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-A Kurtosis Approach

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Higher-Order Statistics Based Estimation of Intersystem Interference in Digital Communication Systems -A Kurtosis Approach

Abstract

The problem to determine the impact on digital communication systems of electromagnetic interference often requires complex calculations and analyses. Therefore, there is a need for approximate methods suitable for computer-based tools for intersystem interference analyses. A commonly used approximation is to treat the interference as if it is additive white Gaussian noise (AWGN) when estimating the resulting bit-error probability (BEP) for the digital communication system. However, the validity of this approximation varies for different types of interference. In this report, we show that it is possible to use higher-order statistics (HOS) as a quality measure of the AWGN-approximation. Furthermore, we also show that HOS can be used to quantify the impact on a digital communication receiver of some electromagnetic interference signals of interest. The results open up the possibility to use HOS as a quality measure of the resulting BEP when using approximate methods in computer-based tools.

Keywords

Intersystem interference, digital communication systems, AWGN-approximation, higher-order statistics, cumulants, kurtosis

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Högre ordningens statistik-baserad estimering av telekonflikters påverkan på digitala kommunikationssystem

Sammanfattning
Att bestämma påverkan på ett digitalt kommunikationssystem som orsakas av en elektromagnetisk interferens kräver ofta relativt komplexa beräkningar och analyser. Det finns därför ett stort behov av approximativa metoder som är lämpade för datorbaserade verktyg avsedda för telekonfliktanalyser. En ofta använd approximation vid estimering av den bitfelshalt som orsakas av interferensen i digitala kommunikationssystem är att behandla interferensen som om den istället vore additivt vitt gaussiskt brus. Approximationens giltighet varierar dock kraftigt för olika typer av interferenser. I den här rapporten visar vi att det är möjligt att använda högre ordningens statistik (HOS) som ett kvalitetsmått på approximationen. Det är dessutom möjligt att använda HOS för att kvantifiera påverkan på ett digitalt kommunikationssystem för några intressanta signaler. Resultaten öppnar för möjligheten att använda HOS som ett kvalitetsmått på den resulterande bitfelshalten vid användning av approximativa metoder i datorbaserade telekonfliktverktyg.

Nyckelord
Telekonflikter, digitala kommunikationssystem, AWGN-approximation, högre ordningens statistik, kumulanter, kurtosis

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Contents

Chapter 1

Introduction

1.1 Background

The use of wireless communications is rapidly increasing both in the civilian and military society. In the military world, the ambition towards information superiority leads to an increased amount of equipment for wireless communications and information processing in mobile military platforms [1]. With this foreseen increase of radiating equipment in the vicinity of radio receivers, caused by intentional transmitters in the form of various wireless communication systems and also by unintentional emitters (computers, printers, etc), it is clear that the radio receivers are going to operate in more difficult environments. The intersystem interference is destined to increase in the future. At the same time, the need for reliable communications increases since information superiority is a key component in the future battle space as well as in future peacekeeping operations. This situation calls for efficient tools to judge the effect from intersystem interference on the communications receiver. It is important to be able to analyze an interference situation, and to plan a co-location of different systems so that the level of intersystem interference is manageable.

The computations necessary to determine the performance degradation on a radio receiver, due to the electromagnetic interference, are often quite complicated. Hence, simplified methods are needed in computer-based tools for interference analysis. One such simplified method is to approximate the interfering signal as additive white Gaussian noise (AWGN) [9]. The problem with this approximation is that it is not always valid. Furthermore, it is often difficult to evaluate the quality of the approximation. One proposal for handling these problems is to use HOS (Higher-Order Statistics). HOS-measures are extensions of second-order measures, such as the autocorrelation and power spectrum.

One application of higher-order statistics is in modulation classification, i.e., it is possible to distinguish between different modulated signals with the use of HOS [7]. Different modulation types of the interfering signal yield different impact on the radio receiver. This implies that it is possible to separate differently modulated signals from each other and thereafter estimate the impact on the receiver.

1.2 Problem Statement

In this report, we examine if higher-order statistics can be used as a tool in estimating the impact of electromagnetic in-band interference on digital radio receivers. Modulated interference signals (PSK, PAM, QAM) as well as pulsed interference are treated.

Our work has focused on achieving the following two objectives:

- Find a low complexity measure with which it is possible to determine, for a particular interference, if the AWGN-approximation leads to an over- or underestimation of the bit-error probability (BEP).

- Find a measure with which it is possible to quantify and estimate the resulting BEP caused by an interfering signal.

It is important to determine if the use of the AWGN-approximation for an interfering signal leads to an over- or underestimation of the BEP. If the use of the AWGN-approximation leads to an underestimation of the BEP the expected system performance will not be achieved. Also, this may have catastrophic effects in scenarios with co-located communication systems. These cases can be avoided if there is a way to estimate the quality of the AWGN-approximation. It would be even better if it was possible to estimate the performance degradation directly for the cases that the AWGN-approximation cannot handle. Such a performance measure would be of great importance in computer-based tools for intersystem interference analyses.

1.3 Assumptions

The following limiting assumptions have been made in order to facilitate the work reported herein. First of all, in this report it is assumed that we have sufficiently large data sets; hence, the effects of Gaussian noise can be neglected. As will be discussed later in this report, Gaussian noise has a zero kurtosis value for sufficiently large data sets. Secondly, we assume that only one interference signal is present. Hence, the receiver will be subjected to the desired signal, one interfering signal, and Gaussian noise. The interfering signal will be either a modulated communication signal or a repetitive pulsed interference.

