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A note on an erosion criterion in AUTODYN

Abstract (not more than 200 words)

Numerical erosion is a technique used in Lagrange codes for handling heavily distorted cells. One of the erosion criteria in AUTODYN uses the so-called "instantaneous geometrical strain". Some properties of this strain is investigated, and we show that this strain can be zero even for cells that have sustained large deviatoric strains.

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Om ett erosionskriterium i AUTODYN

Sammanfattning (högst 200 ord)
Numerisk erosion är en teknik som användes i Lagange-program för att ta om hand kraftigt distorderade celler. Ett av erosionskriterierna i AUTODYN använder en effektivtöjning som kallas “instantaneous geometric strain”. Vissa egenskaper hos denna töjning studeras, och vi visar att den kan vara noll även för celler som genomgått stora deviatoriska deformatiooner.

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1. INTRODUCTION

Numerical erosion is a technique used in Lagrange codes in order to handle the severely distorted zones that appear in, for instance, penetration problems. These zones are deleted (eroded) when a suitably defined effective strain exceeds a pre-set value, the erosion strain. In the AUTODYN code [1-3] from Century Dynamics there are three such effective strains, namely, effective plastic strain, incremental geometrical strain, and instantaneous geometrical strain. The *incremental geometrical strain* is defined by

$$\varepsilon_{\mathrm{incr}} = \int_0^t \sqrt{\tfrac{2}{3}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}\,dt\,, \tag{1}$$

where $\dot{\varepsilon}_{ij}$ is the strain rate tensor or rate of deformation tensor (which can be defined even for finite deformations, although in that case it may not be considered as the time derivative of a strain tensor). The *effective plastic strain* may be defined by the same formula if we, in that case, let $\dot{\varepsilon}_{ij}$ denote the plastic strain rate tensor instead. Both effective plastic strain and incremental geometrical strain are non-decreasing functions of time and can be large even for a quit regular zone, namely, if the zone is subjected to cyclic deformations. If that occurs the zone might be eroded without reason. The *instantaneous geometrical strain* was introduced in AUTODYN in order to avoid this drawback. It is defined by

$$\varepsilon_{\mathtt{inst}} = \tfrac{2}{3}\sqrt{\left|(\varepsilon_{11}^2+\varepsilon_{22}^2+\varepsilon_{33}^2)+5(\varepsilon_{11}\varepsilon_{22}+\varepsilon_{22}\varepsilon_{33}+\varepsilon_{33}\varepsilon_{11})-3(\varepsilon_{12}^2+\varepsilon_{23}^2+\varepsilon_{31}^2)\right|}\,, \tag{2}$$

where ε_{ij} is a strain tensor.

In this note the properties of Eq. (2) will be investigated, especially the surfaces in principle strain space corresponding to a constant value of that strain ($\varepsilon_{\mathrm{inst}}$ = const.). Generally it is not possible to compare the instantaneous geometrical strain with the other two effective strains, but it can be done in special cases. To facilitate that we have included in Appendix B some expressions for effective plastic strain and incremental geometrical strain in the special case where the strain tensor grows proportionally.

2. QUADRATIC FORM

From Eqs (A4) and (A5) in Appendix A, it is seen that the instantaneous geometrical strain, defined by equation (2), can be written as

$$\varepsilon_{inst} = \tfrac{2}{3}\sqrt{\left|K_1^2+3K_2\right|}\,, \tag{3}$$

where K_1 and K_2 are the first and second invariants of the strain tensor, respectively. Since the instantaneous geometrical strain is a function of the invariants it is also a function of the principle strains (ε_1, ε_2, ε_3), and it is expressed as

■

$$\varepsilon_{\mathtt{inst}} = \tfrac{2}{3}\sqrt{\left|(\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^2)+5(\varepsilon_1\varepsilon_2+\varepsilon_2\varepsilon_3+\varepsilon_3\varepsilon_1)\right|}\,, \tag{4}$$

which is readily obtained from equation (2), by setting the non-diagonal elements of the strain tensor to zero. In matrix notation the quadratic form in equation (4) can be written

$$(\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3)\begin{pmatrix} 1 & 5/2 & 5/2 \\ 5/2 & 1 & 5/2 \\ 5/2 & 5/2 & 1 \end{pmatrix}\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}. \tag{5}$$

The eigenvalues of the matrix are 6, -3/2 and -3/2. The eigenvector corresponding to the first eigenvalue is (1,1,1). Eigenvectors corresponding to the other two can be chosen arbitrarily as vectors orthogonal to each other and to (1,1,1). Let ξ be a coordinate along the (1,1,1) line and η be the distance from that line. Then the quadratic form can be written $6\xi^2 - \frac{3}{2}\eta^2$ and we finally obtain

