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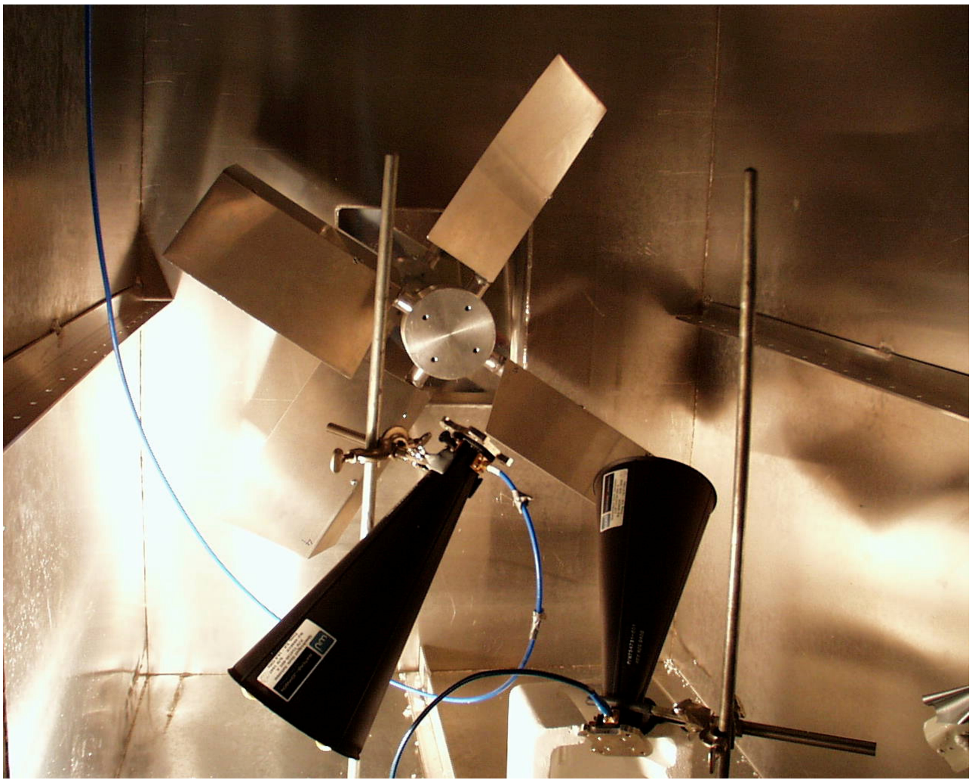
Scientific report

Olof Lundén, Mats Bäckström, Niklas Wellander

Evaluation of Stirrer Efficiency in FOI Mode-Stirred Reverberation Chambers



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Abstract (not more than 200 words)

In this report investigations of the correlation coefficients have been performed using single and dual mode-stirrers independently in FOI large reverberation chamber. Also in focus has been to quantify the fit to the presumed statistical power distribution using goodness-of-fit test techniques. Moreover, investigations have been performed on the maximum value distribution in the chambers and the relations between statistical independence and correlation of measured data. A stringent methodology, using significant tests on stirrer correlation and on the assumed statistical distribution, has been used to evaluate reverberation chamber performance. A conclusion is that the requirement of uncorrelated samples was the limiting factor for the low frequency performance for the chamber, except for the case where two mode-stirrers were used. Investigations using two mode-stirrers instead of one has shown that this will square the number of uncorrelated positions and halve the lowest useful frequency.

Moreover, it seems that the goodness of fit method for estimation of the number of statistically independent stirrer positions is to preferred before the e^{-1} rule since the latter seems to accept data that are statistically dependent.

Keywords

EMC, reverberation chamber, mode stirred chamber

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Rapportens titel

Utvärdering av effektiviteten hos omrörare i FOI modväxlande kammare

Sammanfattning (högst 200 ord)

I denna rapport undersöks korrelationskoefficienter för enkla och dubbla omrörare i FOI modväxlande kammare. Vi har fokuserat på att kvantifiera graden av överensstämmelse med den förmodade statistiska effektfördelningen. Dessutom har maxvärdesfördelningen undersökts och förhållandet mellan statistiskt oberoende och korrelation hos uppmätta data.

En stringent metodik med hjälp av s.k. 'goodness-of-fit' testteknik, för signifikanstester på korrelationen hos omrörarna och på den förmodade statistiska fördelningen, har använts för att utvärdera egenskaperna hos de modväxlande kamrarna. En slutsats är att kravet på okorrelerade sampel var den begränsande faktorn m.a.p. lägsta arbetsfrekvens för kamrarna, utom då två omrörare används. Undersökningar med två omrörare i stället för en visar att detta kvadrerar antalet okorrelerade positioner och halverar den lägsta användbara frekvensen.

Att uppskatta antalet statistiskt oberoende omrörarpositioner är att föredra före e^{-1} regeln eftersom den senare accepterar data som är statistisk beroende.

Nyckelord

EMC, HPM

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INDEX

1. Background 5

2. Introduction 5

Definitions. 5

3. Test of stirrer efficiency by use of correlation coefficients derived from measurements. 6

4. Maximum number of stirrer positions 9

5. Goodness of fit test of assumed distribution. 12

6. Goodness of fit tests of the assumed distribution vs. correlation coefficient < e^{-1} test 13

7. Goodness of fit test for different number of steps vs. correlation tests. 14

8. Good χ^2 distribution repeated 15

9. Statistical independence and linear correlation 16

10. The maximum over mean distribution. 20

11. Conclusions 24

References 25

Appendix A. Results 27

Appendix B. Tables 46

Appendix C. Programs 48

Taylor 48
hemta 49
peta 49
goodness4 50
goodness445 52
korre 53
hallen 57
rho_level 58

Appendix D. 59

Appendix E. 60

Appendix F. Noise reduction 62

1. Background

A Mode-Stirred Reverberation Chamber (RC) is an overmoded cavity used for evaluation of EMC properties at high (mostly microwave) frequencies for electronic equipment. Typical applications are testing of high level radiated susceptibility, radiation emission and shielding effectiveness. In such a chamber one (or more) mechanical mode stirrers are used to change the electromagnetic boundary conditions. The objective is to create a new field pattern for every stirrer position that is different, i.e. statistically independent from all others. The higher number of independent stirrer positions the lower measurement uncertainty can be achieved. This was investigated in a previous report 'Measurements of Stirrer Efficiency in Mode-Stirred Reverberation Chambers' [1] which was focused mainly on evaluating the improvements of stirrer efficiency when using different sizes of stirrers. This was done by calculating the linear correlation coefficients from insertion loss measurements in the two reverberation chambers at FOI (formerly FOA) in Linköping, Sweden. These two, denoted E3 and E4, have the volumes of 37 m^3 and 27 m^3 respectively. Background information about the two chambers, the different stirrers, the measurement set-up and how the correlation coefficients are calculated can be found in [1].

In this report further investigations of the correlation coefficients have been performed using two mode-stirrers independently in the large chamber. Also in focus has been to quantify the fit to the presumed statistical power distribution in the RCs using goodness- of-fit test techniques on old data from [1] as well as data from new measurements. Moreover, investigations have been performed on the maximum value distribution in the chambers and the relations between statistical independence and correlation of measured data.

2. Introduction

Why bother with χ^2 distributed data, statistical independence and correlation coefficients? As described in [1] and chapter 10 it's a matter of measurement uncertainty. To be able to calculate a figure of merit for this, knowledge is needed that the stirrer positions are independent and that power density in the chamber has good exponential distribution. In practise, it is assumed that statistical independence can be demonstrated by showing that the stirrer positions are uncorrelated (or just weekly correlated). This approach is discussed and evaluated later in this report.

A crucial question is if conclusions can be made from an evaluation of correlation coefficients about the fit to a χ^2 distribution, or the opposite, i.e. is it really necessary to calculate both?

How many stirrer steps must I use or how few can I use? Due to the statistical nature of RC testing, many samples are needed for the evaluation [2]. To test a hypothesis of an assumed χ^2-distribution a lot of samples will be needed, not less than 20 – 25 [3], i.e. with less samples you have to accept almost any distribution that is proposed to fit the measured data.

In this report we focus mainly on evaluating chamber and stirrer performance, which is of interest for verifying the chamber specification. The report also includes a study of the statistics for the maximum field.

Definitions.

First we would, like the Greek philosopher Socrates, "*before arguing, make some definitions so we all know what we talk about*".

For evaluation of chamber/stirrer performance we are normally using Vector Network analyzers in the measurements. The absolute Received power and/or the Transmitted power are of limited interest. Instead what is recorded is their complex relationship expressed as:

Insertion loss P_r/P_t is a one parameter measured quantity in the RC which consists of chamber wall losses and losses in/through the antennas. The reference plane for the network analyzer is the antenna input ports where a **response/isolation calibration** is performed. This type of calibration is just a system loss normalization at the antenna ports and a correction of the system isolation.

Transmission Loss $(P_r/P_t = S21)$ is defined as a four parameter measurement (S11;S12;S21 and S22) that includes the insertion loss and the port reflections and requires a **full two-port calibration** at the reference plane. From these measurements it is possible, through zero-doppler filtering [20], to retrieve the Tx and Rx free space antenna mismatch. Moreover if platform stirring is performed it is possible to retrieve and remove the direct antenna to antenna coupling [22]. These type of measurements will give a higher accuracy but is more time consuming.

Normalized received power is the ratio between the Insertion Loss P_r/P_t and the stirrer revolution average $\langle P_r/P_t \rangle$, i.e. the average of the insertion loss at each frequency carried out with respect to all stirrer positions.

For the referenced measurements in this report the Insertion loss is used.

Uncorrelated is defined as when correlation coefficient is less than the 1% and 5% significance levels and the noise in the correlation coefficient calculations, due to undersampling, and is larger than the true correlation.

3. Test of stirrer efficiency by use of correlation coefficients derived from measurements.

The quantity used for evaluation of stirrer efficiency is the linear correlation coefficient r for different increments of stirrer angle [1]. The coefficient is determined using the measured insertion loss P_r/P_t at each angle. The quantitative significance of the measured correlation coefficient ρ_o can be evaluated by calculating the probability that N measurements of two uncorrelated variables (i.e. $r = 0$) would give a result, ρ, as large as or larger than ρ_o. The probability is given by, cf. [3]:

$$\Pr ob_N(|\rho| \geq |\rho_0|) = \frac{2\Gamma[(N-1)/2]}{\sqrt{\pi}\Gamma[(N-2)/2]} \int_{|\rho_0|}^{1} (1-\rho^2)^{(N-4)/2} d\rho \qquad (1)$$

In our case N is equal to the number of stirrer positions used to derive ρ_o. Thus, if we obtain a coefficient ρ_o for which $\Pr ob_N(|\rho| \geq |\rho_0|)$ is small, it is correspondingly unlikely that our variables are uncorrelated, that is, a correlation is indicated. In particular, if $\Pr ob_N(|\rho| \geq |\rho_0|) \leq 5\%$, the correlation is called **significant**, if it is less than 1%, the correlation is called **highly significant** [3]. For practical purposes uncorrelated is sometimes defined as when the measured correlation coefficient $\rho_0 \leq e^{-1} \approx 0.37$, cf. [5]. As shown in the following this is, however, not strictly consistent since the maximum acceptable ρ_o (for a certain level of significance) depends on the number of samples N. The probability given by eq. (1) if $\rho_o = 0.37$ vs. number of samples (stirrer steps) has been plotted in figure 1 where it can be seen that for $N \geq 28$ we have significant evidence that data is correlated. For $N \geq 48$ the evidence is highly significant that data is correlated. On the other hand, for $N < 28$ we see that we may well accept the assumption of having uncorrelated data although $\rho_o > 0.37$. By solving eq.(1), (see Matlab function in appendix C p.C11), we show in figure 2 the maximum value of ρ_o that is permitted if the assumption of uncorrelated data shall be accepted at 1% and 5 % significance levels. Comparisons of correlation coefficients $\rho_0 < e^{-1}$ and calculated probability at 1% and 5 % significance levels can be found in appendix A. Appendix A does also contain the results obtained if data are accepted as uncorrelated if the measured correlation coefficient is less than e^{-1}. It shall be noted that eq. (1) is derived assuming normally distributed variables [19].

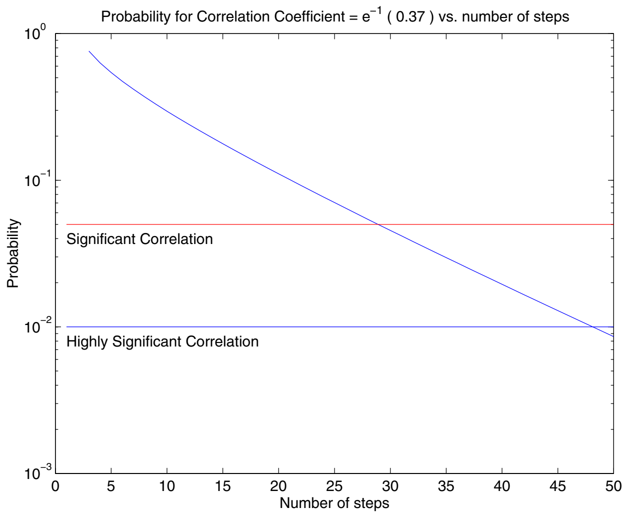


Figure 1. Significance level for correlation for $\rho_0 = e^{-1} \approx 0.37$ for different number of N. If the probability ≤ 5% the correlation is called significant (see text)

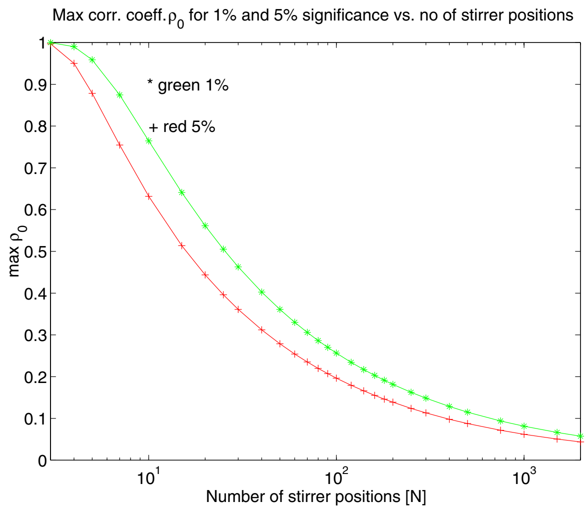


Figure 2. Maximum value of ρ_0 that is permitted if the assumption of uncorrelated data shall be accepted at 1% and 5 % significance level. N is the number of samples i.e. the number the stirrer positions used to calculate ρ_0.

A consequence of using *more samples* is that it gives better base for the evaluation of the true correlation coefficient *r*. This is a straightforward procedure used to indicate if the correlation coefficients are considered to be significant. In figure 3, figure 4, figure 5 and figure 6 this has been illustrated for 50, 250, 500 and 2000 samples (stirrer positions) for 200, 209 and 218 MHz for the chamber E3. The lines are for significant 5% and highly significant 1% probability levels for rejecting the assumption of having uncorrelated data, i.e. $r = 0$, for the respective number of samples. Already in figure 4 a correlation coefficient bounded by ± 0.2 for all stirrer intervals can be suspected. This is further accentuated and verified in figure 5 and figure 6. As can be seen from figure 5 and figure 6 we can see that the measured ρ_0 does not in general converge to zero and thus reject the assumption of uncorrelated for most stirrer intervals. In figure 7-10 the same plots are made at three frequencies well above the lowest usable frequency of the chamber. In this case we can see the expected behaviour for uncorrelated data. All the curves are centred around $\rho_0 = 0$ and the statistical noise decreases for higher values of *N*. We can also see that while the criterion $\rho_0 \leq e^{-1}$ works for the frequencies around 18 GHz it gives the wrong answer for the frequencies 200, 209 and 218 MHz. In the latter case that criterion claims that data are uncorrelated, for stirrer intervals above around 10 degrees, although this is obviously not the case. However, for practical use in testing the method based on eq.(1) might be too laborious. It might also be the case that the statistical distributions for the RC can still be used to estimate measurement uncertainties within an acceptable accuracy although $|r| \neq 0$. The choice of a reasonable value of ρ_0 *max* should be based on a combination of statistical analysis and engineering judgements. After all, the choice of ρ_0 *max* is, for the test engineer, a trade-off between the cost of using more stirrer positions and the merit of measurement uncertainty in the test. Of course, the more $|r|$ approaches one the less will be gained by decreasing the stirrer interval. Awaiting a careful analysis of this matter we are, in this report, using both ρ_0 *max* $= e^{-1}$ and the ρ_0 *max* corresponding to the 1% and 5 % significance levels as shown in figure 2.

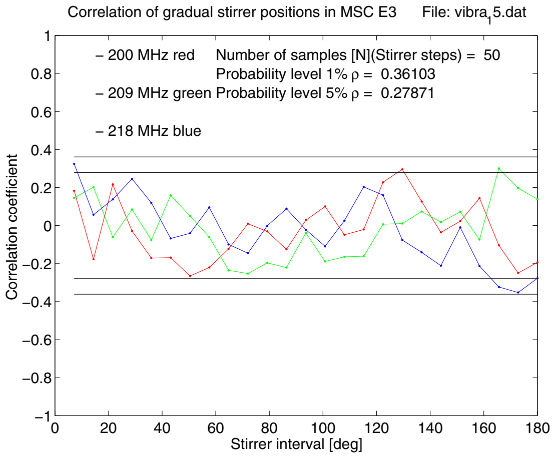


Figure 3. 50 stirrer positions (samples)

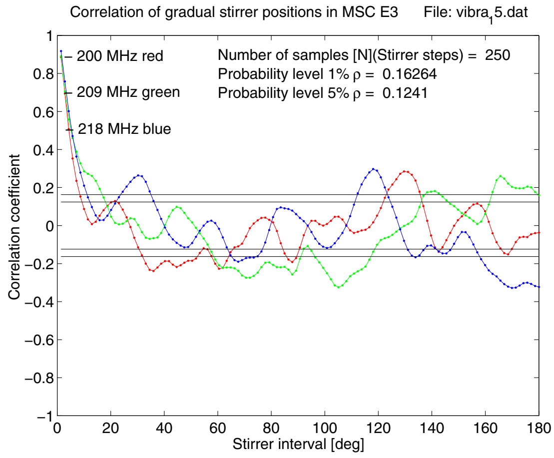


Figure 4. 250 stirrer positions (samples)

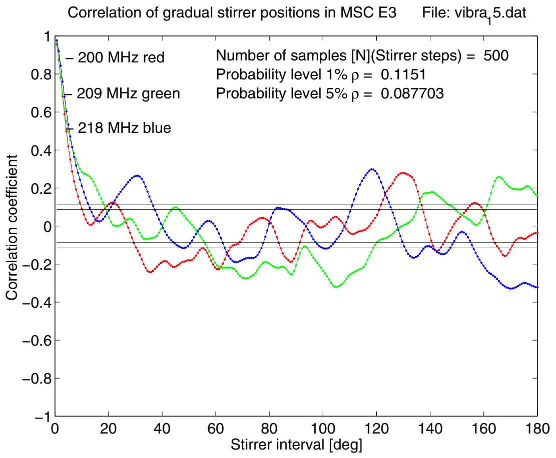


Figure 5. 500 stirrer positions (samples)

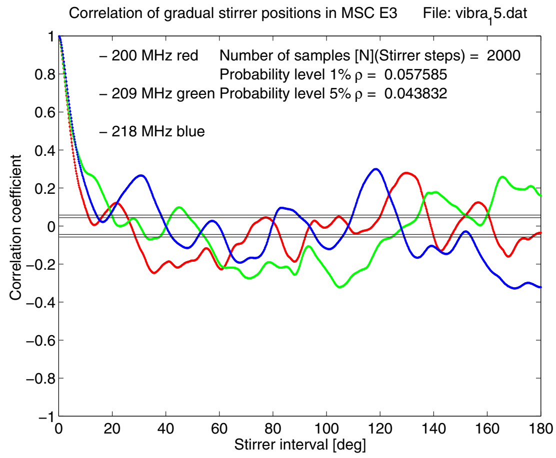


Figure 6. 2000 stirrer positions (samples)

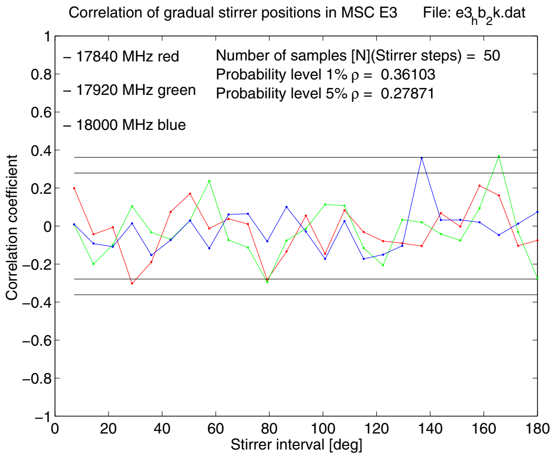


Figure 7. 50 stirrer positions (samples)

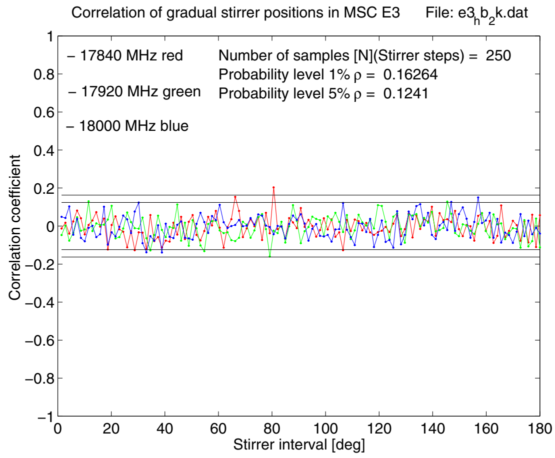


Figure 8. 250 stirrer positions (samples)

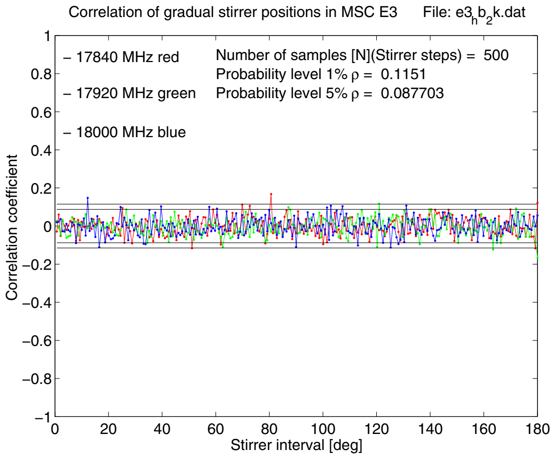


Figure 9. 500 stirrer positions (samples)

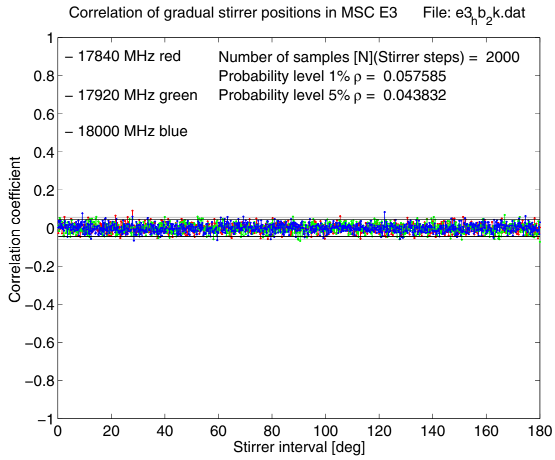


Figure 10. 2000 stirrer positions (samples)

4. Maximum number of stirrer positions

By use of linear correlation coefficient calculations the stirrer performance can be analyzed, see Figure 26, where measurements was performed in the 37 m^3 RC from 200 MHz – 2 GHz in 201 frequency points using one large stirrer and 2000 stirrer steps. The diagram shows, at each frequency, the number of stirrer positions for which the measured correlation coefficient equals the value corresponding to 1% and 5% significance level cf. figure 2 and $\rho_o = e^{-1}$. The cross-shaped stirrer has about 3.5 m^3 rotational volume.

We can calculate the correlated region for one stirrer using the probability significance level of 1% and 5%, see figure 26. It can be seen that one stirrer can be used at 200 MHz at maximum 30 uncorrelated steps per revolution. Using two independent stepped stirrers makes it possible to achieve about 30 * 30 uncorrelated steps, se figure 28. Translated into measurement uncertainty this yields a reduction for a 95% confidence level from ± 1.9 dB to ± 0.3 dB [1].

The lowest useful frequency with only one stirrer and 200 steps is approximately 600 MHz, see Figure 11 and Figure 26. This means that there are not more than 200 independent stirrer steps available due to correlation. To conclude, two stirrers are needed if many stirrer positions are required at low frequencies. It shall be noted that you have to carefully investigate all data, and the step-ratio of the two stirrers, to avoid symmetry effects.

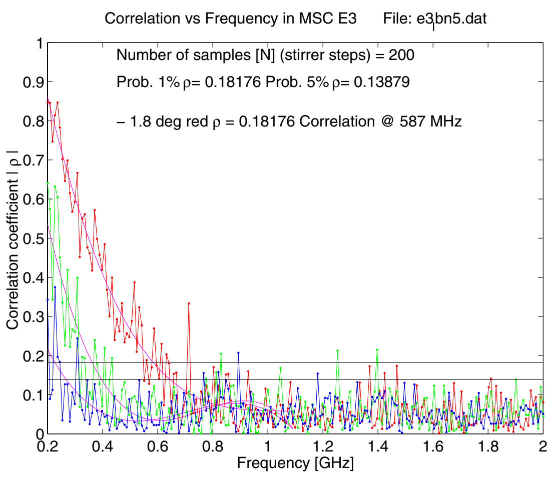


Figure 11. N = 200 steps. One Large Stirrer in the Large Chamber. Red curve shows correlation for 1.8 deg stirrer step, green for 3.6 deg stirrer step and blue for 5.4 deg stirrer step.

In figure 26 the calculations of the correlation coefficients and the 1% and 5% probability levels are based on 2000 samples. But more interesting is to find out how many samples is needed to reveal the true correlation at a certain frequency. This must obviously be less. Considering that we can reveal the true correlation using many samples, this is true at least at low frequencies where in wavelengths the stirrer displacements are small, how many stirrer position is then the maximum that can be used and still improve the measurement uncertainty.

To reveal the true correlation at a certain frequency one has to calculate the correlation coefficient by use of (infinitely) many samples, cf. figures 3 to 10. In practice, if a sufficient number of samples are used all data can, in most cases, be shown to be correlated (although the correlation may be weak), cf. figures 3 to 6. This is an intrinsic shortcoming with using this kind of evaluation. By plotting the "true" correlation coefficient, i.e. the measured correlation coefficient using 2000 samples, together with the curves in figure 2 we can get the expected minimum number of stirrer steps, i.e. samples, necessary to reveal that data are correlated which is equal to the maximum number of samples that supports the hypothesis that data are uncorrelated. Figure 15 show that this number is about 50 for frequencies around 200 MHz. Figure 17 shows that about 700 stirrer steps is needed around 2 GHz to reveal correlation. In practical use of a reverberation chamber a requirement of having, and proving the existence of, strictly uncorrelated data is an unnecessary severe requirement. As an alternative application of the method described above, based on eq.1, is presented in figure 21. In order to get a smooth curve this figure is based (in the same way as figures 15, 17 and 19) for all stirrer steps, on 2000 samples. Corresponding plots for other stirrer configurations are shown in Appendix E.

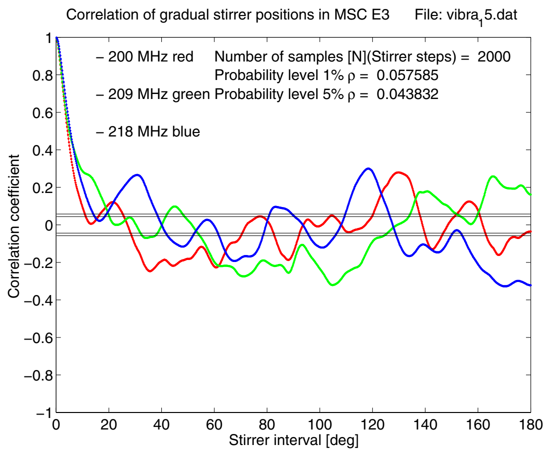


Figure 14. Large stirrer 200, 209 and 218 MHz N=2000

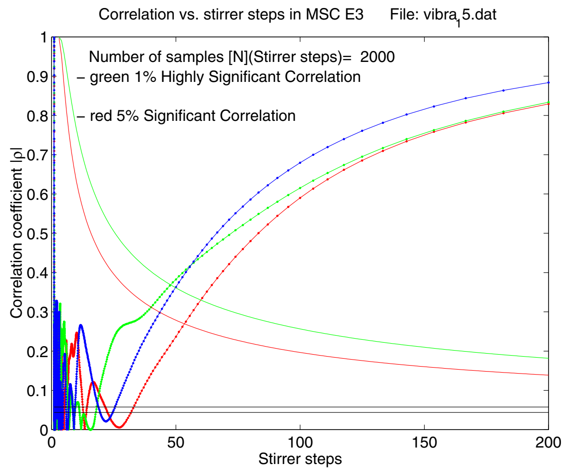


Figure 15. Correlation revealed for > 50 stirrer step/revolution at 200, 209 and 218 MHz N=2000

The figures includes 1% and 5% significant levels for 2000 samples for comparison, the black horizontal lines. Figure 15 will only plot correlation data that corresponds to 1.8 deg to 360 deg stirrer interval and starts of with a high correlation value that corresponds to 360 deg stirrer interval, i.e. correlation coefficient = 1.

Figure 14 shows that data are correlated **independent** of stirrer interval. This is also indicated by the black lines in figure 15. Figure 15 shows how many steps that will be needed to show that data are correlated.

