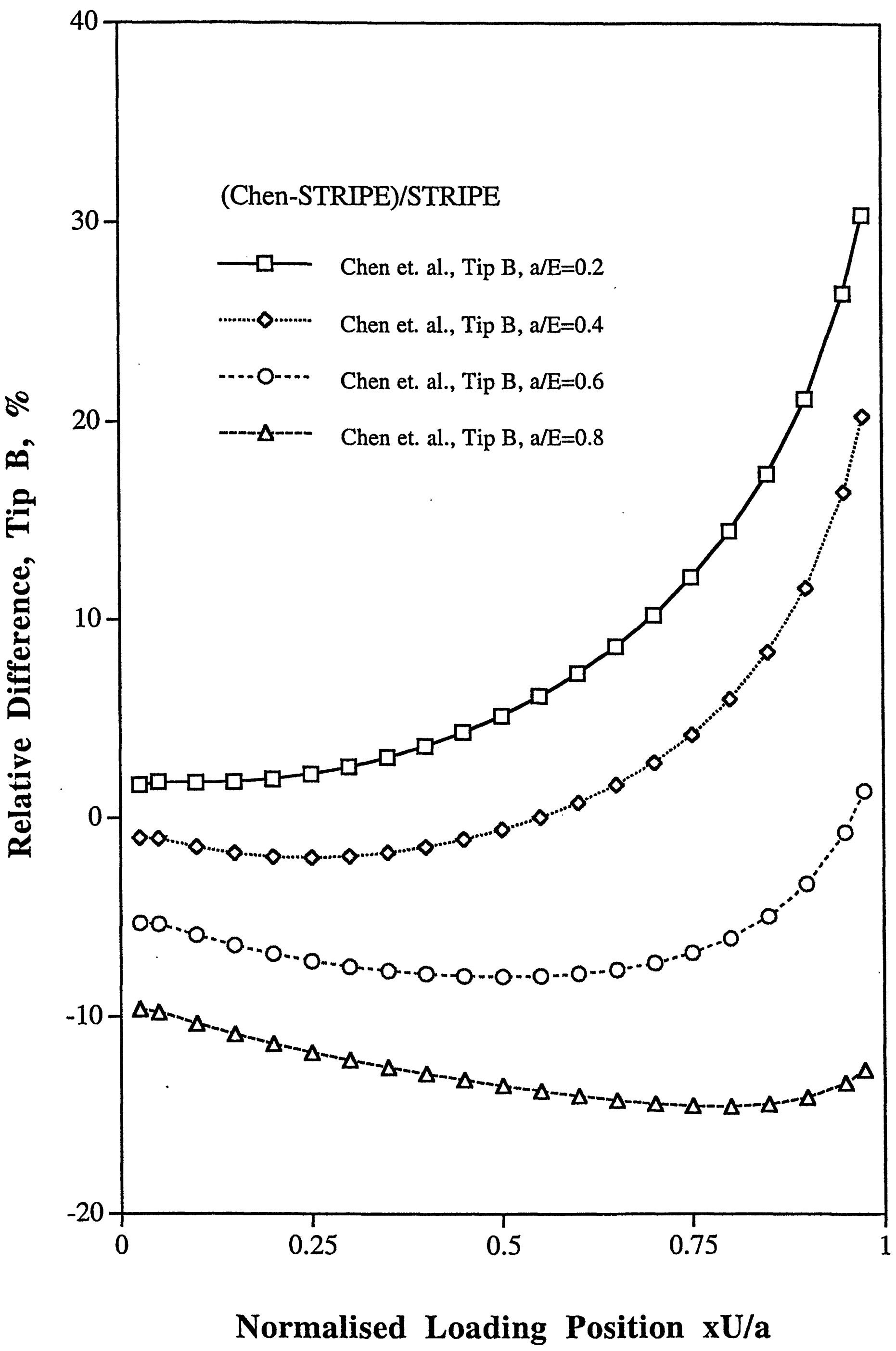

Figure 24. Relative difference between the results, for crack tip B, calculated using the equation by Chen et. al. and the results obtained using the finite element model for a centre cracked strip subjected to a crack surface pressure from 0 to x_U.

