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Finite element analysis of the splitting test

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1 Introduction

At the Swedish Defence Research Agency (FOI) applied research is done on how to assess and optimize the protection level of concrete structures subjected to conventional weapon loadings. The modelling of concrete material is a challenging task, especially when the loading is three dimensional and applied at high rates. Two different material models have been used extensively at the Swedish Defence Research Agency and one of them is used in this study. The scrutinized material model has mainly been used for problems involving impact loading and it is here shown to be inadequate for these purposes. This report presents the behaviour of this model in a splitting test and a simplified non-local continuum approach applied in a tensile test. Based on two years of experience in this field, the report is concluded with thoughts on what is necessary to better model concrete materials.

1.1 Splitting test

Performing a direct tension test on a concrete material is expensive and the preparation of the test specimen is time demanding. The loading rig has to be very stiff in order to capture the complete response curve due to the softening behaviour of the concrete material. The preparation of the specimen consists of manufacturing a notched specimen of the granular material and of gluing grips to the ends, to which the load cell is to be fastened. A simpler way of determining the tensile strength is the splitting test, or the Brazilian test, originating from rock mechanics. This test, on the other hand, is load-controlled and does not provide any information on the softening behaviour. In this method a cubical or cylindrical specimen is subjected to a uniaxial line-load that induces tensile stresses at the centre of the specimen, see Figure 1-1 and Figure 1-2.

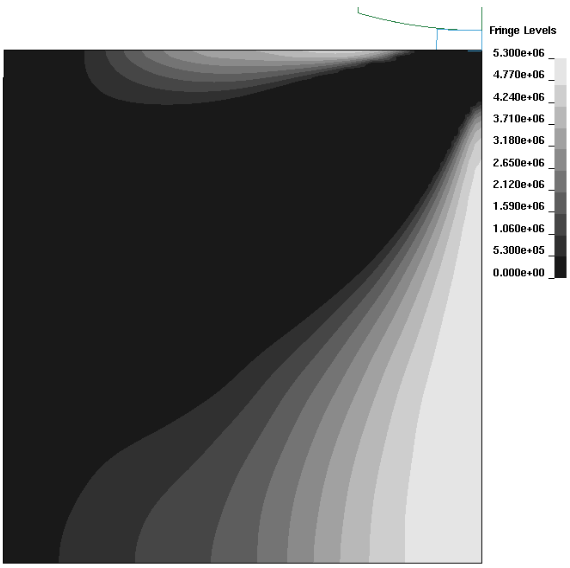

Figure 1-1 Fringes of lateral stresses in an elastic analysis.

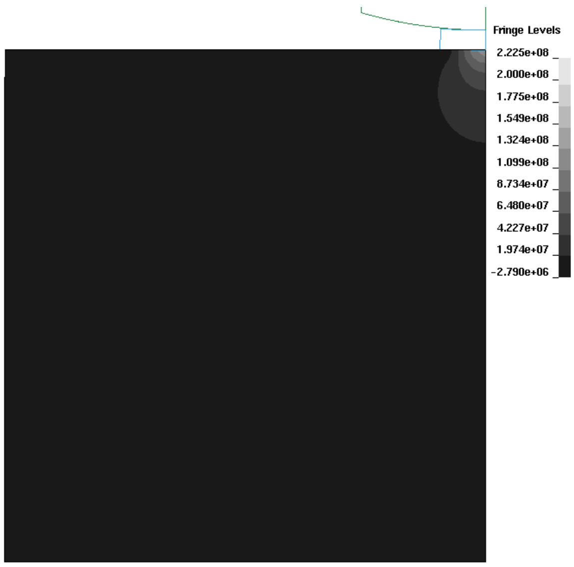

Figure 1-2 Fringes of hydrostatic pressure in an elastic analysis.

The splitting test only works with materials that have an increase in strength with increasing pressure. Otherwise the failure would not occur at the centre of the specimen but in the area under the applied load where large compressive stresses are induced. The test set-up is shown in Figure 1-3. In the test the peak load is registered and the splitting strength is then calculated using an analytical expression and an empirical relation to approximate the tensile strength, cf. Ljungkrantz et al. [1]. At the Swedish Defence Research Agency, this has been the prevailing method when estimating tensile strength input for numerical models. Examples of earlier work where this test has been studied numerically are Feenstra and Borst [2], who used a cubical specimen, Sawamoto et al. [3], who used a cylindrical specimen and a discrete element method and Comi [4], who used a cubical specimen and the Finite Element Method (FEM).