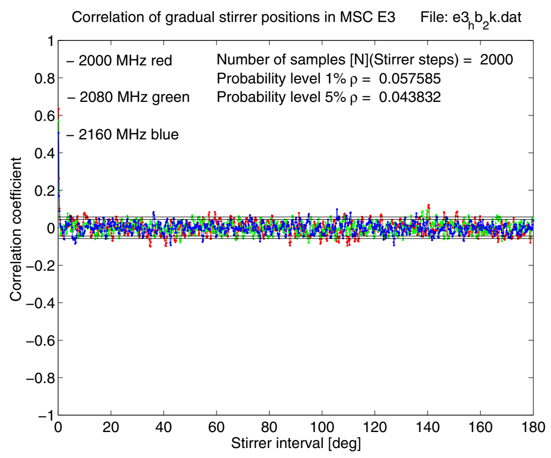


Figure 16. Large stirrer 2000, 2080 and 2160 MHz. N=2000

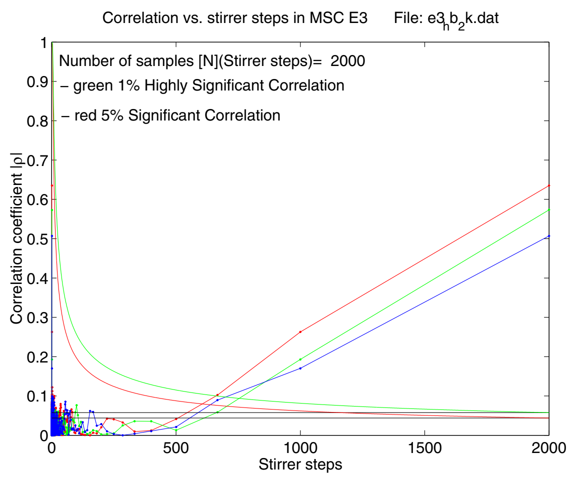


Figure 17. Correlation revealed for > 700 stirrer step/revolution at 2000, 2080 and 2160 MHz. N=2000

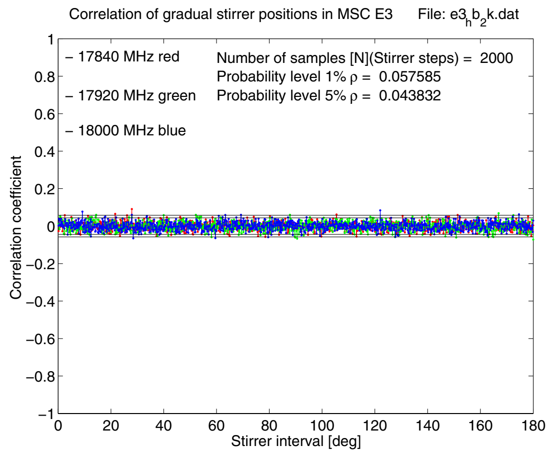


Figure 18. Large stirrer 17840, 17920 and 18000 MHz

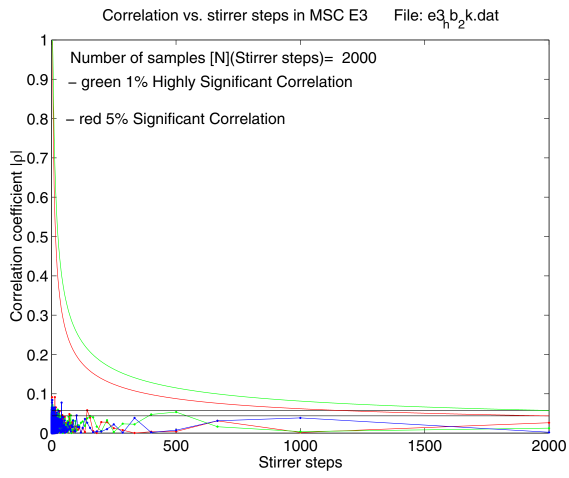


Figure 19. No Correlation revealed for 2000 stirrer step/revolution at 17840, 17920 and 18000 MHz. N=2000.

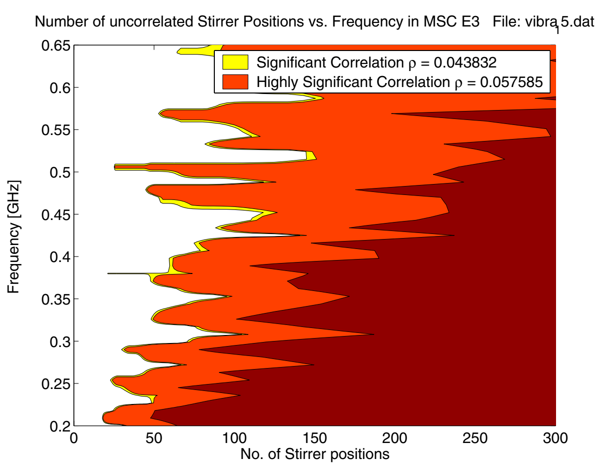


Figure 20. Correlation reveal at each frequency for 1% and 5% significance levels based on 2000 samples.

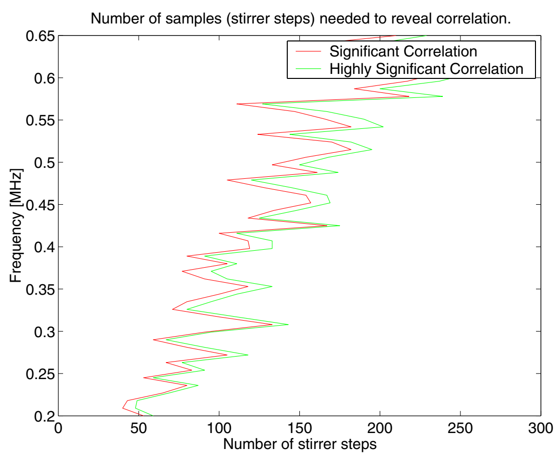


Figure 21. Samples needed to reveal correlation at each frequency. Based on calculations from intercept point as described above.

A summary from the different mode-stirrers and different stirring techniques in the two chambers can be seen in appendix E. The influence of the stirrer size is not dramatic, however the independent stirrers will definitely make a large impact on the performance.

5. Goodness of fit test of assumed distribution.

An often used criterion for a properly working chamber is that a rectangular component of the electric field (E_R) follows a normal distribution, and that the power received by a linear antenna, follows a χ^2- (chi-square) distribution with 2 degrees of freedom (DOF) at each frequency [4]. The data are expressed in terms of the normalized received power i.e. the ratio between the insertion loss P_r/P_t and the average of between the insertion loss $<P_r/P_t>$ at each frequency carried out with respect to all stirrer positions. Since the variance of the normalized χ^2- distribution with 2 DOF is equal to one we can see that RC data seems to support the assumption of data being χ^2-distributed, see Figure 22 and Figure 23 [cf. 14].

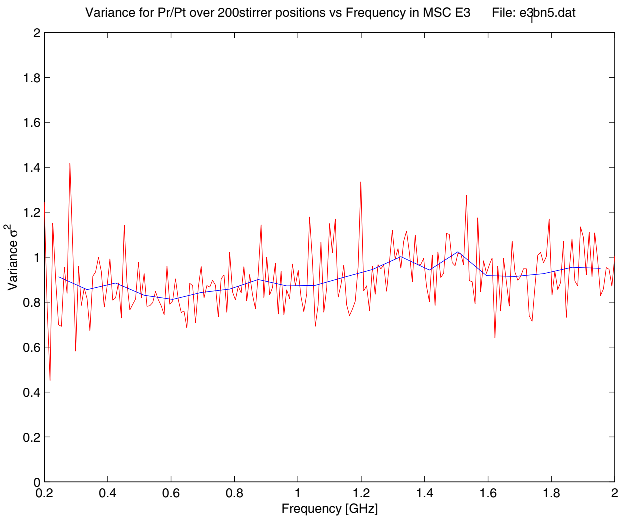


Figure 22. Variance of normalized received power. Large Chamber E3. Large Stirrer N=200

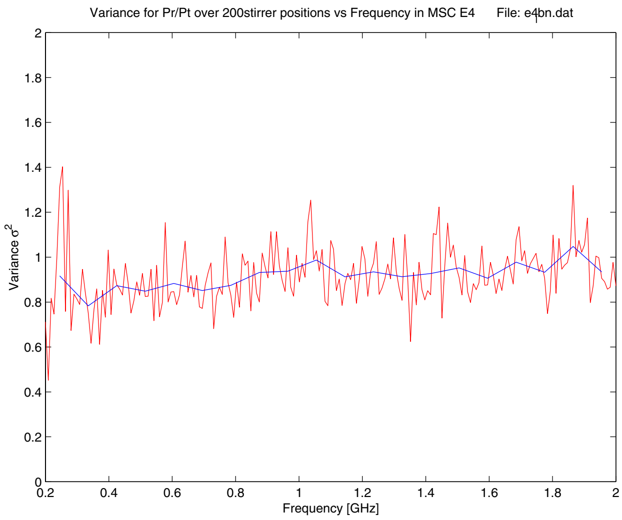


Figure 23. Variance of normalized received power. Small Chamber E4. Large Stirrer N=200

For the RC data there is a small systematic deviation from one at the lower frequency end. Although the variance plots illustrate the RC data we need a more exact, quantitative, approach to evaluate the hypothesis that data are described by a χ^2-distribution with two degrees of freedom. This hypothesis was evaluated by using a Goodness-of-Fit Test, the Chi-Squared Test, [4] on the data.

The percentage probability $\Pr ob_d\left(\tilde{\chi}^2 \geq \tilde{\chi}_0^2\right)$ of obtaining a value of $\tilde{\chi}^2 \geq \tilde{\chi}_0^2$ in a experiment with d degrees of freedom, as a function of d and $\tilde{\chi}_0^2$ can be calculated from the integral [19].

$$\Pr ob_d\left(\tilde{\chi}^2 \geq \tilde{\chi}_0^2\right) = \frac{2}{2^{d/2}\Gamma(d/2)} \int_{\chi_0}^{\infty} \chi^{d-1} e^{-\chi^2/2} d\chi \qquad (2)$$

At each frequency the result is given as a probability, *p*, the *Rejection Significance Level*, yielding the risk that one rejects the assumed distribution even if it should be correct. Normally the hypothesis is rejected if *p* is less than 5 % or 1 %. Rather small differences can be observed in the calculated rejection level for different stirrer configurations and type of stirring. This reflects that the distribution is mainly influenced of the chamber properties and not by the stirrers.

The investigations using goodness of fit tests, supports the assumption that the power density in the chambers has exponential distributions, using 200 stirrer positions, from about 500 MHz and that data is rejected in the test at lower frequencies, see Figure 24 and Figure 25 (cf. *Figure 22* and figure 23). Using 2000 stirrer positions the assumption of exponential distribution is accepted from about 700 MHz, see page A3, A5, A6 and A7 in appendix A. This show that in a goodness-of-fit test one has to specify both the Rejection Significance level and the number of uncorrelated samples.

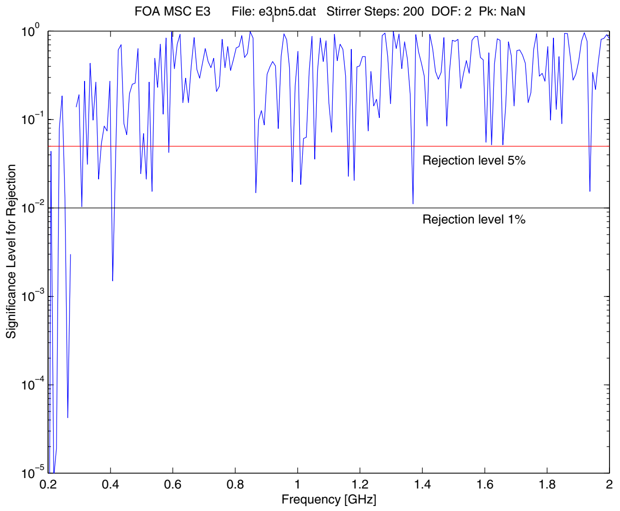


Figure 24. Chi-Squared Test for RC E3, 200 stirrer steps. Large stirrer. Figure shows the 1% and5% Rejection Significance Level.

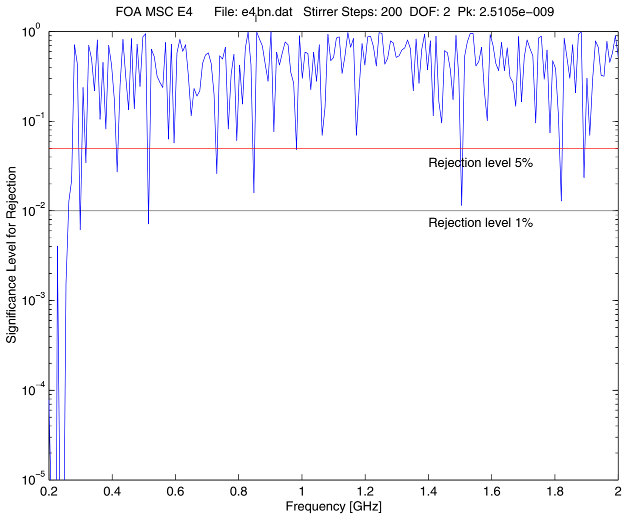


Figure 25. Chi-Squared Test for RC E4, 200 stirrer steps. Large stirrer. Figure shows the 1% and 5% Rejection Significance Level.

Since the goodness-of-fit tests are carried out at many frequencies, some of the *p*-values are expected to fall below e.g., 5 % even if the expected distribution shall not be rejected.

A way to treat *all* the data for the complete frequency range is to calculate a logarithmic sum of all p-values cf. [7], according to:

$$K = -2 \cdot \sum_{i=1}^{N} \ln p_i \qquad (3)$$

where $i = 1$ to N denotes the frequency points and p_i is the above calculated probability of rejection at each frequency. Assuming that the normalized received power is χ^2- distributed at each frequency yields that each p_i is uniformly distributed. From that it can be shown that K is χ^2-distributed with $2N$ degrees of freedom, see appendix D.

Thus, for each value of K a corresponding probability *pk* can be calculated that gives the significance level for rejecting the result for the complete frequency range.

6. Goodness of fit tests of the assumed distribution vs. correlation coefficient < e^{-1} test

For the measured files using 200 stirrer steps and in the frequency range 0.2 – 2 GHz in [1] *pk* was calculated. This was done from a start frequency given by requiring the measured correlation coefficient < 0.37 for 1.8 degrees stirrer interval, to max frequency 2 GHz. Results are given in table 1. The complete frequency intervals from f_{start} to 2 GHz will be accepted as χ^2 distributed with 2 DOF, at 1% significance level, in the goodness of fit test. Thus, we see that, at least in theses cases, uncorrelated data follows a χ^2 distribution /according to a chi square test).

File	frequency where corr. coef. = 0.37 (f_{start})	*pk* calculated from f_{start} to 2 GHz	page in appendix	Chamber	Stirrer
e3lb	578 MHz	0.0940	A2	E3	Basic
e3_lbn5	425 MHz	0.0182	A4	E3	Large
e4lb	677 MHz	0.2310	A8	E4	Basic
e4_lbn	407 MHz	0.4802	A9	E4	Large
e3dual_lbll	371 MHz	0.1003	A6	E3	2 Large

Table 1. Comparison of correlation coefficient test and χ^2 -distribution test. 200 stirrer steps.

7. Goodness of fit test for different number of steps vs. correlation tests.

The tables for the following discussion can be found in appendix B. From measurements in the frequency range 2 – 18 GHz, table B1, it can be seen that the goodness of fit tests will give accepted levels *pk* > 5 % for all stirrer configurations, in both chambers.

For the frequency interval 0.2 to 2 GHz the goodness of fit test will give “accepted” or “rejected” depending on the choice of the number of stirrer positions in one revolution and on the choice of the start frequency.

For the frequency interval f_{start} to 2 GHz, where f_{start} is between 200 MHz and 2 GHz, one example is shown in Figure 27. In this contour plot *pk* is plotted versus f_{start} and versus the number of stirrer positions used in the evaluation. We see that for f_{start} = 200 MHz, i.e. for the interval 200 MHz to 2 GHz, the assumption of having a χ^2 – distribution with 2 DOF is rejected when the number of stirrer positions is greater than around 110. For the interval 0.5 to 2 GHz the assumption is rejected if the number of stirrer position is greater than around 500.

If Figure 27 is compared to the correlation curve in Figure 26 we see that, even for f_{start} = 200 MHz, the assumption of a χ^2 – distribution is accepted if the underlying data come from correlated stirrer positions. We also see that the area of rejection in Figure 27 falls within the correlation region in Figure 26. On the other hand, correlated data can also give “accepted” e.g. f_start = 1.1 GHz and number of stirrer positions > 900. Thus, the criterion $\rho < e^{-1}$ for regarding data as correlated is not contradicted by our results.

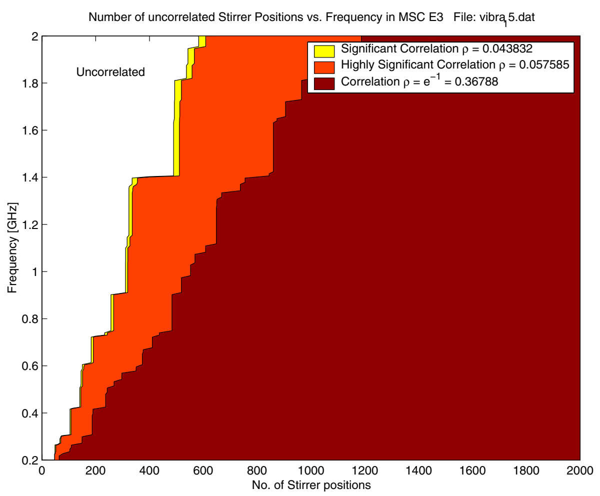


Figure 26. Number of uncorrelated stirrer positions at different frequencies. One stirrer. 2000 steps. Large stirrer.

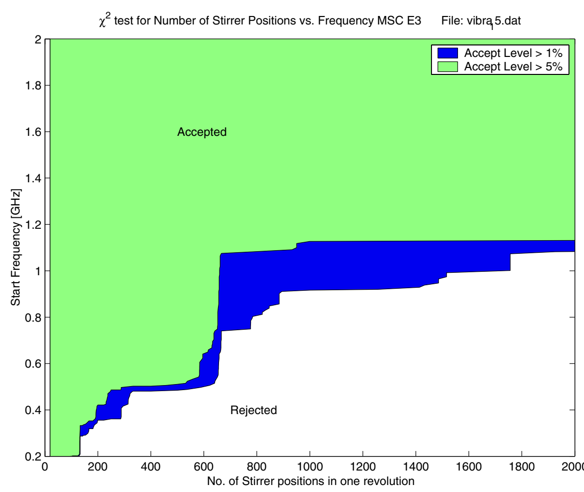


Figure 27. Cumulative Rejection Significance Levels for complete frequency intervals [f_{start}, 2 GHz]. White: $pk \le 1\%$, blue: $1\% < pk < 5\%$, green : $pk \ge 5\%$

However a low correlation coefficient will not necessary give a good χ^2 distribution as can be seen from Figure 28 and Figure 29 where the stirrers have been moved independently in 45 * 45 steps.

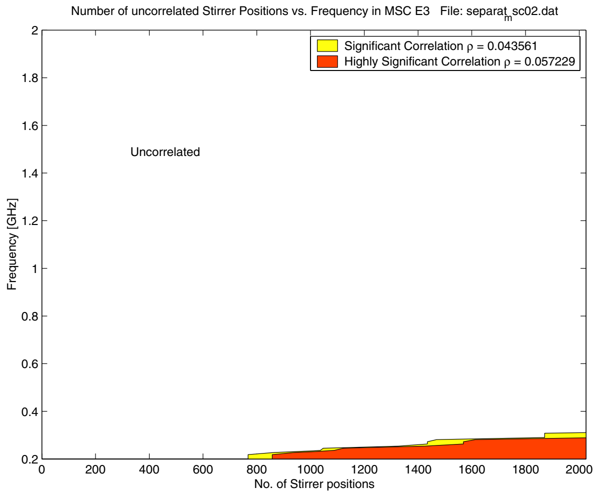


*Figure 28. Number of uncorrelated stirrer positions at different frequencies. Two large stirrers. 45*45 (2025) steps.*

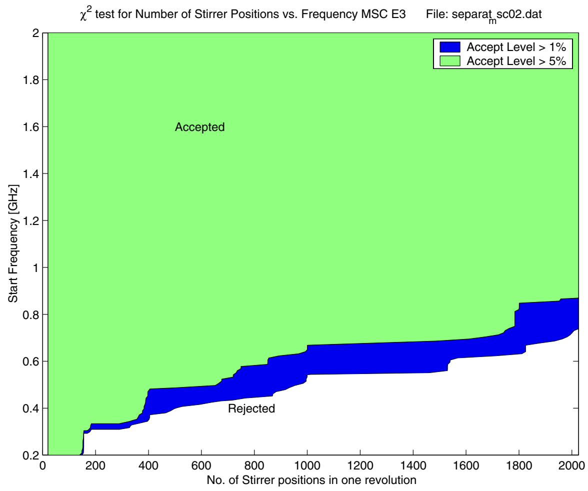


Figure 29. Cumulative Rejection Significance Levels for complete frequency intervals [f_{start}, 2 GHz]. White: pk ≤ 1%, blue: 1%< pk < 5%, green : pk ≥5%

More examples are given in appendix A.

8. Good χ^2 distribution repeated

Will a good χ^2 distribution still be a good χ^2 distribution if data is repeated to intentionally have correlation? To investigate this the file e3_hbn, Figure 30, was repeated to have 400 stirrer positions where of the last 200 are replicas of the 200 first. The correlation coefficient = 1 will occur at the 200:th position as expected, see figure 32. The goodness of fit test will reject data as a χ^2 distribution with 2 DOF if the data is correlated, see figure 31. As the data set now is correlated it will not be statistically independent and shall be rejected by the test according to theory.

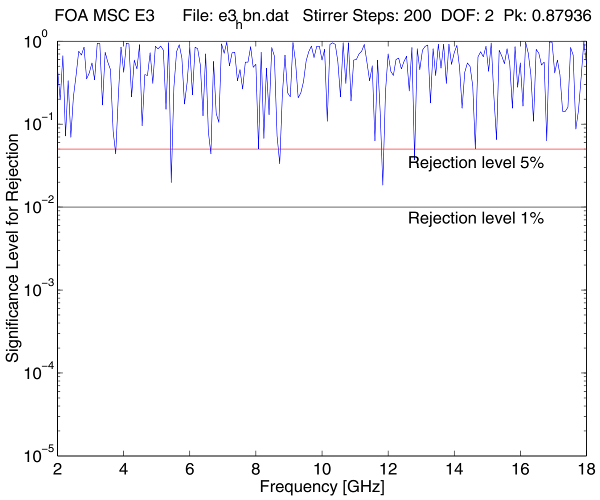


Figure 30. Significance level for rejection. 200 stirrer positions. Large chamber. One Large stirrer.

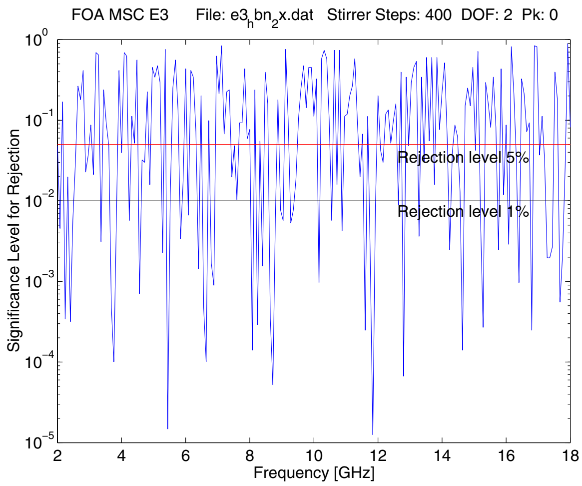


Figure 31. Significance level for rejection for data (as in Figure 30) repeated once.

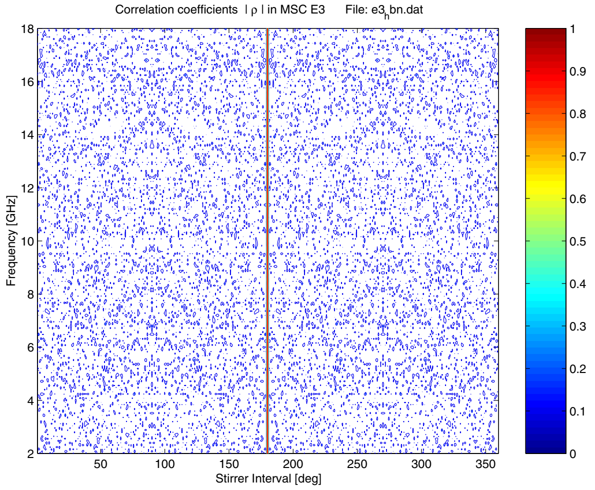


Figure 32. Correlation coefficients for data (as in Figure 30) repeated once.

9. Statistical independence and linear correlation

Here we will present some preliminary results about correlated and statistical independent data. We intend to further investigate and analyze these aspects in the future.

By tradition one usually calculates the linear correlation coefficient for the data. If they are not linearly correlated then one *assumes* that they are statistically independent (e.g. see [13]). It is well known that statistical independence implies uncorrelation but the opposite statement is not true in general, see [16 and 18].

We will investigate the relation between statistical independence and correlation of received power in a reverberation chamber. In short, we will use data that has passed a chi square goodness of fit test for exponential distribution. If the data are independent then a scatter plot of consecutive values should pass a chi-square goodness of fit test for the corresponding product distribution.

Let the array $\{x_1, x_2, \ldots, x_{2n}\}$ be a measured set of data, i.e. the normalized received power in the chamber, Figure 33. We define points in the scatter plot by the set of ordered pairs (x,y), $x= x_i$ and $y= x_{i+1}$ where i = 1, 3, 5 ...2n-1, Figure 34. Further, we will assume that the probability density function for the data is *f*. In case that the samples are statistically independent then the distribution of the scatter plot will be $g(x,y)=f(x)f(y)$, by definition of statistical independence. This is the hypothesis that will be tested in a chi-square goodness of fit test.

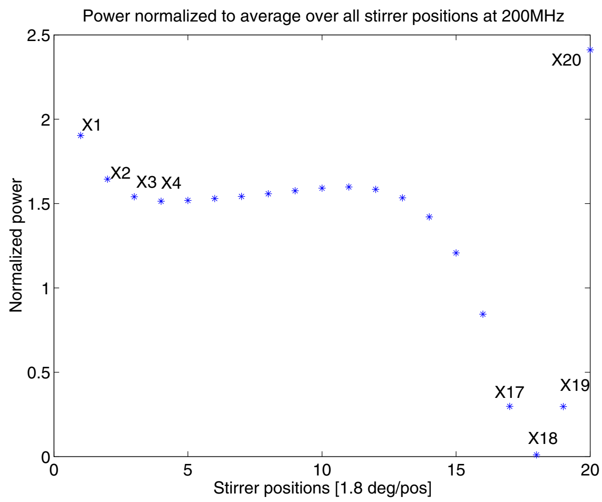


Figure 33. Normalized received power in the large chamber at 200 MHz.

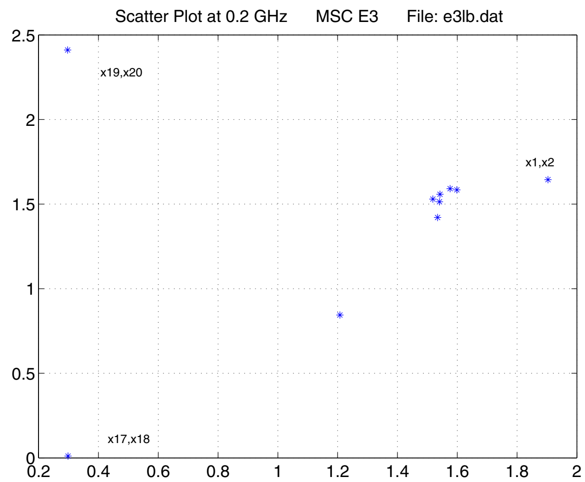


Figure 34. Scatter plot of data in Figure 33.

Two scatter plots of measured data are displayed in Figures 35 and 36, in Figure 35 the linear correlation coefficient is close to one and in Figure 36 it is close to zero.

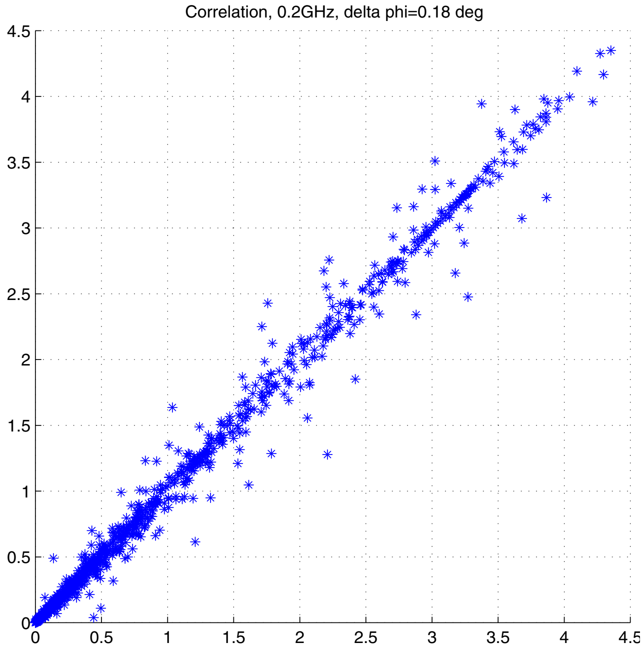


Figure 35. Scatter plot, highly correlated samples. 2000 samples at 200 MHz, single large stirrer. The values on the axis denote normalized received power.

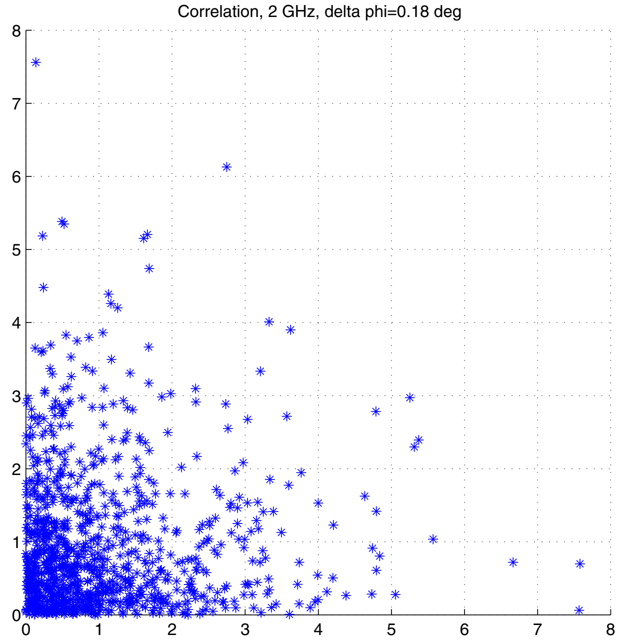


Figure 36. Scatter plot, uncorrelated samples. 2025 samples at 2GHz, dual independent stirrers (45x45 positions).

In the goodness of fit test, the domain x>0, y>0, in the scatter plots, is divided into rectangular sub domains containing expected equal number of samples. It can be shown that if the probability density function f is exponential, i.e. f(x)=exp(-x), and the number of sub-domains is $m^2 + 1$ then the set of points defining the partition of the x and y–axis should consist of the set {-ln(1-sqrt(1/($m^2 + 1$))), -ln(1-2*sqrt(1/($m^2 + 1$))), … , -ln(1-m*sqrt(1/($m^2 + 1$)))}, se Figure 37.

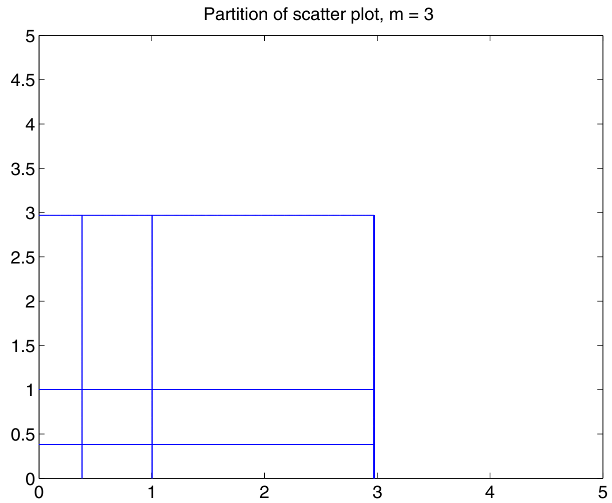


Figure 37. Partition of scatter plot for m=3.

In Figures 38, 41 and 44 correlation is plotted as function of frequency and number of stirrer positions. The derivation of these figures is explained in chapter 4,6 and 8 [13]. In short, the significance of a measured correlation coefficient ρ_0 can be evaluated by calculating the probability that N measurements of two uncorrelated, i.e. $\rho = 0$, variables would give a correlation coefficients, ρ, as large as or larger than ρ_0. This probability is given by eq.(1), cf. [3 and 19]:

Figures 38, 41 and 44 contains the results from the correlation calculations and should be compared with Figures 39, 42 and 45 which are plots of the results from the chi-square goodness of fit test for the probability density distribution g(x,y)= exp(-x)*exp(-y). In Figures 40, 43 and 46 the chi-square goodness of fit test for exponential distributions of the raw data are plotted, cf. [13].