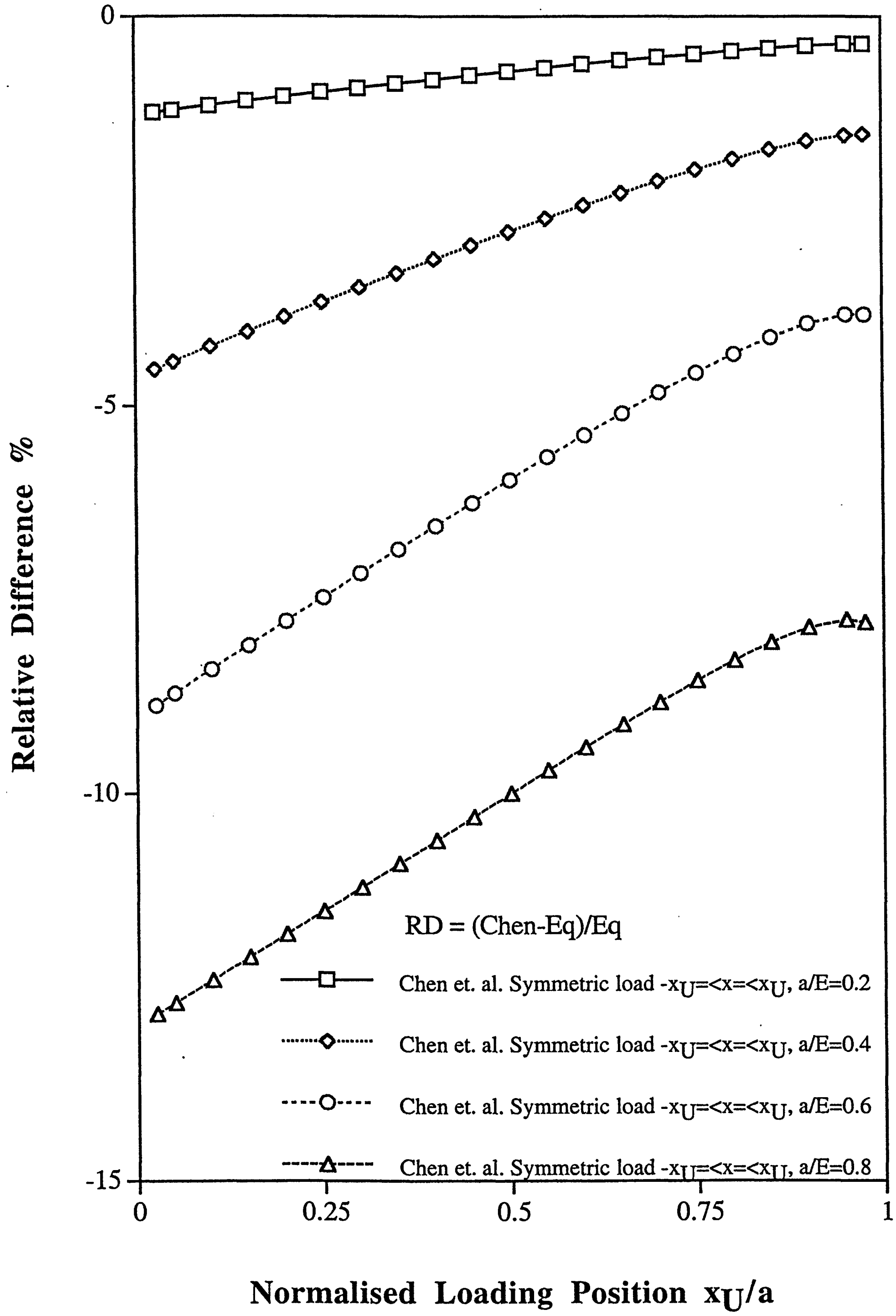

Figure 25. Relative difference between the results calculated using the equation by Chen et. al. and the results obtained using the modified Newman equation for a crack surface pressure from - x_U to x_U.

