SWEDISH DEFENCE
RESEARCH AGENCY

FOI-R--0334--SE
December 2001
ISSN 1650-1942

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Issuing organization	**Report number, ISRN**	**Report type**
FOI – Swedish Defence Research Agency Systems Technology SE-172 90 Stockholm	FOI-R--0334--SE	Scientific report
	Research area code 4. C4ISR	
	Month year December 2001	**Project no.** E6012
	Customers code 1. Research for the Government	
	Sub area code 43 Underwater Surveillance Sensors	
Author/s (editor/s) Bror Jönsson Kristofer Döös Jonas Nycander Peter Lundberg	**Project manager** Ilkka Karasalo	
	Approved by	
	Sponsoring agency Swedish Armed Forces	
	Scientifically and technically responsible Jonas Nycander	

Report title

Fjord seiches in the Gulf of Finland; possible impacts on the global Baltic seiches

Abstract (not more than 200 words)

A spectrum analysis was carried out on sea level data from three stations in the Gulf of Finland, and one in the Archipelago sea. The spectra show amplification of the K_1 and O_1 tidal modes, but not of the M_2 and S_2 modes. The K_1 and O_1 modes have a period around 24 hours, which coincides with the estimated period of a possible fjord sieche in the Gulf of Finland. Despite a stronger astronomical forcing, M_2 and S_2 are much weaker in the spectra. This is consistent with the proposal by Witting (1911) of such a local fjord seiche. A two-dimensional linear shallow-water model of the Baltic Sea was then used to study these amplification fenomena in the Gulf of Finland. The results indicate a strong resonance for the periods 23 and 27 hours, well correlated with the K_1 and O_1 modes. This resonance is most naturally interpreted as a local fjord seiche, rather than as a global eigenmode of the whole Baltic Sea.

Keywords

Fjord seiches, Baltic Sea eigenmodes, Gulf of Finland

Further bibliographic information	**Language** English
ISSN 1650-1942	**Pages** 24 p.
	Price acc. to pricelist
	Security classification

Utgivare	Rapportnummer, ISRN	Klassificering
Totalförsvarets Forskningsinstitut - FOI Systemteknik 172 90 Stockholm	FOI-R--0334--SE	Vetenskaplig rapport
	Forskningsområde 4. Spaning och ledning	
	Månad, år December 2001	**Projektnummer** E6012
	Verksamhetsgren 1. Forskning för regeringens behov	
	Delområde 43 Undervattenssensorer	
Författare/redaktör Bror Jönsson Kristofer Döös Jonas Nycander Peter Lundberg	**Projektledare** Ilkka Karasalo	
	Godkänd av	
	Uppdragsgivare/kundbeteckning Försvarsmakten	
	Tekniskt och/eller vetenskapligt ansvarig Jonas Nycander	

Rapportens titel (i översättning)

Egensvängningar i Finska viken och deras relation till Östersjöns egensvängningar

Sammanfattning (högst 200 ord)

Målet med den här studien är att fastställa huruvida ytvattenståndvariationerna i Finska viken kan beskrivas som en egensvängning av fjordtyp eller som en egensvängning till hela Östersjön. En spektral analys av ytvattendata från tre finska stationer har genomförts och dessutom har Östersjön inklusive Finska viken modellerats med en två-dimensionell linjär ”shallow-water” modell. Resultatet av den spektrala ananlysen visar att de högsta amplituderna erhålls vid en period på 24 timmar. Detta sammanfaller med den uppskattade perioden för en eventuell lokal egensvängning av fjordtyp i Finska viken. Modellstudien visar att resonans sker för perioderna 23 och 27 timmar. Modellen ger således upphov till en frekvensuppdelning av egensvängningarna. Detta fenomen kan dels bero på näraliggande lokala moder i övriga Östersjön, dels vara ett resultat av att icke-linjära effekter och interna vågor inte är representerade i modellen. Slutsatsen av studien är att ytvattensvängningarna som oberveras i Finska viken bör tolkas i termer av en lokal egensvängning av fjordtyp. Denna företeelse kan i sin tur beskrivas som ett antal överlagrade egensvängningar till Östersjön vilka genom resonans skapar höga amplituder vid de nämnda periodtiderna.

Nyckelord

Egensvängningar i fjorder, Östersjöns egensvängningar, Finska viken

Övriga bibliografiska uppgifter	Språk Engelska
ISSN 1650-1942	**Antal sidor:** 24 s.
Distribution enligt missiv	**Pris: Enligt prislista** **Sekretess**

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1 Introduction

A proper knowledge regarding the characteristics of the Baltic seiches is of great practical importance, not least in view of possible flooding of St Petersburg, located at the inner end of the Gulf of Finland.

Ever since Forel in the 1870s described the standing oscillations of Lake Geneva, considerable research efforts have been directed towards the prediction and analysis of seiches. Early last century Chrystal (1905) developed a technique for solving the standing-wave eigenvalue problem for realistic bathymetries, in which variations of the width and depth of the "channel" could be dealt with analytically. It was particularly well adapted to elongated basins, such as the lochs of his native Scotland. Witting (1911) applied this methodology when investigating the tides of the Baltic, and made particular note of the fact that the K_1 and O_1 diurnal tidal modes were amplified in the Gulf of Finland.

Neumann (1941) focused on the global seiche modes of the entire Baltic, using sea-level data from a number of events when particularly pronounced standing oscillations had been observed. His theoretical analysis was based on computational methods due to Defant (1918) and Hidaka (1936). The rapid post-war development of the digital computer, as well as the catastrophic North-Sea floods in 1953, provided a great impetus for this branch of research, and in the late 1950s two important studies of the Baltic were conducted. Svansson (1959) presented the first barotropic numerical numerical model of this almost land-locked sea, and Lisitzin (1959) undertook a detailed study of 17 instances when regular sea-level oscillations (encompassing at least four distinctly recorded periods) had been observed at Haamina in the Gulf of Finland. She determined the associated periods, finding that they tended to be fairly constant, around 26.4 hours, and furthermore undertook a systematic discussion of the possible effects of the Coriolis force on the oscillations of the Gulf of Finland.

The first study properly incorporating the effects of the earth's rotation was conducted by Wübber and Krauss (1979). They used a two-dimensional shallow-water model with a 10-km grid size. The model was forced by continuously applied harmonic oscillations of varying periods to calculate the various standing-wave modes of the basin.

All of these investigations have had a common denominator: the assumption that global seiches are a predominant feature of the oscillations of the Baltic. The aim has primarely been to calculate the period of these seiches, rather than to determine whether they are in fact a predominant feature. This assumption is, however, not well supported by observations.

This is particularly true of the lowest mode, involving oscillations of the system Gulf of Bothnia-Baltic proper. Thus, analysing observations from a timespan of four years, Neumann (1941) found only very few instances of such an oscillation, and even the one he showed in the paper is not very clear, encompassing hardly more than two oscillation periods. Its period was about 40 hours. This agreed with his theoretically calculated value of 39 hours, but this calculation neglected the Coriolis force, and according to the numerical calculations by Wübber and Krauss, the Coriolis force reduces the period from 40 to 31 hours. Metzner et al. (2000) could not find any evidence for this kind of oscillation in a recent study using combined tide-gauge data and satellite-altimetric data.