Comparing Figures 39, 42 and 45 with 40, 43 and 46, respectively, we find that not all samples which pass a chi-square goodness of fit test for exponential distribution does pass the corresponding test for statistical independence. This effect is in particular pronounced in the first two cases, i.e. the cases with single stirrer and dual synchronously stepped stirrers. By synchronously stepped we mean that both stirrers are moved simultaneously using the same step angle.

Comparing the correlation results in Figures 38, 41 and 44 with the statistical independence results in Figures 39, 42 and 45 we find that in the two first cases the uncorrelated sets of data, based on eq. (1) are smaller than the statistically independent sets. This indicates that eq. (1) might be too restrictive in the classification of the data. We also observe that the rule to accept data with a correlation coefficient less than e^{-1} should not be used. On the other hand, comparing Figures 44 and 45 we find, for dual stirrers stepped independently, i.e. the samples consist of different combinations of angular positions of the two stirrers, that the uncorrelated set of data, based on eq. (1), overlaps the set accepted as statistically independent. This is probably explained by the fact that the actual distribution for the measured power, in the low frequency region, is not exponential distributed. This indicates that we in this case should use some other distribution function.

We note that the results in Figures 43 and 46 are similar. Since the number of samples are almost the same in these cases the statistical independent test and the correlation results indicates that two independent stepped stirrers will give us a set of data which are much more uncorrelated and statistically independent, even if the set of accepted exponential distributed raw data almost looks the same.

The procedure described above does of course not prove statistical independence but it is one way to reject data, in a controlled way, that most likely are statistically dependent. The price is that we for some limited probability reject data that are independent. It could also be used to calibrate the correlation coefficient evaluation, as an alternative to the rule to reject data that has a correlation coefficient greater that e^{-1}. After all, the correlation coefficient is simpler to calculate.

For now, it seems that the method based on eq. (1) to estimate the number of statistically independent stirrer positions is preferred before the e^{-1} rule since it seems not to accept data that are statistically dependent.

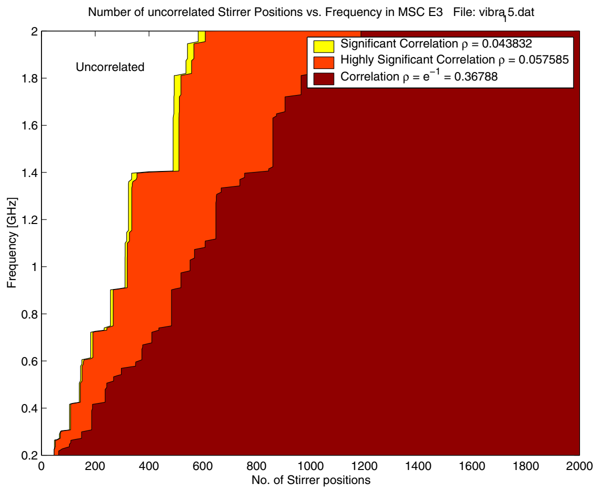


Figure 38. Number of uncorrelated stirrer positions at different frequencies. One large stirrer, 2000 steps.

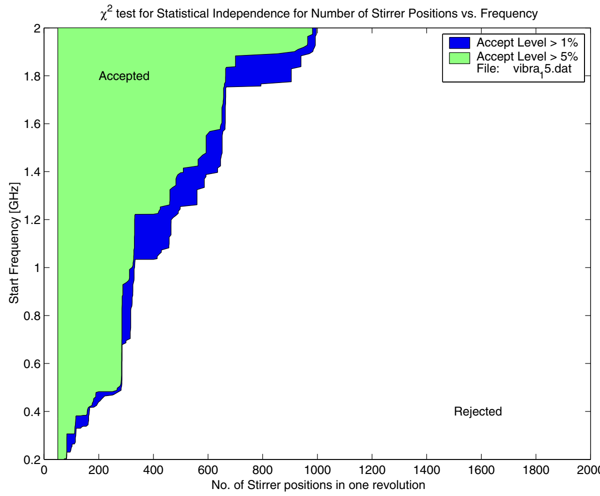


Figure 39. Statistical independence, chi square test result. One large stirrer, 2000 steps.

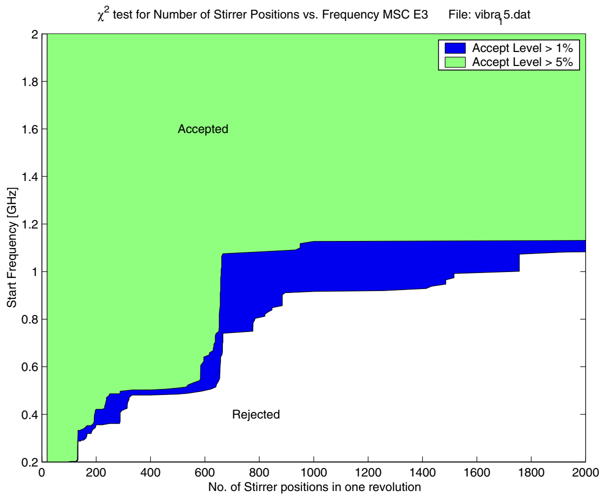


Figure 40. Chi square goodness of fit of exponential distribution. One large stirrer, 2000 steps.

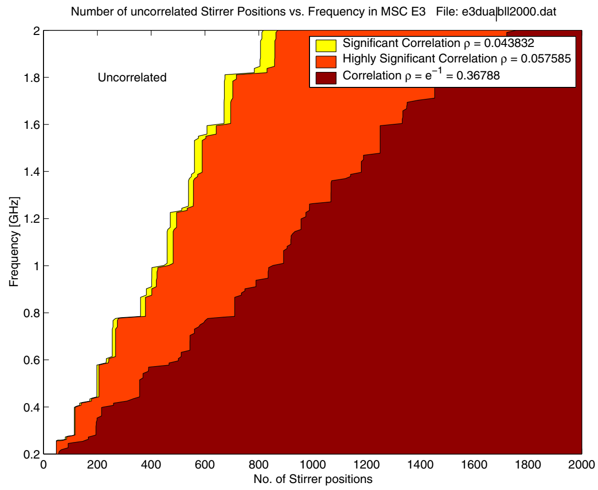


Figure 41. Number of uncorrelated stirrer positions at different frequencies. Two large stirrers stepped synchronously, 2000 steps.

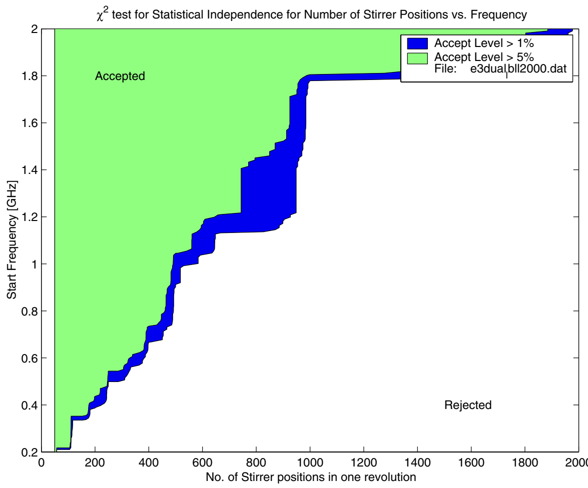


Figure 42. Statistical independence, chi square test result. Two large stirrers stepped synchronously, 2000 steps.

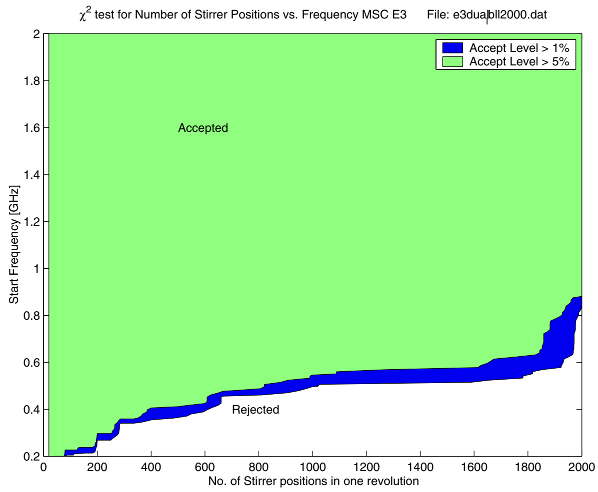


Figure 43. Chi square goodness of fit of exponential distribution. Two large stirrers stepped synchronously, 2000 steps.

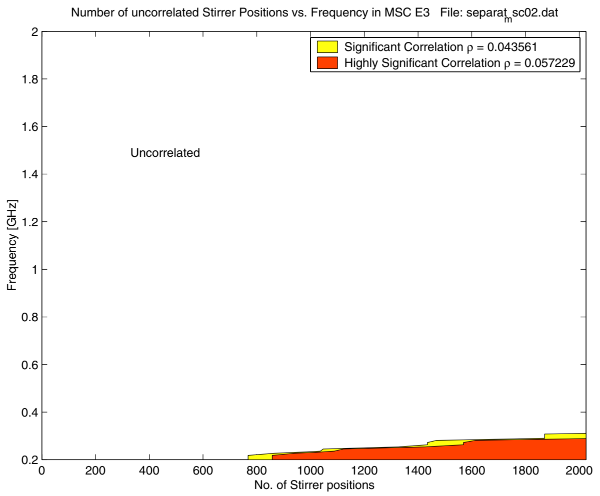


Figure 44. Number of uncorrelated stirrer positions at different frequencies. Two large stirrers stepped independently, 45X45 steps.

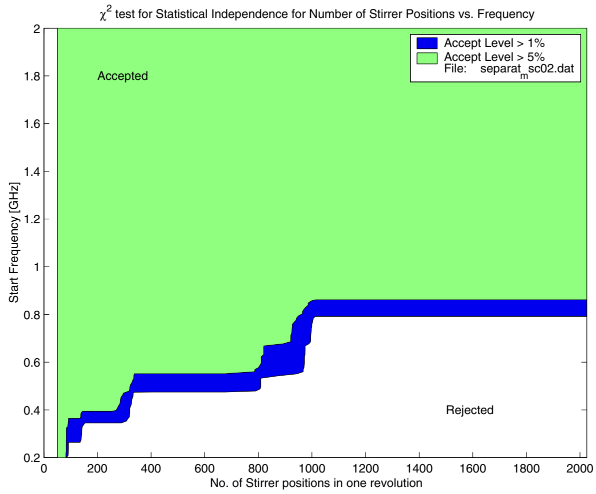


Figure 45. Statistical independence, chi square test result. Two large stirrers stepped independently, 45X45 steps.

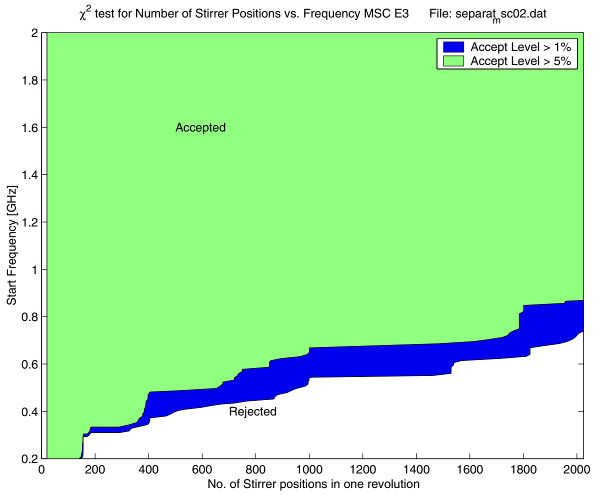


Figure 46. Chi square goodness of fit of exponential distribution. Two large stirrers stepped independently, 45X45 steps.

10. The maximum over mean distribution.

In order to be able to quantify the uncertainty in a test performed in a reverberation chamber (RC) knowledge is needed about the statistical behaviour of the fields in the chamber. Moreover, one has to be sure that the different stirrer positions used in the test correspond to statistically independent field distributions. A typical question when the RC is used for radiated susceptibility testing is: What number of (independent) stirrer positions is needed to state, at a given confidence level, that the *maximum* field level at the equipment under test (EUT) is within ± 3 dB from the *maximum* field detected by the reference antenna?

The maximum over mean distribution of the Insertion Loss P_r/P_t in the FOI reverberation chamber is analyzed below. The experimental results are compared with the theoretical distribution for a chi squared distributed power. For the theoretical distribution the confidence intervals are given as functions of the number of independent measurements for the maximum over mean values. The minimum number of required samples to obtain an acceptable estimate of the maximum over mean is discussed. It turns out that the number of samples should be at least greater than 20.

In immunity testing of electronic equipment in reverberation chambers the equipment under test (EUT) is located in the chamber while the transmitting and reference (calibration) antennas are placed at other positions. The electromagnetic environment for the EUT is varied by changing the frequency of the transmitted field and the boundary conditions in the chamber, which is usually done by one or two stirrers. The purpose of rotating the stirrer is, besides exposing the EUT to different field patterns, to record a sufficient number of statistical independent samples of the field in the chamber at the position for the reference (calibration) antenna. If the numbers of independent samples are large enough then upper and lower bounds for the electromagnetic power stressing the EUT can be given within some prescribed confidence level.

In testing of electronics one is in particular interested in the maximum values of the electric field, i.e. the maximum of a set of statistically independent samples. The distribution of the maximum over mean values has been addressed in [10, 12, 14, 15 and 17]. The results presented in this report can also be found in [21].

Maximum values

In [13] the FOI reverberation chambers were characterised for chi-squared field distribution of P_r/P_t. It turned out that the chamber possesses the property to yield a chi-square distribution as long as the frequency of the transmitted field is high enough. For radiated susceptibility testing it is the maximum power stressing the object under test which is of interest. To be able to make a qualified statement of the maximum powers we take the maximum Insertion Loss P_r/P_t normalized to the mean $<P_r/P_t>$ for a number of uncorrelated samples, i.e. tuner positions. The insertion loss P_r/P_t has been measured in the FOI reverberation chamber at 120 spatial points for 200 uncorrelated stirrer positions at 201 frequencies between 500 MHz and 18 GHz. The maximum values are then extracted for a number of sets containing a specified number of samples, *N*.

In the theoretical model we assume that the average of the samples in each set is close to the expected mean value. This is true for large samples but is obviously not true for small ones. We also use the fact that the insertion loss P_r/P_t for different stirrer positions has passed a goodness of fit test for a chi-square distribution. This means that the normalized received power distribution can be modelled by the following exponential distribution:

$$f(P) = e^{-P} \quad (4)$$

where P stands for the normalized received power [12]. It follows (see [10, 15 and 16]) that the cumulative distribution for the maximum over mean received power of N samples is given by:

$$F_N(P) = (1 - e^{-P})^N \quad (5)$$

In Figure 47 the theoretical eq. (5) and measured cumulative distributions are plotted for N=200. The plot is based on all frequencies and all spatial points. The agreement looks pretty good, but eq. (5) does not pass as the test function on measured data in a chi-square goodness of fit test. In Figure 48 the significance level for rejection is plotted as a function of frequency. Normally, the assumed distribution is rejected if the rejection level is smaller than 1%, or 5%. We note that the lowest frequencies (0.5-1.5 GHz) are rejected in the chi-square test, even at very low levels of rejection and that this is also the case for quite many of the higher frequencies. This illustrates that even if the data passes a chi-square goodness of fit test there is no assurance that the maximum over mean value distribution deduced from a chi-square distribution will pass the corresponding test. This is not surprising since taking the maximum values is a selective process, which could show that the distribution in the tail of the distribution is in disagreement with measurements. After all, we do not expect to get infinite field strengths in the reverberation chamber.

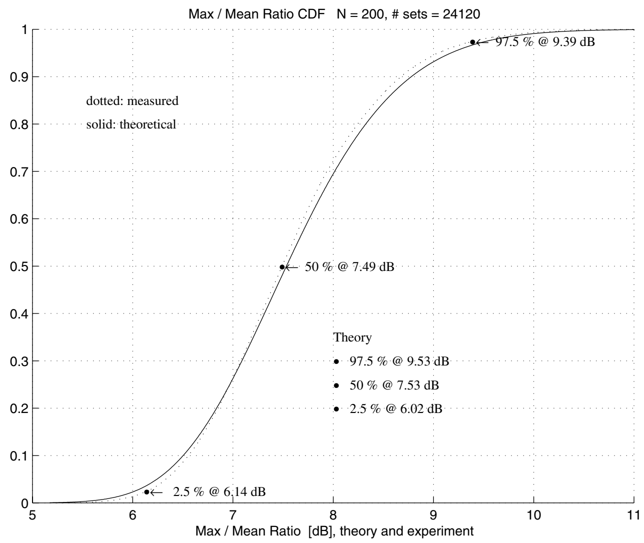


Figure 47. CDF curve of maximum over mean values for measured and theoretical distributions for 200 samples.

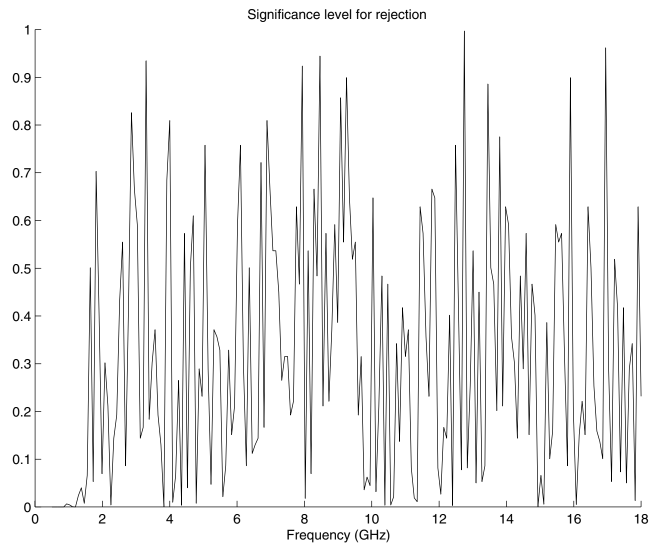


Figure 48. Significance level for rejection, N=200.

Even if the maximum over mean distribution does not fit eq.(5), the theoretical model could be used for estimating confidence intervals. Examining the graphs in figure *47* reveals that the one based on measurements is steeper than the theoretical. Hence, the theoretical curve will give us a slightly wider confidence interval which is neglectable in comparison with the uncertainty. By solving eq. (5) for *P* gives the received power as a function of cumulative probability ($F_N(P)$) and the number of samples, *N*, i.e.

$$P = -\ln(1 - F_N(P)^{1/N}) \qquad (6)$$

Using $F_N(P) = 0.5$ in eq. (6) yields the median maximum over mean received power as a function of *N*. We note that P may take values less than one for small *N* (e.g. for N=2 and $F_N(P) = 0.1$), which is natural since the probability that the maximum of two samples (for instance) is less than the expected power is significant. This is also illustrating that eq. (5) can not be the correct distribution if we divide the maximum in a set by the average value of the same set. The maximum in a set can not be less than the corresponding mean (see also [12]). Another way to illustrate this is by the following example. Let our sample of measurements consist of only two values. If they are equal then the maximum over the mean value will be one, if one of the values are zero then the quotient equals two. All other cases take values in between one and two. This is clearly not consistent with the analysis above, the maximum over the true expected mean can be any value between zero and infinity.

By using eq. (6) we can get an estimate of the confidence interval at any level. For instance, at a 90 percent confidence level the upper and lower bounds are given by:

$P = -\ln(1 - 0.95^{1/N})$ and $P = -\ln(1 - 0.05^{1/N})$,

respectively. We note that

$$\lim_{N\to\infty} P = -\ln(1 - F_N(P)^{1/N}) = \infty \text{ for any } 0 < F_N(P) < 1$$

which once again reflects the unrealistic assumption that the received power is chi-square distributed.
In Figure 49 the confidence bounds are plotted as functions of N for different confidence levels. We note that the width of the confidence intervals are decreasing slowly as N increases. In a linear scale, the limit for the 90% confidence yields:

$$\Delta P = P_{0.95} - P_{0.05} = \ln\left(\frac{(1 - 0.05^{1/N})}{(1 - 0.95^{1/N})}\right) \rightarrow 4.07, N \rightarrow \infty$$

i.e., the length of the confidence interval has a lower limit.

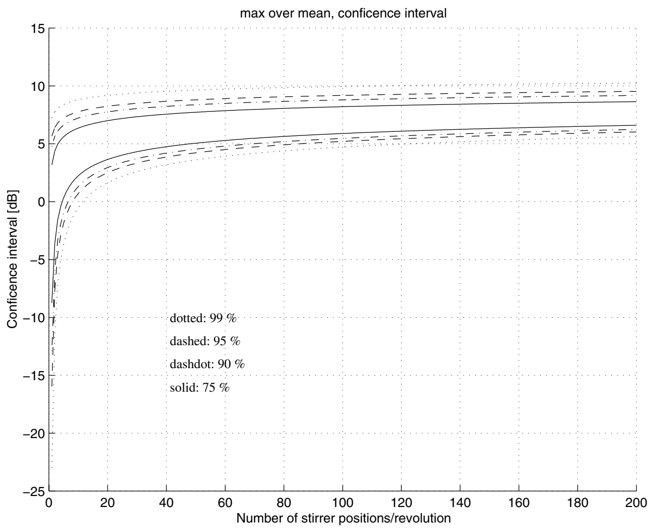


Figure 49. Upper and lower bounds of the maximum over mean values as functions of the number of samples.

This property is explained by the fact that the probability distribution function approaches a distribution with fixed shape that is translated along the axis as N increases. It can be shown that this limit distribution function is given by:

$$f(P) = (e^{-1})^{e^{-P}} e^{-P} \qquad (7)$$

and the corresponding cumulative distribution function is given by

$$F(P) = e^{-e^{-P}} \qquad (8)$$

In a dB-scale, corresponding to Figure *49*, we get the following expression for the 90% confidence interval:

$$\Delta P_{dB} = 10 \lg\left(\frac{\ln(1 - 0.95^{1/N})}{\ln(1 - 0.05^{1/N})}\right) \rightarrow 0, N \rightarrow \infty$$

which is not a contradiction to the previous result, see also discussion in [10].

In this section the distribution of the maximum over the mean value was investigated. The maximum and mean values are taken out of a limited set of measurements. If this set is small then the mean value estimate of the expected mean value usually will be a bad estimate.
We conclude that the cumulative distribution for the maximum over mean value, eq. (5) is a distribution, which does not pass a chi-square test on measured data, but in lack of a better model, it could be used to estimate confidence intervals. It gives slightly larger confidence intervals compared with the true ones. In the small sample case, i.e. $N<20$, one is faced with the problem to estimate the mean value which is crucial for the maximum over mean estimates.

11. Conclusions

A stringent methodology, using significant tests on stirrer correlation and on the assumed statistical distribution, has been used to evaluate reverberation chamber performance. A conclusion is that the requirement of uncorrelated samples was the limiting factor for the low frequency performance for the chamber, except for the case where two mode-stirrers were used. Thus, it is in general necessary to evaluate both correlation coefficients and the expected distribution is followed. Investigations using two mode-stirrers instead of one has shown that this will square the number of uncorrelated positions and halve the lowest useful frequency.

Moreover, it seems that the goodness of fit method based on eq. (1) to estimate the number of statistically independent stirrer positions is preferred before the e^{-1} rule since it seems not to accept data that are statistically dependent.

However, for practical use in testing the method based on eq.(1) might be too laborious. It might also be the case that the statistical distributions for the RC can still be used to estimate measurement uncertainties within an acceptable accuracy although the correlation coefficient $\neq 0$. The choice of a reasonable value of ρ_0,max should thus be based on a combination of statistical analysis and engineering judgements. After all, the choice of ρ_0 *max* is, for the test engineer, a trade-off between the cost of using more stirrer positions and the merit of measurement uncertainty in the test.

We conclude that the cumulative distribution for the maximum over mean value, eq. (5) is a distribution, which does not pass a chi-square test on measured data, but in lack of a better model, it could be used to estimate confidence intervals. It gives slightly larger confidence intervals compared with the true ones.

References

[1]. Lundén O., Jansson L. and Bäckström M., ”Measurements of Stirrer Efficiency in Mode-Stirred Reverberation Chambers”, FOA-R—99-01139-612—SE, May 1999

[2]. Freyer G. J. and Hatfield M. O., “An introduction to Reverberation Chambers for Radiated Emission / Immunity Testing”, ITEM 98, 1998 R&B Enterprises

[3]. Taylor J.R., An Introduction to Error analysis, chapter 9 and 12,2nd ed., University Science Books, Sausalito, California, USA,1997.

[4]. Jansson L., and Bäckström M., “Directivity of Equipment and its Effect on Testing in Mode-Stirred and Anechoic Chambers”, proc. of 1999 IEEE International Symposium on Electromagnetic Compatibility, Seattle, Washington, U.S.A., 2 – 6 Aug. 1999

[5]. Freyer G. J. and Hatfield M. O., Frequency Sampling in Reverberation Chambers, Proc. Of 97 Mode-Stirred, Anechoic Chamber, and OATS User Meeting, April 28 - May 2, 1997, Vail, CO, USA.

[6]. Bäckström M. and Lundén O., Transmission Cross Section of Apertures Measured by Use of Nested Mode-Stirred Chambers, FOA Report FOA-R-96-00359-3.2-SE, FOA Defence Research Establishment, S-581 11 Linköping, Sweden, December 1996.

[7]. D’Agostino R. B. and Stephens M.A., Goodness-of-Fit Techniques, pp.357, Marcel Dekker, Inc., New York, 1986.

[8]. Lundén O and Bäckström M, Design and Characteristics of a New Linked Common Test Aperture MSC, 97 Mode-Stirred Chamber, Anechoic Chamber and OATS Users Meeting, Vail, USA, April 28 - May 2, 1997.

[9]. Statistical Toolbox For Use with Matlab® User’s Guide Version2. February 1999. The MATH WORKS Inc.

[10] J.M. Ladbury, G.H. Koepke and D. Camell, “Evaluation of the NASA Langley Research Center Mode-Stirred Chamber Facility,” NIST Technical Note 1508, 1999.

[11] J. G. Kostas and B. Boverie , “Statistical Model for a Mode-Stirred Chamber”, IEEE Trans. Electromag. Compat., vol. 33, no.4 pp. 366-370, 1991.

[12] J.M. Ladbury and G.H. Koepke, “Reverberation Chamber Relationships: Corrections and Improvements or Three Wrongs Can (almost) Make a Right”, 1999 IEEE International Symposium on EMC, Seattle, WA, August 1999.

[13] O. Lundén and M. Bäckström, “Stirrer Efficiency in FOA Reverberation Chambers. Evaluation of Correlation Coefficients and Chi-Squared Tests.”, 2000 IEEE International Symposium on EMC, Washington D.C., USA, August 2000.

[14] J.M. Ladbury and K. Goldsmith, “Reverberation Chamber Verification Procedures, or, How to Check if Your Chamber Ain’t Broke and Suggestions on How to Fix It if It Is”, 2000 IEEE International Symposium on EMC, Washington D.C., USA, August 2000.

[15] T.H. Lehman and G. J. Freyer, “Characterization of the Maximum Test Level in a Reverberation”, 1997 IEEE International Symposium on EMC, Austin, TX, USA, August 1997.

[16] N. I. Johnson and S. Kotz, “Continuous univariate distributions-1”, Houghton Mifflin Company, Boston, 1970.

[17] G.J. Freyer, T.H. Lehman, J.M. Ladbury, G.H. Koepke and M.O. Hatfield, “Verification of Fields Applied To An EUT in a Reverberation Chamber Using Statistical Theory”, 1998 IEEE International Symposium on EMC, Denver, CO, USA, August 1998.

[18] G. J. Hahn and S. S. Shapiro, “Statistical Models in Engineering”, John Wiley & Sons, Inc., New York, 1994, p.64.

[19] E.M. Pugh and G.H. Winslow, “The Analysis of Physical Measurements”, Addison-Wesley, Reading, Massachusetts, 1966, Section 12-8.

[20] D.L. Mensa, K. Vaccaro, W.T. Yates, “Three-dimensional Imaging from Compact-Range Data”, 20th AMTA Symposium, Montreal, Canada, October 1998.

[21] N. Wellander, O. Lundén and M. Bäckström, “The Maximum Value Distribution in a Reverberation Chamber”, IEEE International Symposium on EMC, Montreal, Canada, August 2001.

[22] K. Rosengren and P-S. Kildal ”Characterization of Antennas for Mobile and Wireless Terminals in Reverberation Chambers : Improved Accuracy by Platform Stirring*”, Microwave and Optical Technology Letters*, Volume 30, number 6. September 20, 2001.

Appendix A. Results

This appendix contains results of the single Basic, Large and dual Large stirrer efficiency measurements performed in the large chamber E3 and the small chamber E4. The measurements were made using different equally spaced stirrer positions per revolution.

The results of each measurement have been plotted in 8 figures (when applicable) according to table A1 below.

Correlation coefficient vs. frequency and stirrer increment, in a colormap-plot. (Partial data.)	Number of statistical independent stirrer positions vs. frequency. *
Significance level for rejection	Variance for insertion loss Pr/Pt vs. frequency.
Number of uncorrelated stirrer positions vs. frequency. *	Pot for accepted / rejected data in χ^2 test *

* These figures have been subjected to noise reduction for clarity, see appendix F

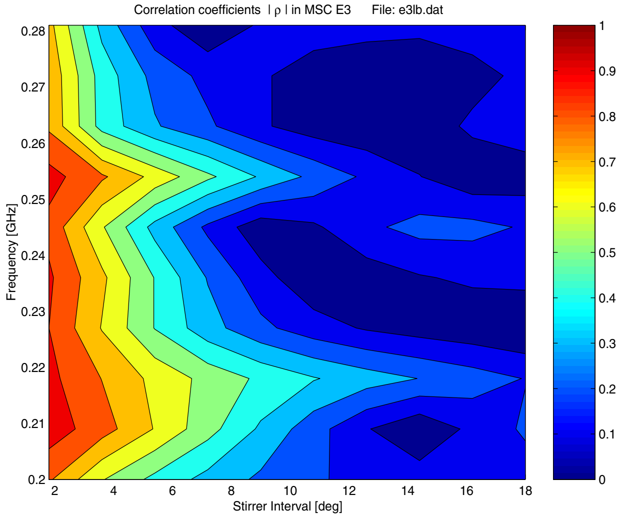
Table A1

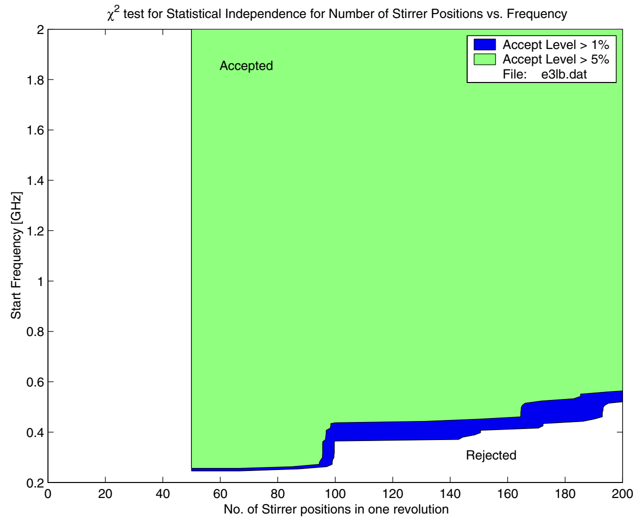
Below in table A2 is a summary of the measured files. Each file is measured using 201 frequency points.