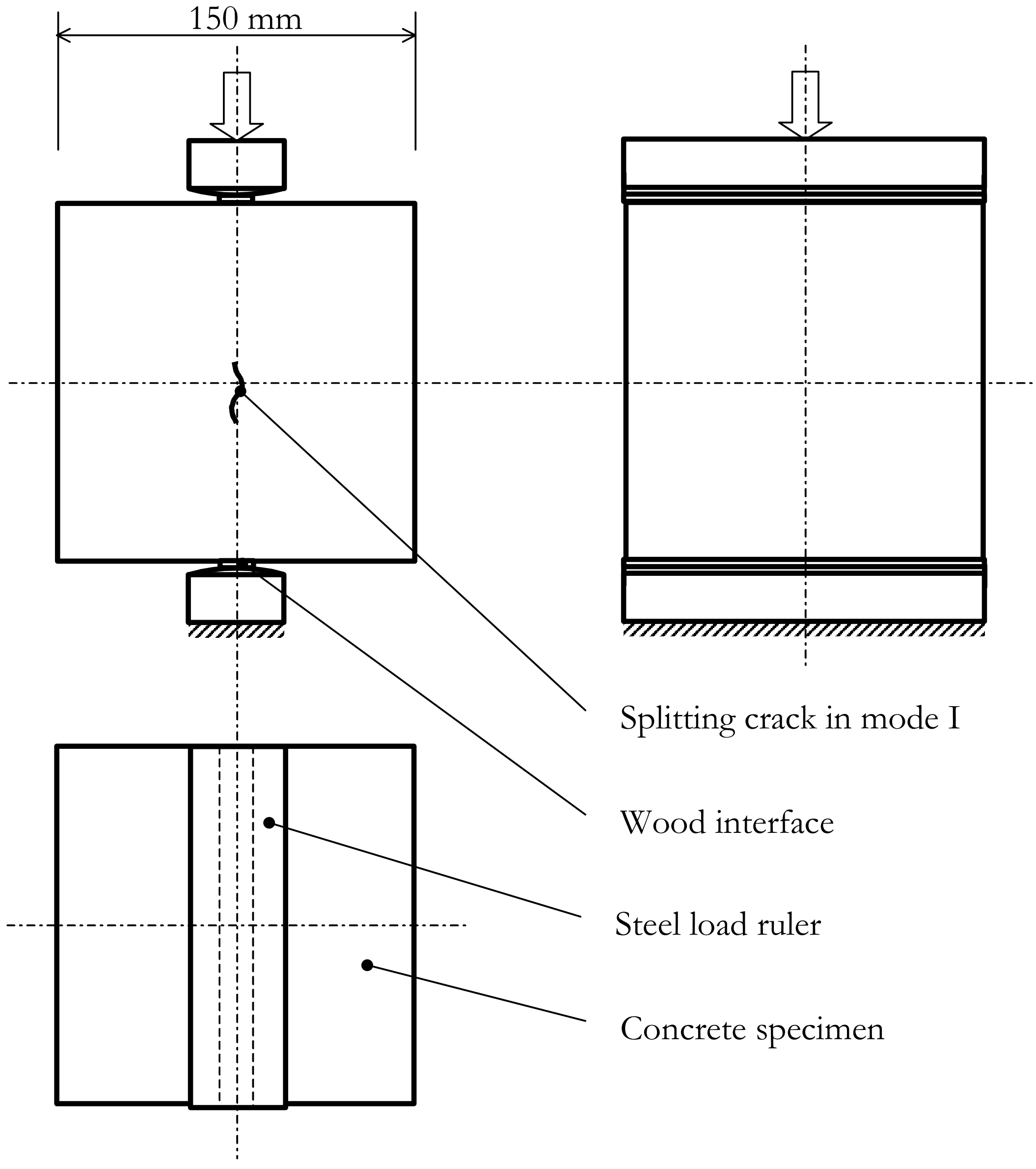

Figure 1-3 Test set-up of splitting test according to the Swedish standard SS 13 72 13 [5].

1.2 Non-local model

The presence of a strain singularity gives rise to mesh sensitivity and in combination with a failure criterion a non-convergent solution. One way to circumvent problems of localization and singularities is to introduce a non-local measure of deformation. In this way the stress at a point does not only depend on the deformation at that point, which is one of the fundamental statements in continuum mechanics, cf. Noll [6], but also on the deformation in a neighbourhood to that point. The idea to use a non-local description of the continuum is found in Eringen [7]. A simple way of introducing non-locality is to define a non-local strain measure based on a weighted average of the local strain field, cf. Bazant and Planas [8].

2 Methods

2.1 Non-local kinematics

In LS-DYNA version 960 a simplified method for non-local treatment has been implemented for solid elements with one-point integration. The local strain measure is weighted and integrated over the element neighbourhood using the following expression, from Bazant and Planas [8], for the non-local rate of evolution of the strain:

$$D(\mathbf{x}_e) = \frac{\sum_{i=1}^{N_r} D_{local} w_{ei} V_i}{\sum_{i=1}^{N_r} w_{ei} V_i}$$

$$w_{ei} = w(\mathbf{x}_e - \mathbf{x}_i) = \frac{1}{\left[1 + \left(\frac{\|\mathbf{x}_e - \mathbf{x}_i\|}{L}\right)^p\right]^q}$$

where D_{local} is the local strain rate measure, w_{ei} is a weight function, $\mathbf{x}_e$ is the position vector of the element integration point, $\mathbf{x}_i$ is the position vector of a neighbouring element and V_i is the corresponding element volume. L is the radius of the element neighbourhood as shown in Figure 2-1. The weight function with the parameters p and q set to 8 and 2, respectively, is shown in Figure 2-2 for the two-dimensional case. In this study the parameters were set to p=8 and q=2 with neighbourhood radius 5 and 2.5 mm. The effect on the weight function of the parameters is shown in Figure 2-3 and Figure 2-4.

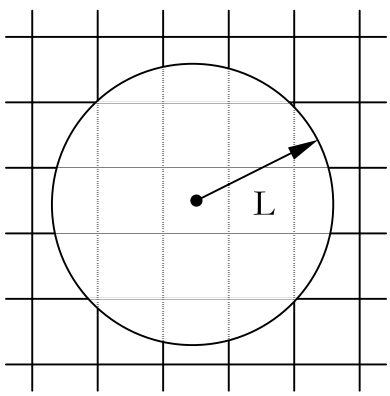

Figure 2-1 Element neighbourhood with radius L.