Neumann found seven instances of a different mode, with its axis from the Gulf of Finland to the Belt Sea, bypassing the Gulf of Bothnia. Its period was about 27 hours, in agreement with his one-dimensional non-rotating calculations. However, even these events were never observed to encompass more than four oscillation periods. Judging from the mode structure, this kind of oscillation mode might correspond to any of the second, third and perhaps fourth eigenmodes found by Wübber and Krauss, which had the approximate periods 26, 22 and 20 hours, respectively.

In general, spectral analysis of tide-gauge observations show no clear peaks at any of the theoretically calculated eigenfrequencies.

Why are the global seiches rarely seen in the Baltic? A possible reason is the complex coastline and bottom topography, which divide it into several bays and sub-basins, separated by straits and sills. This probably means that the global seiches are strongly damped by nonlinear effects and excitation of internal waves, and therefore lose coherence too quickly to be seen in reality.

We propose a different physical picture of the sea-level oscillations of the Baltic: a collection of weakly coupled local oscillators. Each oscillator corresponds to a "fjord mode" or "harbour mode" in a particular bay or sub-basin. These are not proper eigenmodes, since their energy gradually leaks out to the rest of the Baltic Sea, resulting in radiation damping. Nevertheless, their resonance may in fact be sharper than that of the proper global eigenmodes. We find three examples of such local oscillators: the Gulf of Finland, the Belt Sea, and the Gulf of Riga.

Our approach in examining this hypothesis is based on a numerial shallow-water model. The model is run forward in time without forcing, using a large scale initial condition that proves capable of exciting all conceivable oscillations in the main basin of the Baltic. However, we commence the investigation by analysing a set of tide-gauge records from the Gulf of Finland as well as the Baltic proper.

2 Analysis of sea-level data

We first examine the sea-level oscillations in the Gulf of Finland by conducting a spectral analysis of a water-level data set recorded by the Finnish Institute of Marine Research (FIMR) during 1997. Three stations (cf. the map in Figure 1) were used in the analysis: Hamina (60°33.75' N ; 27°10.93'E), Helsinki (60°09.20'N ; 24°57.58'E), and Hanko (59° 49.36'N ; 22°58.79'E). These sites were selected on the basis of their locations: Hamina is in the interior of the gulf, Helsinki in the middle, whereas the Hanko record can be seen as reflecting the open boundary conditions of the elongated basin, since the Osmussaar-Hanko section is frequently taken to delimit the Gulf of Finland from the Baltic proper.

When examining the resulting spectra shown in Figure 2, we first note that there are no signs of the global seiche modes, with the approximate periods 31, 26, 22 and 20 hours (Wübber and Krauss 1979). The most pronounced features in these spectra are instead peaks coinciding with the diurnal K_1 and O_1 tidal periods. This state of affairs may appear somewhat surprising since the semi-diurnal M_2 and S_2 tides are subjected to stronger astronomical forcing than the diurnal component, but, as already recognised by Witting (1911), the K_1 and O_1 tides are in approximate resonance with the gravest-mode “fjord seiche” in the Gulf of Finland. On the basis of the crude predictions provided by Merian’s formula, cf. Defant (1960), the fundamental mode of the oscillation should have a period $T = 4L(gH)^{-1/2}$, where L is the length of a rectangular fjord of depth H. The Gulf of Finland has a length on the order of 400 km from St. Petersburg to Hanko and its average depth is around 40 metres, which thus yields a gravest-mode period of approximately 23 hours. This "fjord-mode interpretation" of the sea-level records in Figure 2 is further reinforced by noting that the spectral amplitude of the pertinent peaks is highest at Hamina and decreases towards the entrance of the gulf.

A number of analogous water-level records taken in the Baltic proper (albeit primarily from the Swedish mainland) have also been subjected to spectral analysis, cf. Figure 3. No signs of “global 30-hour seiching” are visible here, and it is noteworthy that the diurnal tides in the main body of the Baltic are much less pronounced than in the Gulf of Finland. A striking feature of these records is, furthermore, that a semi-diurnal tidal component is so pronounced at Klagshamn in the southern part of Öresund. This is undoubtedly due to the close proximity of this tide-gauge station to the Kattegatt, where the M_2 tide is known to dominate (Defant and Schubert, 1934).

3 Numerical simulations of the seiches

Although water-level recordings at discrete points are valuable when examining the processes described above, they obviously do not suffice for a full dynamical analysis. Tide-gauge data furthermore have the disadvantage that they incorporate purely local as well as meteorologically forced phenomena, and in many cases it may be difficult to discriminate between these “spurious” effects and the signals resulting from the large-scale seiching motion which are the object of the present study. On the other hand, simplified analytical models of the type introduced by Chrystal (1905) tend to represent topographic features in a too crude manner, based as they are on "slow" longitudinal variations of the cross-channel area.

To deal with these problems, a linear shallow-water numerical model of the Baltic Sea incorporating the real bathymetry was employed. Using this, it is possible to examine the outcome of “laboratory runs” where initial conditions and forcing have been manipulated in a controlled fashion. In this way, it is possible to study the system without *a priori* assuming that a global seiche is the primary explanation for the resonant oscillations of the Baltic. The model is a further development of the one originally formulated by Döös (1999). A leapfrog scheme on a C-grid with a grid-size of 2 nautical miles was used, with a sponge zone to handle the open boundary between the Skagerrak and the North Sea proper (cf. the model domain in Figure 4).

The numerical runs were initialised by tilting the sea surface linearly in the west-east direction with respect to its equilibrium position, from -1 m at the western extreme of the Skagerrak to +1 m at St. Petersburg. (As will be seen, this somewhat artificial initial configuration of the free surface not only induces predominantly longitudinal oscillations in the basin but also, to some extent, gives rise to transversal motion.) The initialisation procedure was found to excite oscillatory motion encompassing a wide range of wave-numbers. The model was hereafter run without any forcing for 480 hours using one-hour timesteps. During this time the amplitude of the motion decreased considerably since the model included dissipative processes, parameterized as Rayleigh friction, and the viscosity $A_H = 10^{-3} m s^{-1}$.

3.1 Large-scale oscillations

From the model runs briefly described above, frequency spectra have been calculated at each gridpoint of the domain using FFT. The maps in Fig. 5 show the resulting distribution of the spectral amplitude for the periods from 15 to 37 hours in two-hour intervals. The most striking results from this suite of graphs are the what at first sight appear to be global oscillations with periods of around 23 and 27 hours. Both of these have a large amplitude in the Gulf of Finland, indicating resonance with the local fjord-seiche. At the other extreme of the Baltic basin, viz. the Danish Belt Sea, the 23-hour oscillation only shows weak resonance effects, in contrast to the 27-hour oscillation, which in this region has a large amplitude.