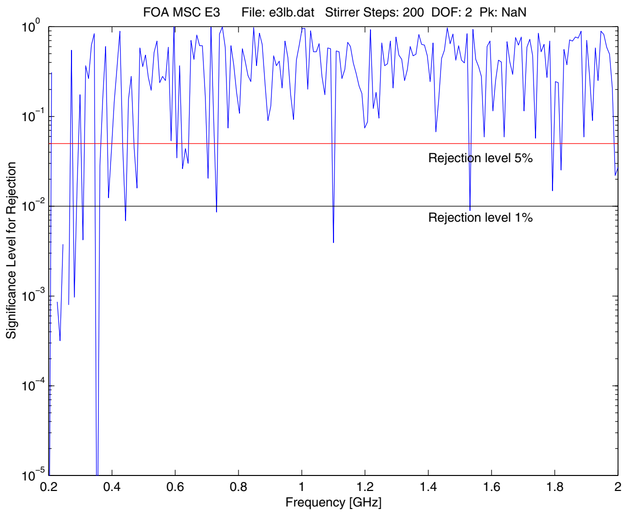
Page	Filename	Stirrer pos.	Stirrer	Frequency	Chamber
A2	e3lb	200	BS	.2 – 2	E3
A3	e3_lb_2kbs	2000	BS	.2 – 2	E3
A4	e3_lbn5	200	LS	.2 – 2	E3
A5	vibra_15	2000	LS	.2 – 2	E3
A6	e3dual_lbll2000	2000	LS LS	.2 – 2	E3
A7	separat_msc02	2025	LS LS	.2 – 2	E3
A8	e4lb	200	BS	.2 – 2	E4
A9	e4_lbn	200	LS	.2 – 2	E4
A10	e3dual_lbll	200	LSLS	.2 – 2	E3
A11	separat_msc01	196	LSLS	.2 – 2	E3
A12	e3hb	200	BS	2 – 18	E3
A13	e3_hbn	200	LS	2 – 18	E3
A14	e4hb	200	BS	2 – 18	E4
A15	e4_hbn	200	LS	2 – 18	E4
A16	e3_hb_2k	2000	LS	2 – 18	E3
A17	e3_hb_2kbs	2000	BS	2 – 18	E3
A18	e3_hb_ls1414	196	LSLS	2 – 18	E3
A19	e3_hb_ls4545	2025	LSLS	2 – 18	E3

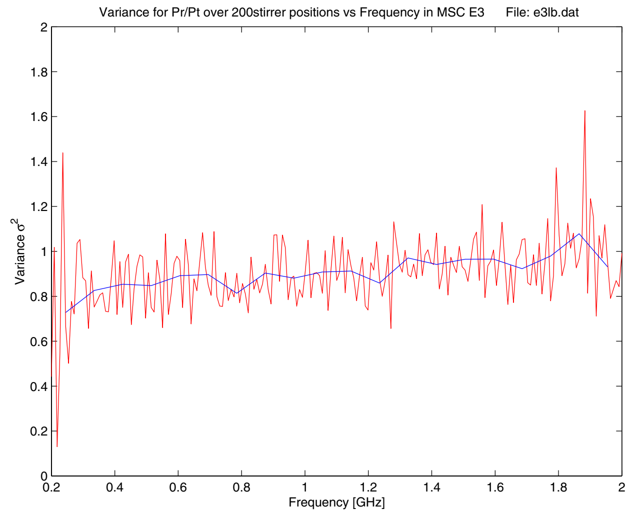
File: e3lb
Stirrer: Basic
Chamber: Large
Stir: 200

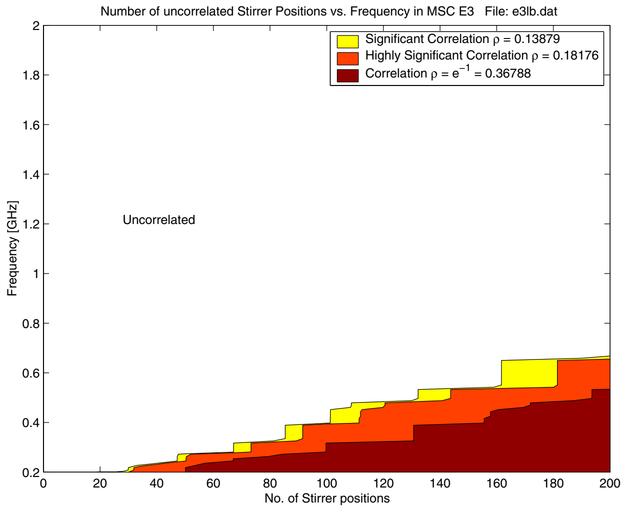
A2

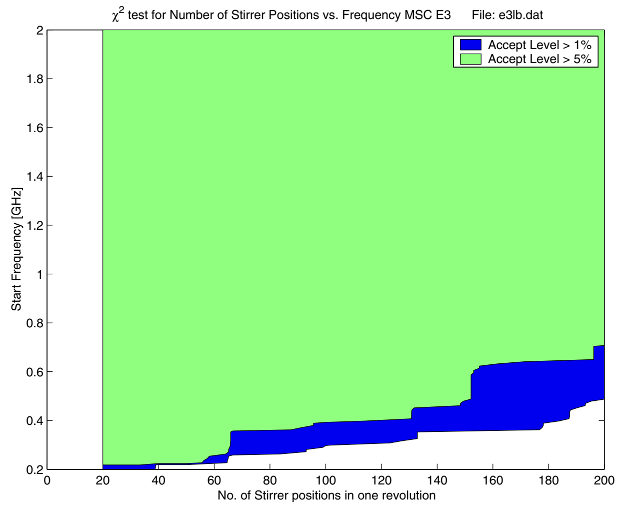












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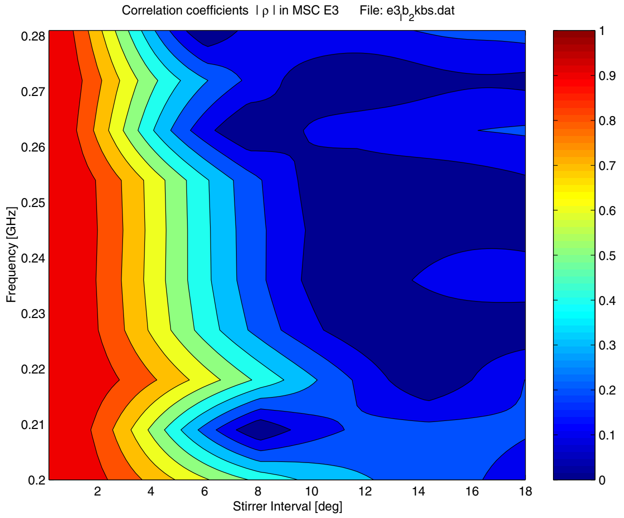
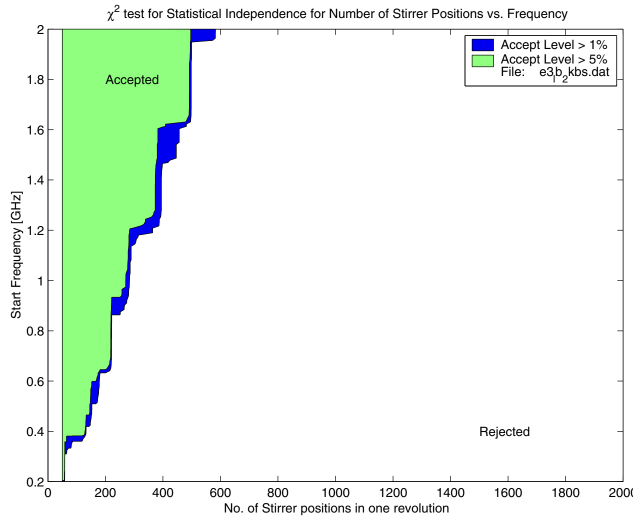
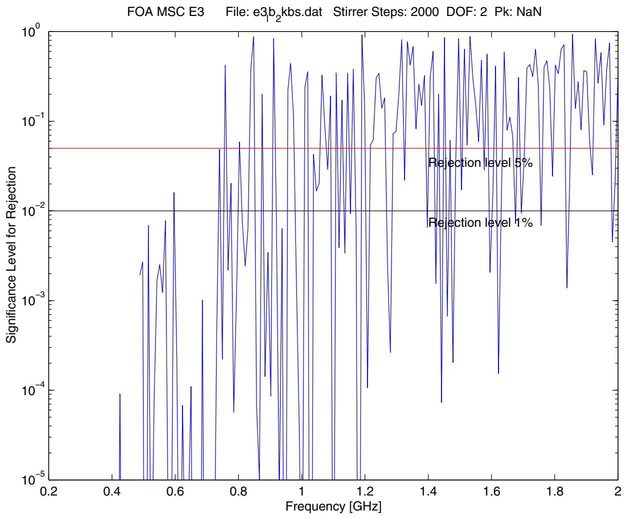
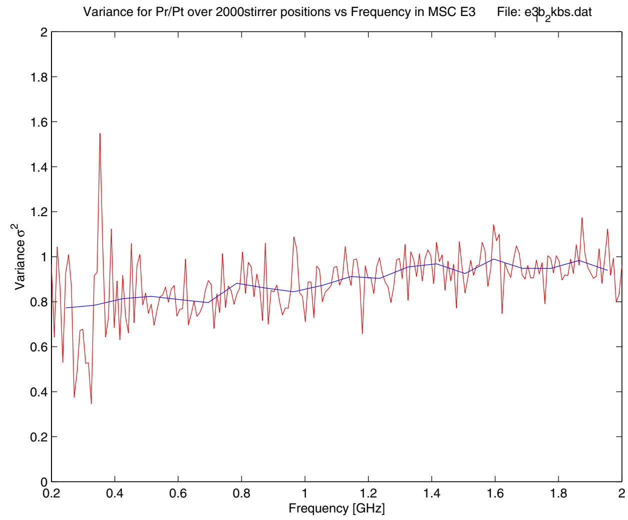
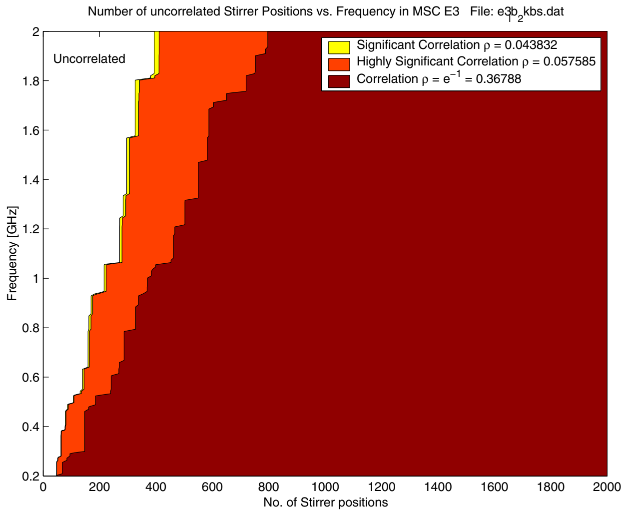
Stirrer: Basic
Chamber: Large
Stir: 2000

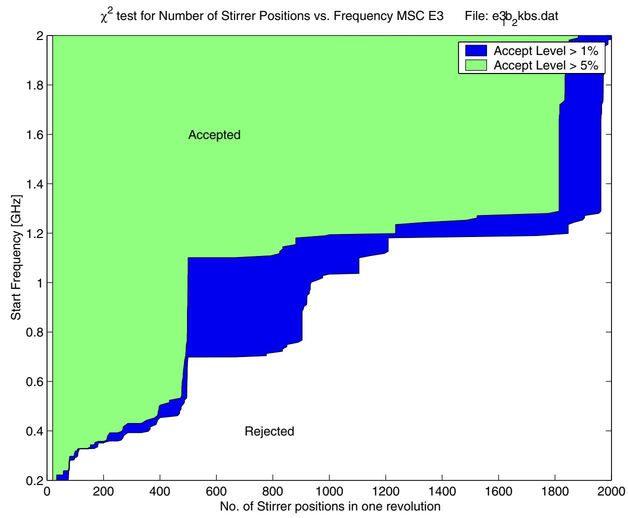












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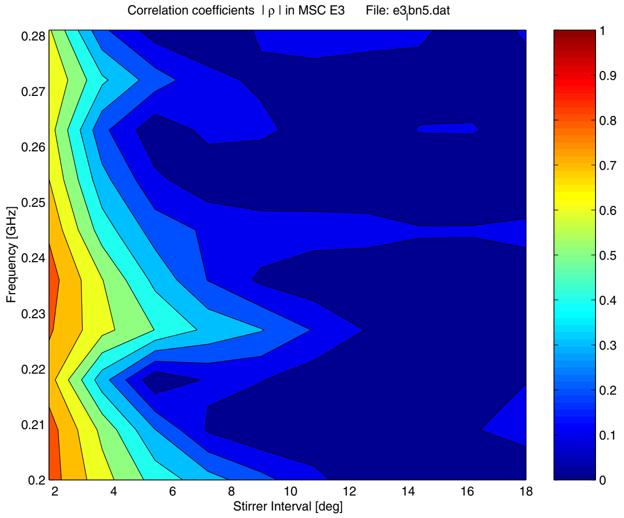
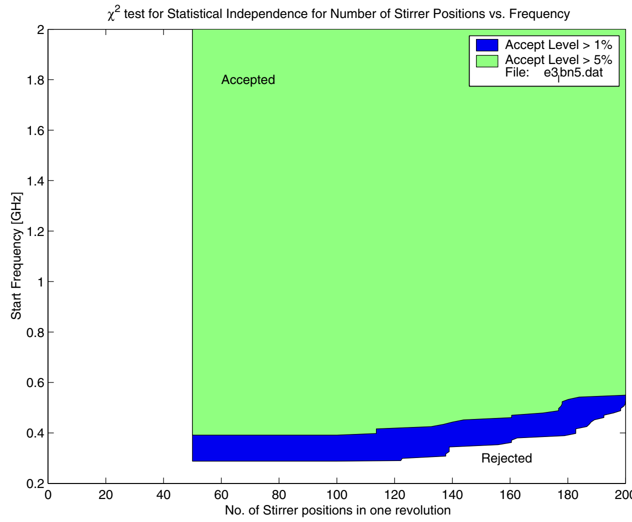
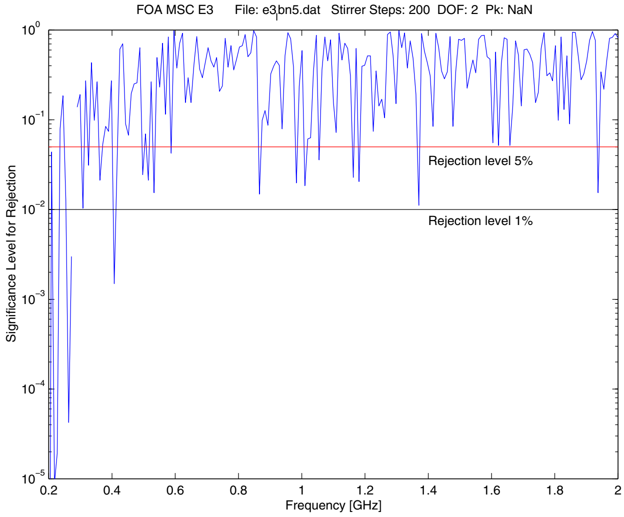
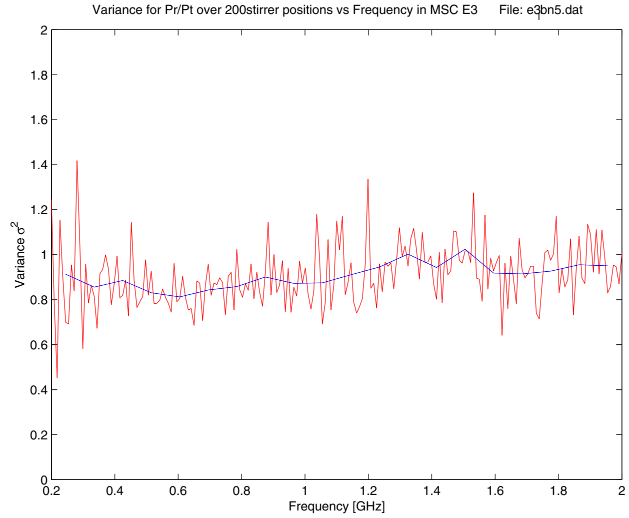
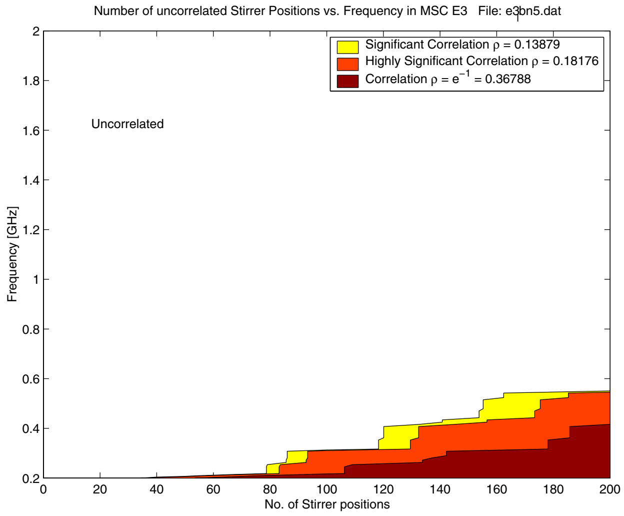
Stirrer: Large
Chamber: Large
Stir: 200

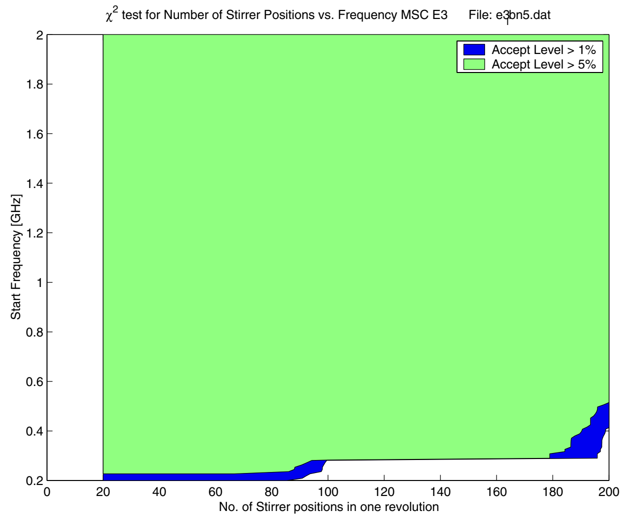




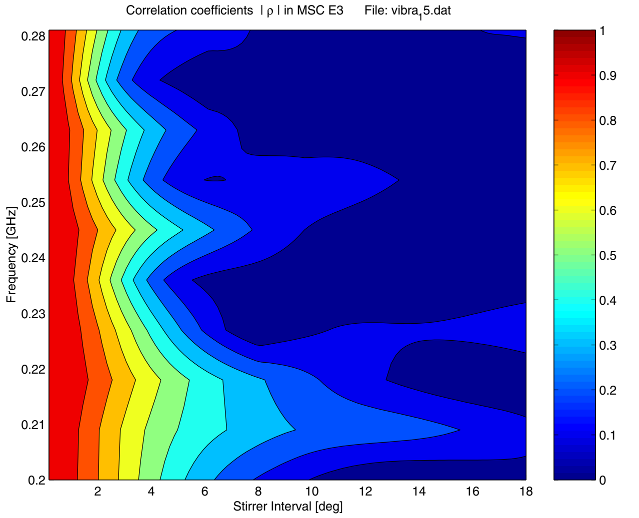


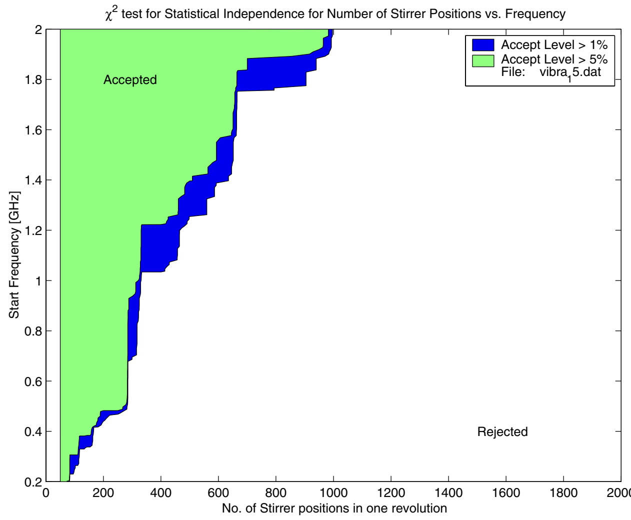


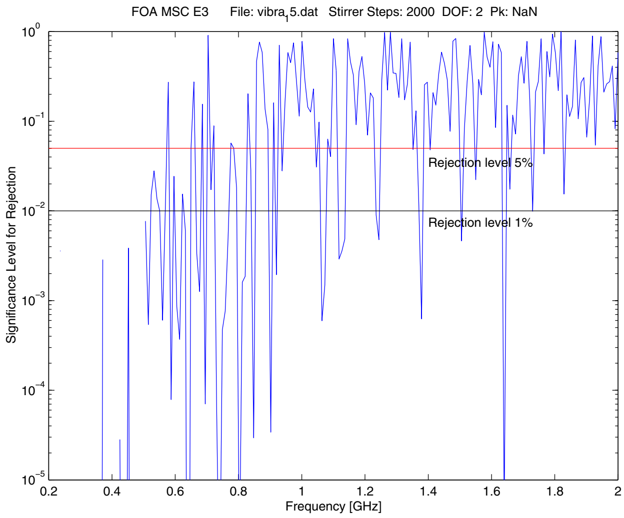


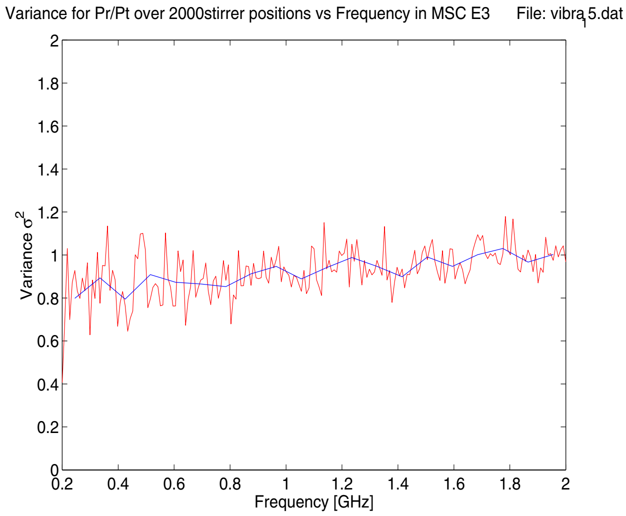


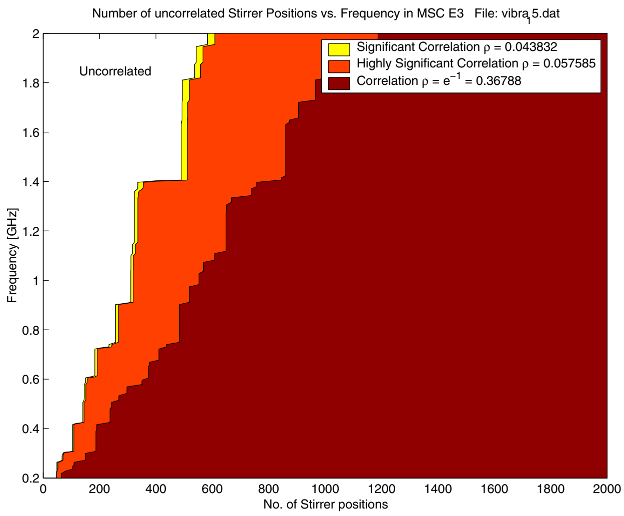
File: vibra_15 A5
Stirrer: Large
Chamber: Large
Stir: 2000