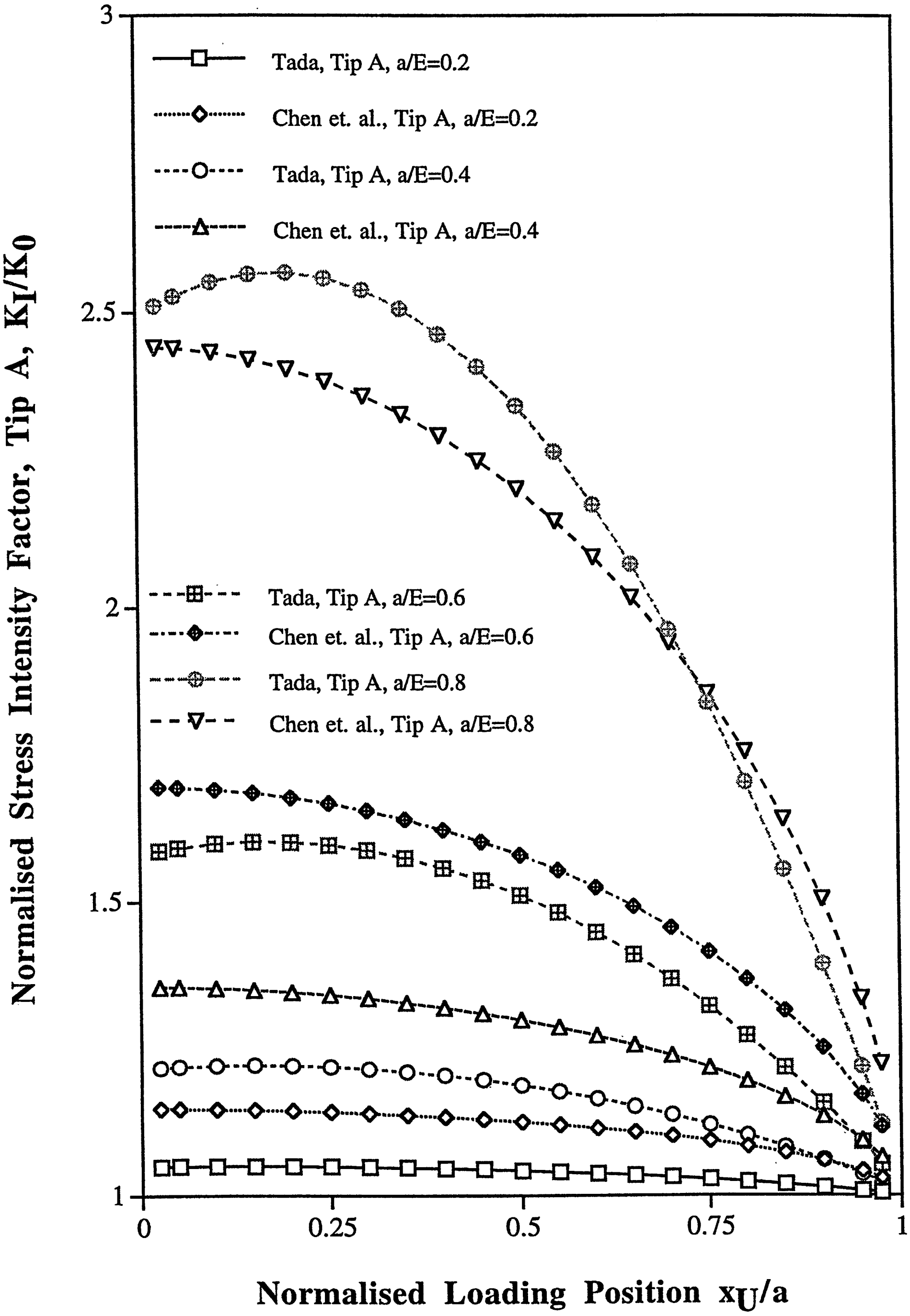

Figure 26. The normalized stress intensity factor at crack tip A for a centre cracked strip subjected to a single pair of splitting forces acting in position x_U. Comparison between the results of the equations by Tada and Chen et.al.