To show that this behaviour is not an artifact arising from the particular initialisation, runs were also made with a different initialial condition:

the surface sloping linearly from +1 m at Tornio (located at the northern extreme of the Gulf of Bothnia) to -1 m at Wladyslavovo on the southern shore of the Baltic. The same phenomena as noted above also proved to characterize this latter set of results, with the exception that the local seiche in the Gulf of Riga, with the period 17 hours, was not excited. This is probably because the main entrance to the gulf, the Irbe Strait, stretches in the East-West direction. The seiche in the Gulf of Riga will be further discussed in subsection 3.3.

To study the large-scale “asymmetric response” in greater detail, the results from a transect following the "Thalweg" from St. Petersburg to the northern Kattegatt, cf. Figure 4, were also examined. A contour plot of the spectral amplitude as a function of distance along the transect and the oscillation period is shown in Figure 6. We see the same overall pattern as previously described. The most prominent features are the strong 27-hour oscillation in the Gulf of Finland and the Belt Sea, and the 23-hour oscillation with only weak resonance in the Belt Sea. It is also noteworthy that the spectral peaks in the Gulf of Finland are considerably sharper than the one found in the Belt Sea, which is broad and has a lower amplitude.

3.2 Runs with modified bathymetry

To determine whether the oscillations seen in the previous runs are caused by local resonance effects or by global seiches, model versions with an artificially modified bathymetry were set up. In the first one the Gulf of Finland was closed off from the Baltic proper by a solid wall. In Figure

7 one can see that, as expected, a normal seiche oscillation develops in the closed gulf of Finland. The period of 10 hours is consistent with an analytic estimate.

In the Baltic proper, on the other hand, the global seiche patterns disappear. The only remaining oscillation of any significance is a local phenomenon in the Danish Belt Sea, with a period of about 25 hours.

To study the fjord seiche in the Gulf of Finland, a model version with a very distorted batyhmetry was used. Only the shoreline of Finland and the Baltic states down to Poland was used, while Sweden and Denmark, etc, were replaced by a rectangular basin having a uniform depth of 90 meters; see Fig. 8. The result in Fig. 9 shows that the oscillations in the Gulf of Finland are only weakly affected by this drastic change of the bathymetry.

These experiments show that the Belt Sea and the Gulf of Finland are both capable of supporting local modes. A probable explanation of the apparent global seiche pattern in Fig. 6, with a realistic bathymetry, is that it is a superposition of these local modes. The two oscillators are only weakly coupled, as shown by the fact that each of them is only slightly affected when the other one is completely removed.

The most important result of the coupling between the two local modes seems to be that the resonance frequency is split into two close but distinct resonance frequencies, corresponding to the periods 23 and 27 hours. This is seen particularly clearly in the Gulf of Finland, cf. Fig. 6. When only one of the two oscillators is present, there is only one resonance frequency, cf. Figs 7 and 9.

Such frequency splitting occurs very generally when two oscillators with close frequencies are coupled. Describing the oscillators by the simple equations

$$\frac{d^2\xi_1}{dt^2} + \omega_1^2\xi_1 = \alpha\xi_2,$$

$$\frac{d^2\xi_2}{dt^2} + \omega_2^2\xi_2 = \alpha\xi_1,$$

we obtain the eigenfrequencies

$$\omega^2 = \frac{\omega_1^2 + \omega_2^2}{2} \pm \left[\left(\frac{\omega_1^2 - \omega_2^2}{2}\right)^2 + \alpha^2\right]^{1/2}.$$

For $\omega_1^2 = \omega_2^2$ this reduces to $\omega^2 = \omega_1^2 \pm \alpha$, illustrating the frequency splitting.

3.3 Harbour seiche in the gulf of Riga

One peculiar detail is the indication of an oscillation with a period of 17 hours in the Gulf of Riga, as can be seen in Fig. 5. The fact that the period is fairly long although this gulf is small is a consequence of the narrow straits that connect it to the rest of the Baltic.

Such a low-frequency mode is often referred to as a Helmhotz mode, and is well known in harbour seiching (Miles, 1974). The Helmholtz mode in the Gulf of Riga has recently been studied in detail by Otsmann et al. (2001). They constructed a simple theoretical model which was calibrated using current observations in the Suur Strait, at the northern extreme on the Gulf, and concluded that the main resonance period of the system is 24 hours. Oscillations with this period are also the most prominant feature of the current observed in the Irbe Strait, the main strait connecting the Gulf of Riga with the Baltic proper.

The difference between this value and our result of 17 hours is probably explained by numerical errors. To describe the Helmholtz mode accurately it is essential to have a good resolution in the straits connecting the sub-basin to the rest of the sea, which is not the case in our model. Even the coupling to the tiny Väinameri basin just north of the Gulf of Riga was found to increase the resonant period by one hour (Otsmann et al. 2001).

4 Oscillations in a long channel

To explain and interpret oscillations in a closed basin, it is customary to compute the global eigenmodes. However, in a basin with complex coastline and bottom topography, like the Baltic Sea, an interpretation in terms of local "quasi-modes" may be more useful. These are localised modes that couple weakly to the rest of the basin, gradually losing their energy by wave radiation. The resulting damping is described by an imaginary part of the eigenfrequency. Harbour seiches (Miles 1974) are an example of such quasi-modes.

In the present section we illustrate the relation between the two points of view by examining the modes of oscillation in a long and narrow basin, which will be treated as a one-dimensional channel. This example is not chosen to be a realistic model of the Baltic Sea. It is rather an easily solvable toy model, meant to illustrate the basic ideas.

The channel consists of two sub-basins, a short one with the length L_1 and the depth H_1, and a long one with the length L_2 and the depth H_2.

Conceptually, we can think of the shallow sub-basin as analogous to the Gulf of Finland, while the long sub-basin is analogous to the Baltic proper.

Neglecting the Coriolis force, the equation for barotropic waves is

$$\frac{\partial}{\partial x}\left(gH\frac{\partial h}{\partial x}\right) + \omega^2 h = 0, \quad (1)$$

where $H(x)$ is the equilibrium depth, h the depth perturbation, and we have assumed that the time variation is proportional to $\exp(-i\omega t)$. The boundary condition at the "coastal points" $x = -L_1$ and $x = L_2$ is the usual one of no normal flow, implying $\partial h/\partial x = 0$. At the step in bottom topography at $x = 0$ we must require continuity of the surface elevation h, and of the mass flow $H\partial h/\partial x$.