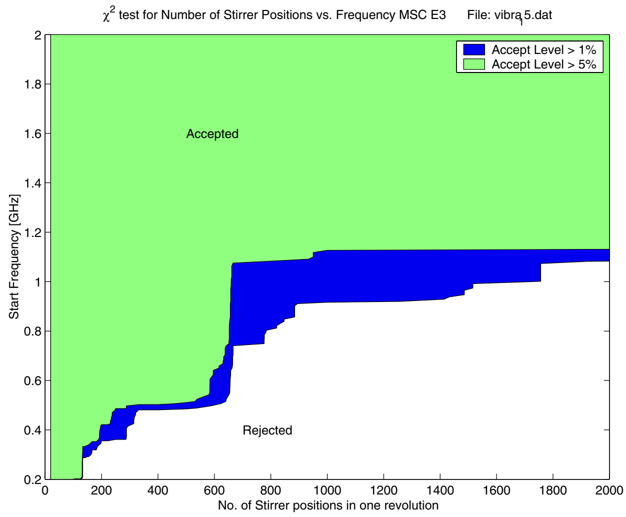










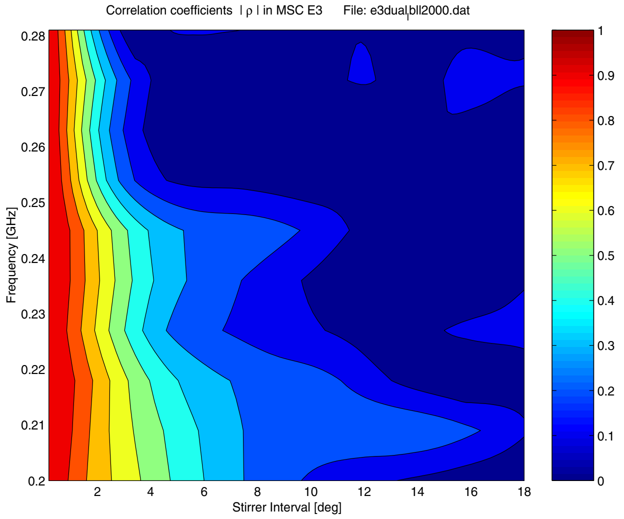


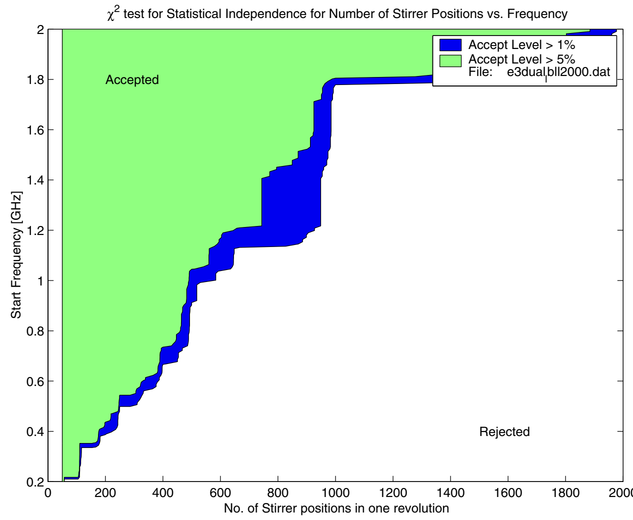
File: e3dual_lbll2000 A6

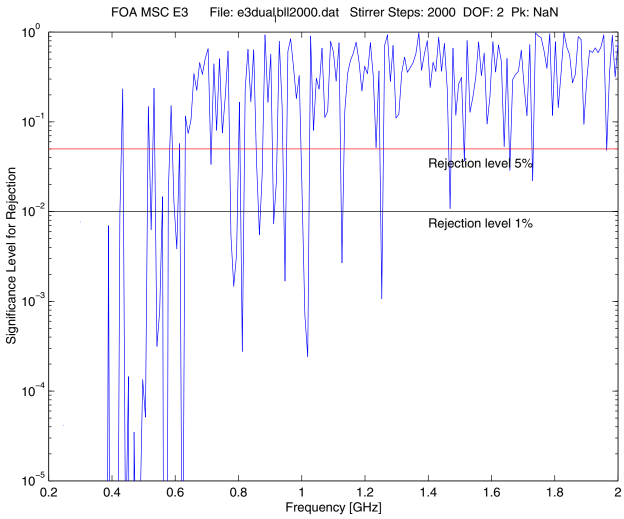
Stirrer: Large Large Synchronously

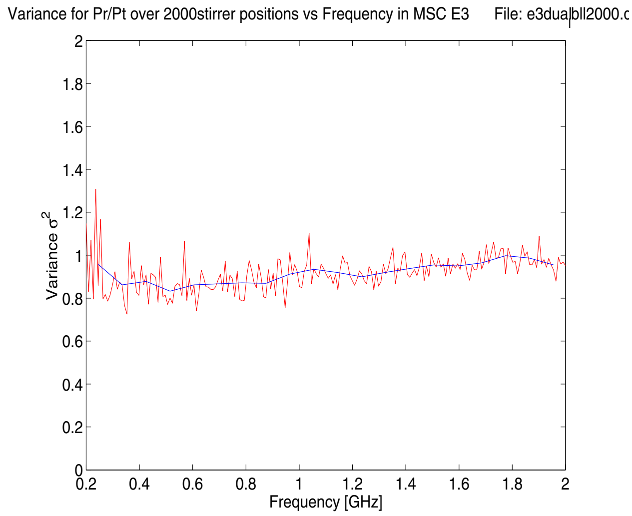
Chamber: Large

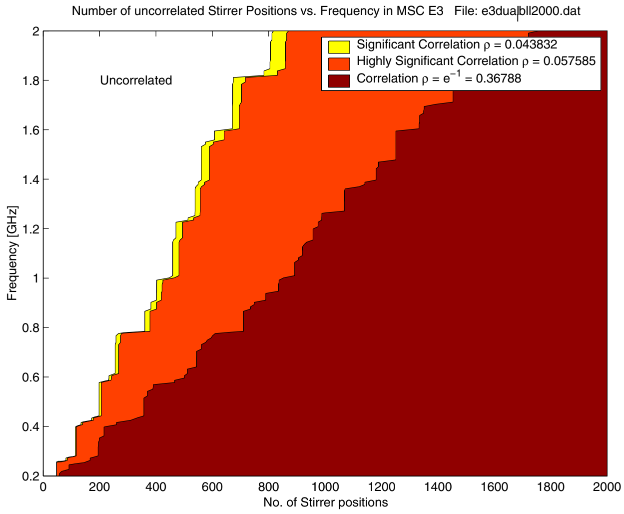
Stir: 2000

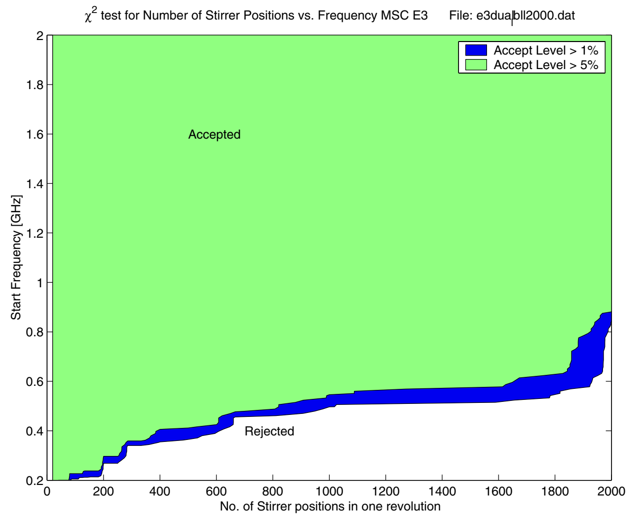




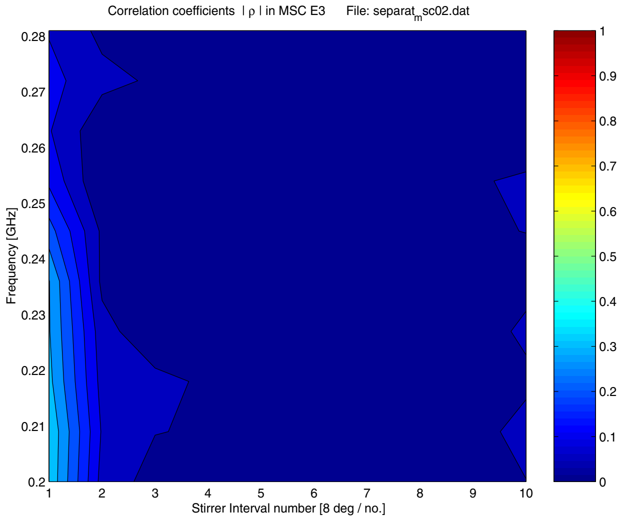


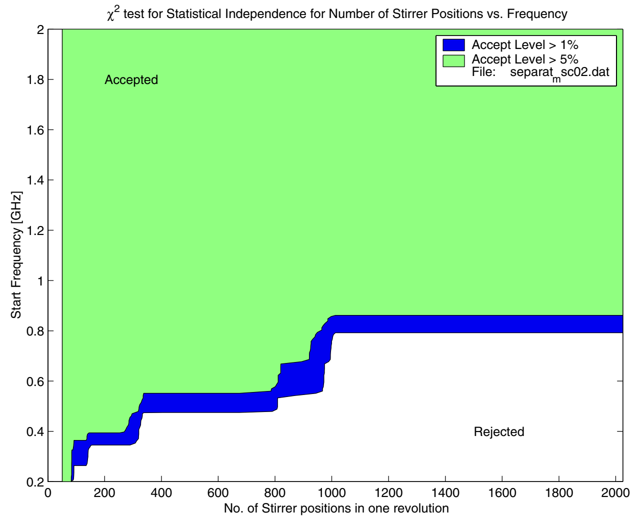


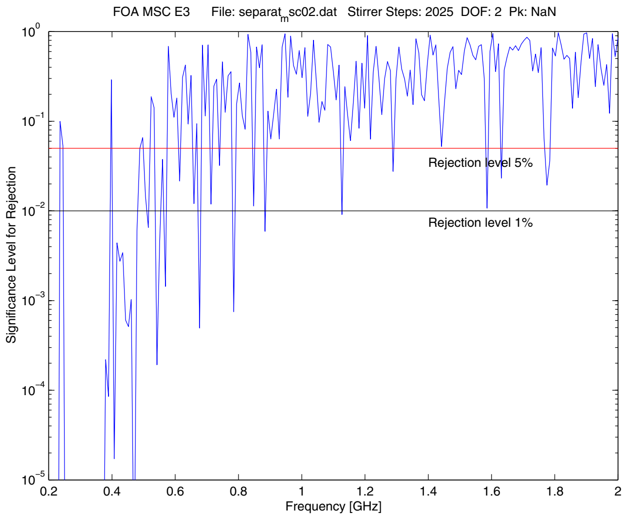


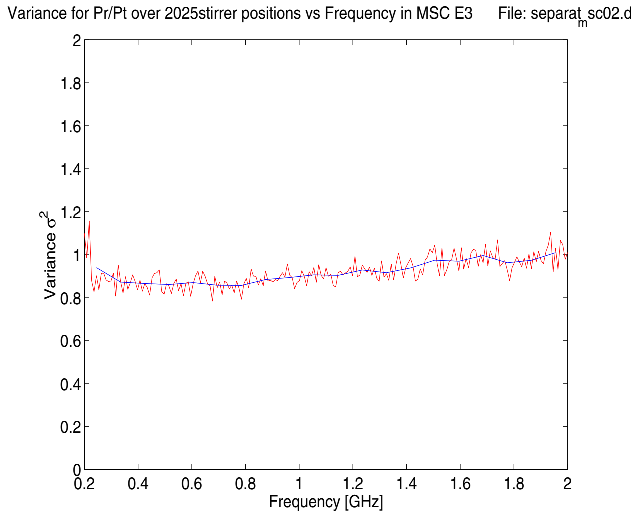


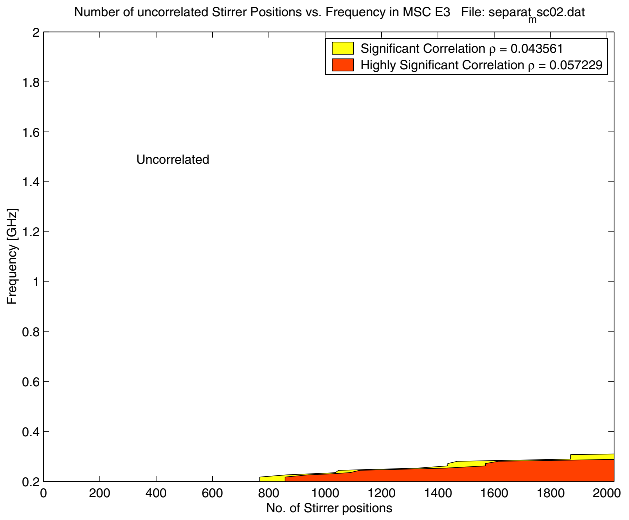
File: separat_msc02 A7
Stirrer: Large Large Independently
Chamber: Large
Stir: 2 * 45 = 2025

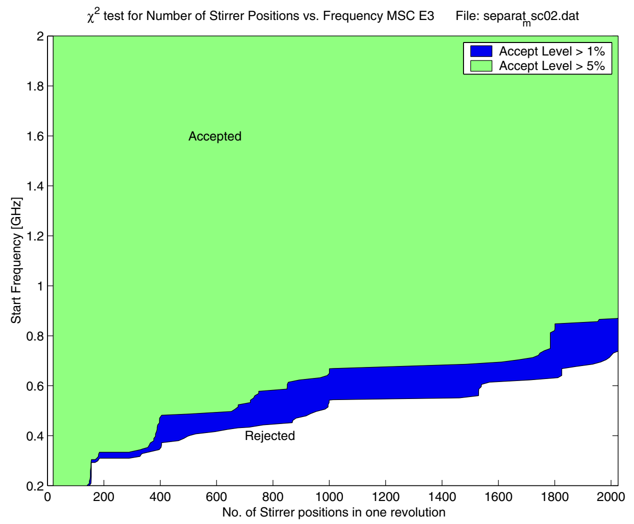






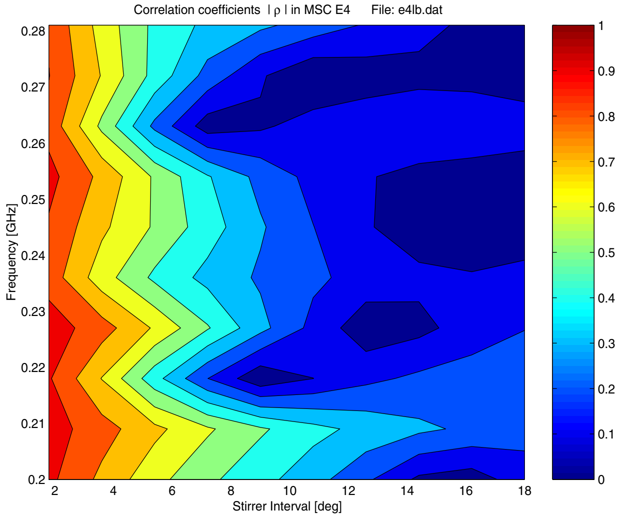


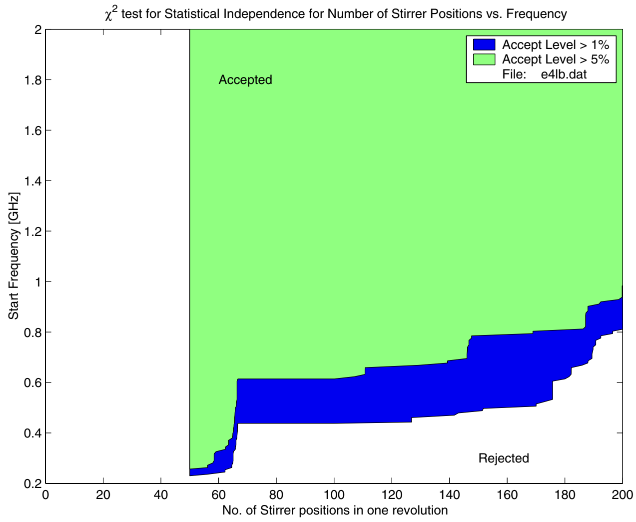


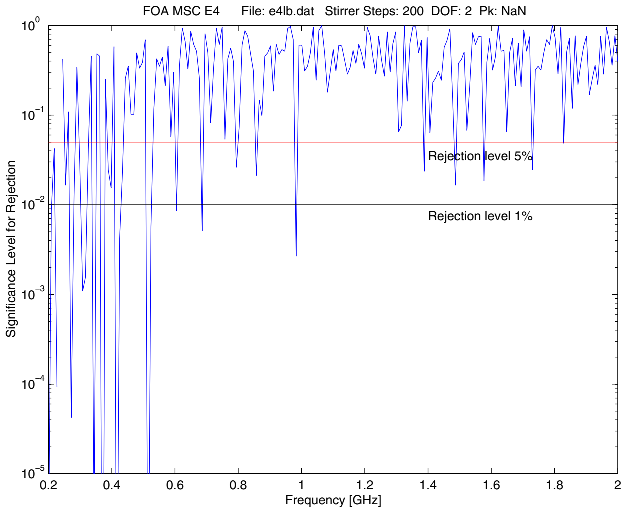


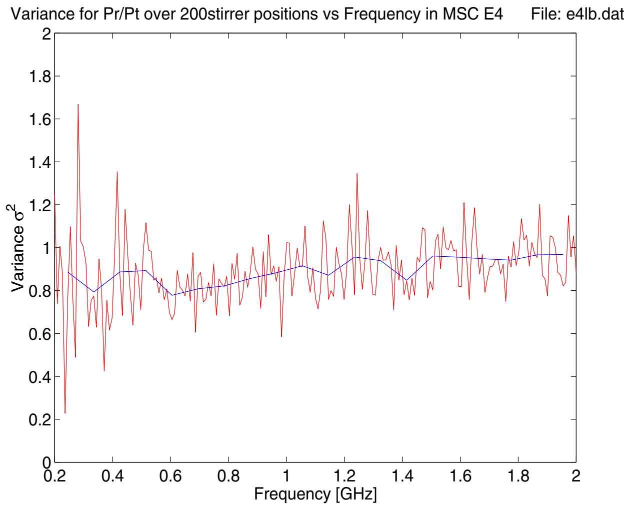
File: e4lb
Stirrer: Basic
Chamber: Small
Stir: 200

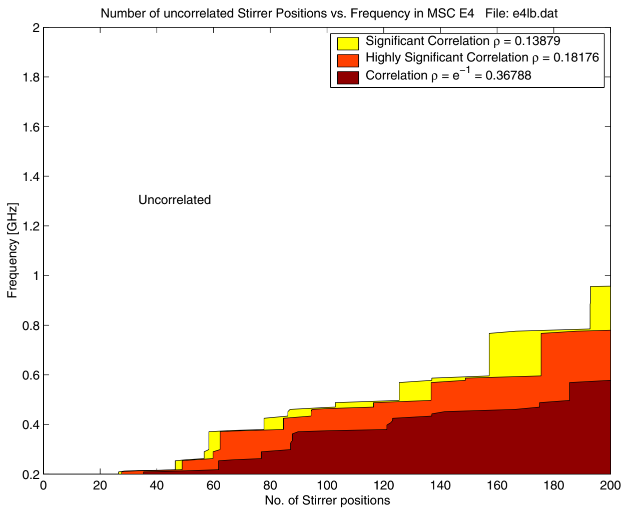
A8

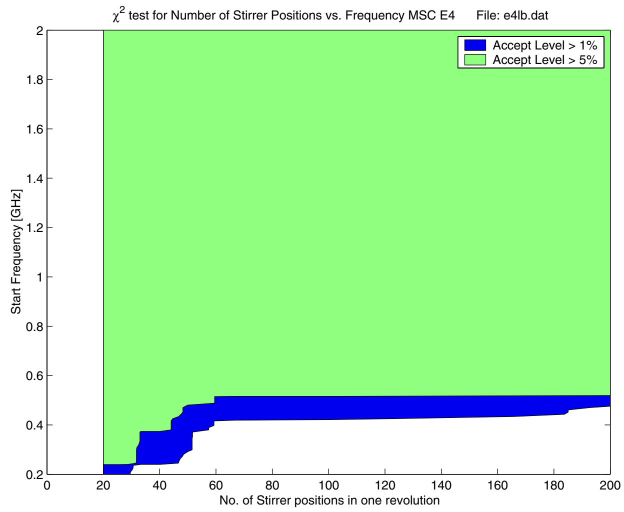












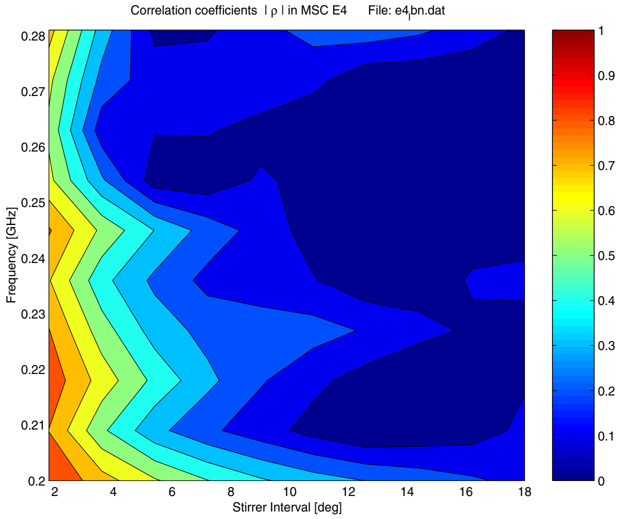
File: e4_lbn

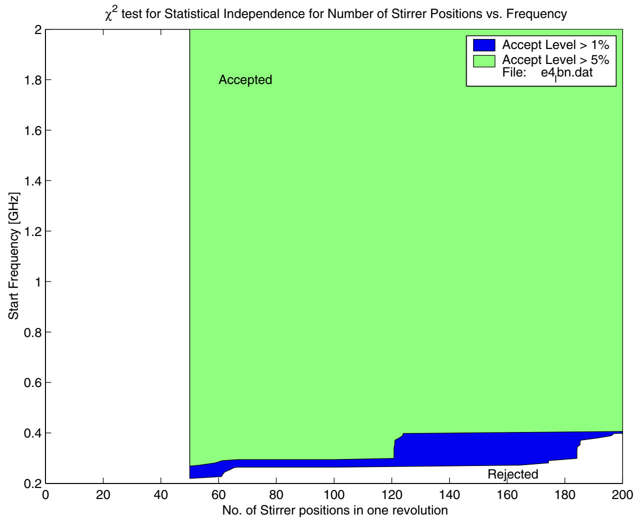
Stirrer: Large

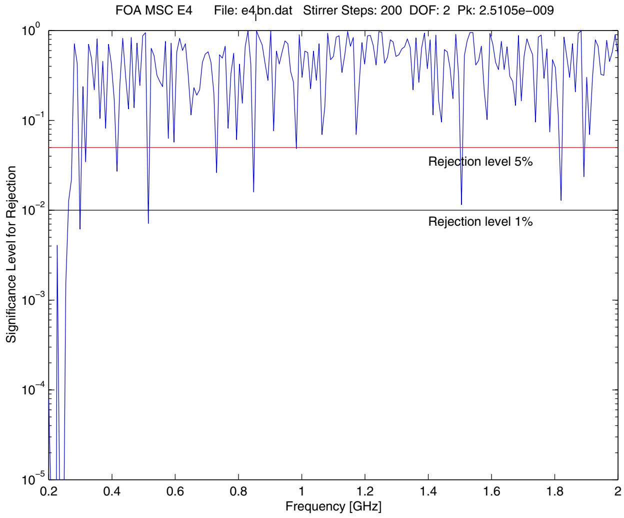
Chamber: Small

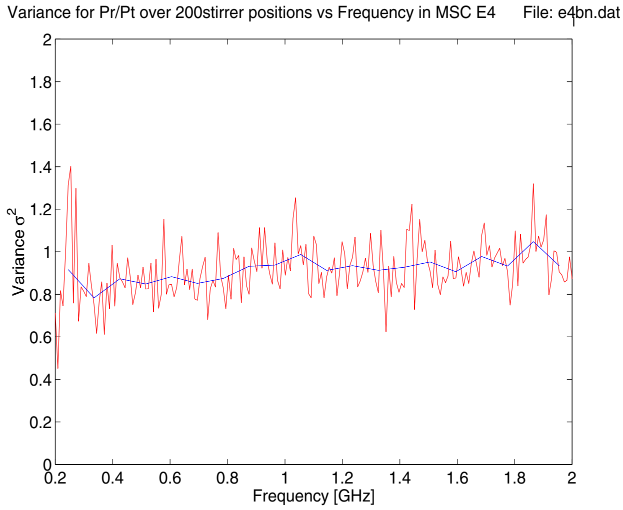
Stir: 200

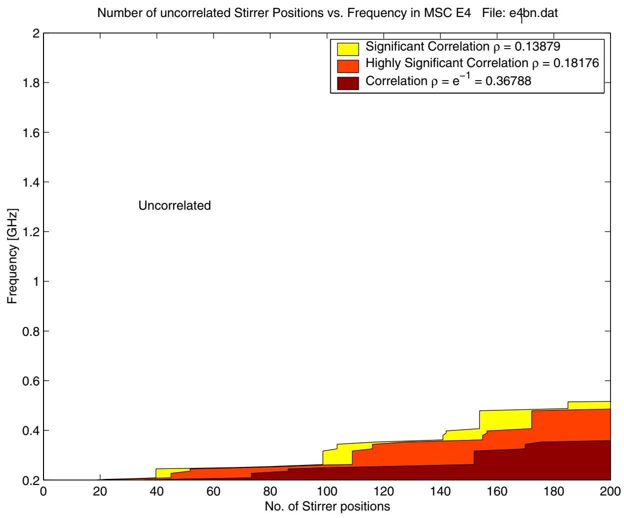
A9

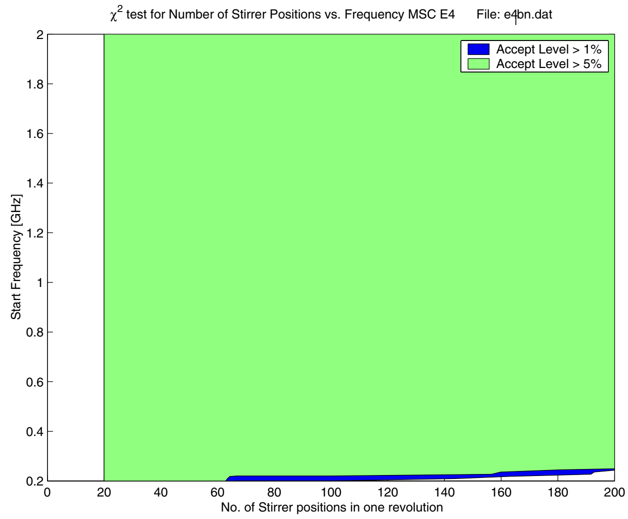










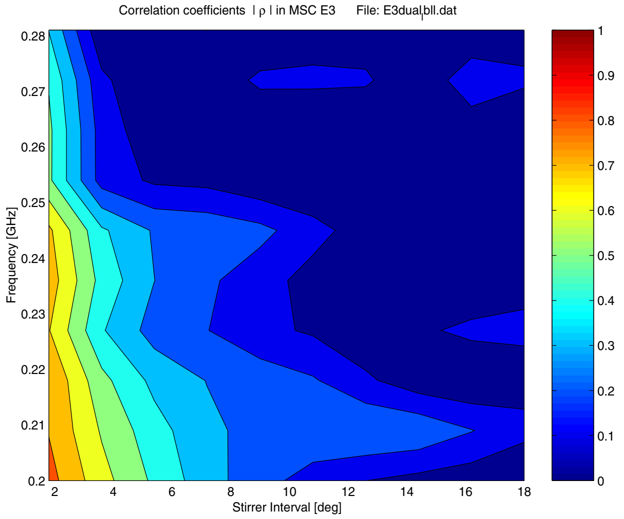


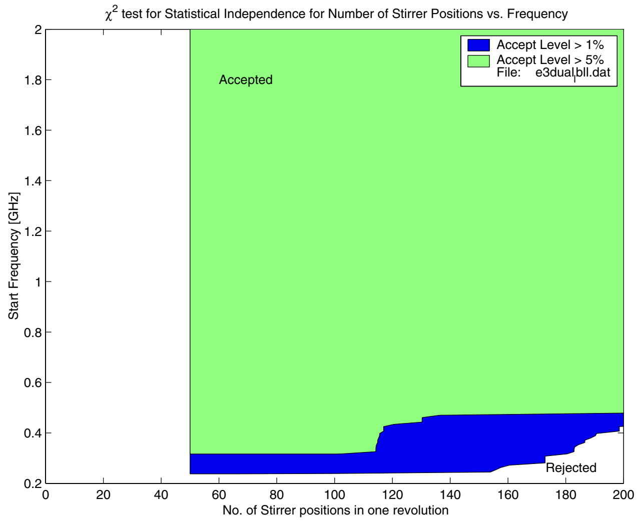
File: e3dual_lbll A10

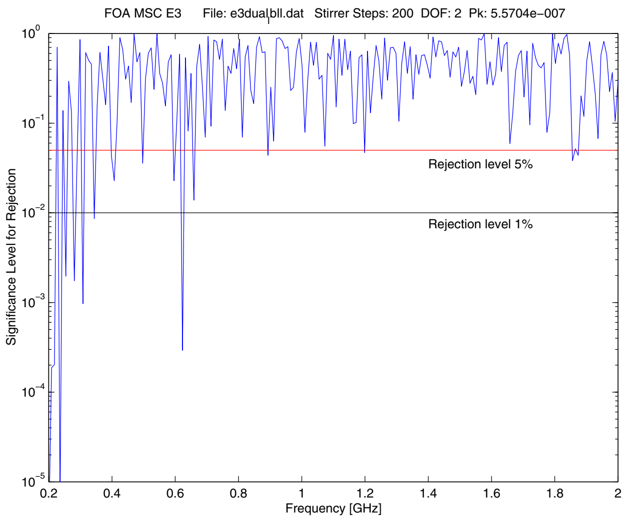
Stirrer: Large + Large Synchronously

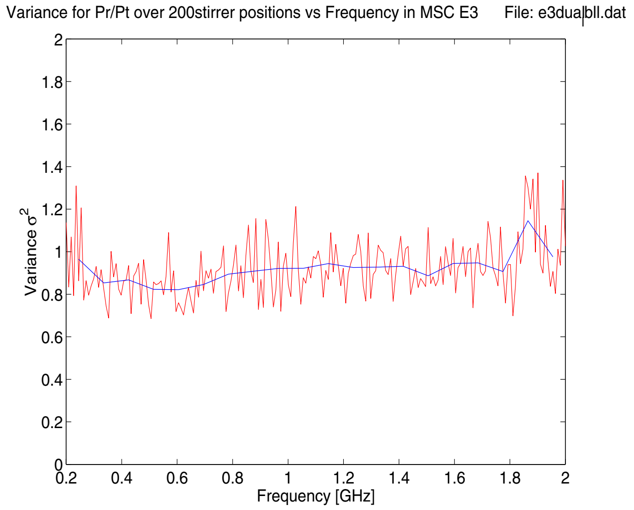
Chamber: Large

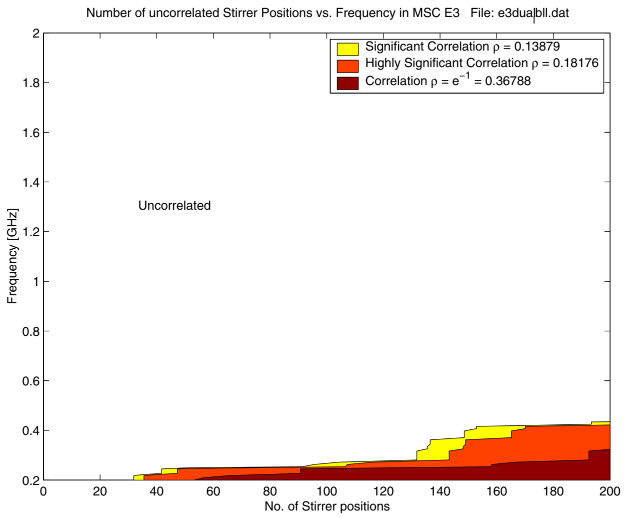
Stir: 200

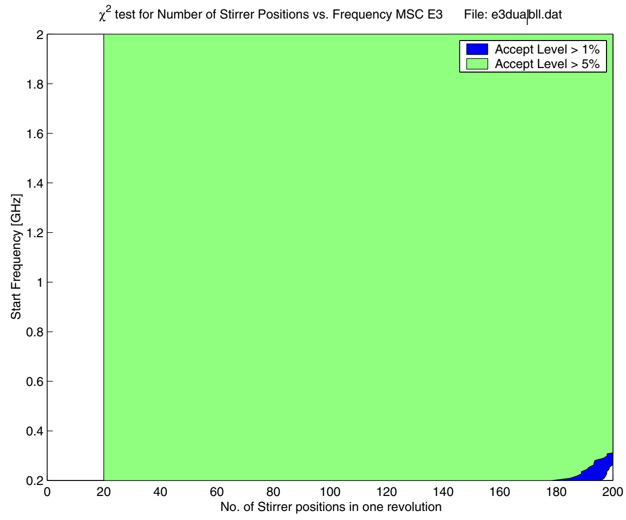










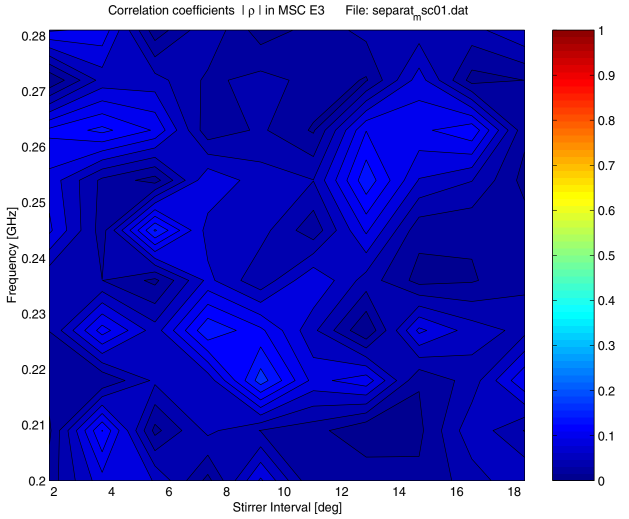


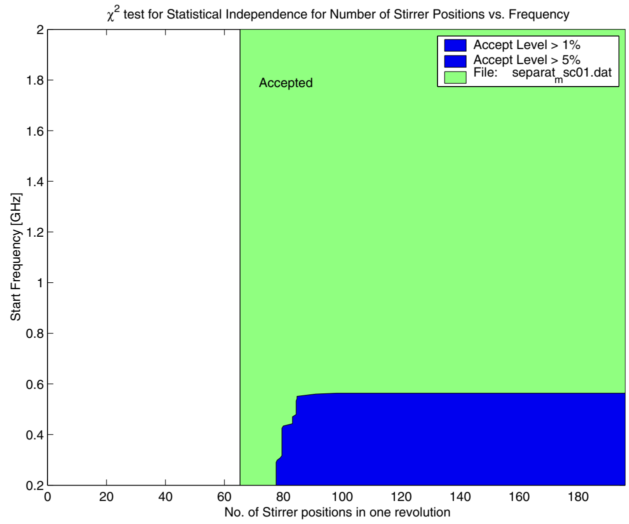
File: separat_msc01 A11

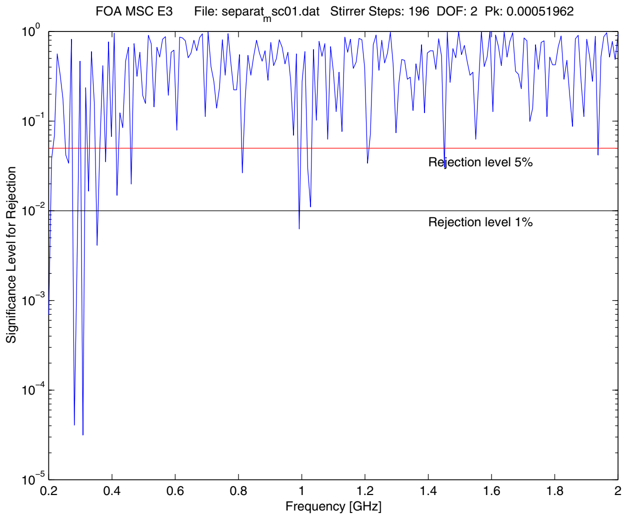
Stirrer: Large + Large Independently

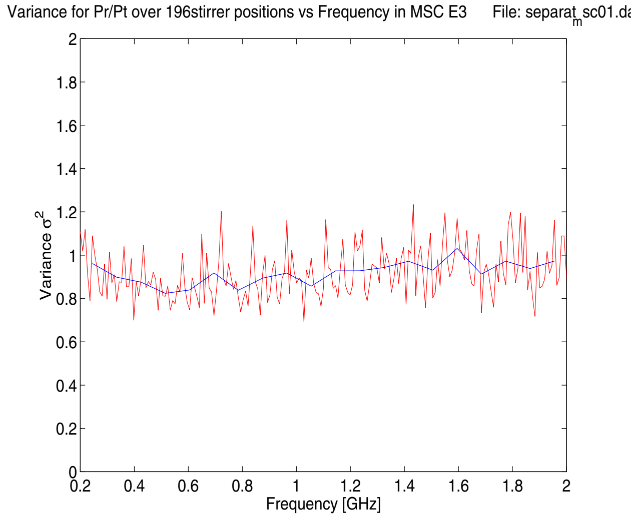
Chamber: Large

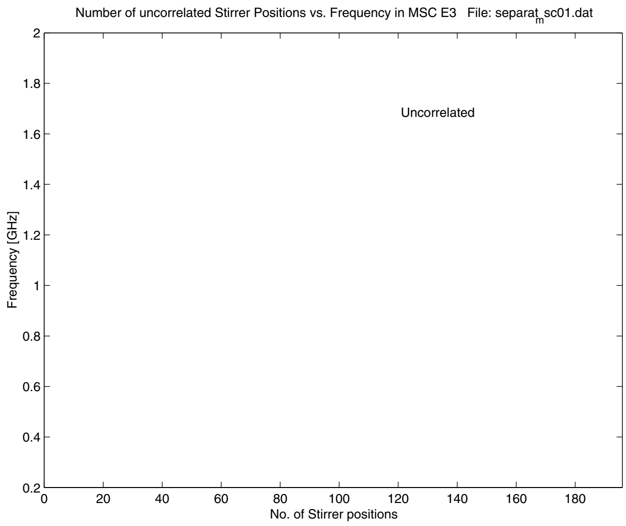
Stir: 2 * 14 = 196

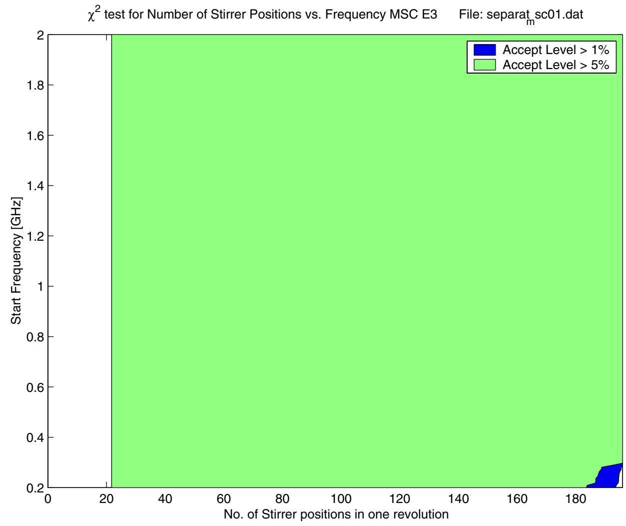






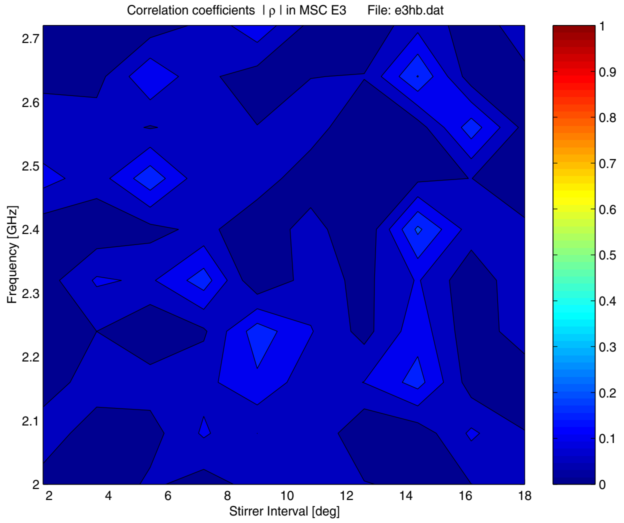


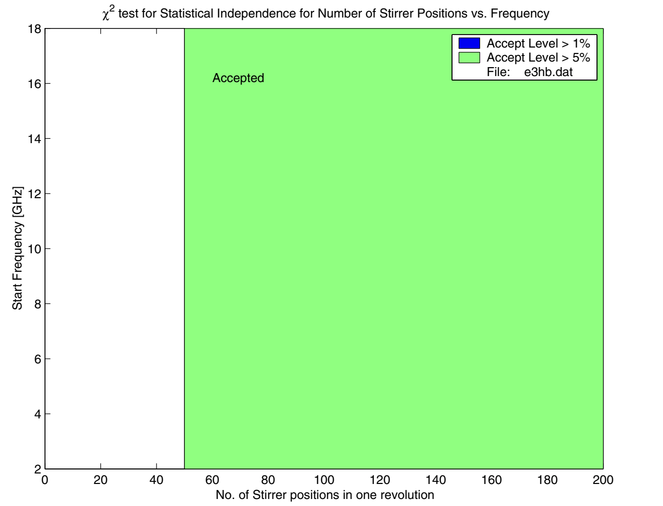


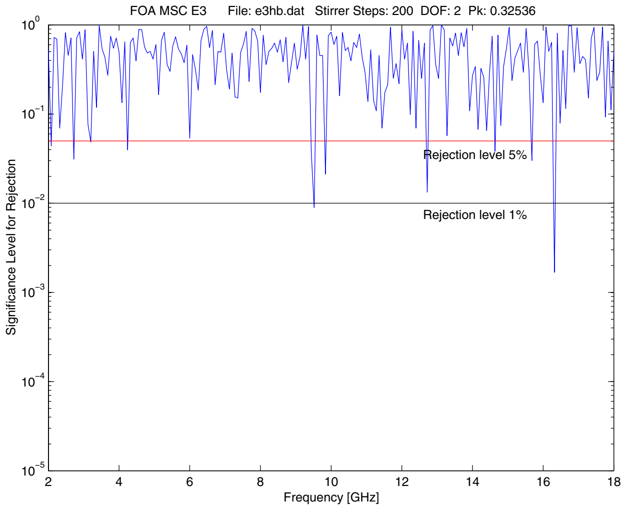


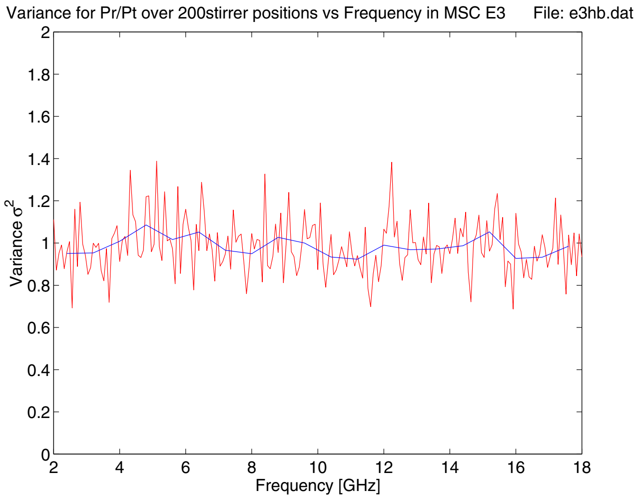
File: e3hb
Stirrer: Basic
Chamber: Large
Stir: 200