■ $$\varepsilon_{inst} = \tfrac{2}{3}\sqrt{\left|6\xi^2 - \tfrac{3}{2}\eta^2\right|} = \tfrac{2}{3}\sqrt{\left|\tfrac{3}{2}\eta^2 - 6\xi^2\right|} = \sqrt{\tfrac{2}{3}\left|\eta^2 - 4\xi^2\right|}. \tag{6}$$

From this it can be concluded that the surface $\varepsilon_{inst} = const.$ is a hyperbolic surface that is rotationally symmetric around the (1,1,1)-line through origin. Note that $\varepsilon_{inst} = 0$ at the double cone $\eta = 2|\xi|$, which contains states with arbitrarily large deviatoric strains (which are proportional to η).

3. INVARIANTS

The results obtained in the preceding section may also be obtained by using invariants. The second invariant of the deviatoric part of the strain tensor is [4]

$$M_2 = K_2 - \tfrac{1}{3}K_1^2. \tag{7}$$

Using that, one can rewrite (3) in the following way

■ $$\varepsilon_{inst} = \tfrac{2}{3}\sqrt{\left|K_1^2 + 3(M_2 + \tfrac{1}{3}K_1^2)\right|} = \tfrac{2}{3}\sqrt{\left|2K_1^2 + 3M_2\right|}. \tag{8}$$

When interpreting this formula it is helpful to compare with von Mises theory for stresses. In this theory the effective stress is equal to $\sqrt{-3J_2}$, where J_2 is the second invariant of the stress deviator, cf. Eq. (11.91) in Ref. [1] (where a different sign convention for J_2 is used). The effective stress is proportional to the distance from the (1,1,1) line in principle stress space and the mean stress is proportional to the projection on that line. From these facts it is obvious that K_1 and $\sqrt{-M_2}$ are proportional to the volumetric strain and the deviatoric strain, respectively, and also proportional to the coordinates ξ and η (introduced in Section 2), respectively. So we may write $K_1 = a\xi$ and $\sqrt{-M_2} = b\eta$. If these expressions are substituted into Eq. (8), we obtain

$$\varepsilon_{inst} = \tfrac{2}{3}\sqrt{\left|2a^2\xi^2 - 3b^2\eta^2\right|}, \tag{9}$$

which is consistent with Eq. (6). Even without knowing the values of the proportionality factors ($a = \sqrt{3}$, $b = 1/\sqrt{2}$) the hyperbolic nature of the expression under the root sign is apparent.

4. EXAMPLE

A simulation in planar symmetry (plain strain) was carried out with one Lagrange cell filled with 4340 steel in order to show the behaviour of the instantaneous geometrical strain. The units were millimetre, milligram and microsecond. The cell, which was quadratic with side length L_0 = 1 mm at time zero, was elongated in the *x*-direction and compressed in the *y*-direction, see Figure 1. For the left and right sides of the cell we prescribed *x*-velocities to zero and v_x = 0.01 km/s, respectively. The y-velocities for the bottom and top sides were zero and v_y = -0.001347 km/s, respectively. For erosion we used "instantaneous geometrical strain" set to 0.20 (20% strain).

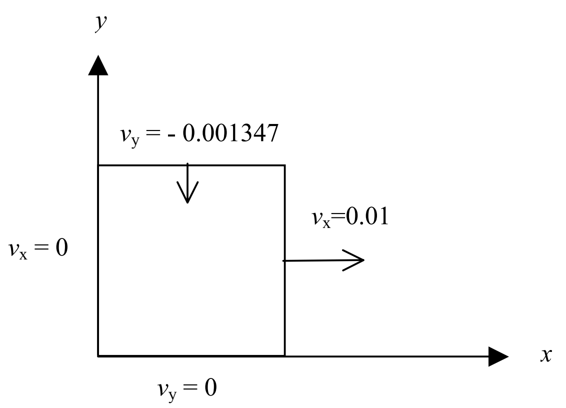

FIGURE 1. Boundary conditions.