We first consider a quasi-mode in the shallow sub-basin, taking the long sub-basin to be infinitely long (analogously to the open ocean). Thus, instead of using the boundary condition at $x = L_2$, we require the wave in $x > 0$ to be purely rightward-propagating, which gives the following ansatz:

$$h(x) = \begin{cases} a_+ e^{i\omega x/c_1} + a_- e^{-i\omega x/c_1}, & x < 0 \\ b_+ e^{i\omega x/c_2}, & x > 0 \end{cases} \quad (2)$$

The phase velocity in the two regions is given by $c_1 = (gH_1)^{1/2}$ and $c_2 = (gH_2)^{1/2}$, respectively, and we assume that ω is positive, so that the terms proportional to a_+ and b_+ represent rightward propagating wave components, while the term proportional to a_- represents a leftward propagating component. The continuity conditions at $x = 0$ give the reflexion coefficient at the bottom step:

$$\frac{a_-}{a_+} = \frac{c_1 - c_2}{c_1 + c_2}. \tag{3}$$

Note that if $H_1/H_2 \to 0$, so that $c_1/c_2 \to 0$, then $|a_-/a_+| = 1$, i.e. a wave travelling toward an inifinitely deep ocean from the shallow sub-basin is totally reflected. We then get a standing wave in the shallow sub-basin, a "fjord mode".

The boundary condition at $x = -L_1$ gives another relation between a_+ and a_-. Combining it with eq. (3) we obtain the dispersion relation for the quasi-mode:

$$e^{-2i\omega L_1/c_1} = \frac{c_1 - c_2}{c_1 + c_2}. \tag{4}$$

If we assume that $H_1 \ll H_2$ the right-hand side of this equation is close to -1, and we obtain approximately

$$\omega = \frac{c_1}{L_1}\left(\frac{\pi}{2} + n\pi - i\frac{c_1}{c_2}\right) \quad n = 0, 1, 2.... \tag{5}$$

These eigenfrequencies represent damped oscillations. The real part is the same as the eigenfrequency of the fjord modes, and the imaginary part describes the damping due to wave radiation.

To see how this quasi-mode relates to the global eigenmodes, we will then solve the problem with a finite value of L_2. The ansatz (2) is then replaced by

$$h(x) = \begin{cases} a_+ e^{i\omega x/c_1} + a_- e^{-i\omega x/c_1}, & x < 0 \\ b_+ e^{i\omega x/c_2} + b_- e^{-i\omega x/c_2}, & x > 0 \end{cases} \tag{6}$$

and we must also use the boundary condition at $x = L_2$. After some standard calculations, the following dispersion relation can be obtained:

$$\tan \frac{\omega L_1}{c_1} = -\frac{c_2}{c_1} \tan \frac{\omega L_2}{c_2}. \tag{7}$$

It has two sets of roots. If c_2/c_1 is large, the first set is approximately given by $\cos(\omega L_1/c_1) = 0$, i.e. the frequencies of the fjord mode in the shallow sub-basin. The second set is approximately given by $\sin(\omega L_2/c_2) = 0$, corresponding to the eigenmodes of the deep sub-basin.

To understand how these global eigenmodes relate to the quasi-mode, it is useful to calculate the ratio between the energy density in the shallow and deep sub-basins. The energy density of a harmonic oscillation is in general given by $E = (\omega^2 + Hk^2)|h|^2/2 = \omega^2|h|^2$. Using the fact that $|a_+| = |a_-|$ and $|b_+| = |b_-|$, the ratio between the energy density in the two sub-basins can then be calculated as $E_1/E_2 = \cos^2(\omega L_2/c_2)/\cos^2(\omega L_1/c_1)$. With the help of eq. (4) this can be written as

$$\frac{E_1}{E_2} = \frac{1}{\cos^2(\omega L_1/c_1) + (H_1/H_2)\sin^2(\omega L_1/c_1)}. \tag{8}$$

This function is plotted in Fig. 10, setting $H_2/H_1 = 25$. For $H_2 > H_1$ its maximum value is $E_1/E_2 = H_2/H_1$, and the maximum points are at $\omega = (c_1/L_1)(\pi/2 + n\pi)$, $n = 0, 1, 2, \ldots$. If H_2/H_1 is large, the maxima are narrow resonance peaks, with the half-width $\Delta\omega \approx (2H_1/H_2)(c_1/L_1)$. At these peaks, the global eigenmodes are in resonance with the fjord mode in the shallow sub-basin.

In Fig. 10 we also show the energy ratio for the eigenfrequencies obtained from eq. (7) for two different values of L_2. The points all lie on the curve defined by eq. (8). Thus, the only effect of increasing the value of L_2 is that the eigenmodes sample this curve more densely, while the curve itself is independent of L_2. Its shape entirely reflects the properties of the quasi-mode in the shallow sub-basin: the location of the resonance peaks is equal to the real part of the eigenfrequency of the quasi-mode, and their width is determined by the imaginary part, i.e. by the reflexion coefficient at the bottom step.

With the parameters used in Fig. 10, the frequency of the fjord mode coincides with one of the eigenfrequencies of one of the deep sub-basin. This causes frequency splitting, as discussed in the previous section, which gives two symmetrically situated global eigenfrequencies at the resonance peak.

We have also solved the time-dependent shallow-water equations corresponding to eq. (1) numerically, using a uniformly sloping surface h as initial condition. The result is shown in Fig. 11. It has the appearance of a global oscillation superimposed with a local damped oscillation in the shallow sub-basin, with the period T=4 given by eq. (5) with $n = 0$.

How should we understand this local oscillation? One answer is that it is a superposition of those global eigenmodes that coincide with in the resonance peak of Fig. 10, and therfore have a large amplitude in this sub-basin. There are two (or perhaps four) such modes for the parameters used in the simulation. The damping of the oscillation is then a result of these modes gradually coming out of phase.

Another answer, which is mathematically equivalent but physically more natural, is that this is a local quasi-mode. This point of view also gives a natural explanation of the waves that can be seen to radiate away from the shallow sub-basin in Fig. 11. They qualitatively resemble the Kelvin waves that were seen to radiate away from the Gulf of Finland in our simulations.

5 Conclusion

In our shallow-water simulations of the Baltic Sea, we could identify three different local oscillatory modes: one in the Gulf of Finland, with the two distinct periods 23 and 27 hours, one in the Belt Sea, with a less distinct period in the range 23-27 hours, and one in the Gulf of Riga, with the period 17 hours.

The strongest mode is the one in the Gulf of Finland. This agrees with the frequency analysis of sea level observations, showing that the amplitude there is highest for periods in the range 23-30 hours, and also that the tidal components K_1 and O_1 are much stronger in the Gulf of Finland than elsewhere, cf. Fig. 2.

The local Helmholtz mode in the Gulf of Riga also exists in reality; however, as shown recently in the detailed study by Otsmann et al. (2001), its real period is 24 houors rather than 17 hours. The discrepancy is most likely caused by numerical errors. In order to describe this mode accurately, one must have a better resolution than in our model of the straits connecting the Gulf of Riga to the Baltic proper.

We are also not certain about the accuracy with which the local mode in the Belt Sea is described on our simulations. The Belt Sea, too, is characterised by a complex bathymetry with narrow straits, moreover, it is likely to be affected by the open boundary, which is a potential problem in any model.

It has sometimes been asked whether the sea level oscillations observed in the Gulf of Finland are caused by a local fjord mode or by global eigenmodes (Neumann 1941). As illustrated in section 4, these two alternatives are really two sides of the same coin. Mathematically speaking, a local "fjord mode" is a superposition of several global eigenmodes with close frequencies.