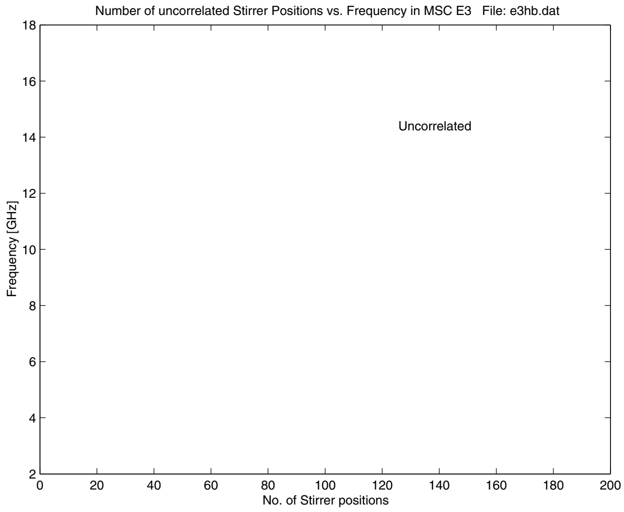
A12

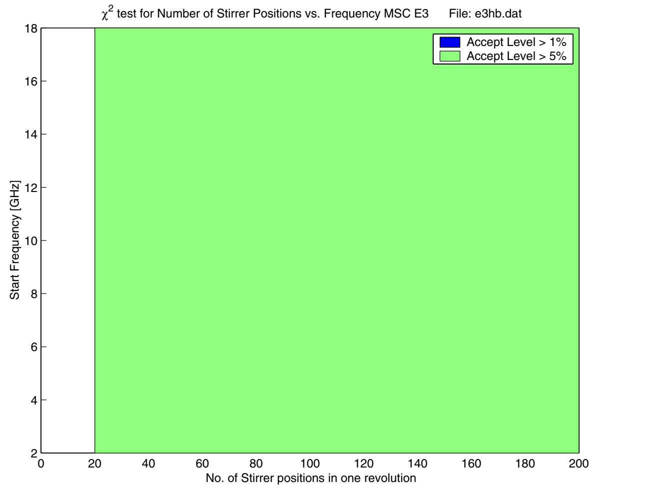






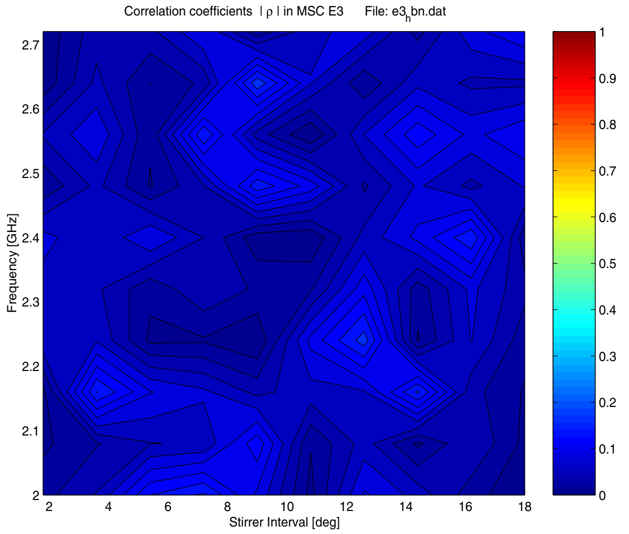


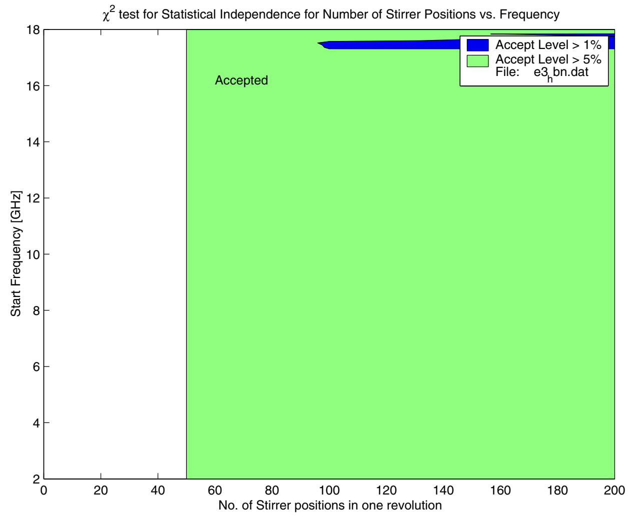


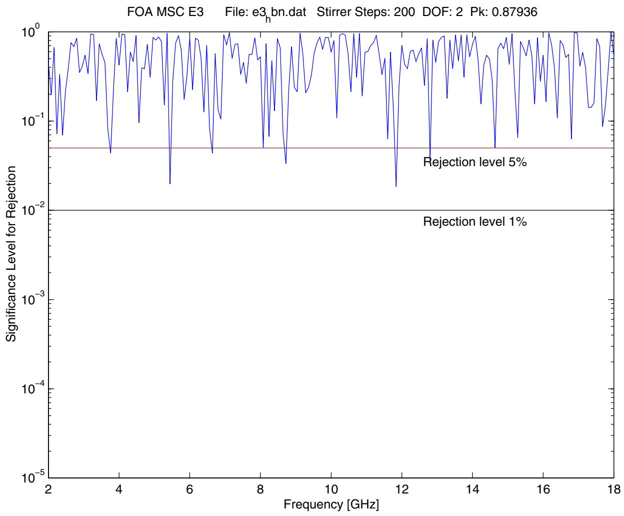


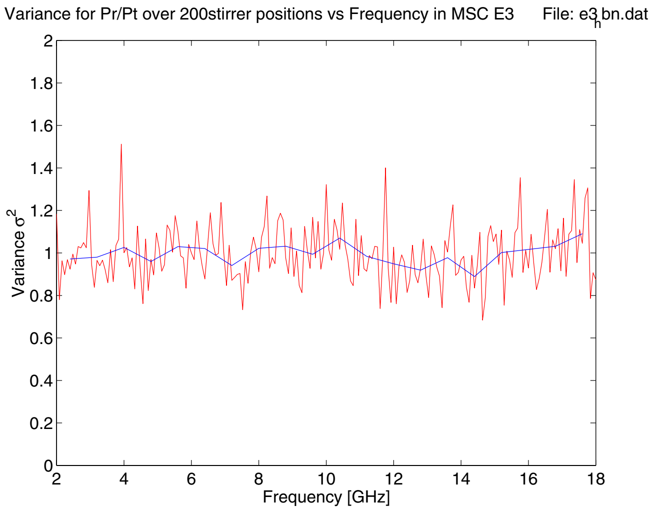
File: e3_hbn
Stirrer: Large
Chamber: Large
Stir: 200

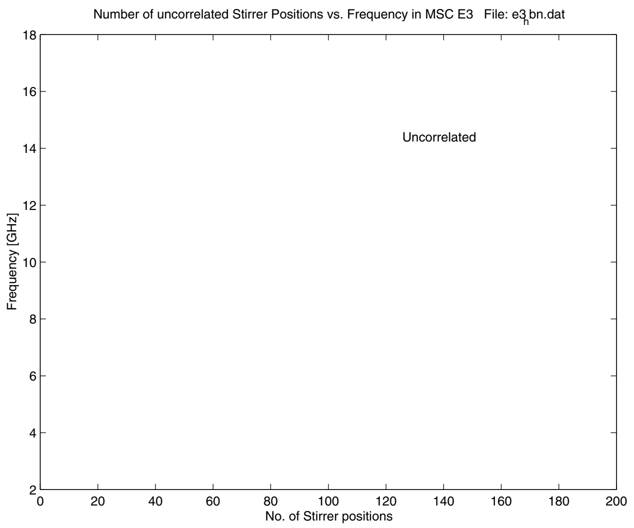
A13

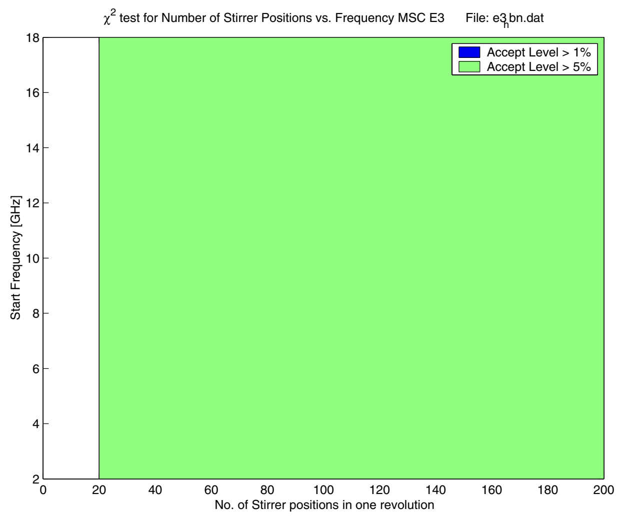






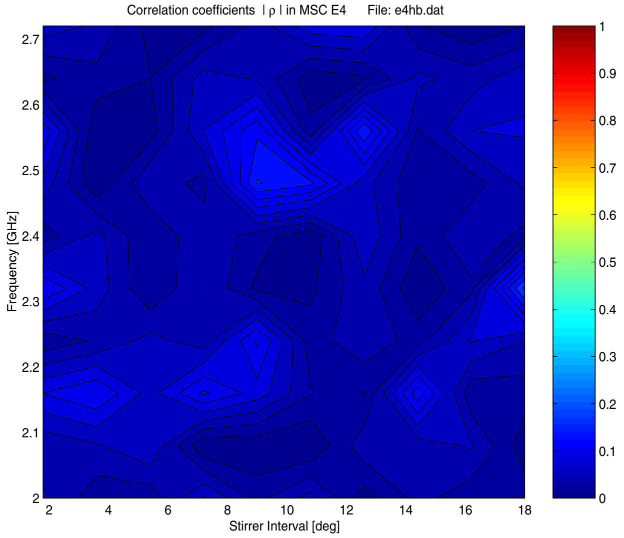


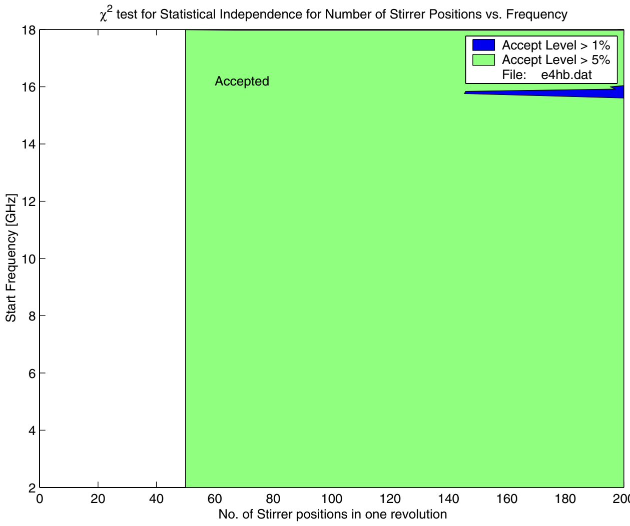


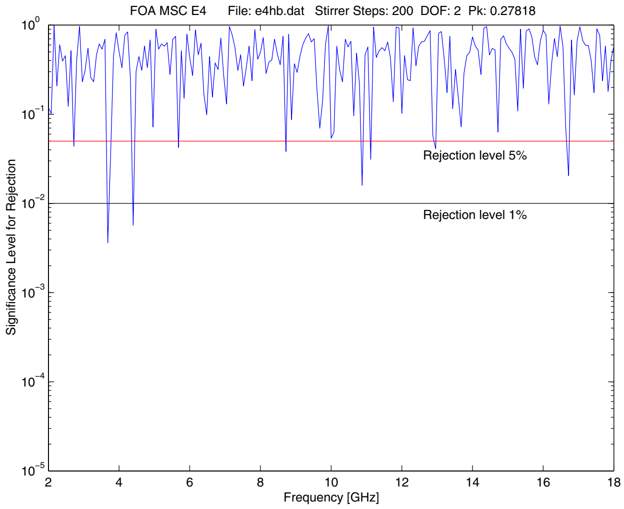


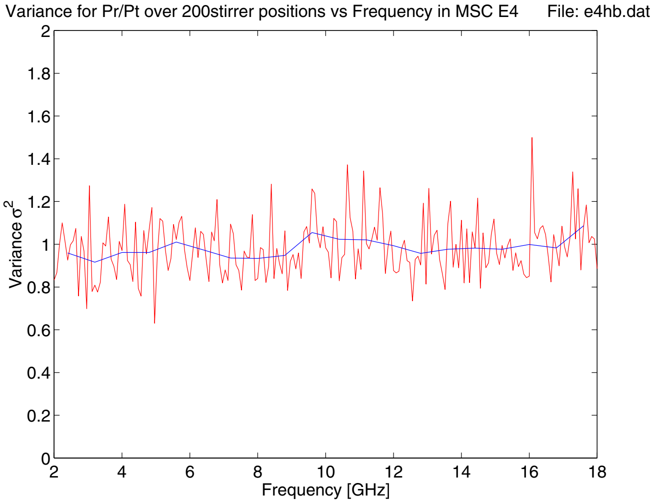
File: e4hb
Stirrer: Basic
Chamber: Small
Stir: 200

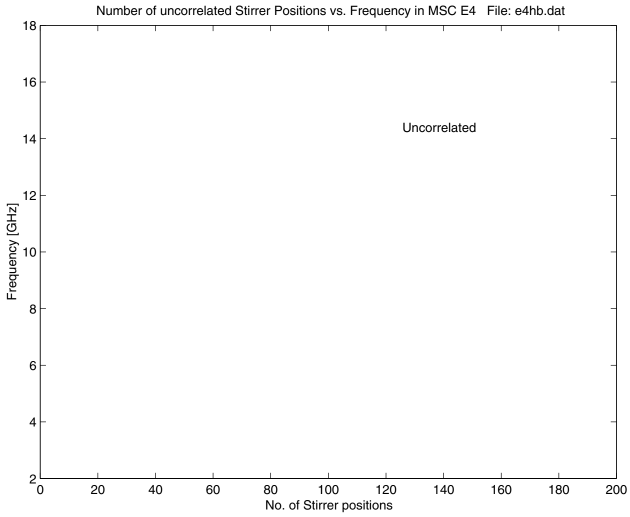
A14

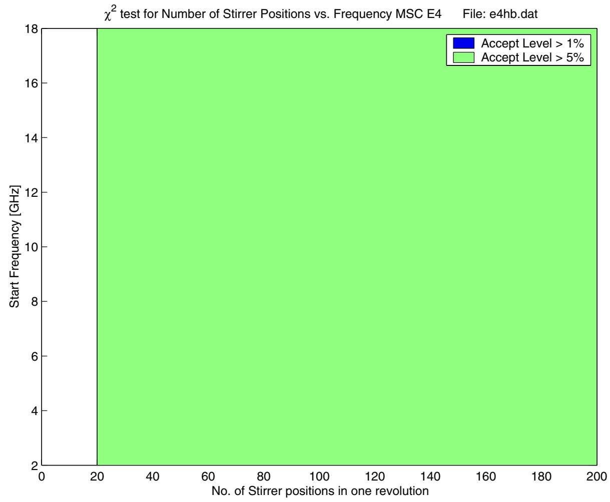






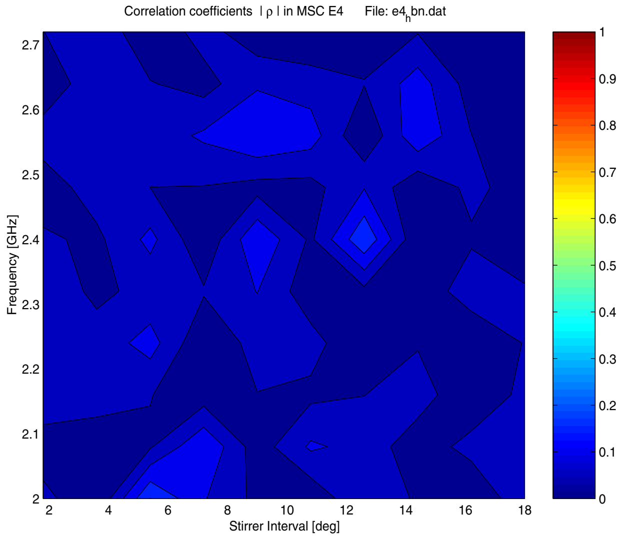


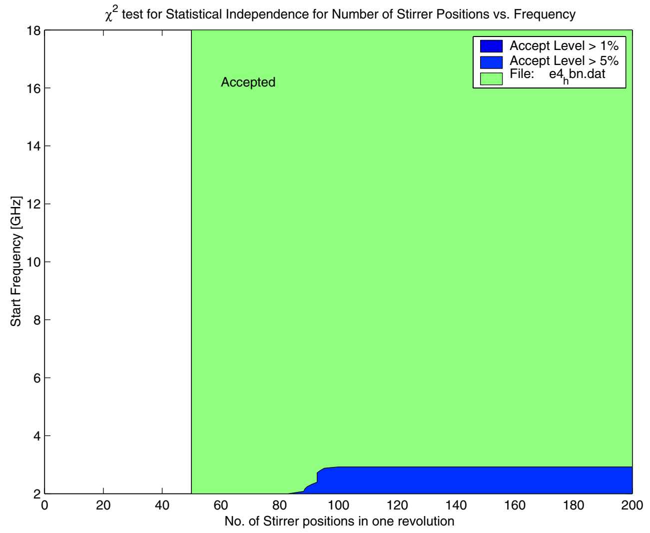


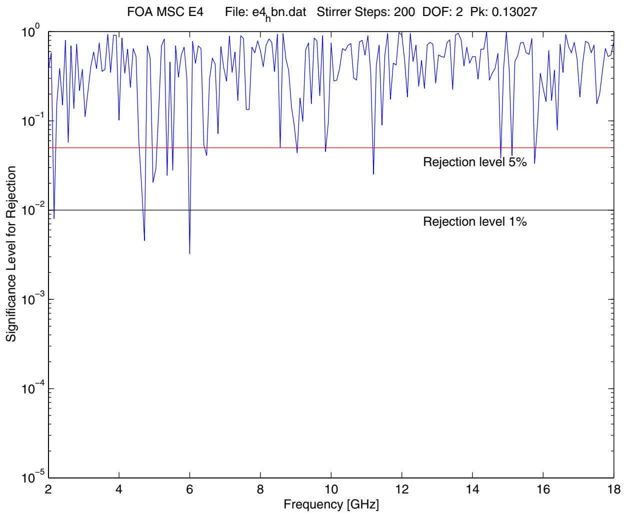


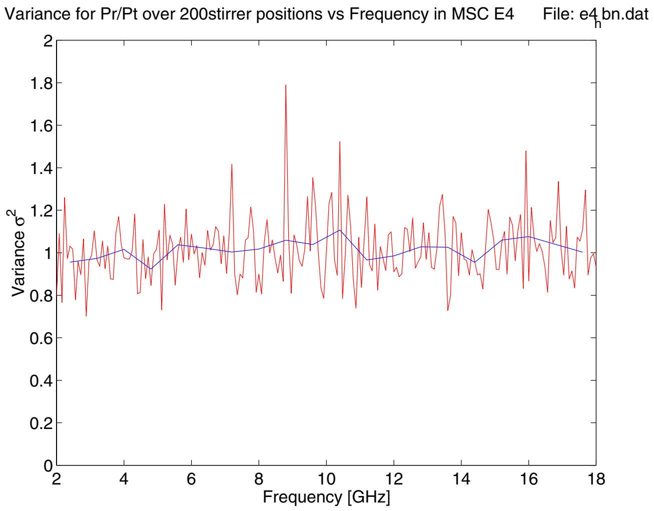
File: e4_hbn
Stirrer: Large
Chamber: Small
Stir: 200

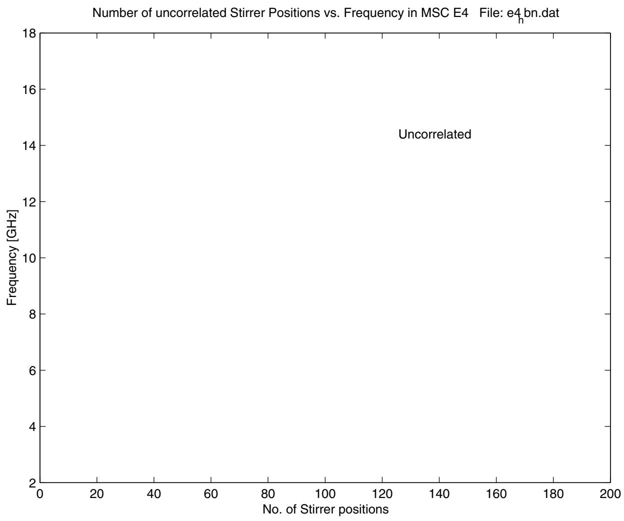
A15

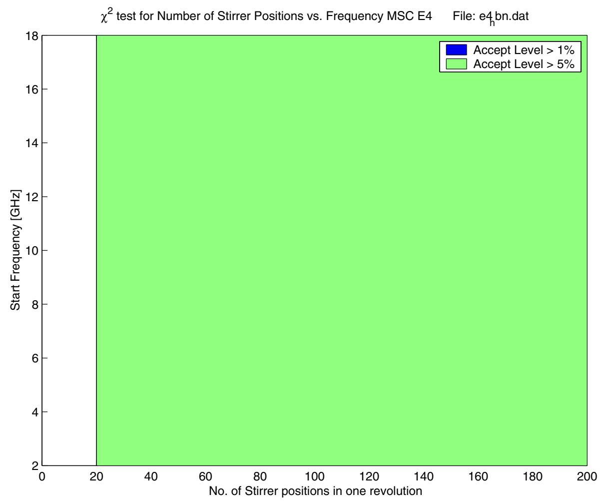






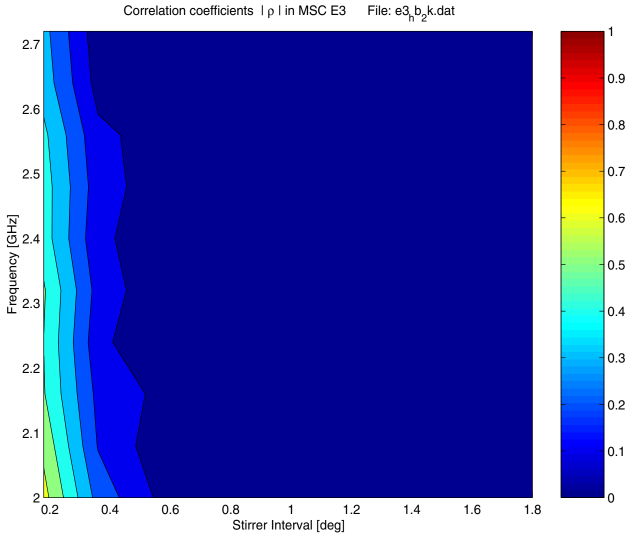


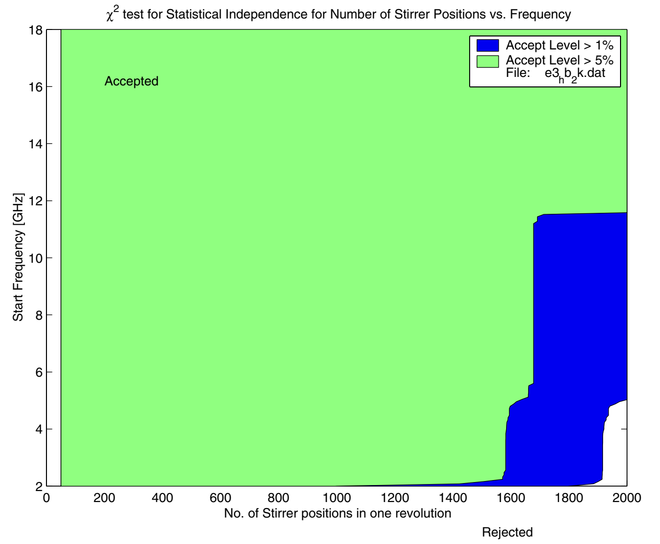


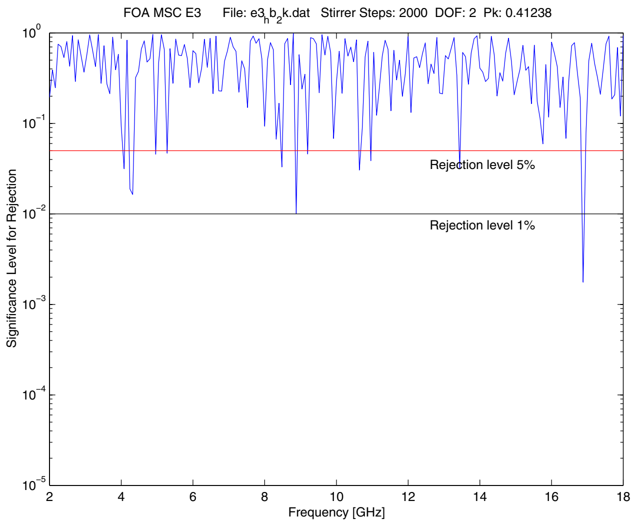


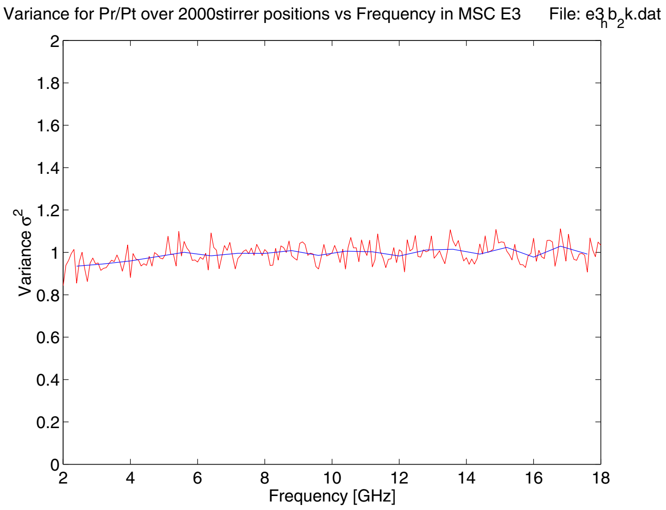
File: e3_hb_2k
Stirrer: Large
Chamber: Large
Stir: 2000