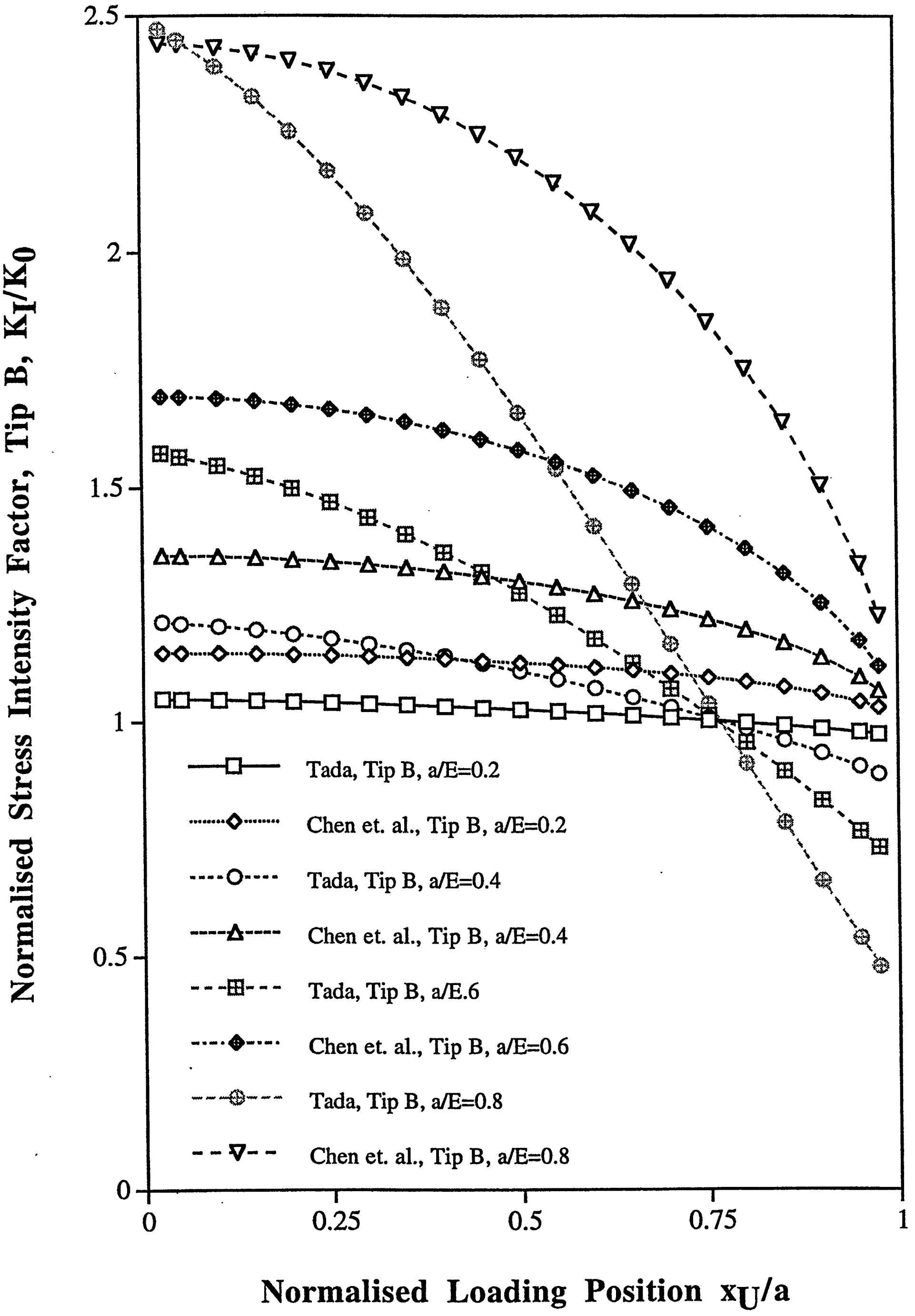

Figure 27. The normalized stress intensity factor at crack tip B for a centre cracked strip subjected to a single pair of splitting forces acting in position x_U. Comparison between the results of the equations by Tada and Chen et.al.

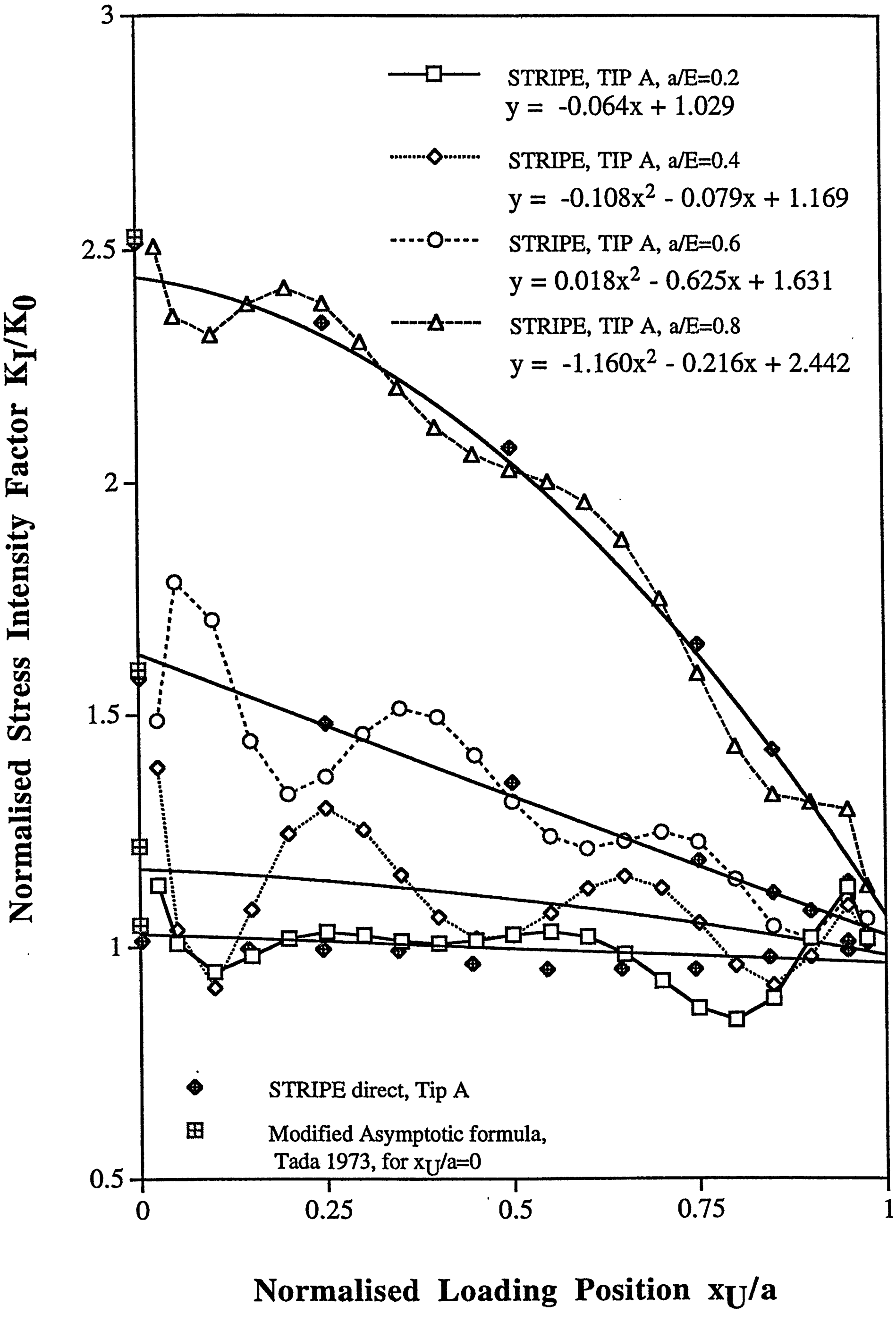

Figure 28. The normalized stress intensity factor at crack tip A for a centre cracked strip subjected to a single pair of splitting forces acting in position x_U. Results obtained using the finite element model.