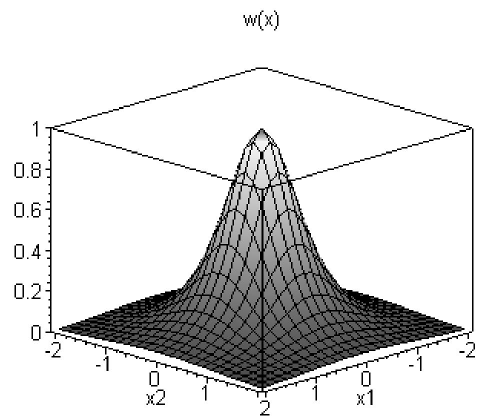

Figure 2-2 Weight function over element neighbourhood with p=8 and q=2.

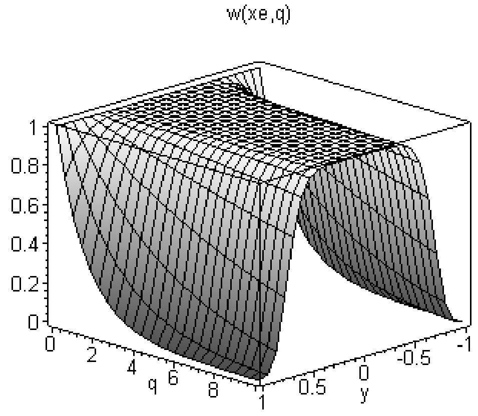

Figure 2-3 Weight function as a function of neighbouring element position and q with p=8.

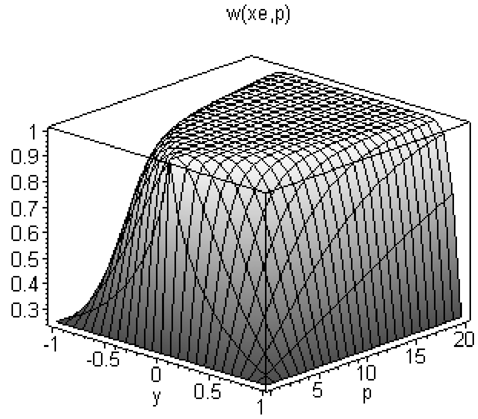

Figure 2-4 Weight function as a function of neighbouring element position and p with q=2.

2.2 Constitutive equations

The concrete material was modelled with a user-modified version of the K&C concrete model by Malvar et al. [9], where a scaling of the softening behaviour based on the element size was introduced. The material input parameters were taken the same as in Unosson [10] and [11]. The wood interface and the steel load ruler were modelled using an isotropic elastic model with the parameters given in Table 2-1.

Table 2-1 Isotropic elastic parameters for the wood interface and the steel load ruler.

Material	Mass density [kg m^{-3}]	Modulus of elasticity [GPa]	Poisson's ratio [-]
Wood	500	17	0.45
Steel	7 800	200	0.2

The non-local treatment of strains is not implemented in LS-DYNA version 960 for all material models, why an elastic-plastic model with the von Mises yield criterion and a kinematic hardening with the parameters given in Table 2-2 was used in a tensile test instead.

Table 2-2 Parameters for the elastic-plastic model with isotropic hardening.

Mass density [kg m^{-3}]	Modulus of elasticity [GPa]	Poisson's ratio [-]	Yield strength [MPa]	Hardening modulus [GPa]
2 420	44	0.16	5.3	1.0

2.3 Finite element analysis

For the spatial discretization reduced integrated eight node brick elements with viscous hourglass control were used. The contact definition relied on a penalty based surface-to-surface algorithm with friction. The loading was applied through prescribed nodal displacements. The finite element analysis was carried out using LS-DYNA version 960.

3 Results

3.1 Splitting test

The current version of the software is incapable of treating the K&C concrete model with a non-local theory. Consequently the splitting test has only been analyzed with a local formulation. Three different models were defined for the simulations; a three-dimensional model, a plane strain model and a plane stress model. The geometry for the three-dimensional model is shown in Figure 3-1 and the geometry for the plane stress and plane strain models in Figure 3-2. Four levels of spatial discretization were used with every geometry.