1.4 Outline

The report begins with a review of the impact of intersystem interference on digital communication systems. Different sources of intersystem interference are described and the resulting bit-error probability (BEP) on a BPSK-receiver, caused by different modulated signals or from a repetitive pulsed interference, is presented.

Thereafter, in Chapter 3, the basic theory behind higher-order statistics is presented. In particular, the fourth-order cumulants are described for complex-valued stationary processes. Furthermore, the estimation of the cumulants from a finite set of data samples is described.

In Chapter 4, the simulation tool and simulation parameters are described.

Chapter 5 contains the results from the simulations. The impact on the digital radio receiver for different interference types is compared with the kurtosis for these interferences. We show that it is possible to separate modulated signals from repetitive pulsed interference through the kurtosis. Furthermore, a lower normalized pulse repetition frequency results both in an increased impact on the receiver and a larger kurtosis value. Hence, for pulsed interference, it is possible to use the kurtosis to determine if the AWGN-approximation yields an underestimation of the bit-error probability.

The conclusions are summarized in Chapter 6. The main conclusion is that it can be possible to use the kurtosis as a quality measure in computer-based tools for intersystem interference analyses. Furthermore, for pulsed interference, the kurtosis can be used to quantify the impact on the communication receiver.

Finally, suggestions for further work are presented in Chapter 7.

Chapter 2

Intersystem Interference in Digital Communication Systems

In wireless communications, the radio receiver does not only receive the desired signal, but also noise and other signals. These other signals can originate from other radio transmitters, but also from co-located electronic equipment such as computers, see figure 2.1. In military scenarios, hostile jamming is also a potential threat. Noise and undesired signals can seriously deteriorate the receiver's performance. This performance degradation can, for example, result in a decreased capacity or a reduced operating range [2].

For a digital radio receiver, the bit-error probability (BEP) is a widely used measure of the receiver's performance. For an additive white Gaussian noise (AWGN) interference there are well known equations that can be used to determine the resulting BEP. For other interference types it is necessary to calculate the BEP through a comprehensive analysis of the digital radio receiver. This analysis can become very difficult. One assumption commonly used to make these calculations easier is to assume that the noise, or interfering signal, can be modeled as AWGN with the same average power as the interfering signal. This simplifies the calculations, but the approximation is not always valid. Furthermore, it is often difficult to evaluate the quality of the AWGN-approximation.

When estimating the BEP, key parameters are the power ratios between the desired signal and the noise or the interference. The relationship between the desired signal and the noise is called the signal-to-noise ratio, SNR, and is defined as the ratio between the power of the desired signal and the power of the

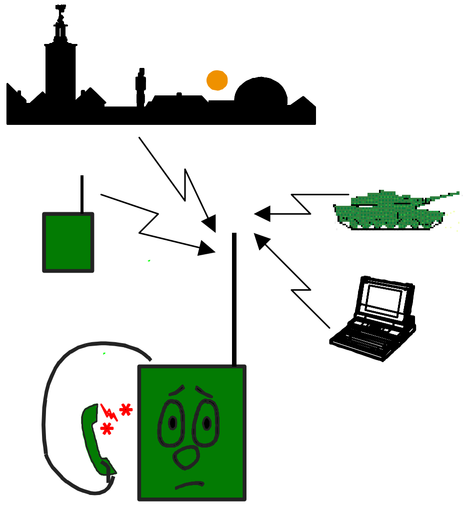

Figure 2.1: Common sources of intersystem interference in communication receivers are: “man-made” noise, radiating electronic equipment, as well as other (friendly) transmitters. Also, in military applications jamming is a potential threat.

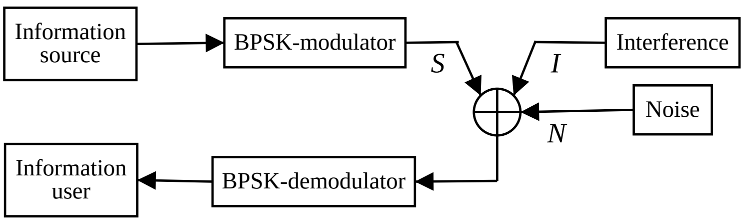

Figure 2.2: The BPSK-receiver is subjected to the desired signal, interfering signals, and additive white Gaussian noise. In this work, the interference can either consist of different modulated signals or of a repetitive pulsed interference.

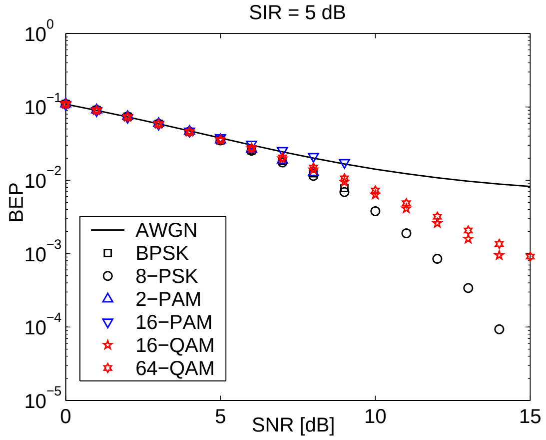

Figure 2.3: Bit-error probability for a BPSK receiver, as a function of SNR, for different modulated interference signals with SIR = 5 dB.

noise. When using the notation in figure 2.2, the signal-to-noise ratio can be expressed as SNR = S/N. The signal-to-interference ratio, SIR, can in the same way be defined as SIR = S/I. However, as discussed above, it is not sufficient to only compare the power of the different signals when determining their impact on a communication receiver. The modulation type of the signals is also of great importance for the impact on the receiver.