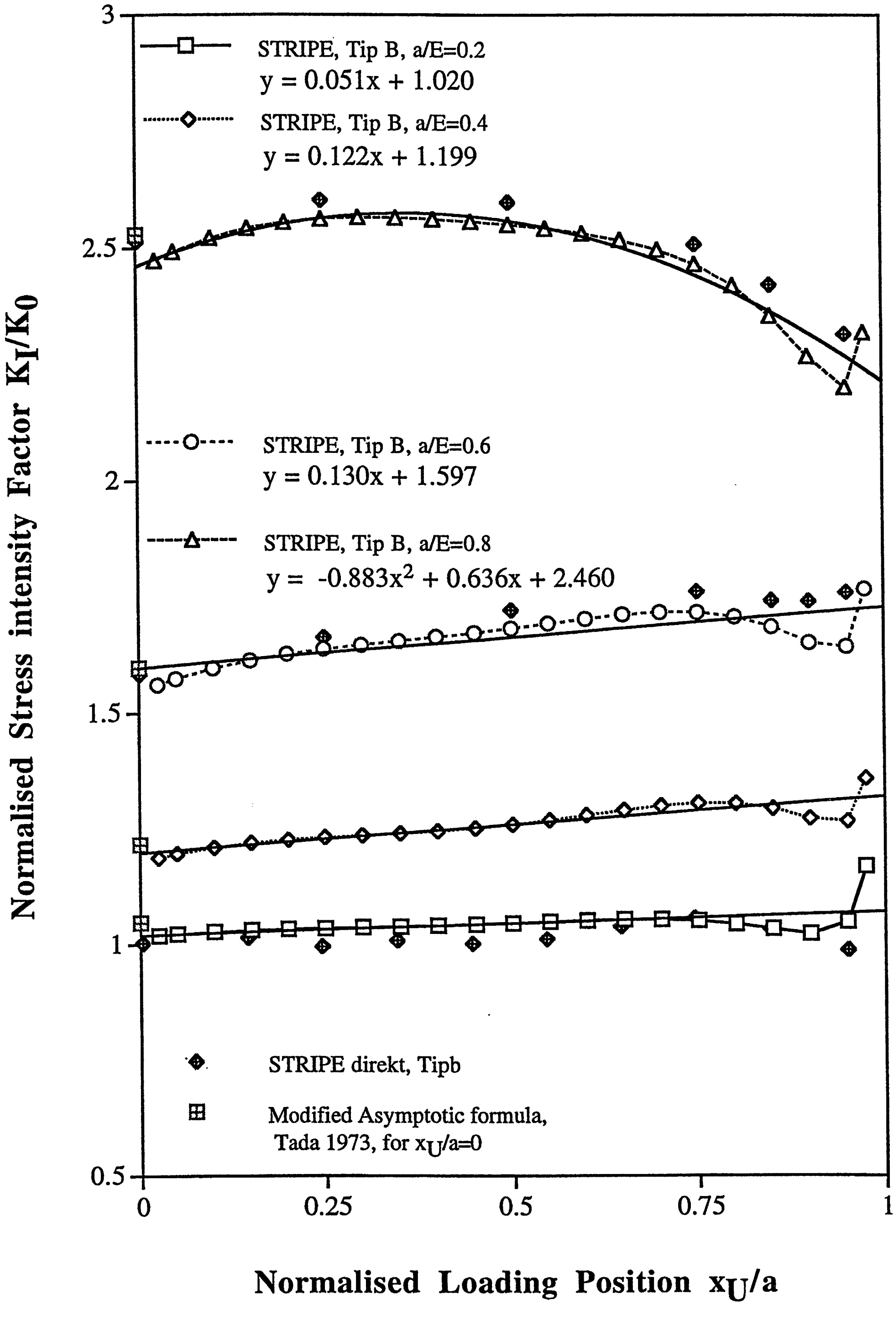

Figure 29. The normalized stress intensity factor at crack tip B for a centre cracked strip subjected to a single pair of splitting forces acting in position x_U. Results obtained using the finite element model.