However, the interpretation as a local mode focuses on the most rubust aspect of the problem. If for example, the bathymetry is modified outside of the Gulf of Finland, the local mode there remains almost the same, and so does the temporal evolution of a sea-level perturbation in the gulf. Yet the global eigenmodes of which this quasi-mode consists may have changed strongly.

The most pronounced non-local effect seen in our simulations is the frequency splitting that appears to be caused by a coupling between the local modes in the Gulf of Bothnia and the Belt Sea. In the analysis of observed oscillatory events by Neumann (1941) there is in fact indications of a similar double peak, but this cannot be seen in the spectra of sea-level observations in the Gulf of Finland that we have analysed. There are several reasons why one would expect this double peak to be less pronounced in observations than in our model.

One is that this splitting is an effect of these two modes coincidentally having almost the same resonant periods, and as already remarked, we are not confident about the accuracy of our value of the period of the Belt Sea mode. Another reason is that some effects, such as nonlinearities and coupling to internal waves, that would tend to decorrelate the global modes, are not present in our model. Also note that the oscillatory events found be Neumann encompass at most four oscillation periods, which by itself indicates a spectral width comparable to the difference between the two periods we observed the Gulf of Finland, and also comparable to the difference between the periods of first few global eigenmodes found by Wübber and Krauss (1979).

It is curious that the periods of all the three most distinct local modes are all so similar (using the value 24 hours in the Gulf of Riga, as given by Otsmann et al.). This of course enhances the interaction between them. Theoretically this should lead to frequency splitting, but in practice the result is most likely that the response is broadened, so that no distinct resonance at all is observed.

This also means that if one wants to compute the global eigenmodes with periods in this range, one must have a good description of all these local modes. In particular, it is necessary to have a good resolution in the straits of the Gulf of Riga. This shows that the local modes are also the computationally robust aspect of the problem, while the global eigenmodes, involving the interaction of the local modes, are not.

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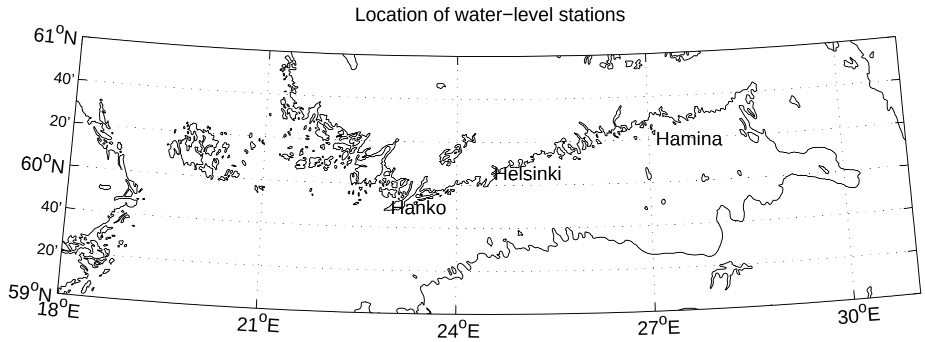

Figure 1: Location of three Finnish water-level stations in the Gulf of Finland

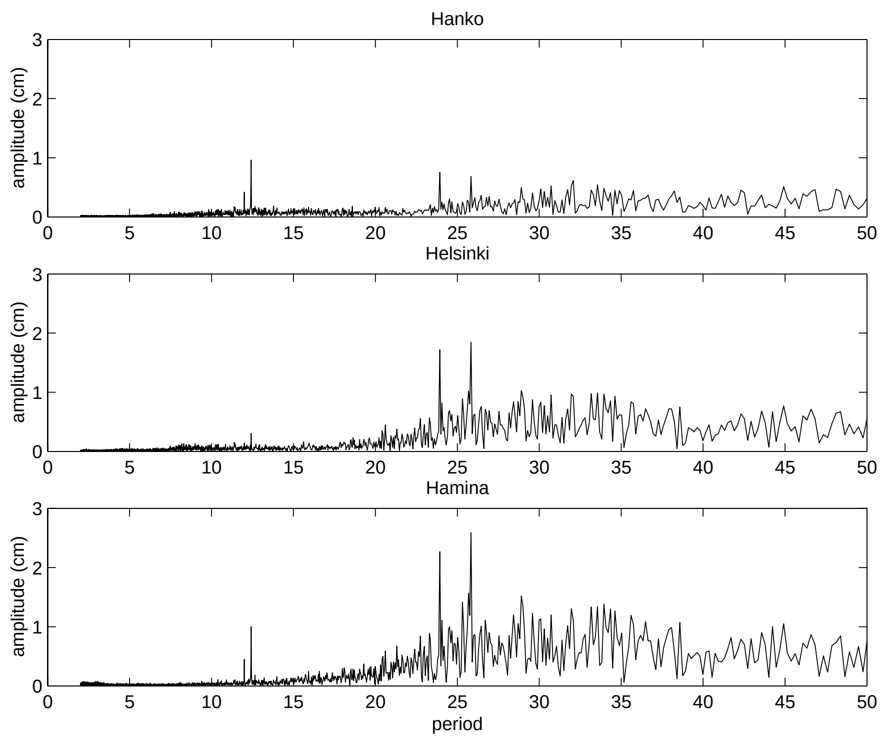

Figure 2: Spectral diagram of water-level timeseries sampled at three stations in the Gulf of Finland 1997.

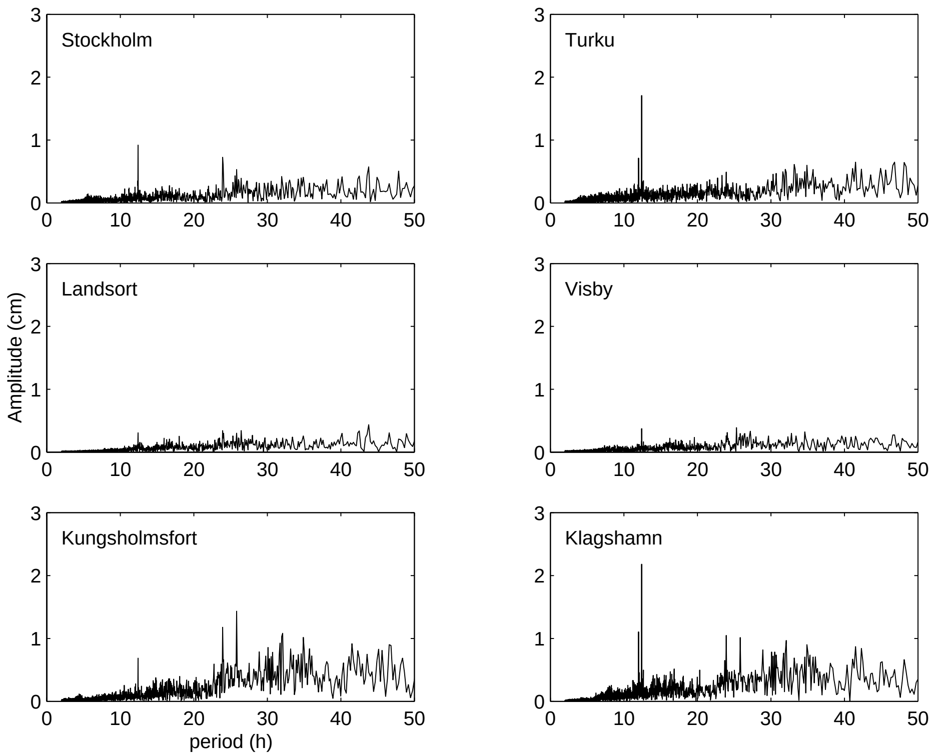

Figure 3: Spectral diagram of water-level timeseries sampled at different locations in the Baltic sea 1997.