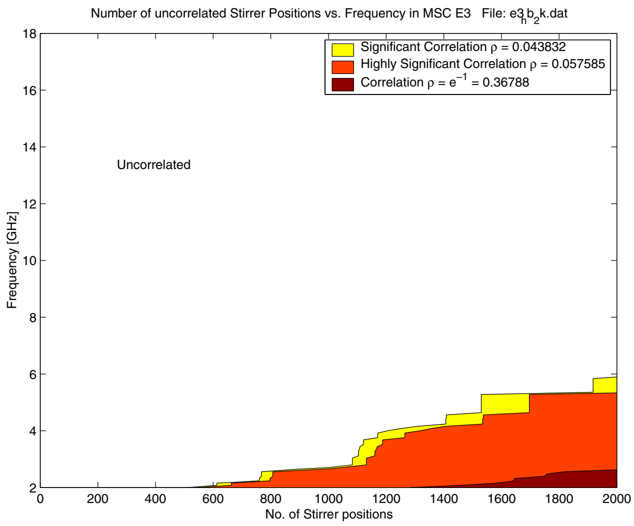
A16

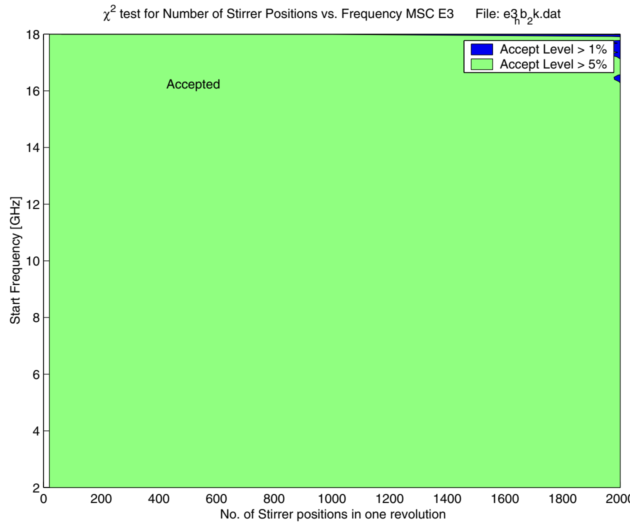






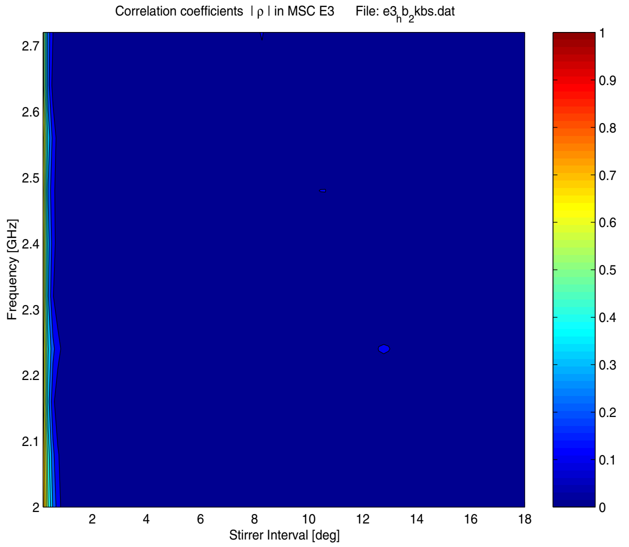


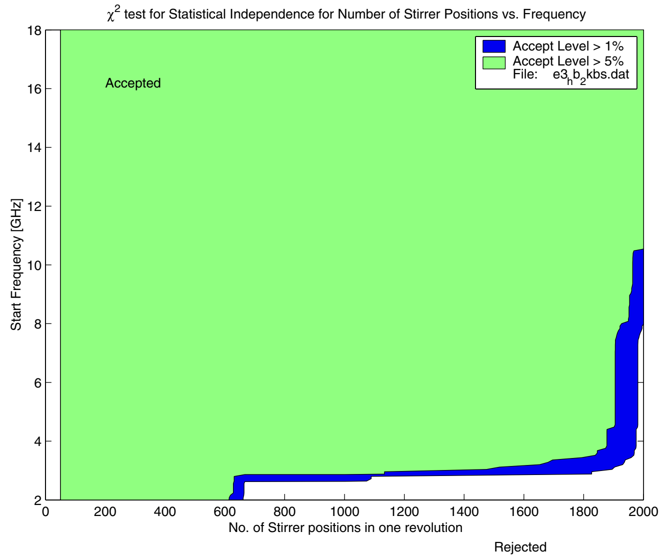


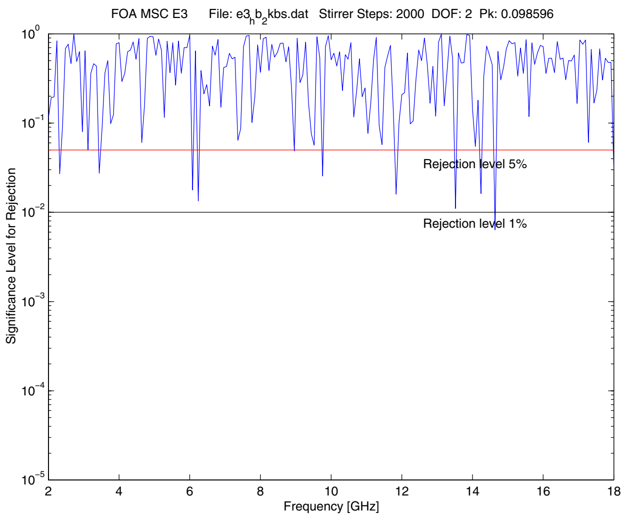


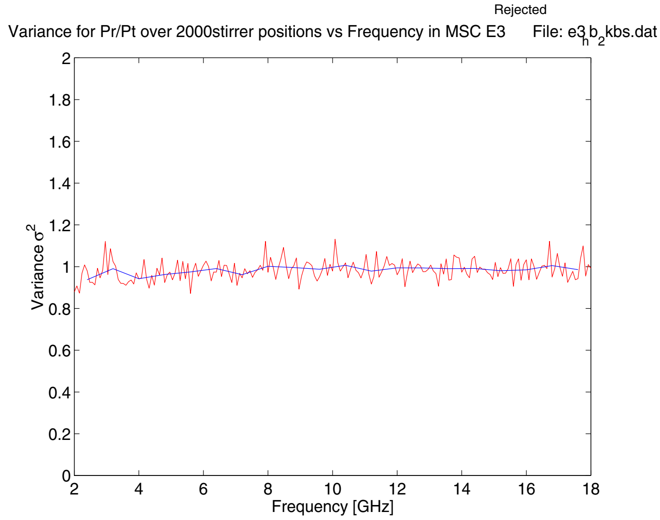
File: e3_hb_2kbs
Stirrer: Basic
Chamber: Large
Stir: 2000

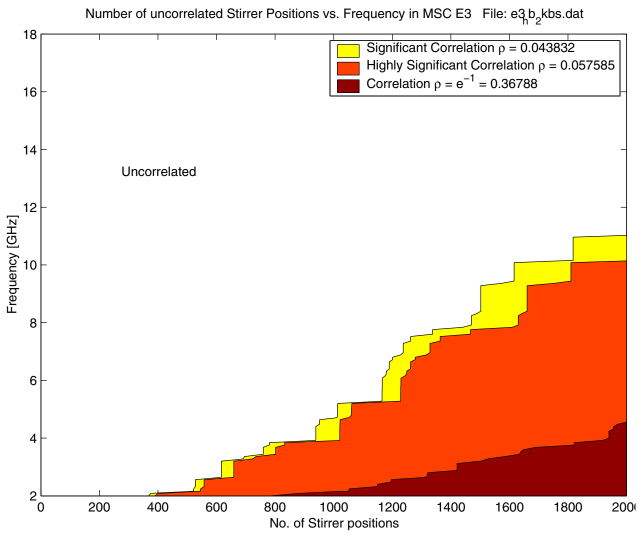
A17

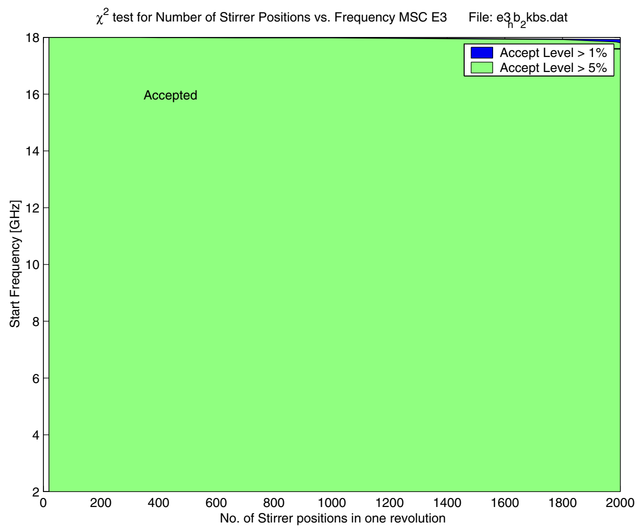




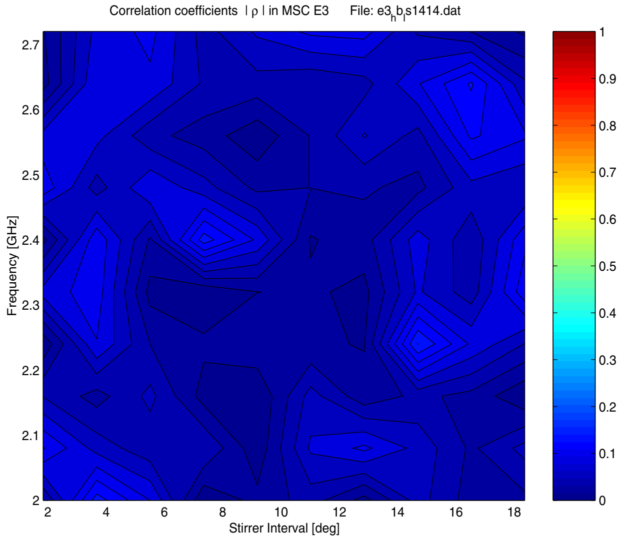


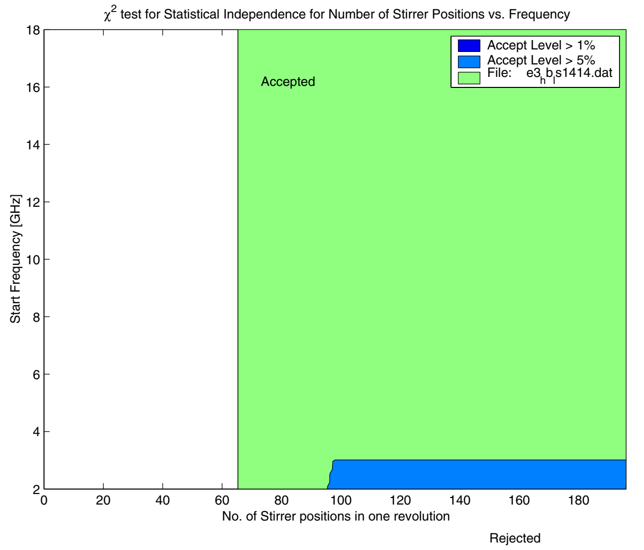


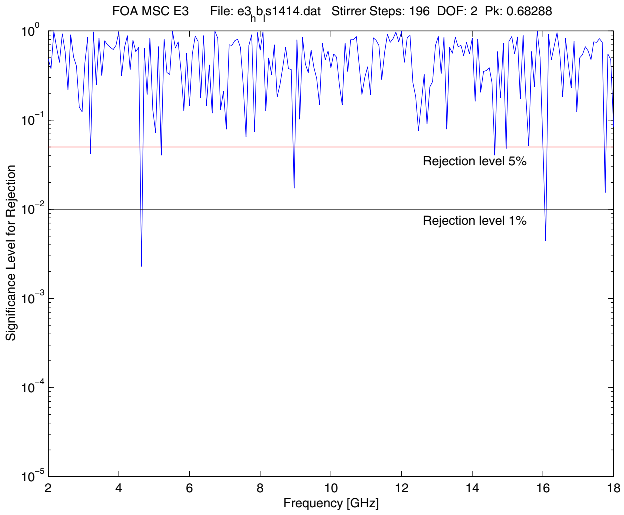


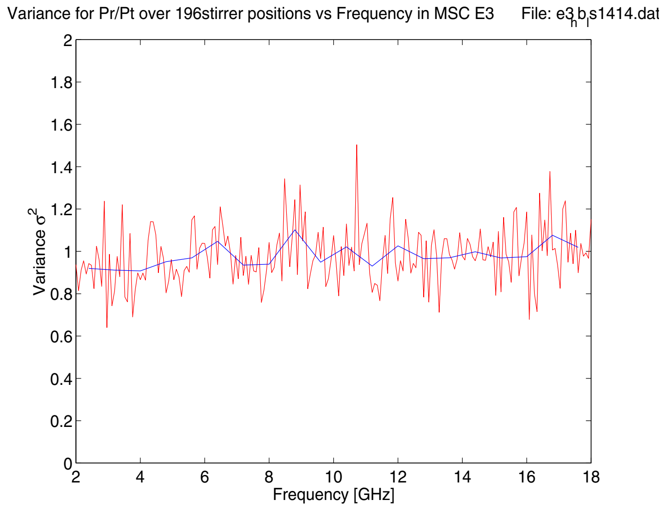


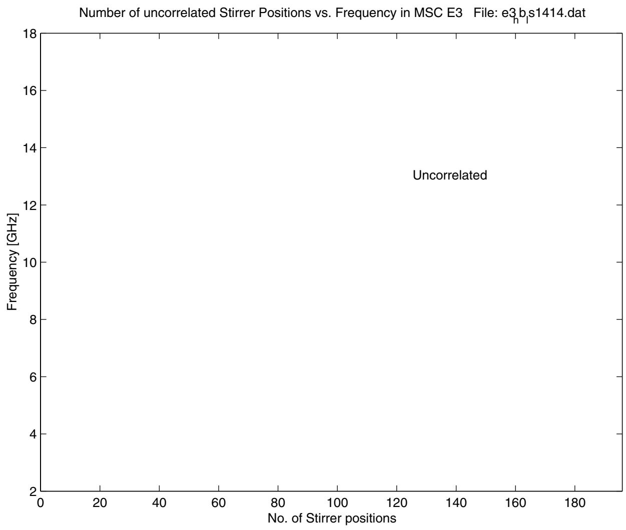
File: e3_hb_ls1414 A18
Stirrer: Large Large Independently
Chamber: Large
Stir: 2 * 14 = 196

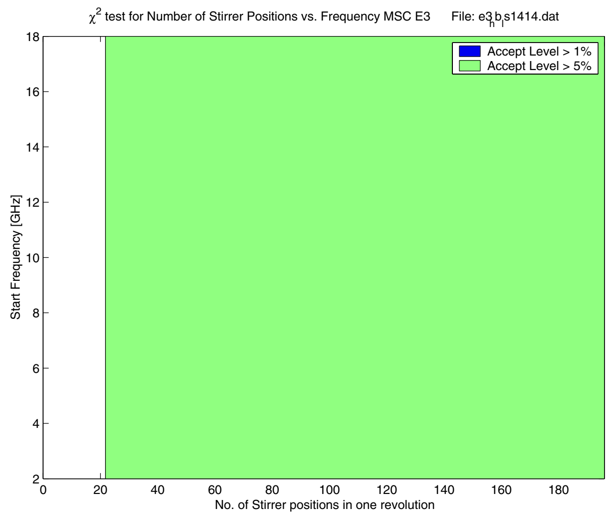




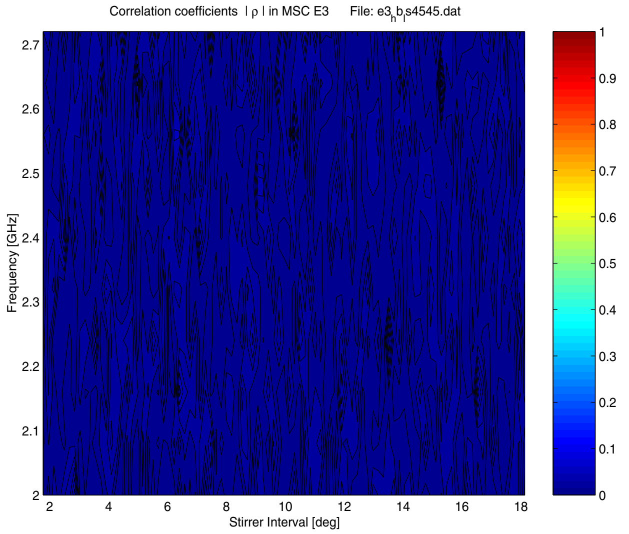


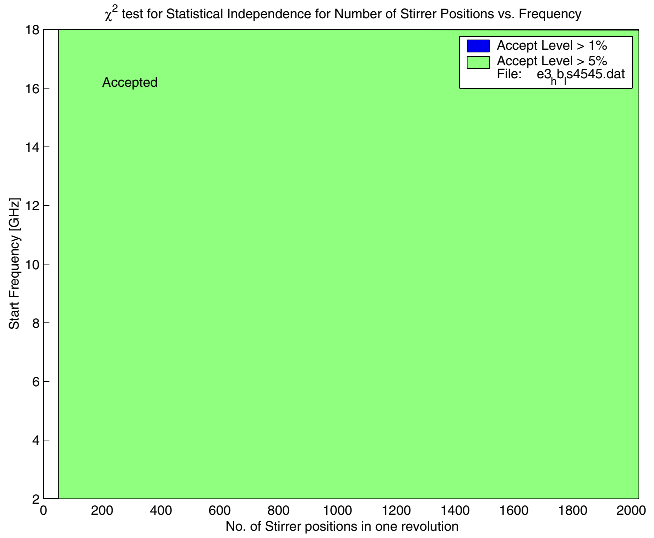


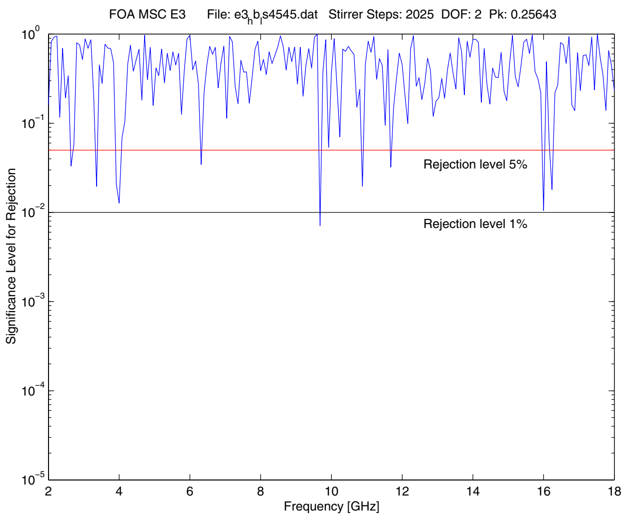


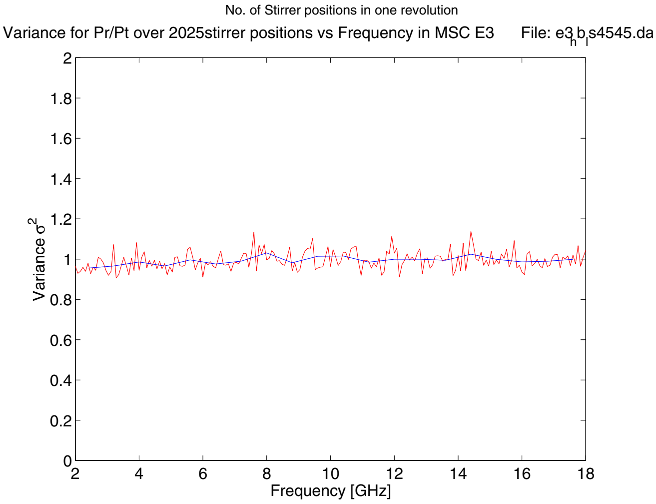


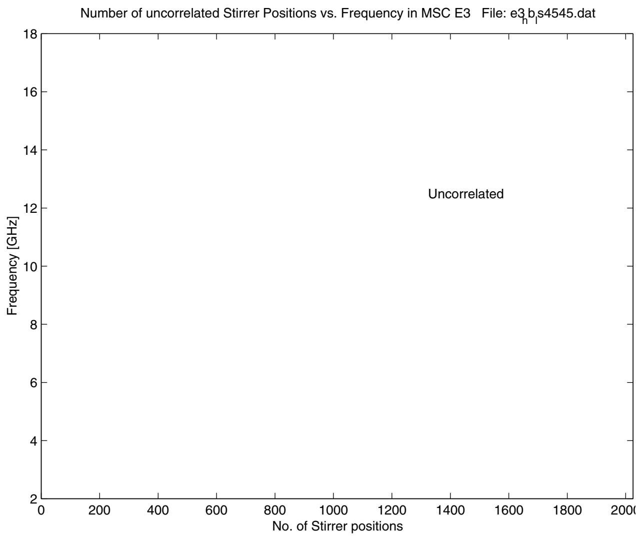
File: e3_hb_ls4545 A19
Stirrer: Large Large Independently
Chamber: Large
Stir: 2 * 45 = 2025

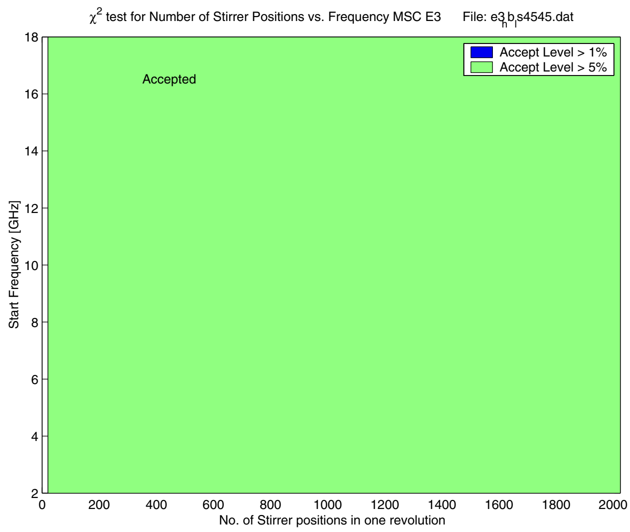












Appendix B. Tables

This appendix contains tables of all the measured files and the Significance Level for Rejection from Goodness of fit test of the assumed χ^2 distribution. Effect of reducing number of stirrer positions and effect of reducing both number of stirrer positions and number of frequencies

Filename	*freq GHz*	*Stir[N]*	*pk*	*Stirrer*	*Chamber*	*Page*
e3lb	.2 – 2	200	NaN	BS (*basic*)	E3	A2
e3_lbn5	.2 – 2	200	NaN	LS (*large*)	E3	A4
e4lb	.2 – 2	200	NaN	BS	E4	A8
e4_lbn	.2 – 2	200	2.5*E-9	LS	E4	A9
e3dual_lbll	.2 – 2	200	5.5*E-7	LSLS	E3	A10
separat_msc01	.2 – 2	196	5.2*E-4	LSLS	E3	A11
separat_msc02	.2 – 2	2025	NaN	LSLS	E3	A7
e3dual_lbll2000	.2 – 2	2000	NaN	LSLS	E3	A6
e3_lb_2kbs	.2 – 2	2000	NaN	BS	E3	A3
vibra_15	.2 – 2	2000	NaN	LS	E3	A5
e3hb	2 – 18	200	0.3254	BS	E3	A12
e3_hbn	2 – 18	200	0.8794	LS	E3	A13
e4hb	2 – 18	200	0.2780	BS	E4	A14
e4_hbn	2 – 18	200	0.1303	LS	E4	A15
e3_hb_2k	2 – 18	2000	0.4124	LS	E3	A16
e3_hb_2kbs	2 – 18	2000	0.0986	BS	E3	A17
e3_hb_ls1414	2 – 18	196	0.6829	LSLS	E3	A18
e3_hb_ls4545	2 – 18	2025	0.2564	LSLS	E3	A19

Table B1. Significance Level for Rejection from Goodness of fit test. NaN (Not a Number) indicates that pk is very small and that the hypothesis of chi-square distribution is rejected.

Filename	*freq GHz*	*Stir[N]*	*pk*	*corr. 0.37*	*step*	*Stirrer*
e3_lbn5	.2 – 2	200	NaN	443	1	LS (*large*)
		100	3.4*E-5	263	2	
		66.7	0.0267		3	
		50	0.0047	NC>200	4	
		40	0.2497		5	
		33.3	0.0034		6	
		28.6	0.0166		7	
		25	0.0390		8	
		22.2	0.0738		9	
		20	0.0699		10	
		18.2	0.0019		11	
		10	0.0075		20	

Table B2. Reducing number of stirrer positions

Filename: vibra_15 *Frequency range:* 0.2 – 2 GHz

	F_{start} *MHz*	200	371	551	731	911	1091	1271	1451	1631	1811
Step	*Stir[N]*	*pk*	*20*	*40*	*60*	*80*	*100*	*120*	*140*	*160*	*180*
1	2000	NaN	NaN	0	0	0	0	0	0	0	0.1070
2	1000	NaN	0	0	0	0.0126	0.0359	0.1701	0.0886	0.0201	0.5975
3	666.7	NaN	0	0.0068	0.0344	0.3345	0.2162	0.3177	0.4037	0.2083	0.4336
4	500	NaN	0	0.1106	0.3122	0.5441	0.3802	0.7631	0.7535	0.3195	0.7504
5	400	NaN	0	0.0159	0.0502	0.0225	0.0473	0.0750	0.0163	0.0359	0.0806
6	333.3	NaN	0.0002	0.0945	0.0736	0.1546	0.1105	0.3222	0.5677	0.3179	0.7355
7	285.7	NaN	0.0066	0.1590	0.1561	0.3483	0.2181	0.3056	0.1418	0.1126	0.3270
8	250	NaN	0.1395	0.5609	0.5652	0.5285	0.5667	0.4899	0.5790	0.5621	0.3785
9	222.2	5.7*E-9	0.1554	0.5748	0.5374	0.4235	0.5570	0.5202	0.7364	0.2477	0.2086
10	200	1.9*E-8	0.0233	0.3776	0.3663	0.2782	0.3160	0.2771	0.0887	0.0483	0.2502
11	181.8	0.0001	0.4227	0.7618	0.6961	0.7965	0.5821	0.5535	0.3884	0.1184	0.1162
12	166.7	7*E-10	0.0022	0.0758	0.0621	0.0460	0.1017	0.3382	0.5898	0.4939	0.4980
13	153.8	3*E-5	0.1239	0.4677	0.3272	0.2895	0.2349	0.2885	0.1181	0.5829	0.7220
14	142.9	0.0002	0.2142	0.3087	0.2977	0.2873	0.1842	0.1635	0.0676	0.0272	0.1256
15	133.3	1*E-5	0.0075	0.0783	0.0950	0.1258	0.1824	0.3426	0.0898	0.2870	0.8592
16	125	0.0170	0.6133	0.4969	0.4372	0.3794	0.6821	0.7487	0.7126	0.4882	0.3440
17	117.6	7*E-5	0.0242	0.1827	0.1017	0.0415	0.1491	0.1861	0.3838	0.4816	0.2766
18	111.1	8*E-5	0.0115	0.0385	0.0141	0.0054	0.0170	0.1106	0.4437	0.1380	0.4615
19	105.3	4*E-6	0.0067	0.0110	0.0065	0.0123	0.0642	0.0903	0.0307	0.1516	0.4945
20	100	0.0562	0.5398	0.7310	0.6576	0.5634	0.6967	0.8901	0.8860	0.6943	0.8347
21	95.2	0.0006	0.0093	0.0191	0.0183	0.1175	0.1522	0.1126	0.1602	0.3768	0.3085
22	90.9	0.1761	0.6997	0.5934	0.3760	0.4305	0.4416	0.2411	0.2422	0.1834	0.0174
23	86.9	0.0160	0.2067	0.3349	0.1809	0.1669	0.3703	0.2235	0.2168	0.3172	0.5458
40	50	0.6429	0.7496	0.8298	0.7528	0.5406	0.8896	0.6742	0.3658	0.3004	0.0782
80	25	0.5273	0.4358	0.4215	0.4742	0.2888	0.3221	0.2556	0.0822	0.0182	0.0291

Table B3. Reducing number of stirrer positions and number of frequencies for file vibra_15. The column Step in table B3 is the sparse factor for reducing the number of stirrer positions in the data file. If Step is = 1 we calculate χ^2 for 2000 stirrer steps (column Stir). If Step is = 2 we calculate χ^2 for 1000 stirrer steps (Stir). Grey areas indicate accepted values as χ^2 distributed.

Appendix C. Programs

This appendix contains some of the programs used for evaluation.

Taylor ….. … calculates the Significance Level for Rejection from Goodness of fit test.
korre ……… calculates the correlation coefficients.