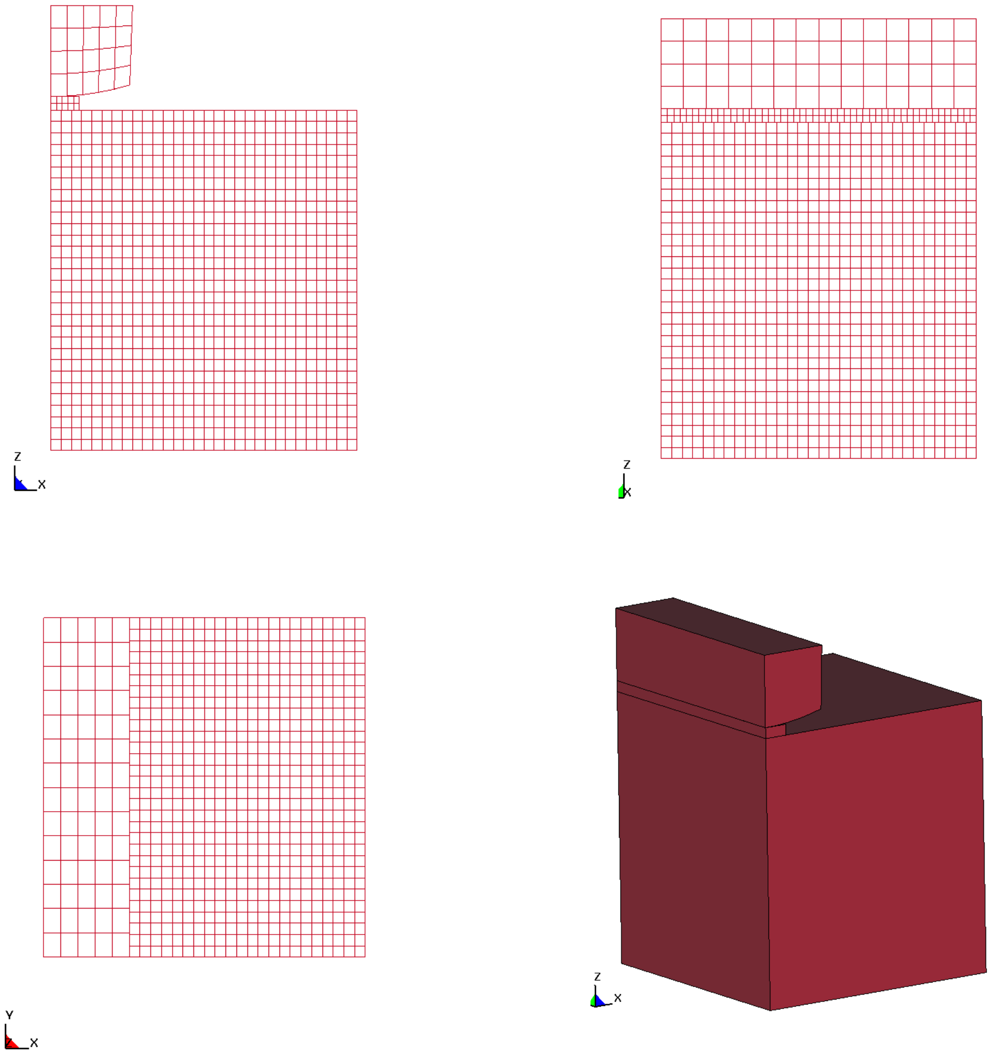

Figure 3-1 Three-dimensional geometry with symmetry in the xy-, yz- and xz-planes.

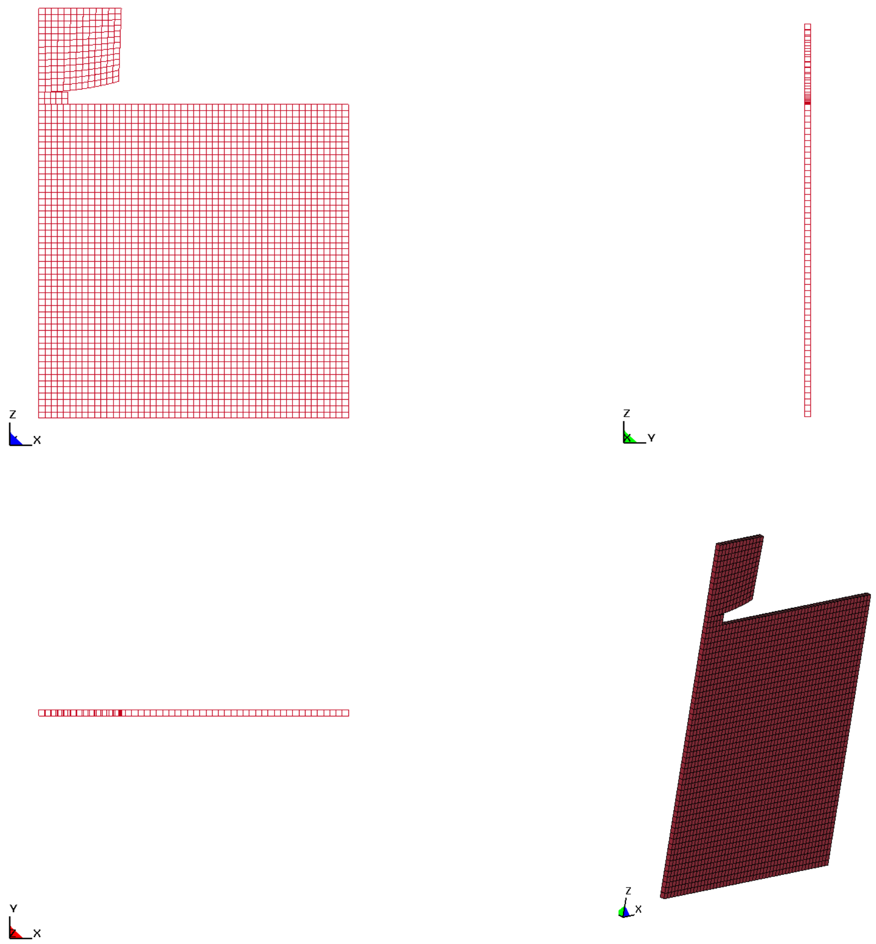

Figure 3-2 Plane strain and stress geometry with symmetry in the xy- and yz- planes.

The global convergence properties and the resulting load-displacement relations for these three models are shown in Figure 3-3 to Figure 3-8. In the figures *h* is the finite element size.

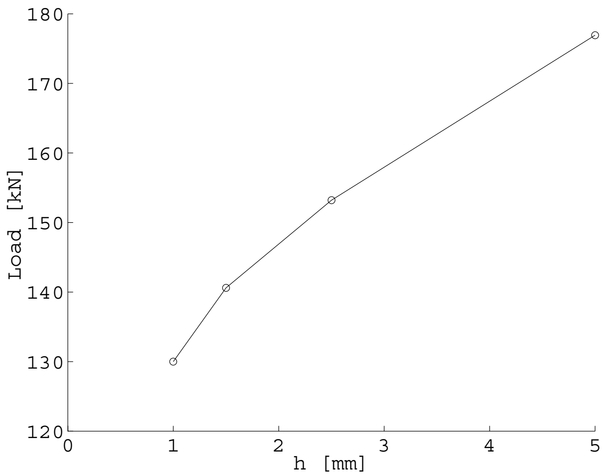

Figure 3-3 Peak load in the 3D-model versus finite element size.