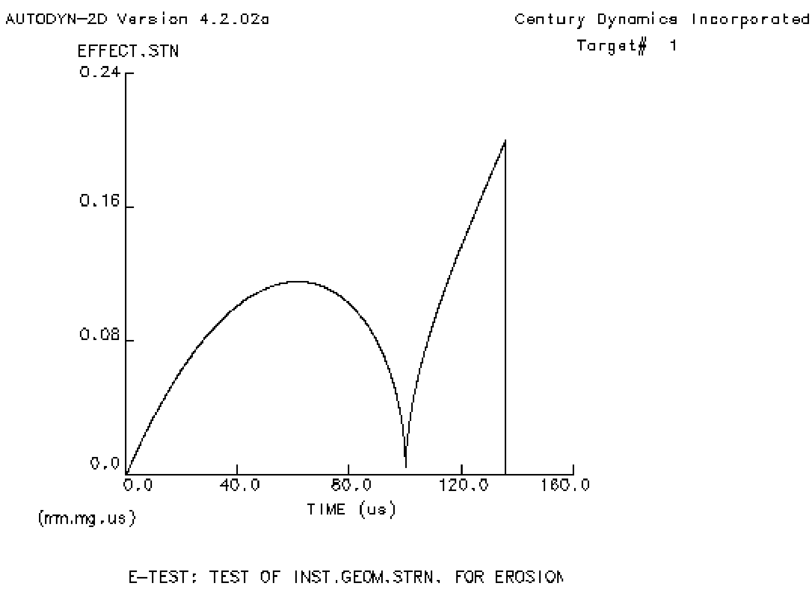

FIGURE 2. Instantaneous geometrical strain vs. time..

As is seen from Figure 2, the instantaneous geometrical strain increases to about 0.12 at *t* = 60 μs and then decreases to almost zero at t = 100 μs. Finally it increases to the erosion strain at which point the cell is removed by erosion. This behaviour occurs despite the fact that the prescribed boundary velocities do not vary with time. At 100 μs, when the instantaneous geometrical strain is almost zero, the engineering strain in the *x*-direction is $v_x t/L_0 = 1.000$ and in the *y*-direction $v_y t/L_0 = -0.135$, i.e. the cell is elongated to its double original length and compressed 13.5% in the perpendicular direction.

By setting $\varepsilon_3 = 0$ in Eq. (4), it is easily obtained that the instantaneous geometrical strain is zero if the quotient

$$\varepsilon_2/\varepsilon_1 = \tfrac{1}{2}\left(-5+\sqrt{21}\right) = -0.2087\,. \tag{10}$$

The behaviour of the curve in Figure 2 can be understood if the strain tensor, which is used in AUTODYN for calculation of the instantaneous geometrical strain is updated incrementally using the strain rates $\dot{\varepsilon}_x = v_x/L_x$ and $\dot{\varepsilon}_y = v_y/L_y$, where $L_x = L_0 + v_x t$ and $L_y = L_0 + v_y t$ are the current lengths of the respective sides of the cell. Because, then the strains

$$\varepsilon_x = \ln\left(L_x/L_0\right) = \ln\left(1 + v_x t/L_0\right) \tag{11}$$

$$\varepsilon_y = \ln\left(L_y/L_0\right) = \ln\left(1 + v_y t/L_0\right) \tag{12}$$

are effectively logarithmic strains, and their quotient will vary with time and attain the value $\varepsilon_y/\varepsilon_x = -0.2087$, cf. Eq. (10), at $t = 100$ µs. This explains why the instantaneous geometrical strain in Figure 2 remains comparatively small for times less than 100 µs and turns back towards zero at 100 µs.

5. CONCLUSIONS

One of the strain measures used in the erosion criteria in AUTODYN is called “instantaneous geometrical strain”. It can be expressed as a function of the invariants of a strain tensor, used for that purpose. In principle strain space the surfaces corresponding to constant instantaneous geometrical strains are hyperbolic surfaces, rotationally symmetric around the (1,1,1) line through origin. The surface corresponding to the value zero for the instantaneous geometrical strain is a double cone, rotationally symmetric in the same way. This cone contains points not only far from origin but also far from the symmetry line. The latter means that deformation states with large deviatoric strains may have zero instantaneous geometrical strain. This was also demonstrated by a simulation with AUTODYN-2D, where a cell which was strained to its double length in one direction and slightly compressed in the perpendicular direction, had a value of only a few tenth of a percent for the instantaneous geometrical strain.

6. REFERENCES

1. Century Dynamics: Theory Manual for AUTODYN.
2. N. K. Birnbaum, M. S. Cowler, M. Itoh, M. Katayama and H. Obata: AUTODYN - an interactive non-linear dynamic analysis program for microcomputers through supercomputers. *9th Int. Conf. on Structural Mechanics in Reactor Technology,* August 1987, Lausanne, Switzerland.
3. N. K. Birnbaum and M. S. Cowler: Numerical simulation of impact phenomena in an interactive computing environment. In *Impact Loading and Dynamic Behavior of Materials* , Vol. 2, pp. 881-888 (Edited by C. Y. Chiem, H.-D. Kunze and L. W. Meyer), DGM Informationsgesellschaft mbH, Oberursel (1988).
4. B. Sundström (Ed.): Handbok och formelsamling i Hållfasthetslära. *Institutionen för hållfasthetslära KTH*, 1999.