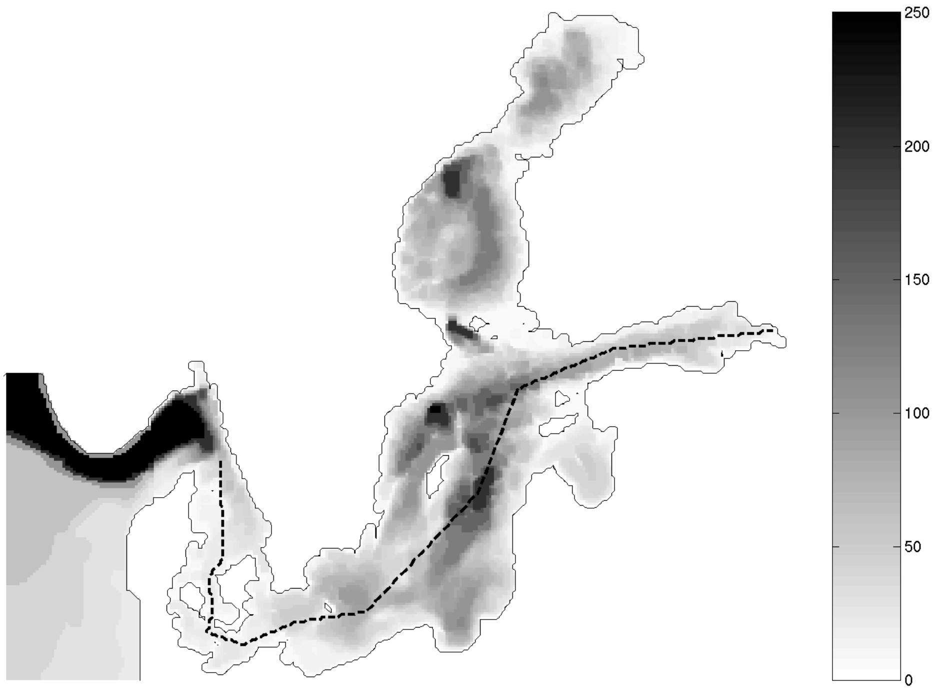

Figure 4: Two nautical-miles bathymetry of the Baltic sea. Transect for analysing spectral amplitudes plotted.

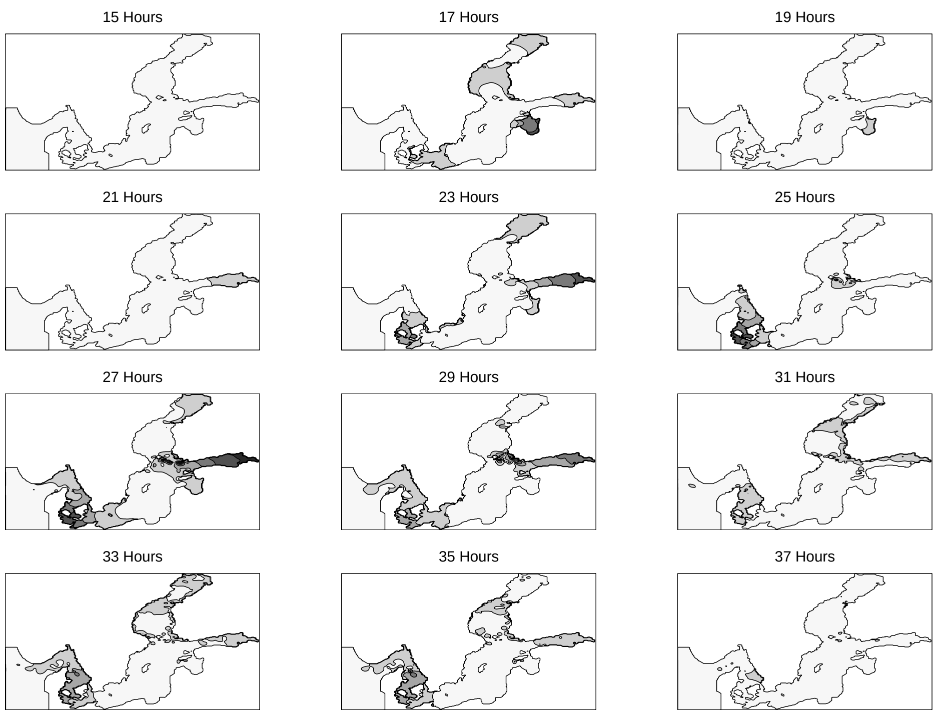

Figure 5: Spatial plots of harmonic amplitudes in the baltic sea. Each isoline represents 5 centimeters.

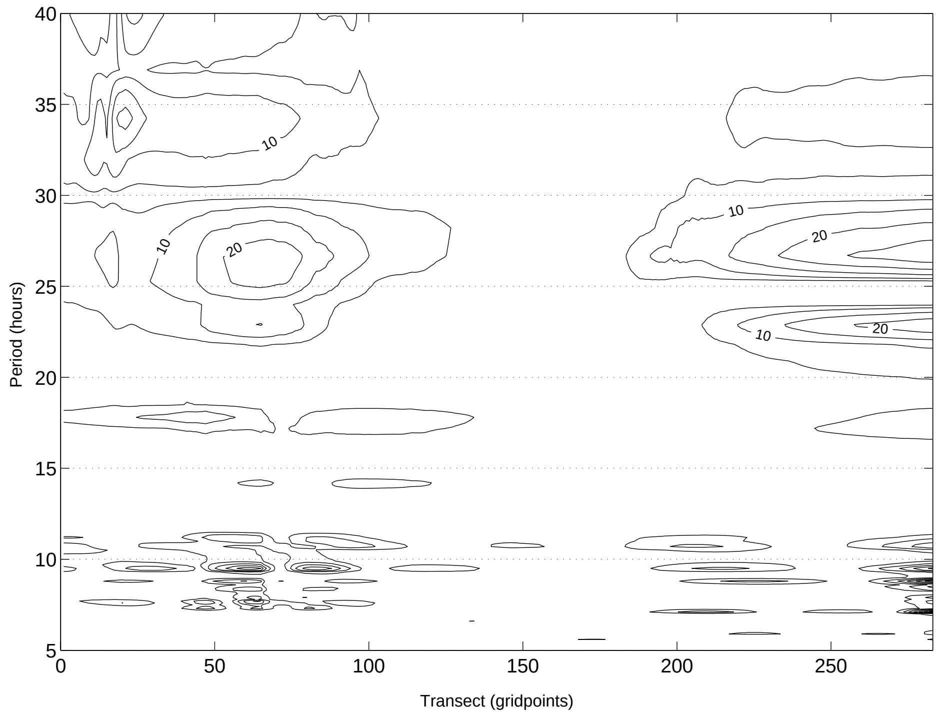

Figure 6: Spectral amplitudes of all gridpoints on the transect in Figure 4. Normal bathymetry. Each isoline represents 5 centimeters.