2.1 The Effects of Intersystem Interference on a BPSK receiver

Binary phase-shift keying (BPSK) can be seen as a kind of fundamental modulation scheme. Furthermore, other popular modulation schemes such as minimum-shift keying (MSK) and quadrature phase-shift keying (QPSK) are based on decision-making algorithms that can be related to the decision-making algorithms of BPSK. Finally, the performance of BPSK in terms of BEP does not differ much from other modulation schemes, why conclusions made on results for BPSK can often be extended to other related schemes.

The BEP performance for an interference with a SIR of 5 dB is shown, as a function of SNR, in figure 2.3 for different modulated signals (BPSK, 8-PSK, 16-PAM, 16-QAM, and 64-QAM). The results are average values from Monte Carlo simulations with different relative phases between the BPSK receiver and the interference. Due to the simulation setup, the BEP is not simulated for all values of SNR for all modulated signals. The simulations are described in more detail in Chapter 4. In figure 2.3, the resulting BEP for an AWGN interference is also shown. It can be seen that, when approximating the interfering modulated signals as AWGN, the BEP is overestimated. Furthermore, it is better to make an overestimation of the BEP since underestimating the BEP could lead to serious communication problems. Hence, the AWGN-approximation is valid, for practical purposes, for modulated signals.

In [9] it is shown, through a comprehensive theoretical analysis, that the AWGN-approximation is valid also for an MSK receiver when subjected to an interfering MSK signal.

We have also studied the impact on a BPSK receiver from repetitive pulsed interference. The interference pulses were chosen to have a pulse duration that is short compared to the digital symbols transmitted. This type of pulsed interference is also used in the present CISPR/ITU work concerning future emission standards for the protection of digital communication services [6].

The impact of pulsed interference on a BPSK receiver is shown in figure 2.4. In the simulations, the pulse length is 22 % of the bit duration. The energy of the pulses is normalized so that the average power of the interference is equal for all cases. The normalized pulse repetition frequency is normally lower than the data rate, i.e., not every data bit is subjected to the interference. In figure 2.4 we show the BEP when every bit, every tenth bit and every fiftieth bit are subjected to the pulses. The simulation parameters are described more thoroughly in Chapter 4.

In figure 2.4, the BEP reaches a maximum value for low values of SIR, dependent on the normalized pulse repetition frequency. For high values of SIR the BEP reaches a minimum value dependent on SNR.

As mentioned previously, the AWGN-approximation should not be used in situations when the result is an underestimation of the BEP. In figure 2.4, the AWGN-approximation yields an overestimation of the BEP, for low values of SIR, which is acceptable. However, for some higher values of SIR the BEP is largely underestimated, for pulses with low normalized pulse repetition frequency, when using the AWGN-approximation. An underestimation of BEP can have serious consequences and should be avoided. Hence, from figure 2.4 it is

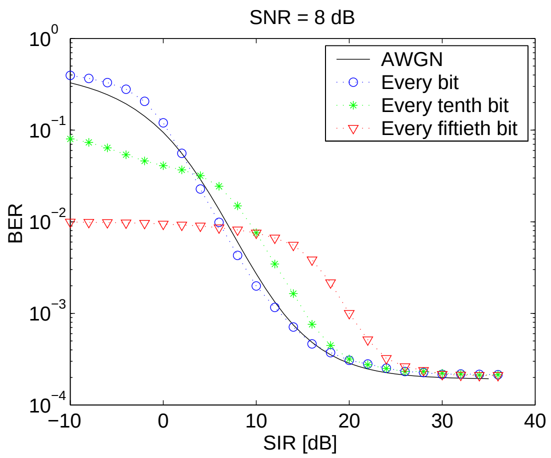

Figure 2.4: Comparison of bit-error probability for different pulsed interference signals in an BPSK receiver with SNR = 8 dB.

evident that the AWGN-approximation cannot be used for this type of repetitive pulsed interference.

In conclusion, for many intersystem interference analysis applications the AWGN-approximation is a decent approximation for modulated interference signals. However, in the case of pulsed interference the AWGN-approximation can lead to large underestimations of the resulting BEP. This may in some situations result in a severe degradation of the communication system.

Chapter 3

Higher-Order Statistics Theory

Higher-order statistics (HOS), also known as cumulants, are an important tool in many different fields [3]; e.g., image reconstruction, time-delay estimation, adaptive filtering, array processing [8], blind equalization and modulation classification [7]. Cumulants are related to the more familiar concept of moments and may be expressed in terms of them. As will be shown below, higher-order moments are generalizations of the autocorrelation, while higher-order cumulants can be seen as specific nonlinear combinations of the higher-order moments.

3.1 Introduction to Higher-Order Statistics

Higher-order statistics is a field of statistical signal processing which has been studied extensively for the last two decades. The interest in HOS stems from the fact that there is considerably more information in a stochastic non-Gaussian or deterministic signal than is conveyed by the autocorrelation or the corresponding power spectrum. Higher-order statistics contain this information. Cumulants reveal phase information as well as amplitude information. This is an important advantage as second-order statistics (i.e., correlation) are phase blind. Furthermore, cumulants are insensitive to any kind of Gaussian process, whereas correlation is not; hence, cumulant-based methods are favorable in situations with colored Gaussian noise [3]. These properties of higher-order statistics make them a potentially valuable tool for intersystem interference analysis applications.

However, although HOS is a promising tool in many applications, it also has drawbacks. One of the most important limitations with HOS-based methods is that they generally require larger data sets compared to correlation-based methods. The reason is that the variance of cumulant estimates increases rapidly with the order of the cumulant. Long data sequences are needed in order to reduce the variance associated with estimating the higher-order statistics from real data using sample-averaging techniques [3]. Sufficiently long data sequences may not always be present in practical situations. Furthermore, real-world signals are generally non-stationary in nature, which means that the stationarity assumption may become invalid if too large data sets are required for accurate cumulant estimation.