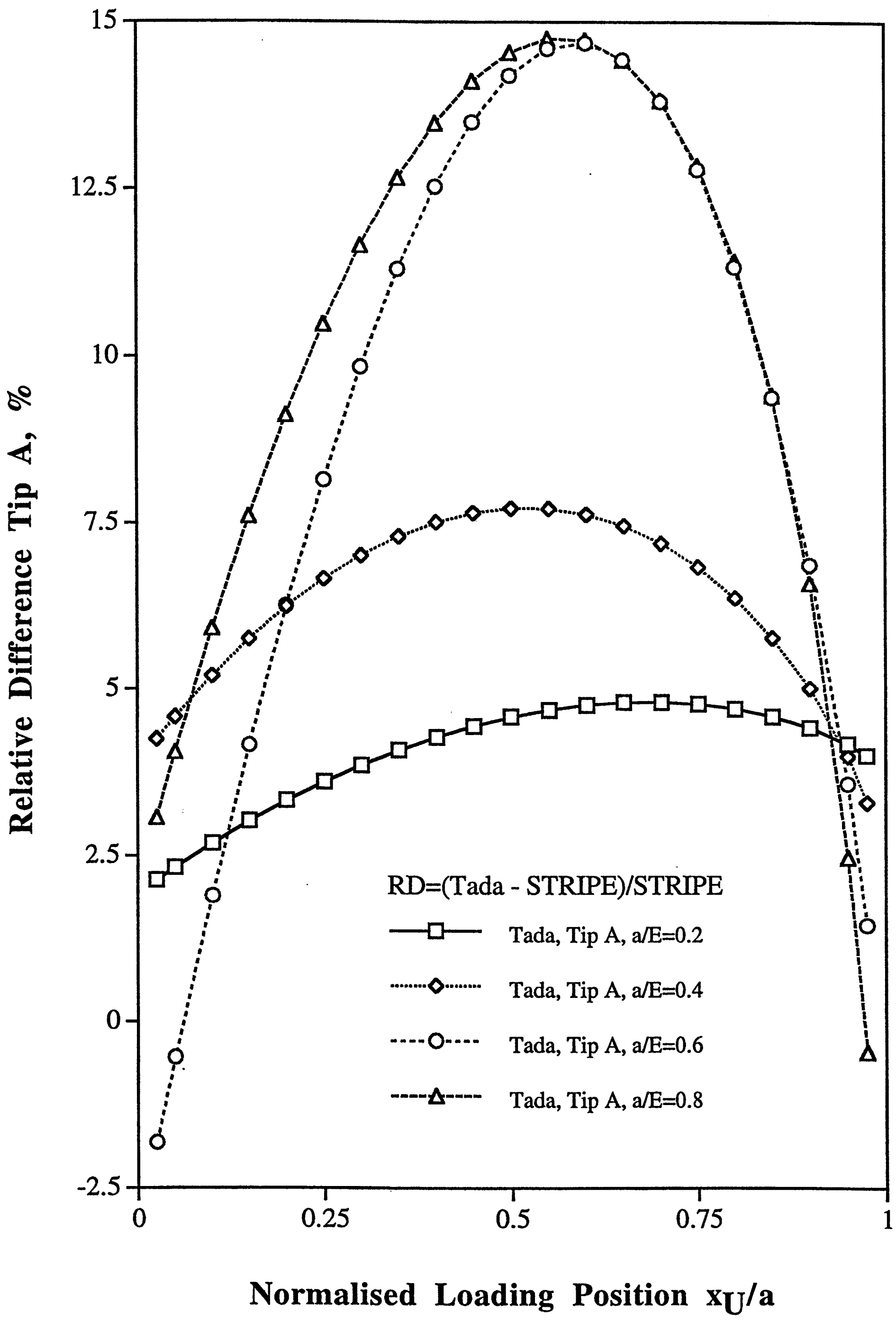

Figure 30. Relative difference in stress intensity factor for crack tip A between the equation by Tada and the solution by STRIPE.