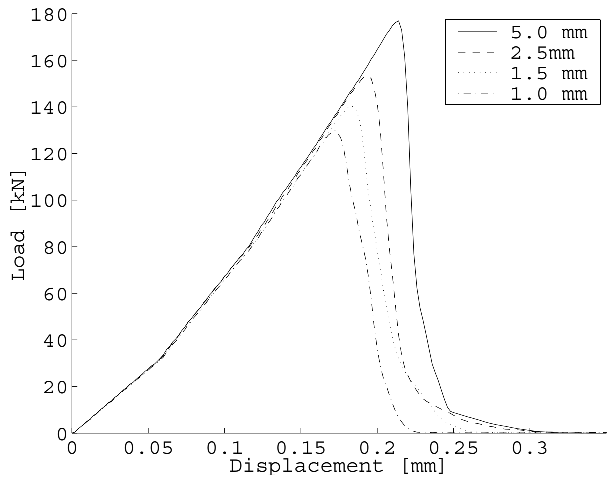

Figure 3-4 Load versus displacement in the 3D-model with four different finite element sizes.

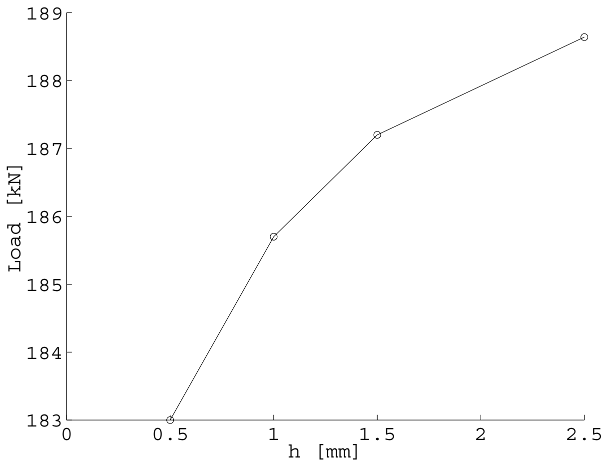

Figure 3-5 Peak load in the plane strain model versus finite element size.

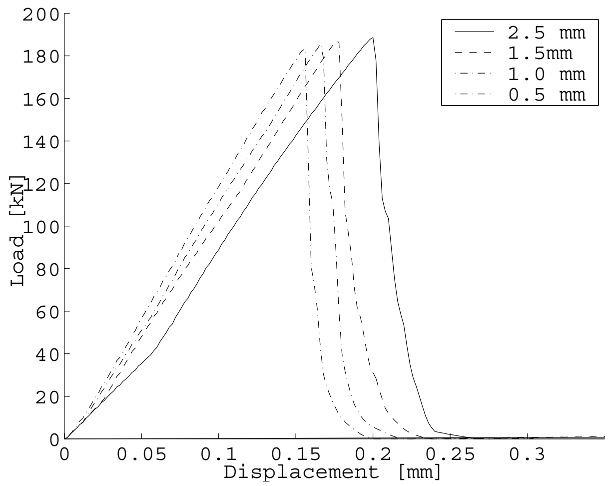

Figure 3-6 Load versus displacement in the plane strain model with four different finite element sizes.

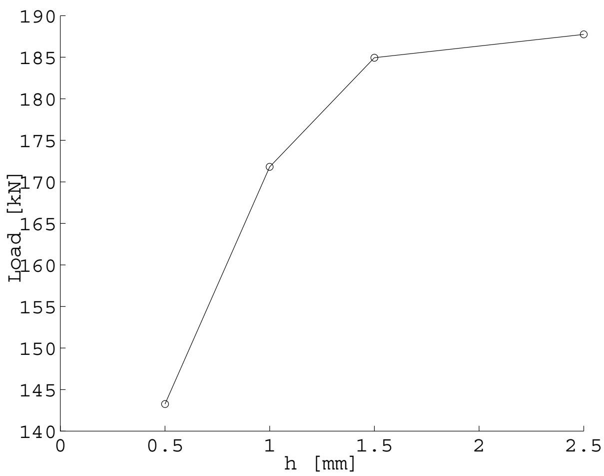

Figure 3-7 Peak load in the plane stress model versus finite element size.

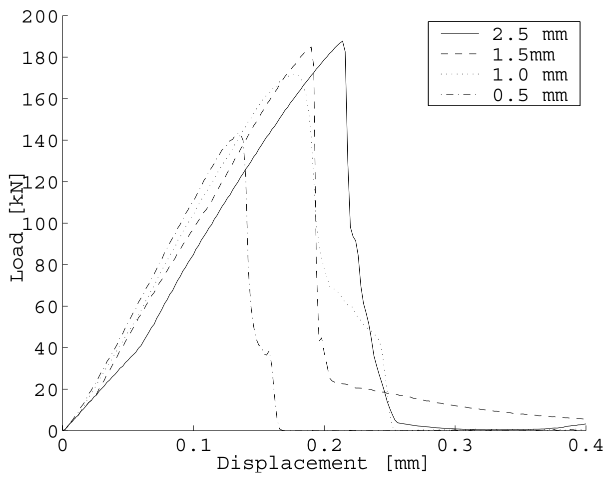

Figure 3-8 Load versus displacement in the plane stress model with four different finite element sizes.