3.2 Moments and Cumulants

The moments are an important, widely used, set of statistical averages. The *k*th moments of a random variable are the expectations of the *k*th powers of the random variable. For a random variable x_1 the moments of order one to four are [5]

$$m_1 = \text{mom}(x_1) = E\{x_1\} \tag{3.1}$$
$$m_2 = \text{mom}(x_1, x_1) = E\{x_1^2\} \tag{3.2}$$
$$m_3 = \text{mom}(x_1, x_1, x_1) = E\{x_1^3\} \tag{3.3}$$
$$m_4 = \text{mom}(x_1, x_1, x_1, x_1) = E\{x_1^4\}. \tag{3.4}$$

The *k*th-order cumulant can be defined in terms of a weighted sum of joint moments of orders up to *k*, see [5] for a definition of the cumulant-generating function. The cumulants of order one to four are

$$c_1 = \text{cum}(x_1) = m_1 \tag{3.5}$$
$$c_2 = \text{cum}(x_1, x_1) = m_2 - m_1^2 \tag{3.6}$$
$$c_3 = \text{cum}(x_1, x_1, x_1) = m_3 - 3m_2m_1 + 2m_1^3 \tag{3.7}$$
$$c_4 = \text{cum}(x_1, x_1, x_1, x_1) = m_4 - 4m_3m_1 - 3m_2^2 + 12m_2m_1^2 - 6m_1^4. \tag{3.8}$$

Hence, for a set of zero-mean real-valued random variables $\{x_1, x_2, x_3, x_4\}$

the first-, second-, third- and fourth-order cumulants are [3]

$$\text{cum}(x_1) = E\{x_1\} = 0 \tag{3.9}$$

$$\text{cum}(x_1, x_2) = E\{x_1 x_2\} \tag{3.10}$$

$$\text{cum}(x_1, x_2, x_3) = E\{x_1 x_2 x_3\} \tag{3.11}$$

$$\begin{aligned} \text{cum}(x_1, x_2, x_3, x_4) = & E\{x_1 x_2 x_3 x_4\} \\ & - E\{x_1 x_2\}E\{x_3 x_4\} \\ & - E\{x_1 x_3\}E\{x_2 x_4\} \\ & - E\{x_1 x_4\}E\{x_2 x_3\}. \end{aligned} \tag{3.12}$$

3.3 Moments and Cumulants of Stationary Processes

The moments and cumulants of stationary processes are most easily understood for real-valued stationary processes and these will be described herein. However, in communication applications the received data is normally complex-valued (baseband data with I- and Q-channels). Therefore, the cumulants for complex-valued stationary processes are also defined.

3.3.1 Real-Valued Stationary Processes

If $\{x(n)\}$, $n = \pm 1, \pm 2, \pm 3, \ldots$ is a real-valued stationary random process and its moments up to order *k* exists, then the moments depend only of the time differences [5]

$$\begin{aligned} m_k(\tau_1, \tau_2, \ldots, \tau_{k-1}) &= \text{mom}[x(n), x(n+\tau_1), \ldots, x(n+\tau_{k-1})] \\ &= E\{x(n) \cdot x(n+\tau_1) \cdots x(n+\tau_{k-1})\}. \end{aligned} \tag{3.13}$$

Similarly, we write the cumulants in the form

$$c_k(\tau_1, \tau_2, \ldots, \tau_{k-1}) = \text{cum}[x(n), x(n+\tau_1), \ldots, x(n+\tau_{k-1})]. \tag{3.14}$$

Hence, for stationary processes, the *k*th-order moments and cumulants are only functions of the lags $\tau_1, \tau_2, \ldots, \tau_{k-1}$.
First-order cumulants (mean value):

$$c_1 = m_1 = E\{x(n)\}. \tag{3.15}$$

Second-order cumulants (covariance sequence):

$$c_2(\tau_1) = m_2(\tau_1) - (m_1)^2, \tag{3.16}$$

where $m_2(\tau_1)$ is the autocorrelation sequence. Hence, for zero-mean processes the first- and second-order cumulants equal their corresponding moments.
Third-order cumulants:

$$c_3(\tau_1, \tau_2) = m_3(\tau_1, \tau_2) - m_1[m_2(\tau_1) + m_2(\tau_2) + m_2(\tau_2 - \tau_1)] + 2(m_1)^3. \tag{3.17}$$

Fourth-order cumulants, derived from equation 2.7 in [5]:

$$\begin{aligned} c_4(\tau_1, \tau_2, \tau_3) = m_4(\tau_1, \tau_2, \tau_3) &- m_2(\tau_1) \cdot m_2(\tau_3 - \tau_2) \\ &- m_2(\tau_2) \cdot m_2(\tau_3 - \tau_1) - m_2(\tau_3) \cdot m_2(\tau_2 - \tau_1) \\ &- m_1[m_3(\tau_2 - \tau_1, \tau_3 - \tau_1) + m_3(\tau_2, \tau_3) \\ &\quad + m_3(\tau_1, \tau_3) + m_3(\tau_1, \tau_2)] \\ &+ 2(m_1)^2[m_2(\tau_1) + m_2(\tau_2) + m_2(\tau_3) + m_2(\tau_3 - \tau_1) \\ &\quad + m_2(\tau_3 - \tau_2) + m_2(\tau_2 - \tau_1)] \\ &- 6(m_1)^4. \end{aligned} \tag{3.18}$$

Thus, by assuming that the (real-valued) process is zero-mean ($m_1 = 0$) and setting $\tau_1 = \tau_2 = \tau_3 = 0$ we get [5]

$$C_1 = c_1 = E\{x(n)\} = 0 \qquad \text{(mean)} \tag{3.19}$$

$$C_2 = c_2(0) = E\{x^2(n)\} \qquad \text{(variance)} \tag{3.20}$$

$$C_3 = c_3(0,0) = E\{x^3(n)\} \qquad \text{(skewness)} \tag{3.21}$$

$$C_4 = c_4(0,0,0) = E\{x^4(n)\} - 3[C_2]^2 \qquad \text{(kurtosis)} \tag{3.22}$$

Normalized kurtosis is defined as $C_4/[C_2]^2$.