APPENDIX A. FORMULAS FOR INVARIANTS

The invariants for the strain tensor ε_{ij} (or for that matter any 3 by 3 matrix) are defined as

$$K_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \tag{A1}$$

$$K_2 = \varepsilon_1\varepsilon_2 + \varepsilon_2\varepsilon_3 + \varepsilon_3\varepsilon_1 \tag{A2}$$

$$K_3 = \varepsilon_1\varepsilon_2\varepsilon_3 \tag{A3}$$

where ε_1, ε_2, and ε_3 are the eigenvalues of ε_{ij}. From textbooks in the field e.g. [4], it is seen that the first and second invariants can be expressed in terms of the tensor elements (utilising the symmetry of the strain tensor) as

$$K_1 = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \tag{A4}$$

$$K_2 = \varepsilon_{11}\varepsilon_{22} + \varepsilon_{22}\varepsilon_{33} + \varepsilon_{33}\varepsilon_{11} - \varepsilon_{12}^2 - \varepsilon_{23}^2 - \varepsilon_{13}^2, \tag{A5}$$

respectively. The third invariant is equal to the determinant.

The invariants of the deviatoric strain tensor $e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_{kk}\delta_{oj}$ are defined by formulas analogous to those for the strain tensor. The first invariant is always zero and the second can be expressed as

$$M_2 = -\tfrac{1}{2}e_{ij}e_{ij} = K_2 - \tfrac{1}{3}K_1^2 \tag{A6}$$

in terms of the elements of the deviator and of the invariants for the strain tensor.

APPENDIX B. COMPARISONS WITH THE INCREMENTAL STRAINS

In order to be able to compare the incrementally defined strains with the instantaneous one we look at a case where the incremental strains can be integrated. Let therefore the strain tensor increase proportionally from zero to its final value. Then the incremental geometrical strain, defined by equation (1) can be integrated to

$$\varepsilon_{\mathtt{incr}} = \sqrt{\tfrac{2}{3}\varepsilon_{ij}\varepsilon_{ij}} = \sqrt{\tfrac{2}{3}\left(\varepsilon_{11}^2 + \varepsilon_{22}^2 + \varepsilon_{33}^2 + 2\varepsilon_{12}^2 + 2\varepsilon_{23}^2 + 2\varepsilon_{13}^2\right)}, \tag{B1}$$

which is also a function of the invariants. In terms of the principle strains it is

$$\varepsilon_{\mathtt{incr}} = \sqrt{\tfrac{2}{3}(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2)}\,. \tag{B2}$$

We also compare with a similar incremental strain based on the deviatoric part, namely,

$$\varepsilon'_{\mathtt{incr}} = \sqrt{\tfrac{2}{3}(e_1^2 + e_2^2 + e_3^2)} = \sqrt{-\tfrac{4}{3}(e_1e_2 + e_2e_3 + e_1e_3)}\,, \tag{B3}$$

which is approximately equal to the effective plastic strain if the plastic strains are much larger than the elastic ones. The last equal sign in Eq. (B3) depends on the following calculation:

$$0 = (e_1 + e_2 + e_3)^2 = e_1^2 + e_2^2 + e_3^2 + 2(e_1e_2 + e_2e_3 + e_1e_3)\,. \tag{B4}$$

In terms of the invariants we have

$$\varepsilon_{\mathtt{incr}} = \sqrt{\tfrac{2}{3}(K_1^2 - 2K_2)} = \sqrt{\tfrac{2}{3}(\tfrac{1}{3}K_1^2 - 2M_2)} = \tfrac{2}{3}\sqrt{\tfrac{1}{2}K_1^2 - 3M_2} \tag{B5}$$

$$\varepsilon'_{\mathtt{incr}} = \sqrt{-\tfrac{4}{3}M_2} = \tfrac{2}{3}\sqrt{-3M_2} \tag{B6}$$

and in terms of the $\xi\eta-$ coordinates

$$\varepsilon_{\text{incr}} = \tfrac{2}{3}\sqrt{\tfrac{1}{2}a^2\xi^2 + 3b^2\eta^2)} = \tfrac{2}{3}\sqrt{\tfrac{3}{2}(\xi^2 + \eta^2)} = \sqrt{\tfrac{2}{3}(\xi^2 + \eta^2)} \tag{B7}$$

$$\varepsilon'_{\text{incr}} = \tfrac{2}{3}\sqrt{3b^2\eta^2} = \tfrac{2}{3}\sqrt{\tfrac{3}{2}\eta^2} = \sqrt{\tfrac{2}{3}}\eta \tag{B8}$$