For the 2.5 mm mesh a mode I crack is initiated at the centre of the specimen, in accordance with test results, see Figure 3-9. As the mesh is refined the point of crack initiation is moved to the loading zone, see Figure 3-10. Arrows in the figures indicates the points of crack initiation.

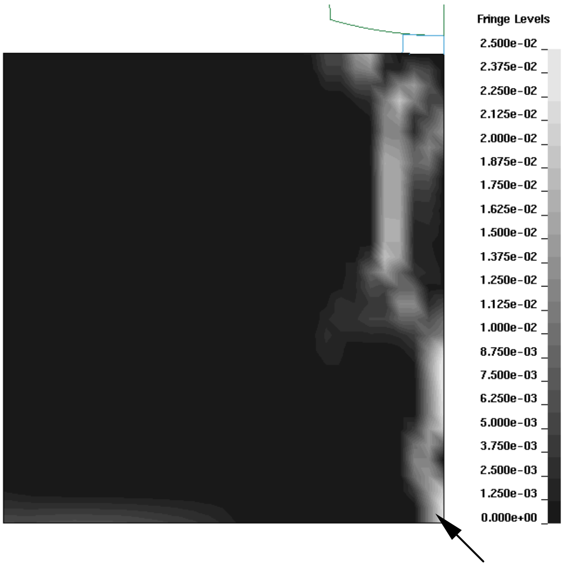

Figure 3-9 Fracture for the 2.5 mm mesh at 0.20 mm displacement. Representation by a scalar valued damage parameter.

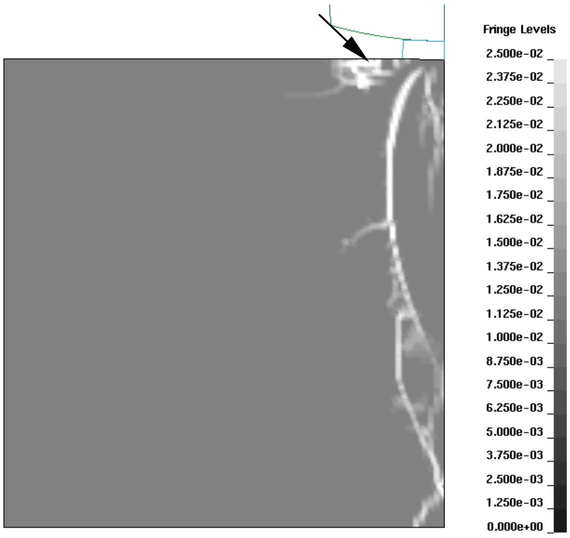

Figure 3-10 Fracture for the 0.5 mm mesh at 0.16 mm displacement. Representation by a scalar valued damage parameter.

3.2 Direct tension test with non-local theory

The splitting test only works with brittle materials that have pressure dependent strength. The non-local treatment is not available for any appropriate model in the current version of the software. To investigate the possibility of using a non-local theory when dealing with singularities the elastic-plastic model described in Section 2.2 was used in a direct tension test. A 100x50xh mm plate in plane stress was set up with a 2x2 mm notch introducing a singularity (see Figure 3-11). Computations were carried out with both local and non-local theory and the resulting plastic strain fields are shown in Figure 3-12 and Figure 3-13.

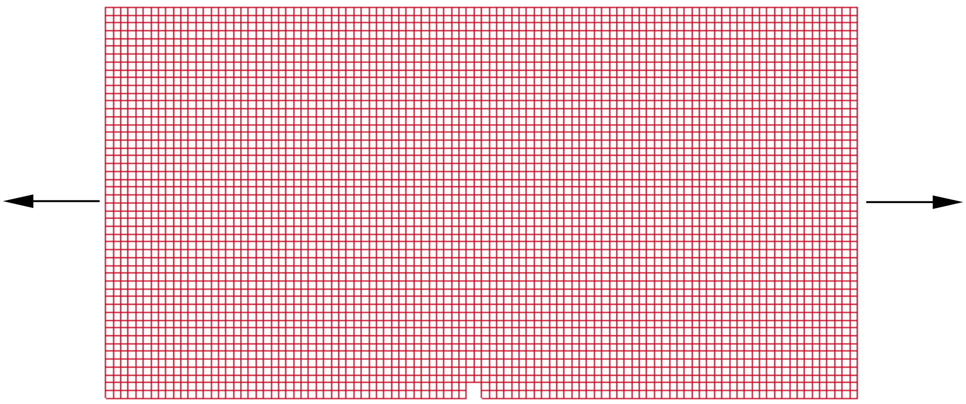

Figure 3-11 Notched geometry for the direct tension test. (h=1 mm)