Also, various slices can be calculated from the cumulants by freezing some of the indices, e.g., a diagonal slice can be obtained by setting $\tau_i = \tau, i = 1, \ldots, k-1$. These slices contain different information compared with the skewness and kurtosis.

If a random process is symmetrically distributed it's third-order cumulant equals zero [4]. Communication signals are generally symmetrically distributed; therefore, the fourth-order cumulants are used in communications applications. The kurtosis measure is based on the size of the distribution's tail. Hence, the kurtosis characterize the shape of the distribution of the signal [7]. The impact of an interfering signal on a digital radio receiver depends on the distribution of the interfering signal. This is the motivation for examining the potential use of kurtosis-based measures in intersystem interference applications.

3.3.2 Complex-Valued Stationary Processes

So far we have treated a real-valued stationary random process. However, for the application at hand the data is complex-valued. Therefore, the cumulant definitions are extended to the case of complex-valued stationary processes.

For a complex-valued zero-mean stationary random process $y(n)$ the second-order moments can be defined in two different ways depending on placement of conjugation. The variance can be defined as [3]

$$C_{20} = E\{y^2(n)\} \tag{3.23}$$

$$C_{21} = E\{|y(n)|^2\}. \tag{3.24}$$

Similarly, the kurtosis measure can be defined in three ways

$$\begin{aligned} C_{40} &= \text{cum}(y(n), y(n), y(n), y(n)) \\ &= E\{y^4(n)\} - 3E^2\{y^2(n)\} \end{aligned} \tag{3.25}$$

$$\begin{aligned} C_{41} &= \text{cum}(y(n), y(n), y(n), y^*(n)) \\ &= E\{y^3(n)y^*(n)\} - 3E\{y^2(n)\}E\{|y(n)|^2\} \end{aligned} \tag{3.26}$$

$$\begin{aligned} C_{42} &= \text{cum}(y(n), y(n), y^*(n), y^*(n)) \\ &= E\{|y(n)|^4\} - |E\{y^2(n)\}|^2 - 2E^2\{|y(n)|^2\}. \end{aligned} \tag{3.27}$$

These three different versions of the kurtosis describe the complex process in different ways. For our application C_{42} is the most interesting measure, the others are included for completeness.

3.4 Sample Estimates

In practice, the cumulants must be estimated from a finite set of sample data. The sample estimates of the correlations (with zero-lag) of a zero-mean complex-valued process, $y(n), n = 1, 2, \ldots, N$, can be calculated as [7]

$$\hat{C}_{20} = \frac{1}{N} \sum_{n=1}^{N} y^2(n), \tag{3.28}$$

$$\hat{C}_{21} = \frac{1}{N} \sum_{n=1}^{N} |y(n)|^2, \tag{3.29}$$

where $\hat{}$ denotes a sample average. The fourth-order cumulants can be defined in three different ways, depending on the placement of conjugation. The different fourth-order cumulant values for zero-lags (the kurtosis) can be calculated from the sample data through one of the following equations [7]

$$\hat{C}_{40} = \frac{1}{N} \sum_{n=1}^{N} y^4(n) - 3\hat{C}_{20}^2 \tag{3.30}$$

$$\hat{C}_{41} = \frac{1}{N} \sum_{n=1}^{N} y^3(n) y^*(n) - 3\hat{C}_{20}\hat{C}_{21} \tag{3.31}$$

$$\hat{C}_{42} = \frac{1}{N} \sum_{n=1}^{N} |y(n)|^4 - |\hat{C}_{20}|^2 - 2\hat{C}_{21}^2. \tag{3.32}$$

Furthermore, the kurtosis estimate is normalized in order to remove any scaling problem in the data. The normalized kurtosis estimate is calculated as

$$\tilde{C}_{4k} = \frac{\hat{C}_{4k}}{\hat{C}_{21}^2}, \tag{3.33}$$

where $k = 0, 1, 2$ and $\tilde{}$ is used to denote the *normalized* kurtosis estimate. In situations where additive Gaussian noise is present in the data, the variance, $\hat{C}_{21}$, should be subtracted by an estimate of the variance of the noise, denoted $\hat{C}_{21,\mathrm{G}}$, i.e.,

$$\tilde{C}_{4k} = \frac{\hat{C}_{4k}}{(\hat{C}_{21} - \hat{C}_{21,\mathrm{G}})^2}. \tag{3.34}$$

The normalized kurtosis has a low computational complexity, which is of order N.

In practice, the sample mean is calculated and subtracted from the data prior to the cumulant estimation in order to get a zero-mean process. Finally, although higher-order cumulants are in theory unaffected by AWGN, this is only true for sufficiently large data sequences. Consistency is warranted for the cumulant estimates in the presence of AWGN; however, the variance of finite sample cumulant estimates increases.