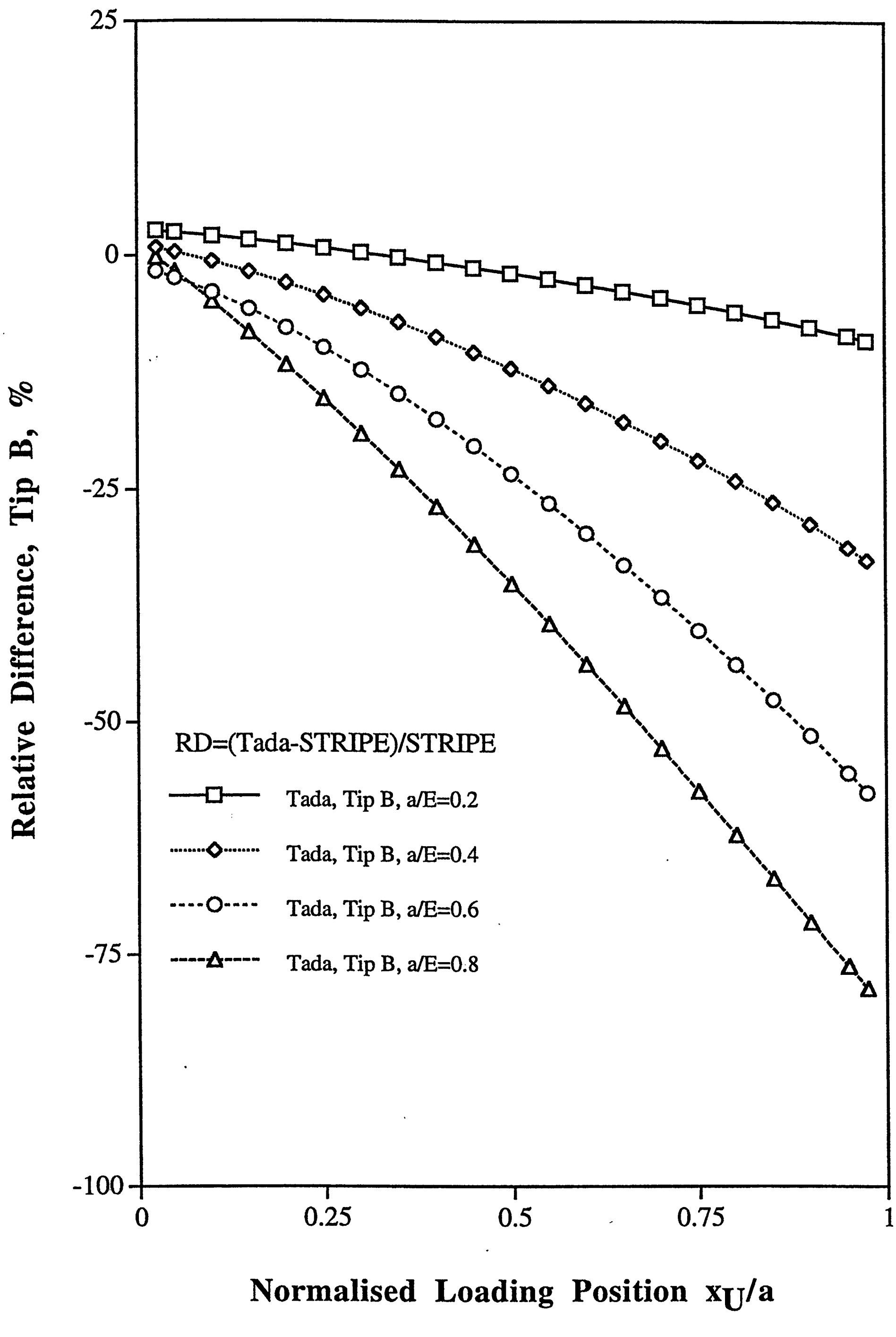

Figure 31. Relative difference in stress intensity factor for crack tip B between the equation by Tada and the solution by STRIPE.