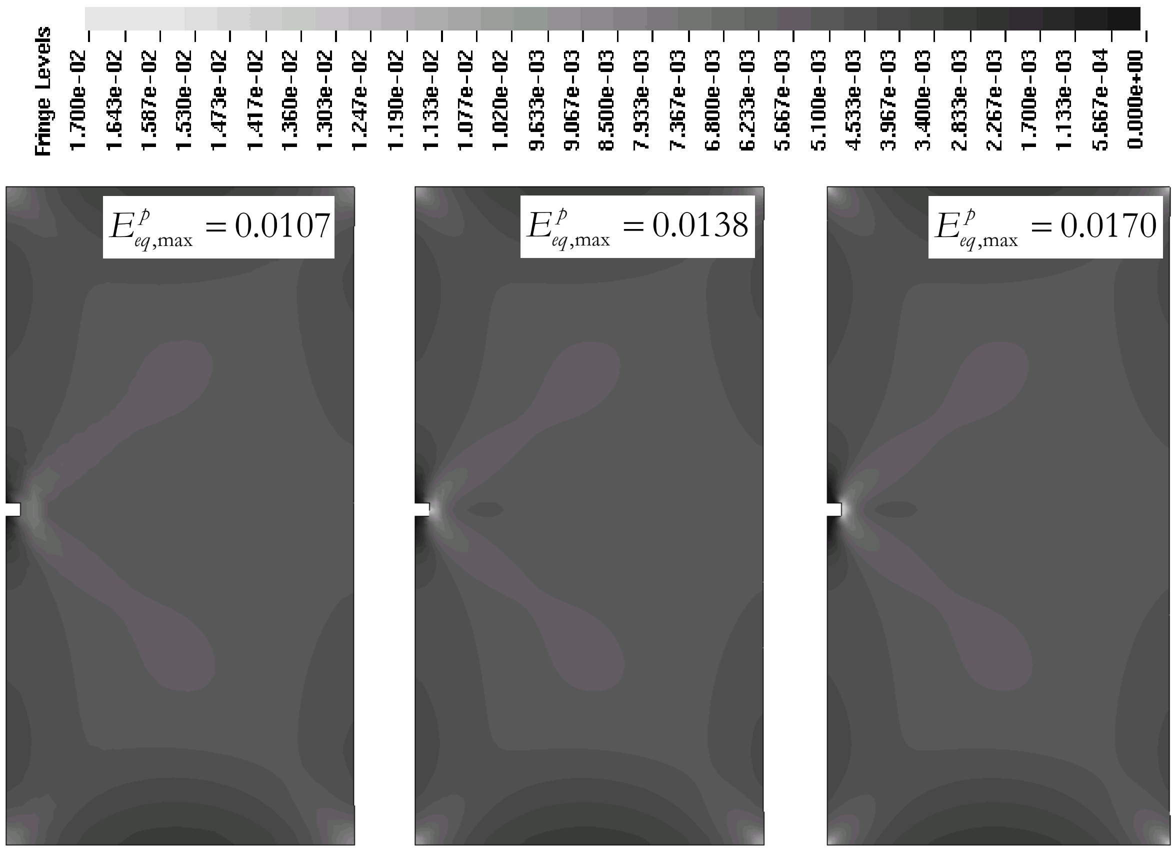

Figure 3-12 Local plastic strain fields (h=2.0, 1.0 and 0.5 mm).

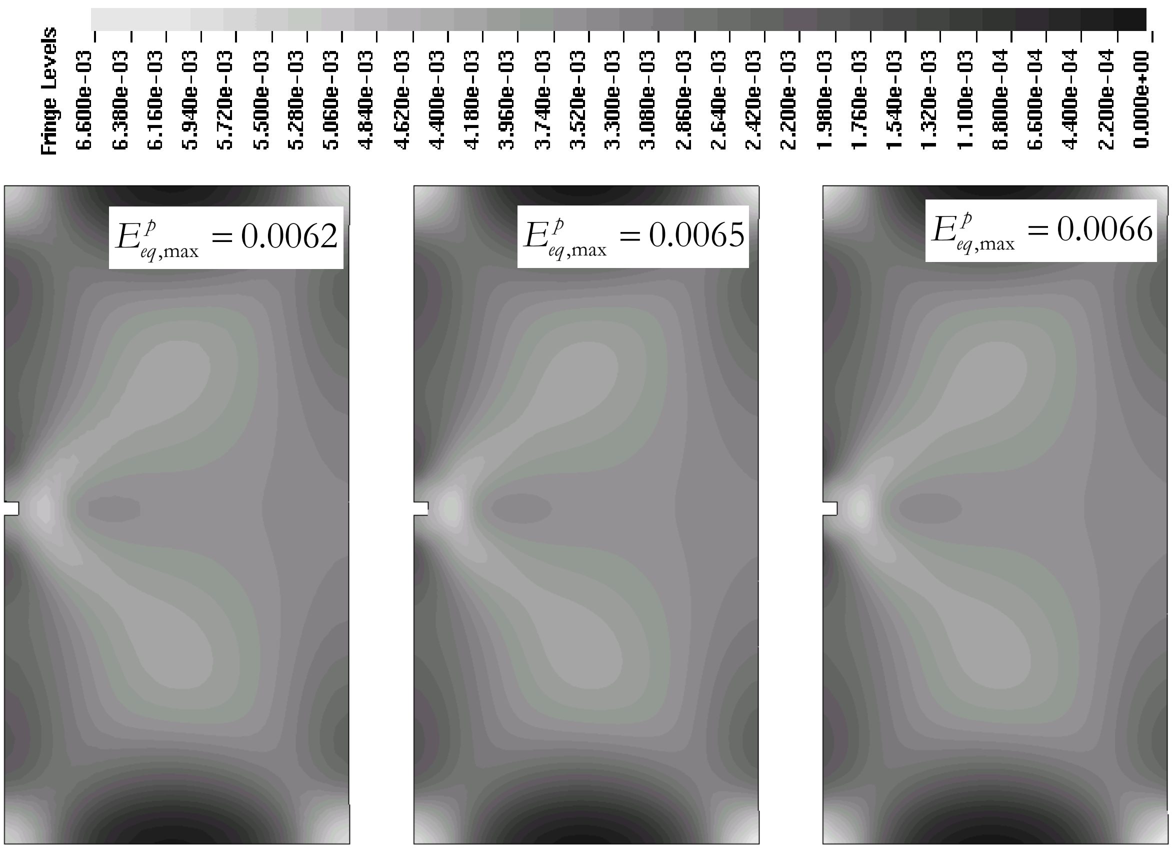

Figure 3-13 Non-local (L=5 mm) plastic strain fields (h=2.0, 1.0 and 0.5 mm).