Chapter 4

Simulations

The simulations on the performance of the BPSK receiver for different interference signals (some of which were presented earlier) were performed in ACOLADE (Advanced COmmunication Link Analysis and Design simulation Environment). Further, we use MATLAB to calculate the kurtosis-based measures for the different interfering signals.

4.1 ACOLADE

ACOLADE is a simulation tool for simulating communication systems. ACOLADE provides a number of models of different parts of a communication system such as different channel models, modulators, encoders, data sources etc. These models can then be connected in order to run a simulation. During a simulation, ACOLADE sends data between the different models and each model processes the data sample, consuming and generating data samples as necessary. It is also possible to monitor and analyze the data stream.

We have used ACOLADE for two different purposes. Firstly, ACOLADE is used to simulate the bit-error probability (BEP) for a communication system when subjected to different kinds of interference. From these simulations we obtain the BEP as a function of SIR or SNR as shown previously, see figure 2.3 and 2.4. Secondly, ACOLADE is used to produce waveform data that we import into MATLAB where we calculate the kurtosis value for the waveform. In figure 4.1 it is shown how the simulations are performed.

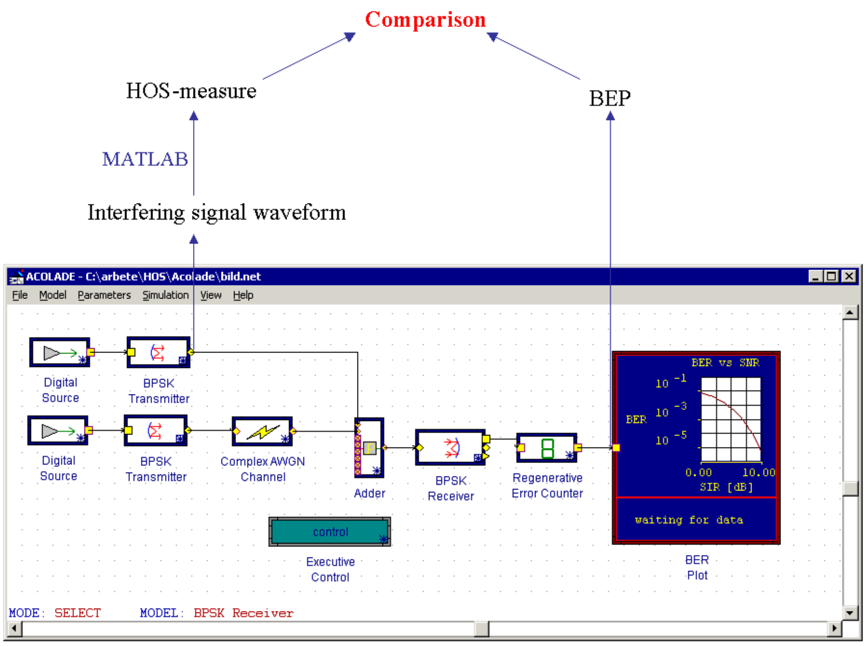

Figure 4.1: The simulation setup in ACOLADE and the connection to MATLAB.

4.2 Communication systems

We have used ACOLADE to simulate a communication system that uses binary phase-shift keying (BPSK) modulation and coherent detection. The signals are represented in the baseband.

In ACOLADE a digital source generates the data that is going to be transmitted. This data is then modulated using a BPSK modulator. Noise and interference are then added to the modulated signal. The received signal is then demodulated and the number of bit errors are calculated. The signal levels can be changed in order to simulate different levels of SIR or SNR.

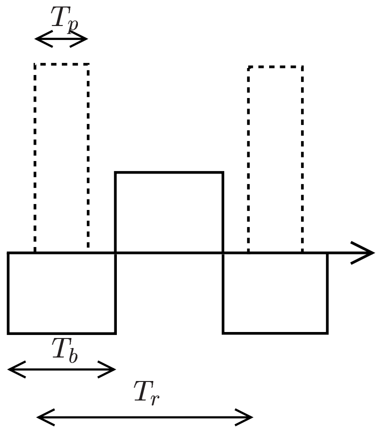

Figure 4.2: Definition of the pulse time, T_p, bit time, T_b, and the time between two consecutive pulses, T_r.

4.3 Modulated Interfering Signals

In a co-location scenario, it is common that the interfering signal is another communication signal from a radio transmitter. The interference then consists of a modulated signal. Therefore, we have simulated the performance with different modulations on the interference, i.e., BPSK, 8-PSK, 16-PAM, 16-QAM, and 64-QAM. The BEP depends on the relative phase between the BPSK receiver and the interference. This can easily bee understood by looking at the signal-space diagram for the desired and interfering signals. The results shown in this report are the average values from Monte Carlo simulations with different relative phases between the desired BPSK signal and the interference.

Each time interval is sampled a number of times, here 8 samples per bit, which is the standard value in ACOLADE. When the waveform is saved in order to calculate the kurtosis, 5.000 time intervals are saved.

4.4 Pulsed Interference

Radio receivers are quite often subjected to pulsed interference. These pulses could come from electrical equipment that repeatedly radiates for short periods.

A typical example is a local oscillator in information technology equipment. In this report, we examine the impact on radio receivers from pulsed interference with different normalized pulse repetition frequencies that are much lower than the data rate, i.e., not every data bit is subjected to the interference. The pulse duration, T_p, used in the simulations is 22 % of the duration of the information bit, T_b, see figure 4.2. Also, we define the normalized pulse repetition frequency as $f_r = T_b/T_r$. In order to enable a fair comparison, the average power of the interference is equal for all cases, which means that the energy of the pulses is scaled in the simulations.