Taylor

```
% Program for calculations of Reduced Chi Square Test and Significance Level
% for Rejection
% Ref. John R. Taylor. An introduction to Error Analysis.
% FOA 34 OL Rev. 990721
clear
tic
% **************** Retrieve Parameters and Data from Filename ****************
fil=input('Filename file? (e3_lbn5): ','s');
if isempty(fil)
 fil=('e3_lbn5');
end
filename=[fil '.par'];
eval(['load ' filename]);
x=eval(fil);
[ave,nop,start,stop,stir,att_cal,atten,par,pwr,cham]=peta(x);% get parameters
filename=[fil '.dat'];
eval(['load ' filename]);
a=eval(fil);
clear (fil);

[pow,medel,stand,att,fre,vinkel]=hemta(a,nop,start,stop,stir,filename);% get
data
% ****************************************************************************
ny=2;% DOF
[Pmvka,varmvka,pmvka,rmvka,pk,pk1]=goodness445(pow,medel,fre,start,stop,nop,sti
r,ny,cham,filename);
figure (1)
subplot(2,1,2)
title(['
pk: ',num2str(pk)])
hold off
figure (4)
title (['FOA MSC E',num2str(cham),'      File: ',filename,'   Stirrer Steps:
',num2str(stir)])
hold off

toc
% ****************************** END taylor **********************************
```

hemta

```
function[pow,medel,stand,att,fre,vinkel]=hemta(a,nop,start,stop,stir,filename);

a=reshape (a,801,stir*2);
ga=a(1:nop,1:stir*2);
clear a;
%-------------------------------------------------------------------------------
gaa=ga.*ga;
j=0;
for i=1:2:stir*2-1,
   j=j+1;
   pow(:,j)=gaa(:,i)+gaa(:,i+1);% convert real and imag to power
end
%-------------------------------------------------------------------------------
%*************************************** calculate average and std deviation
for i=1:nop,
   medel(i)=mean(pow(i,:));% average over all stirrer positions
   stand(i)=std(pow(i,:));% standard deviation over all stirrer positions
end
for i=1:nop,
   att(i)=10*log10(medel(i));
   std_log(i)=10*log10(stand(i));
end
%-------------------------------------------------------------------------------
% ************************************** create frequency and stirrer vectors
j=0;
step=(stop-start)/(nop-1);
for i=start:step:stop
   j=j+1;
   fre(j)=i/1000;% frequency vector
end

for i=1:stir
   vinkel(i)=i*360/stir;% stirrer vector
end
%-------------------------------------------------------------------------------
% ***************************** END hemta ************************************
```

peta

```
function [ave,nop,start,stop,stir,att_cal,atten,par,pwr,cham]=peta(a)
  % retrieve parameters from MSC measurement
   ave=a(1);
   nop=a(2);
   start=a(3)/1e6;
   stop=a(4)/1e6;
   stir=a(5);
   att_cal=a(6);
   atten=a(7);
   par=a(8);
   pwr=a(9);
   cham=a(10);
% ***************************** END peta ***********************************
```

C3

goodness4

```
function[p,redchi2,freq,rr1,rr5]=goodness4(x,a,m,N,f1,f2,ny,cham,filename)
%The program calculates a goodness-of-fit calculation on input data
%assuming the data is chi2 distributed.
%x is a m,N matrix where each row corresponds to a frequency
%a=number of equal probability bins and N is the total number of data
%at each frequency.
%f1 is the start and f2 the stop frequency. ny is the number of the
%degrees of freedom for the assumed chi2 distribution.
%A rule of thumb is that N/a shall be greater than 5 and that a shall
%not be too small, in any case greater than two (cf. Taylor "An
%Introduction to Error Analysis, 2nd ed., chapter 12).
%hold off
f1=f1/1000;
f2=f2/1000;
for ii=1:m
   n=0;
   xx=x(ii,:);
   mu=nanmean(xx);
   %mu is the average value of the data at the specific frequency
   xx=xx.*ny./mu;
   %the measured data are adjusted to have the same average value as the
   %supposed chi2 distribution
   b=1/a;
   SS=chi2inv(b:b:(1-b),ny);
   %a bins of equal probability are created
   ee=find(xx<SS(1));
   n(1)=length(ee);
   for i=2:(a-1);
      ff=find(xx<SS(i));
      nn=sum(n);
      n(i)=length(ff)-nn;
   end
   gg=find(xx>=SS(a-1));
   n(a)=length(gg);
   %the number of data points in each bin was calculated
   zero(ii)=N-sum(n);
   %a check that the total number of data in all bins are equal to N
   d=a-2;
   %the number of degrees of freedom for the significance test is calculated
   redchi2(ii)=sum((n-N/a).^2./(N/a))/d;
   %the reduced test parameter chi2 is calculated, its expectation value is
   %one.
   %If it is much greater than one the assumption of chi2 distributed
   %data is probably wrong
   p(ii)=1-chi2cdf(redchi2(ii)*d,d);
   freq(ii)=f1+(ii-1)*(f2-f1)/(m-1);
end
%p is the probability of obtaining a value of redchi2
%greater than or equal to the one calculated from the
%measurements, assuming the measurements concerned are governed by
%the expected  distribution. If p is small, say below 5%,
%the assumption should be rejected. If p is between 5% and 1% the assumption
%is said to be rejected at the 5% level, if p is less than 1% at
%the 1% level etc.
zerocheck=sum(zero)
subplot(2,1,1)
semilogy(freq,p)
%calculate part of probability values, i.e. part of p-values, that
%is less than 1 percent or 5 percent.
rr=find(p<=0.01);
rrr=length(rr);
```

```
rr1=rrr/m;
```



```
rejectionratioAt1percentlevel=rrr/m
tt=find(p<=0.05);
ttt=length(tt);
rr5=ttt/m;
rejectionratioAt5percentlevel=ttt/m
xlabel('Frequency (GHz)')
ylabel('Significance Level for Rejection')
title (['FOA MSC E',num2str(cham),'      File: ',filename,'   Stirrer Steps:
',num2str(N),'  DOF: ',num2str(ny)])
hold on
plot(freq,0.05,'r-')
hold on
plot(freq,0.01,'k-')
hold off
subplot(2,1,2)
plot(freq,redchi2)
ylabel('Reduced Chi Square (calculated)')
xlabel('Frequency (GHz)')
hold off
% ****************************** END goodness4 ******************************
```

C5

goodness445

```
function[Pmvka,varmvka,pmvka,rmvka,...
pk,pk1]=goodness445(filea,P,fre,start,stop,na,stir,ny,cham,filename)
%The program calculates a goodness-of-fit calculation on input data from
% a measurement in FOA Reverberation Chamber.
%The program uses as hypothesis that data are chi2 distributed.
%The program calculates the variances. Finally, after using goodness4.m for
%calculating goodness-of-fit data it calculates a final probability for
%rejecting
%all data in the complete frequency interval, denoted pk. pk1 is the same but
%for the reduced frequency.
% Then goodness4.m can be used to
%calculate the p's used for pk (the frequency intervals have to be
% added before calculating pk).
%
N=stir;
Pmvka=filea;

freqa=fre;
hold off

figure(4)
varmvka=(nanstd(Pmvka')./nanmean(Pmvka')).^2;
plot(freqa,varmvka)

xlabel('Frequency (GHz)')
ylabel('Variance')
hold off
% All below requires Statistical Toolbox !!!!!!
figure(1)
steg=1;

hold off
[pmvka,rmvka,fa,rr1mvka,rr5mvka]=goodness4(Pmvka(:,1:steg:N),10,na,N/steg,start
,stop,ny,cham,filename);

psum=[pmvka];
logsumpsum=-2*sum(log(psum))
Ntot=na;
pk=1-chi2cdf(logsumpsum,2*Ntot)

fp=20;% frequency point
psum1=[pmvka(fp:na)];
logsumpsum1=-2*sum(log(psum1));
pk1=1-chi2cdf(logsumpsum1,2*(Ntot-fp));

%pk is the significance level for rejection for the complete frequency
%interval. Normally one rejects if pk is less than 0.01 (or less than 0.05).
%pk1 is for the frequency interval from freqa(fp) to freqmax
% ******************** END goodness445 ***************************************
```

C6

korre

```
% plot files measured in MSC correlation coefficient ************************
% FOA 34 OL rev 980826 ******************************************************
clear
tic
% **************** get  file name *******************************************
fil=input('Filename fil? (e3lb): ','s');
if isempty(fil)
   fil=('e3lb');
end
filename=[fil '.par'];
eval(['load ' filename]);
x=eval(fil);
[ave,nop,start,stop,stir,att_cal,atten,par,pwr,cham]=peta(x);% get parameters
filename=[fil '.dat'];
eval(['load ' filename]);
a=eval(fil);
clear (fil);
[pow,medel,stand,att,fre,vinkel]=hemta(a,nop,start,stop,stir,filename);% get
data
[teo_loss]=hallen (cham,start,stop,nop);% calculate teor.loss according hallén
clear a;
nos=360./vinkel;
%---------------------------------------------------------------------------------
%****************************************** calculate correlation coefficients
j=0;
summa(nop,stir)=0;

for r=1:stir-1
   j=j+1;
   for i=1:stir-1
      indy=(i+r)-floor((i+r)/(stir+1))*stir;% ___________________modulo Stir-1
      summa(:,j)=summa(:,j)+(pow(:,i)-medel').*(pow(:,indy)-medel');
   end
end
sigma2=stand.*stand;%_________________________________________________variance
for i=1:stir
   rho(:,i)=(1/(stir-1)).*summa(:,i)./sigma2';
end
ny=abs(rho);% ___________________absolute value of correlation coefficients
%---------------------------------------------------------------------------------
% ********************************************************* calculate Q - value
lambda=.3./fre;
switch cham
case 3
   laengd=5.1;
   bredd=2.457;
   hoejd=3.00;
case 4
   laengd=3.6;
   bredd=2.457;
   hoejd=3.067;
end
volym=laengd*bredd*hoejd;

Q=16*pi^2*volym./lambda.^3.*medel;
%---------------------------------------------------------------------------------
%********************************************************* calculate
variance
for i=1:nop
   npow(i,:)=pow(i,:)./(medel(i));% ______________________normalize to average
   nstd(i)=std(npow(i,:));% ________________________________stand. of norm.avg.
end
```

C7

```
%kvot=(stand./medel).*(stand./medel);% __________________equivialent to nstd^2
kvot=nstd.^2;
j=1;
k=10;
for i= 1:nop/k
   rull(i)=mean(kvot(j:k));% __________________________moving average 10 points
   frekv(i)=fre(j+5);
   j=j+10;
   k=k+10;
end
% ***************************************************** calculate probabilities
ja=1;
que=input('Calculate Probabilities ? (N): ','s');
if isempty(que)
   ja=0;
else
   for j=1:nop
      for i=1:stir
         p(j,i)=1-betacdf(ny(j,i)^2,0.5,stir/2-1);% requires STATISTICS TOOLBOX
      end
   end
end
%--------------------------------------------------------------------------------
%polynomial fit at 1.8 deg

ko=polyfit(fre(1:100),ny(1:100,1)',3)
for i=1:nop
   anpa(i)=ko(1)*fre(i)^3+ko(2)*fre(i)^2+ko(3)*fre(i)+ko(4);
end
j=0;
for i=1:nop
   if anpa(i)<=0.37,
      if j==0,
         j=1;
         frq=fre(i);
      end
   end
end

if anpa(1)<0.37
   frq=0;
end

% polynomial fit at 3.6 deg
ko=polyfit(fre(1:100),ny(1:100,2)',3)
for i=1:nop
   anpa2(i)=ko(1)*fre(i)^3+ko(2)*fre(i)^2+ko(3)*fre(i)+ko(4);
end
j=0;
for i=1:nop
   if anpa2(i)<=0.37,
      if j==0,
         j=1;
         frq2=fre(i);
      end
   end
end
if anpa2(1)<0.37
   frq2=0;
end
```

C8

```
% polynomial fit at 5.4 deg
ko=polyfit(fre(1:100),ny(1:100,3)',3)
for i=1:nop
   anpa3(i)=ko(1)*fre(i)^3+ko(2)*fre(i)^2+ko(3)*fre(i)+ko(4);
end
j=0;
for i=1:nop
   if anpa3(i)<=0.37,
      if j==0,
         j=1;
         frq3=fre(i);
      end
   end
end
if anpa3(1)<0.37
   frq3=0;
end

%-------------------------------------------------------------------------------
%*********************************************************************** plot
figure (1)
plot (fre,att,'b')
hold on
plot (fre,teo_loss,'r')
xlabel('Frequency [GHz]')
ylabel ('Loss [dB]')
title (['Transmission loss Pr/Pt in MSC E',num2str(cham),'      File:
',filename])
av=mean(att);
mx=max(att);
mn=min(att);
if stop <= 2000
   golv=-20;
else
   golv=-50
end
axis ([0 stop/1000 golv 0])

hold off

figure (2)
mesh (vinkel,fre,ny)
axis ([0 360 0 stop/1000 0 1])
view (-127,28)
ylabel('Frequency [GHz]')
xlabel ('Stirrer Interval [deg]')
title (['Correlation coefficients | \rho |  in MSC E',num2str(cham),'
File: ',filename])
hold off

figure (3)
plot (vinkel,rho(1,:),'r')
axis ([0 180 -1 1])
hold on
plot (vinkel,rho(1,:),'r.')
plot (vinkel,rho(2,:),'g')
plot (vinkel,rho(2,:),'g.')
plot (vinkel,rho(3,:),'b')
plot (vinkel,rho(3,:),'b.')
title (['Correlation of gradual stirrer positions in MSC E',num2str(cham),'
File: ',filename])
ylabel ('Correlation coefficient ')
xlabel ('Angle [deg]')
```

```
text (15,.9,['- ',num2str(fre(1)*1000),' MHz red'])
```

C9

```
text (15,.7,['- ',num2str(fre(2)*1000),' MHz green'])
text (15,.5,['- ',num2str(fre(3)*1000),' MHz blue'])
hold off

figure (4)
contour (vinkel,fre,ny)
caxis ([0 1])
colorbar('vert')
ylabel('Frequency [GHz]')
xlabel ('Stirrer Interval [deg]')
title (['Correlation coefficients  | \rho | in MSC E',num2str(cham),'
File: ',filename])


figure (8)
plot (fre(1:100),anpa(1:100),'m')
hold on
plot (fre(1:100),anpa2(1:100),'m')
plot (fre(1:100),anpa3(1:100),'m')
plot (fre,ny(:,1),'r')
axis ([0 stop/1000 0 1])
hold on
plot (fre,ny(:,1),'r.')
plot (fre,ny(:,2),'g')
plot (fre,ny(:,2),'g.')
plot (fre,ny(:,3),'b')
plot (fre,ny(:,3),'b.')
title (['Correlation vs. Frequency in MSC E',num2str(cham),'      File:
',filename])
ylabel ('Correlation coefficient | \rho | ')
xlabel ('Frequency [GHz]')
if anpa(1)>=0.37,
   text (start/1000+.25,.9,['- ',num2str(vinkel(1)),' deg red  Correlation @
',num2str(frq*1000),' MHz'])
else
   text (start/1000+.25,.9,['- ',num2str(vinkel(1)),' deg red  NO Correlation >
',num2str(fre(1)*1000),' MHz'])
end
if anpa2(1)>=0.37,
   text (start/1000+.25,.8,['- ',num2str(vinkel(2)),' deg green Correlation @
',num2str(frq2*1000),' MHz'])
else
   text (start/1000+.25,.8,['- ',num2str(vinkel(2)),' deg green NO Correlation
> ',num2str(fre(1)*1000),' MHz'])
end
if anpa3(1)>=0.37,
   text (start/1000+.25,.7,['- ',num2str(vinkel(3)),' deg blue Correlation @
',num2str(frq3*1000),' MHz'])
else
   text (start/1000+.25,.7,['- ',num2str(vinkel(3)),' deg blue NO Correlation >
',num2str(fre(1)*1000),' MHz'])
end
hold off
figure (6)
plot (fre,kvot,'r')
hold on
plot (frekv,rull,'b')
axis([0 stop/1000 0 2])
title (['Variance for Pr/Pt over ',num2str(stir),'stirrer positions vs.
Frequency in MSC E',num2str(cham),'      File: ',filename])
ylabel ('Variance \sigma^2')
xlabel ('Frequency [GHz]')
hold off
```

```
figure (7)
plot (fre,Q,'r')
title (['Q-value vs. Frequency in MSC E',num2str(cham),'      File:
',filename])
ylabel ('Q-value ')
xlabel ('Frequency [GHz]')
hold off

%*******************************************************************************
toc
% **************************** END korre ************************************
```

hallen

```
%calculate chamber wall losses w.r.t. antenna losses
function [teo_loss]=hallen(cham,start,stop,nop)

if cham == 0,

   a=1.19;% kiosk_measure
   b=0.87;% kiosk_measure
   d=1.05;% kiosk_measure
   ch2='kiosk';
   mtrl='aluminium';

elseif cham == 3,
   a=5.105;% E3
   b=2.460;% E3
   d=2.926;% E3
   ch2='E3';
   mtrl='zinc';

elseif cham == 4,
   a=3.600;% E4
   b=2.457;% E4
   d=3.067;% E4
   ch2='E4'
   mtrl='zinc';

else
   (['**** Chamber ? ****']), break, end% _______________check chamber

yta=a*b*2+a*d*2+b*d*2;%___________________________________surface area
volym=a*b*d;%___________________________________________chamber volume

sigma_cu=5.8e7;


switch mtrl
   case 'zinc'
      sigma=.26;
   case 'iron'
      sigma=.17;
   case 'aluminium'
      sigma=.69;
end

my_r=1;
c0=3E8;
sigma=sigma*sigma_cu;
my_0=4e-7*pi;
my=my_r*my_0;
j=0;
```

```
step=(stop-start)/(nop-1);
```



```
for i=start:step:stop
j=j+1;
   lambda=300/i;
   ae=lambda^2/(8*pi);
   area(j)=10*log10(ae);
   tau=4*sqrt(pi*my_r)/(sqrt(120*pi*sigma*lambda))*yta;
   vz(j,1)=(10*log10(ae/(tau+2*ae)));
   vz(j,2)=i/1000;
   ch_loss(j)=10*log10(tau);
   fre(j)=i/1000;
   f_=i/1000*1e9;
   ga(j)=1/(2+32*yta*pi*f_^(5/2)/(c0^(5/2))*sqrt(1/(120*sigma)));
end
ga_loss=10*log10(ga);
teo_loss=vz(:,1);
% ****************************** END hallen **********************************
```

rho_level

```
function [one_percent,five_percent]=rho_level(n);
% calculate probability vs. number of samples N. Requires
% MATLAB STATISTICAL TOOLBOX.
p=1; % one percent level
  one_percent=sqrt(betainv((1-p/100),.5,n/2-1));
p=5; % five percent level
  five_percent=sqrt(betainv((1-p/100),.5,n/2-1));
%********************************** END rho_ level ***********************
```

N	1%	5%	N	1%	5%
3	0.9999	0.9969	100	0.2565	0.1966
4	0.9900	0.9500	120	0.2343	0.1793
5	0.9587	0.8783	140	0.2170	0.1660
7	0.8745	0.7545	160	0.2031	0.1552
10	0.7646	0.6319	180	0.1915	0.1463
15	0.6411	0.5140	200	0.1818	0.1388
20	0.5614	0.4438	250	0.1626	0.1241
25	0.5052	0.3961	300	0.1485	0.1133
30	0.4629	0.3610	400	0.1287	0.0981
40	0.4026	0.3120	500	0.1151	0.0877
50	0.3610	0.2787	750	0.0940	0.0716
60	0.3301	0.2542	1000	0.0814	0.0620
70	0.3060	0.2352	1500	0.0665	0.0506
80	0.2864	0.2199	2000	0.0576	0.0438

Appendix D.

Goodness-of-fit evaluation of a complete frequency interval.

A way to treat all the data for a complete frequency interval is to calculate a logarithmic sum of all the probabilities of rejection, p_i, given at each frequency. The sum is:

$$K = -2 \cdot \sum_{i=1}^{N} \ln(p_i)$$

where i denotes the frequency points and p_i is the probability of rejection at each frequency.

If the assumed distribution used in the goodness-of-fit test is true then each p_i is uniformly distributed. As an example, a goodness-of-fit test on, e.g., 1000 samples gives in such a case statistically 50 values of p_i that is smaller than 0.05.

Now define $y = -2 \cdot \ln(p)$, where $y \in [0, \infty]$. The cumulative distribution function:

$F_Y(y) = P(-2 \cdot \ln X < y) = P(-y/2 < \ln X) = P(X > e^{-y/2}) = 1 - P(X < e^{-y/2}) = 1 - e^{-y/2}$, see note below, i.e. the frequency function = $f_Y(y) = \frac{1}{2} \cdot e^{-y/2}$, thus $f_Y(y) \in \chi_2^2$.

For $K = -2 \cdot \sum_{i=1}^{N} \ln(p_i)$ we then get that $K \in \chi_{2N}^2$

Note: $P(X < e^{-y/2}) = e^{-y/2}$. This is explained by: Since $y \in [0, \infty] \Rightarrow e^{-y/2} \in [0,1]$. Furthermore, since X is uniformly distributed we get that $P(X < a) = a \Rightarrow P(X < e^{-y/2}) = e^{-y/2}$

Appendix E.

This appendix contains, on page E1, plots of the intercept points, the blue curves, for all 201 frequencies from 200 MHz to 2 GHz for the single Basic, Large and dual Large stirrer efficiency measurements performed in the small chamber E4 and the large chamber E3. On page E2 the samples needed to reveal correlation is plotted for the corresponding plot on this page.

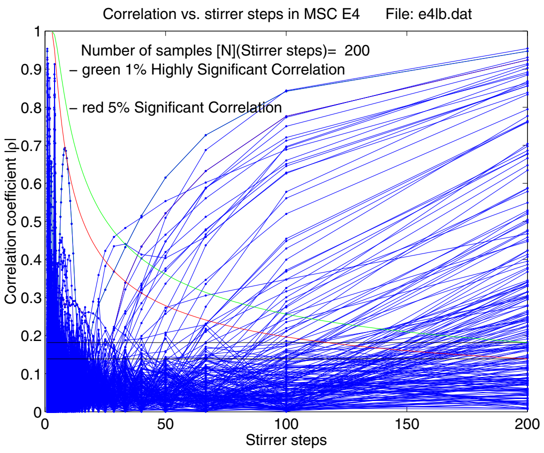


Figure E1. E4 Basic Stirrer
Max stirrerpos/rev: 29.8

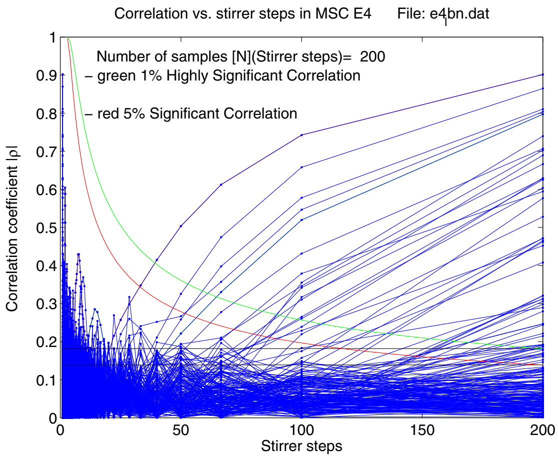


Figure E2. E4 Large Stirrer
Max stirrerpos/rev: 39.2

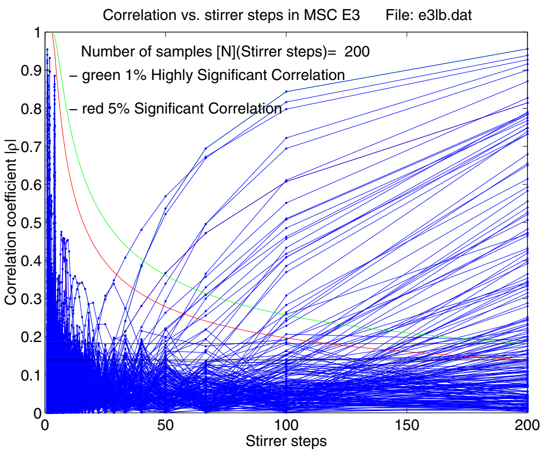


Figure E3. E3 Basic Stirrer
Max stirrerpos/rev: 35.2

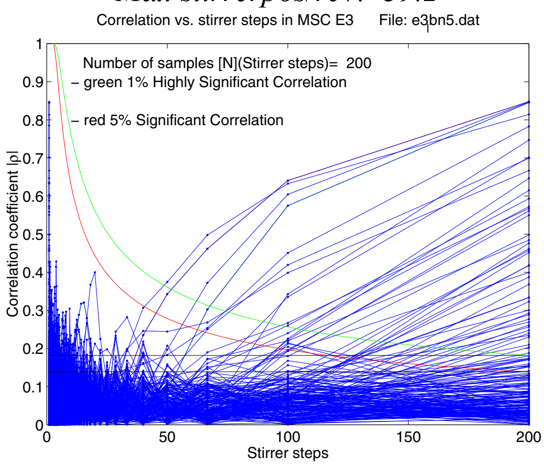


Figure E4. E3 Large Stirrer
Max stirrerpos/rev: 48.7

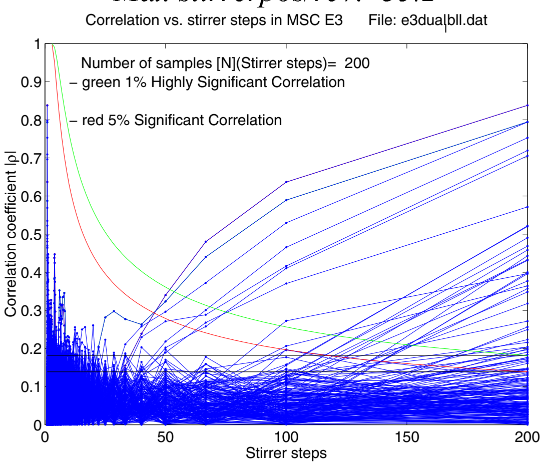


*Figure E5. E3 2*Large Stirrer Syncr.*
Max stirrerpos/rev: 51.8

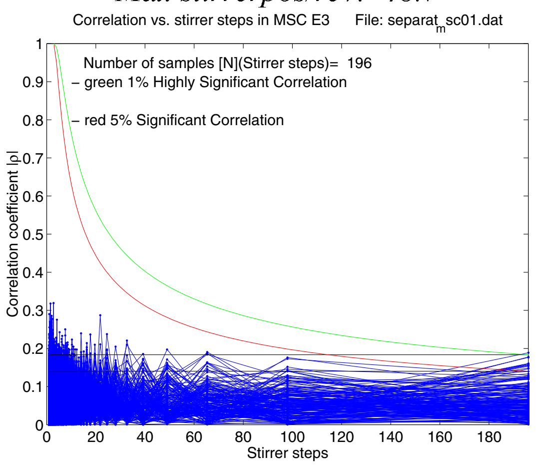


*Figure E6. E3 2*Large Stirrer Ind.*
Max stirrerpos/rev: 193.2

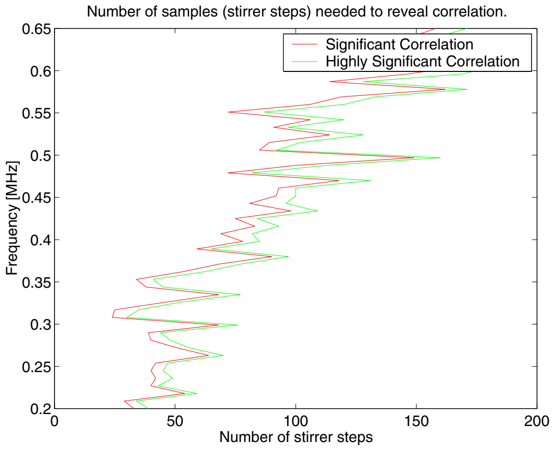


Figure E7. E4 Basic Stirrer

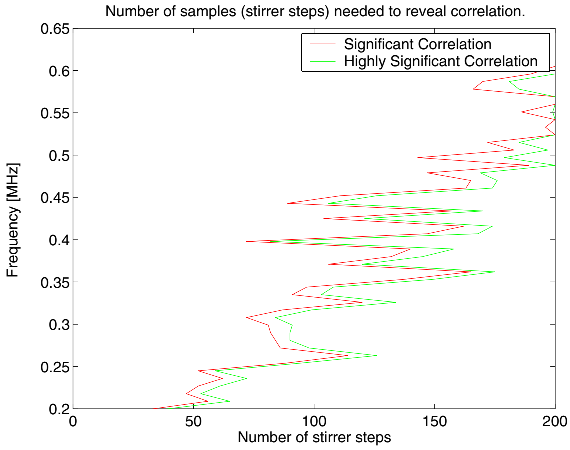


Figure E8. E4 Large Stirrer

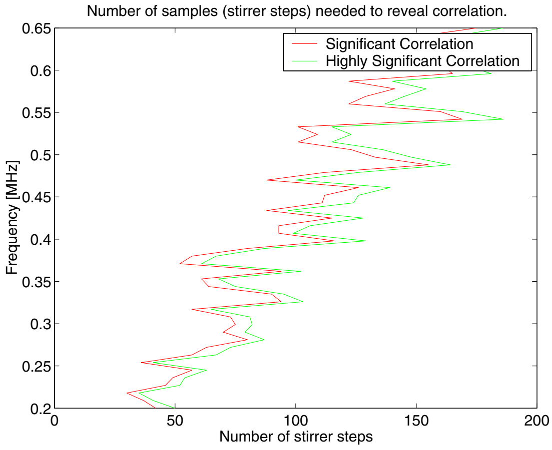


Figure E9. E3 Basic Stirrer

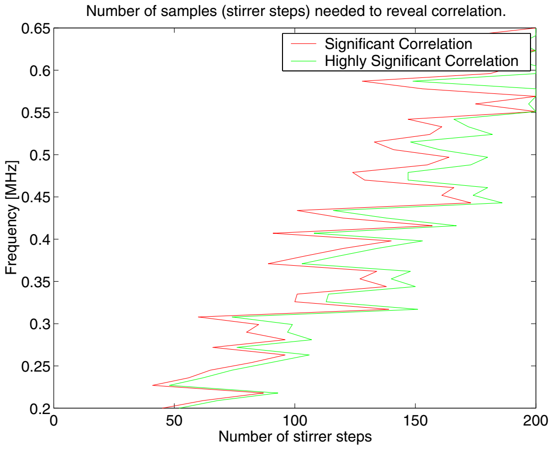


Figure E10. E3 Large Stirrer

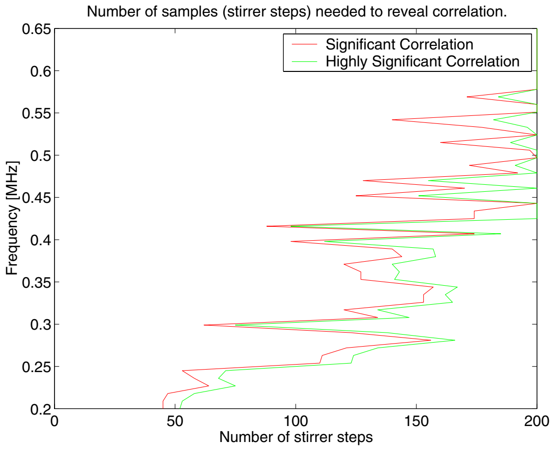


*Figure E11. E3 2*Large Stirrer Synchronous*

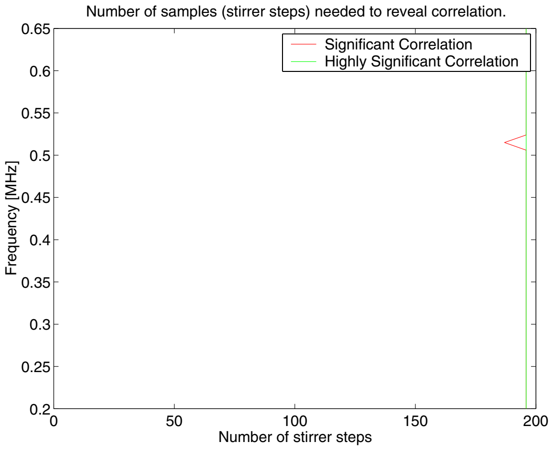


*Figure E12. E3 2*Large Stirrer Independently stirred 14*14 steps*

Appendix F. Noise reduction

The figures has been subjected to noise reduction, to get rid of the clutter. This as we are mainly interested in the boundaries between the uncorrelated and the correlated regions. This is performed like the following. In the matrix that is plotted, see figure F1, we start at the lowest frequency and at max number of stirrer steps and compare the values and ensuring a gradual decrease in the correlation coefficients going from right to left. This is then repeated for all frequencies, see figure F2. The procedure is the repeated in the vertical direction starting from the same spot and compare the values and ensuring a gradual decrease from bottom to the top, see figure F3.

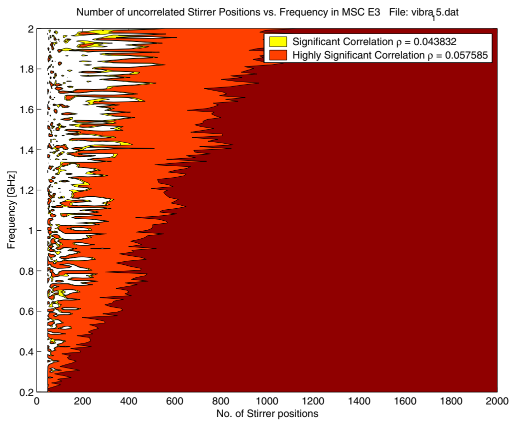


Figure F1. No noise reduction

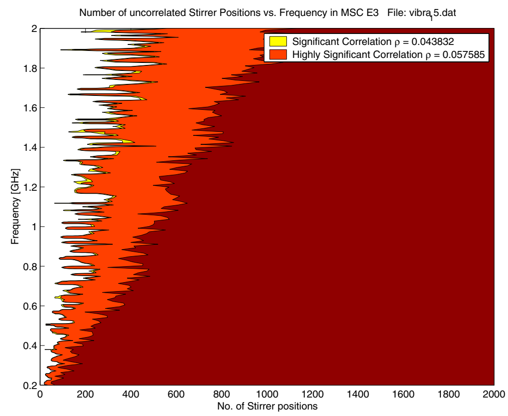


Figure F2. One dimensional reduction

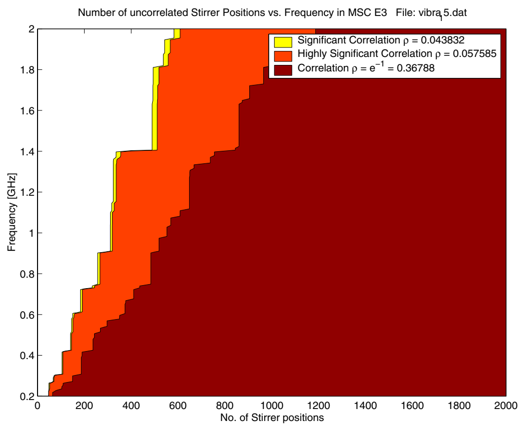


Figure F3. Two dimensional reduction