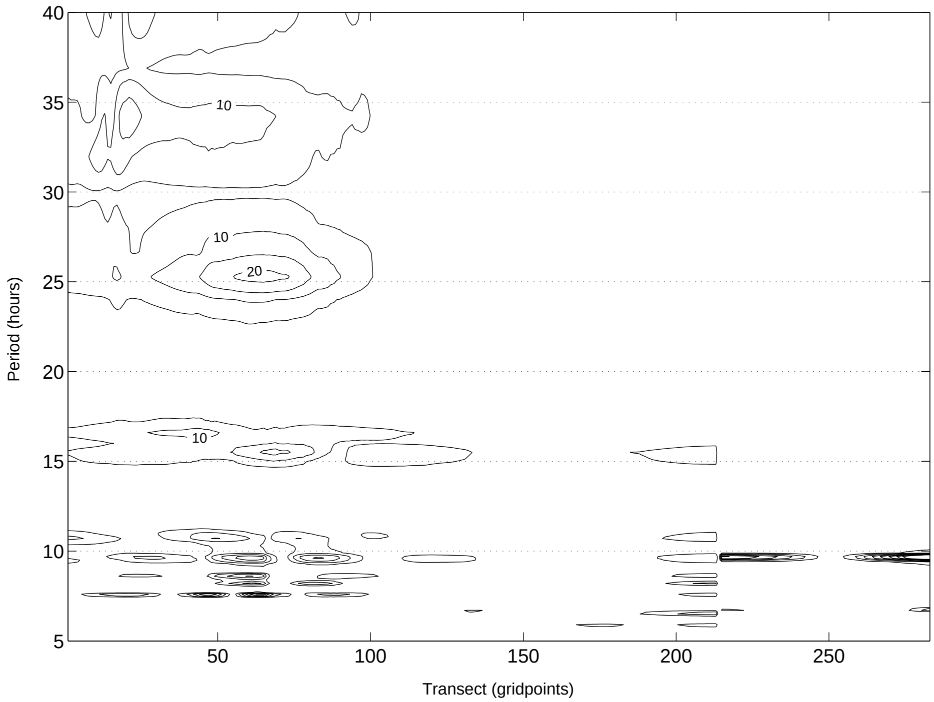

Figure 7: Spectral amplitudes of all gridpoints on the transect in Figure 4. Bathymetry with the Gulf of Finland closed. Each isoline represents 5 centimeters.

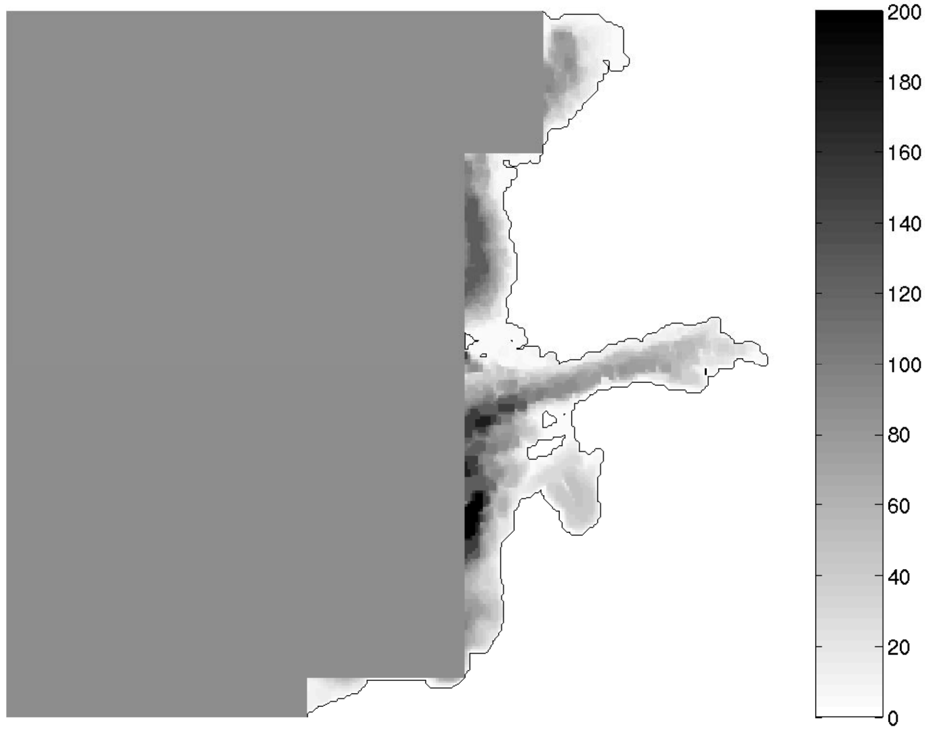

Figure 8: Bathymetry where Sweden have been replaced with a 90 metre deep basin.

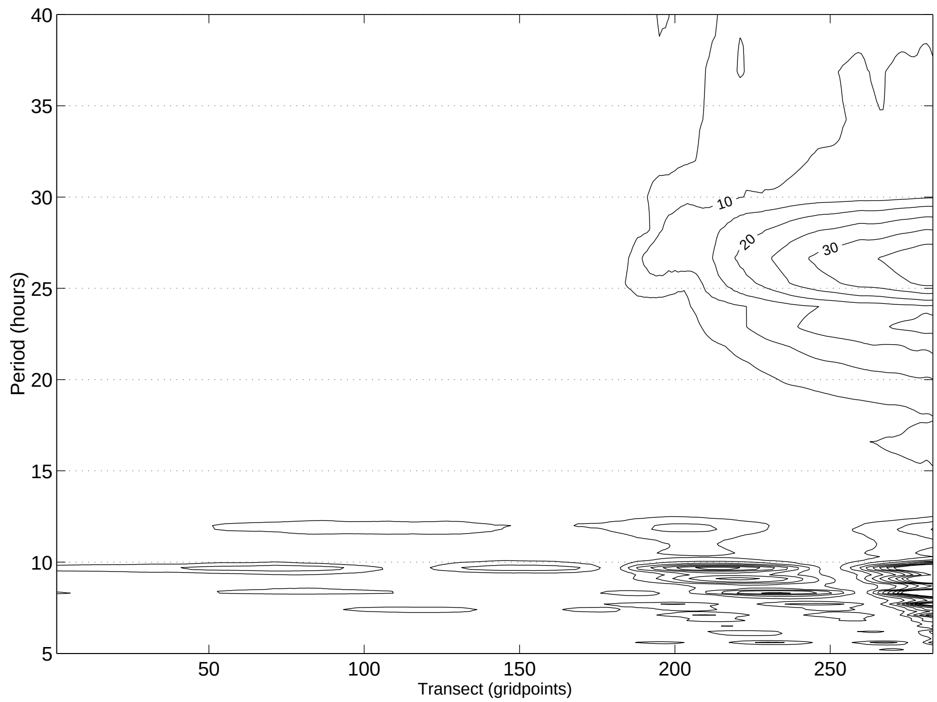

Figure 9: Spectral amplitudes of all gridpoints on the transect in Figure 4. Bathymetry in Figure 8 used. Each isoline represents 5 centimeters.