The phase of the interfering pulse is uniformly distributed in the interval $[0, 2\pi]$. Each time interval is sampled a number of times, here 50 samples per bit. When saving the waveform, for calculations of the kurtosis, 20.000 time intervals are saved. We use a larger number of samples for the pulsed interference than for the modulated interfering signals. The pulse is only non-zero for a short interval in each time interval, hence the larger number of samples in each interval. For pulsed interference, with low normalized pulse repetition frequency, a number of time intervals are zero, hence the larger number of time intervals that are required for the kurtosis calculation.

Chapter 5

Results

Some results concerning the performance of the BPSK receiver when subjected to different interference signals were presented already in chapter 2 (figure 2.3 and 2.4). Here we will present the corresponding HOS-measures for the interfering signals; hence, the results are repeated here, in figure 5.1 and 5.3, together with the bit-error probabilities for different pulsed interference signals with an SNR of 10 dB, see figure 5.4.

In section 3.4 three different versions of sample estimates of the kurtosis for a complex-valued process where defined. These three different kurtosis estimates describes the process differently. From our simulations and comparisons we have seen that $\tilde{C}_{42}$ is the most interesting measure for our application and is the only one treated hereafter.

5.1 Kurtosis for Modulated Interference Signals

In order to calculate the kurtosis we first create the interfering signals in ACOLADE and thereafter calculate the kurtosis of the interfering signals in MATLAB, as was shown in figure 4.1.

In figure 5.2 the estimated normalized kurtosis value, $\tilde{C}_{42}$, see equation 3.33, is shown for several modulated signals. From figure 5.2, one can see that the kurtosis-based measure is appropriate for modulation classification. Signals within the same modulation class have similar kurtosis value. Kurtosis-based measures have been used by Swami and Sadler for modulation classification in [7].

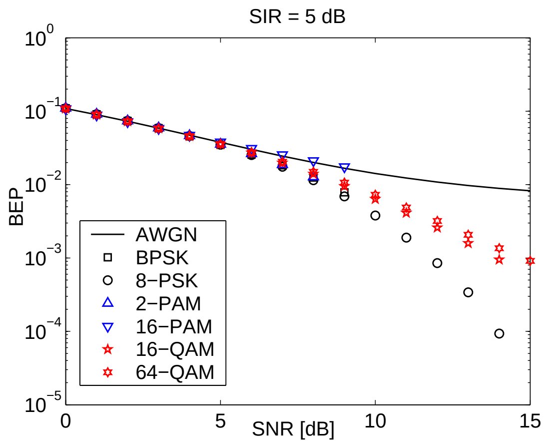

Figure 5.1: BEP for a BPSK receiver as a function of SNR, for different interfering modulated signals with SIR = 5 dB.

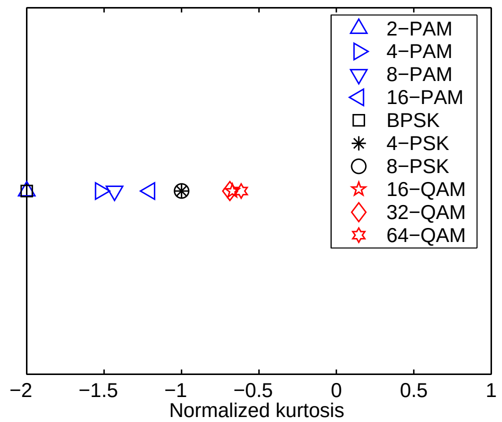

Figure 5.2: The estimated normalized kurtosis measure, $\tilde{C}_{42}$, calculated for different modulated signals.

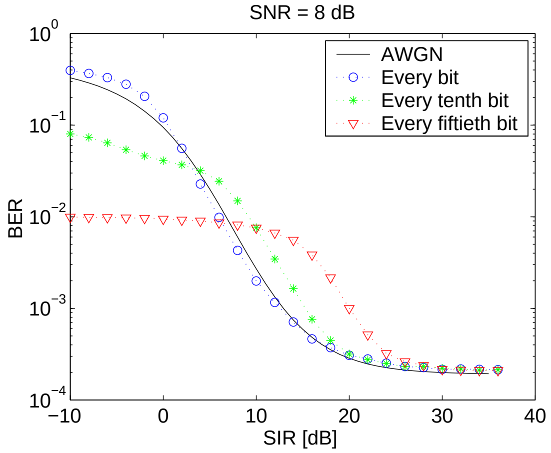

Figure 5.3: Comparison of bit-error probability for different pulsed interference signals in an BPSK receiver with SNR = 8 dB.

In figure 5.1 we can see that the type of modulation of an interfering signal has impact on the receiver performance. Unfortunately, when comparing the results in figure 5.1 and figure 5.2, it seems as a single kurtosis measure can not be used to directly estimate the impact of the electromagnetic in-band interference in terms of BEP for modulated signals. However, as concluded in chapter 2, the AWGN-approximation is valid for practical purposes for these modulated signals. Thus, a negative kurtosis value close to zero suggests that the AWGN-approximation is valid.

5.2 Kurtosis for Pulsed Interference

The estimated normalized kurtosis measure $\tilde{C}_{42}$, for pulsed interference with different normalized pulse repetition frequencies, f_r, is shown in figure 5.5 and 5.6. The pulse length, T_p, is 22 % of the bit duration, T_b. The energy of the pulses is normalized so that the mean power of the interference is equal for all cases.