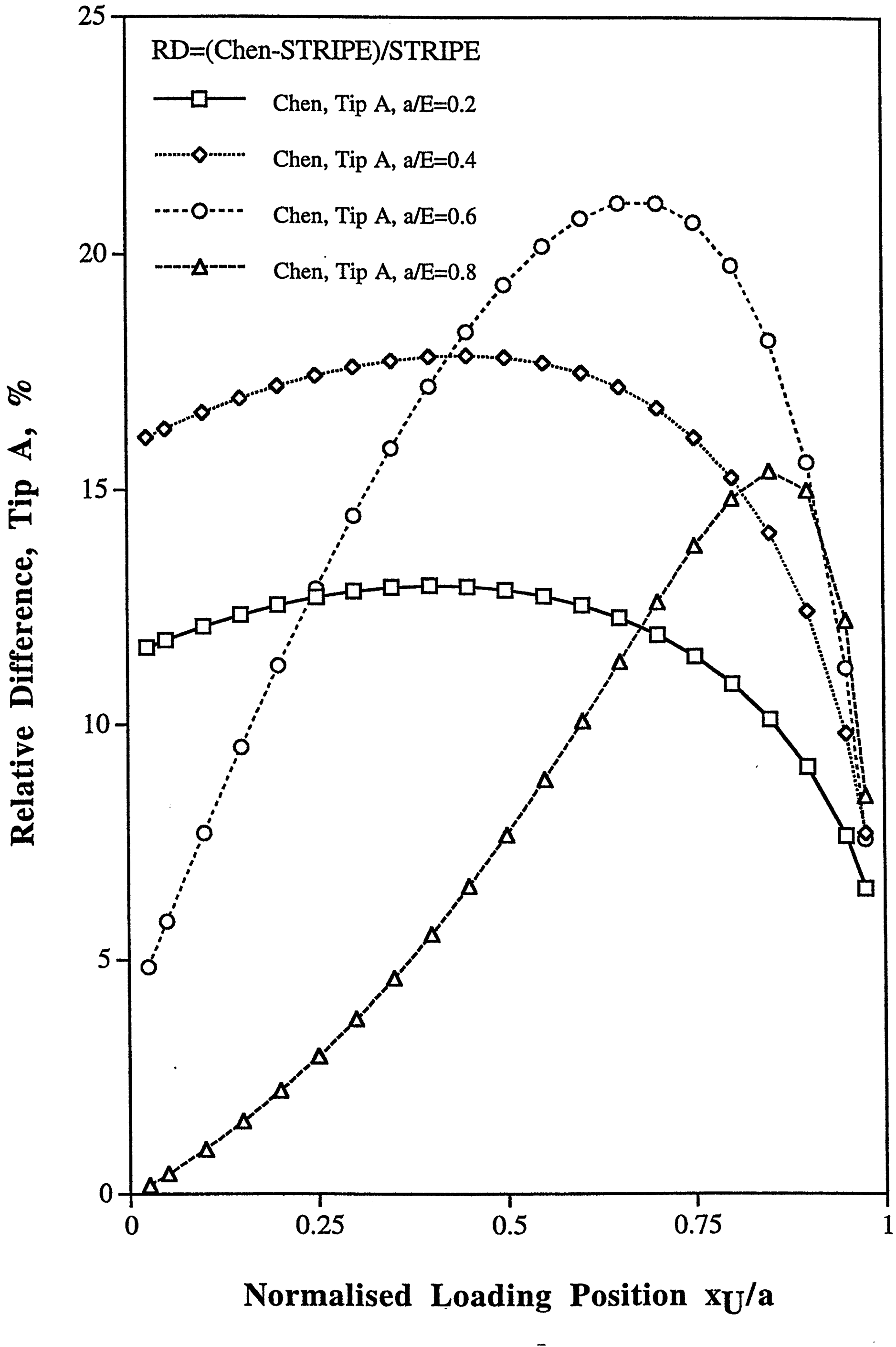

Figure 32. Relative difference in stress intensity factor for crack tip A between the equation by Chen et. al. and the solution by STRIPE.

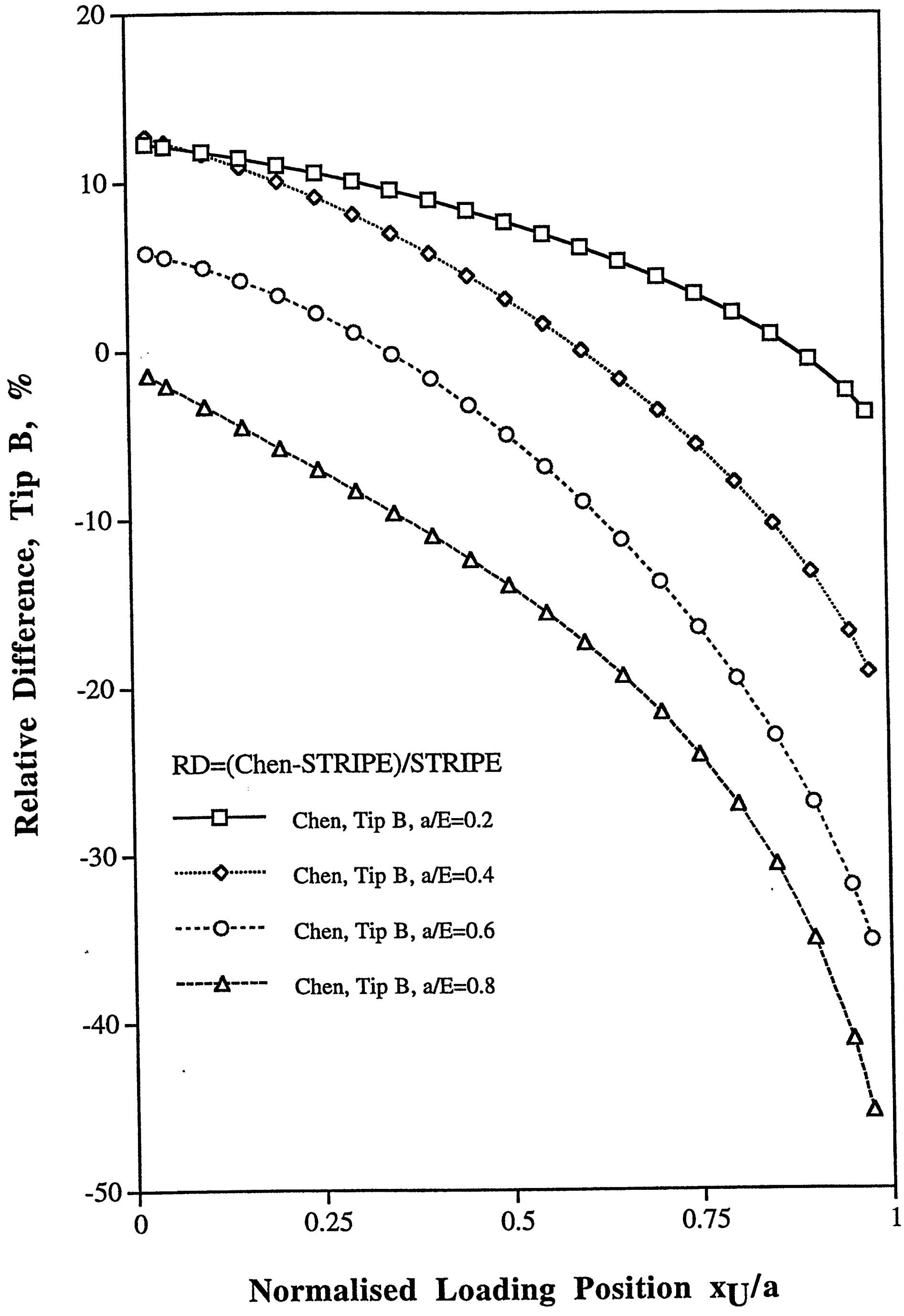

Figure 33. Relative difference in stress intensity factor for crack tip B between the equation by Chen et. al. and the solution by STRIPE.