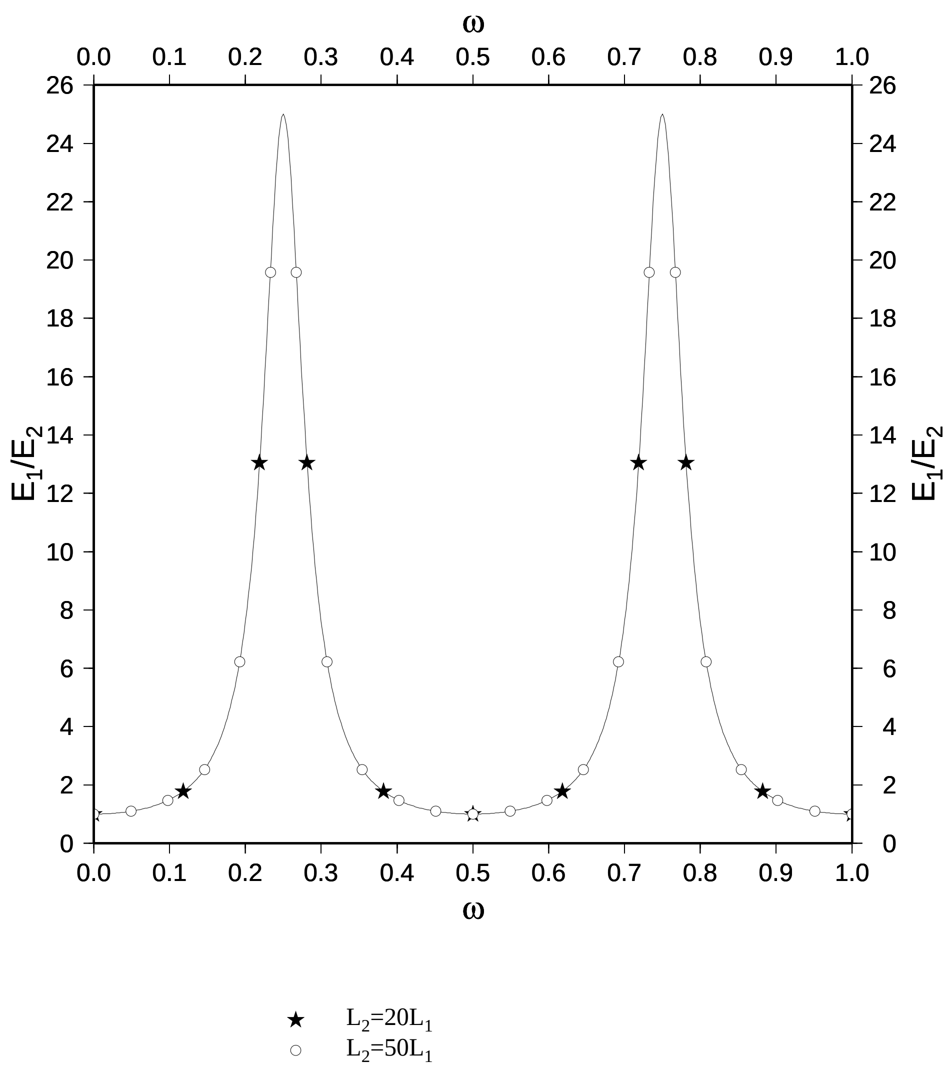

Figure 10: Ratio of the energy density in the shallow sub-basin to that in the deep one as a function of frequency. The curve is eq. (8), and the dots represent the global eigenfrequencies for two different values of L_2.

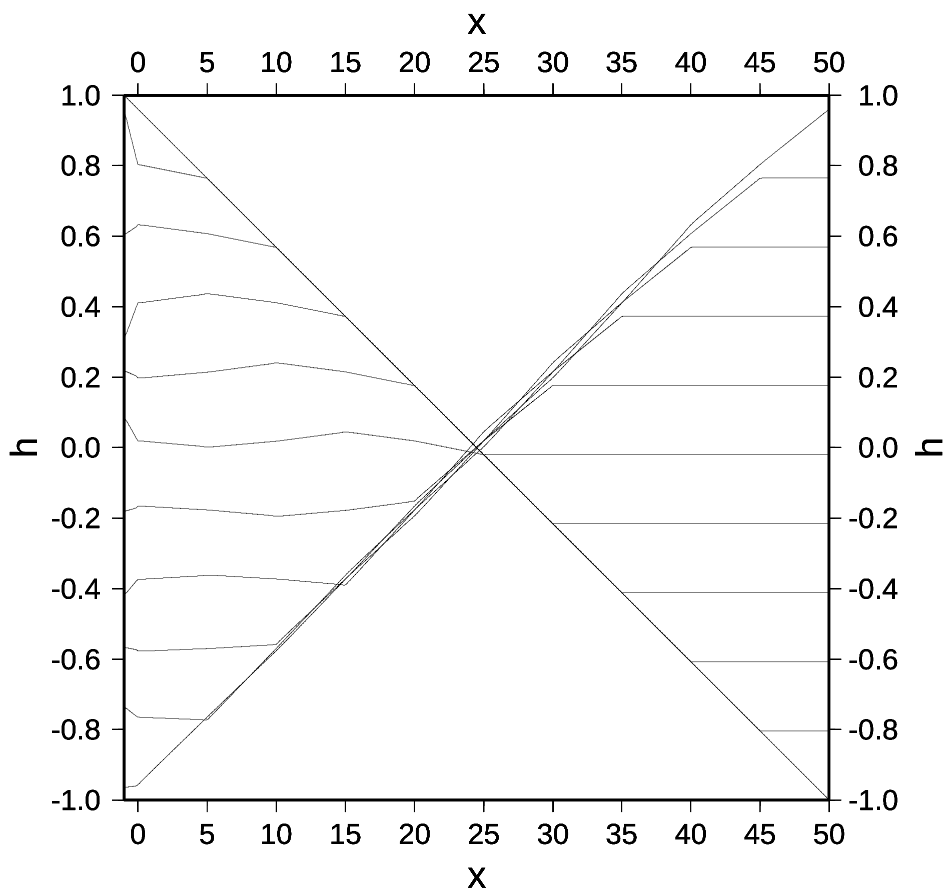

Figure 11: Time development of the free surface, with one time unit between each curve. The elevation is decreasing in the left half of the figure and increasing in the right half. The shallow sub-basin is in -1<x<0.