With the local theory the maximum plastic strain in the finest mesh is 1.6 times the plastic strain in the 2.0 mm-mesh, whereas with the non-local theory the same comparison gives a factor 1.06. The global load-displacement relations are the same for all models as can be seen in Figure 3-14. The CPU-cost is shown graphically in Figure 3-15 for different levels of spatial discretization and non-local neighbourhood radius.

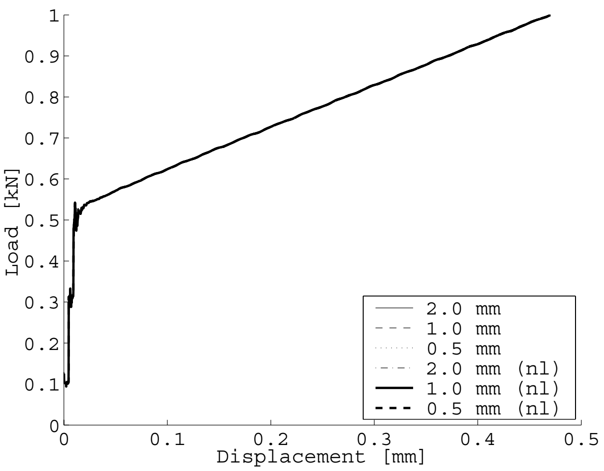

Figure 3-14 Global load-displacement relation for local and non-local theory with different finite element sizes.

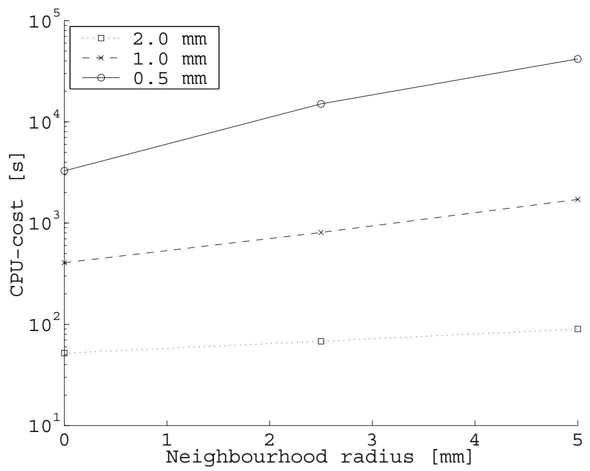

Figure 3-15 CPU-cost versus neighbourhood radius with different finite element sizes.

4 Discussion

Splitting test and the K&C concrete material model

The change of the fracture behaviour in the splitting test as the mesh is refined, is due to the occurrence of a singularity in the loading zone. At this singularity the solution converges towards immediate failure as the mesh density is increased. This is not in accordance with observations of experimental results. The overall experience from this material model gives that it will not be used in future work at FOI.

Non-local deformation

The use of a non-local theory results in a converging solution even though strain singularities are present. However, how large should the domain of influence, the element neighbourhood, be chosen? The neighbourhood corresponds to a statistical representative volume, i.e. the smallest volume for which the statistics do not change. In Bazant and Planas [8] it is suggested that tests of geometrically similar notched specimens with different sizes should be used. Iterative computations are then used to determine the size of the representative volume.

A discussion on mechanical constitutive equations for concrete

The following notes apply to the mechanical modelling of concrete materials subjected to monotonically increasing loads at high rates.

Elastic domain: It is sufficient to use isotropic hypoelasticity with the additative split of the rate-of-deformation tensor. The anisotropic elastic domain should be convex and defined in strain space in order to have a stable material description according to Lubliner [12]. In addition, from a theoretical point of view a closed elastic domain is more appealing than an open deviatoric domain combined with a separate pressure-compaction curve. Careful modelling of the elastic domain at low pressures is very important to correctly model spalling and to reproduce standard material characterization loading paths.

Inelastic domain: Plasticity is the theory of time-independent inelastic deformations, cf. Hill [13]. Strain-rate scaled plasticity, used in many material models, is thus a contradiction and can lead to numerical oscillations. Instead, a viscoplastic theory with a non-associated flow rule coupled to isotropic damage should be used, cf. Lemaitre and Chaboche [14]. Available experimental response curves can be used when defining relations governing inelastic deformations, both volumetric and deviatoric.

Implementation: For air blast loading applications, with relatively small deformations, a material description of the motion of the structure is the best approach. A material description of motion is computationally efficient and accurate at moderate strain levels. A target penetrated by a projectile should be described in a spatial reference frame so that the need for numerical erosion is eliminated. This calls for an implementation that allows both material (Lagrangian) and spatial (Eulerian) descriptions of motion.

A non-local theory such as the one used in this report should be used in the presence of singularities, keeping in mind that this calls for further size effect testing.

For the applications at hand solid elements are sufficient. However, the implementation should allow for full integration of elements near the impact zone.

Model input: Obtaining model parameters for the full range of loading magnitudes and loading rates in defence applications is not yet possible. However, registrations from high explosive planar wave set-ups enable the extraction of material data for uniaxial compaction at high strain rates and pressures. This method will be employed at the Swedish Defence Research Agency for different materials.

Fitting of material model parameters is possible by comparing simulation results to real projectile velocity history data. The velocity history through the target can be accurately registered with a Doppler radar or with accelerometers mounted inside the projectile.

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