In appendix A, equation A.10, it is shown that the normalized kurtosis measure, for this type of pulsed interference, can be calculated by the following

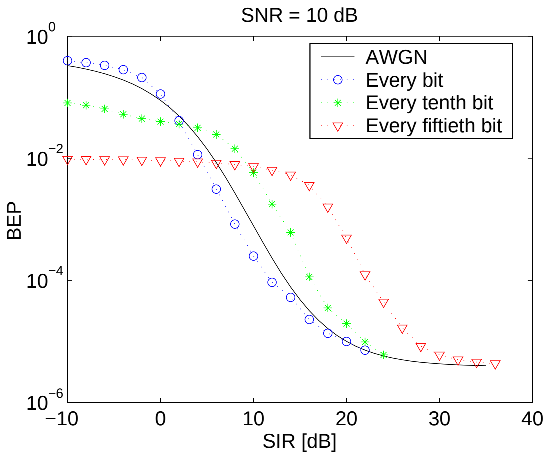

Figure 5.4: Comparison of bit-error probability for different pulsed interference signals in an BPSK receiver with SNR = 10 dB.

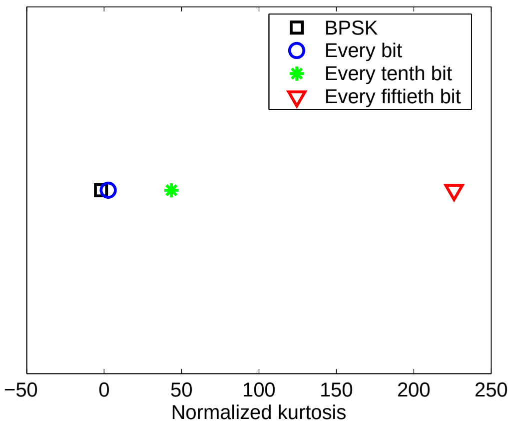

Figure 5.5: The estimated normalized kurtosis measure, $\tilde{C}_{42}$, for pulsed interference with different normalized pulse repetition frequency.

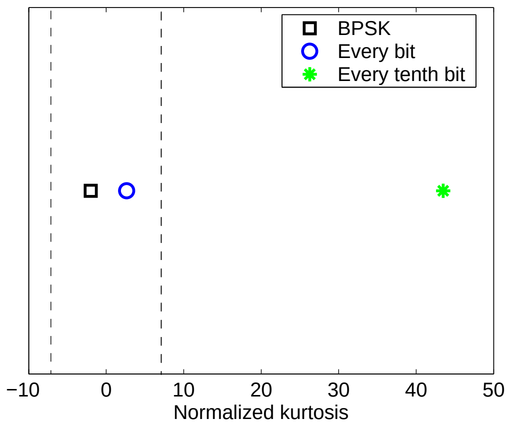

Figure 5.6: The estimated normalized kurtosis measure, $\tilde{C}_{42}$, for pulsed interference compared to the same measure for a BPSK modulated signal.

simple expression

$$\tilde{C}_{42} = \frac{1}{f_r}\frac{T_b}{T_p} - 2, \tag{5.1}$$

where $f_r = T_b/T_r$ is the normalized pulse repetition frequency, see also figure 4.2. For $T_p/T_b = 0.22$ and $f_r = 1/50$ this results in a kurtosis value of $\tilde{C}_{42} = 225.3$, and $\tilde{C}_{42} = 43.4$ if $f_r = 1/10$. These results supports the kurtosis-results shown in figure 5.5. As can be seen from figure 5.3 to 5.6, the AWGN-approximation is valid also for positive kurtosis values which are close to zero. For instance, if the pulsed interference is present in every data bit, the deviation from the AWGN-approximation is negligible and the kurtosis value is also close to zero.

The kurtosis value is higher for pulses with low normalized pulse repetition frequency. From figure 5.3 and 5.4 it is clear that the BEP from interference with low normalized pulse repetition frequency is underestimated, for some values of SIR, by the AWGN-approximation. Hence, the kurtosis value can be used to estimate the receiver performance, i.e., a large kurtosis value typically means a larger impact on the receiver (higher BEP).

5.3 Discussion

With the kurtosis measure it is possible to separate the pulsed interference from the modulated signals, see figure 5.2 to 5.6. The modulated signals can be approximated as AWGN without underestimating the impact on a receiver [9], but that is not the case for pulsed interference with low normalized pulse repetition frequency. Hence, it is possible to use a simple threshold on the kurtosis value to determine whether the interference can be approximated as AWGN. For a modulated signal, the kurtosis value is negative and for a pulsed interference the kurtosis value is normally positive. Hence, using the AWGN-approximation when having a negative kurtosis value results in an overestimation of the impact on the receiver for the signals examined here. For kurtosis values near zero, both positive and negative, the AWGN-approximation can be used for practical purposes. For large positive kurtosis values the approximation may yield an underestimation of the BEP, which can have serious consequences. Hence, this opens the possibility to use a simple threshold on the kurtosis value, as illustrated in figure 5.6, to determine if the AWGN-approximation can be used.

The kurtosis also gives an indication of the size of the errors in the AWGN-approximation. For the pulsed interference, a lower normalized pulse repetition frequency results in a larger kurtosis value. The impact on the receiver is also larger for pulses with a lower normalized pulse repetition frequency, provided that the average interference power is the same. Therefore, the kurtosis value can be used to estimate the receiver performance for pulsed interference.

However, it seems as a single kurtosis measure cannot be used to directly estimate the impact of the electromagnetic in-band interference in terms of BEP for modulated signals. However, as shown in [7], different variants of them can be used to classify the signal modulations through a hierarchical scheme. The scheme used thresholds based on kurtosis-measures, which facilitated classification between several modulation types at relatively low SNR's. Further, the impact from different signal modulations on a radio receiver can be determined on beforehand, e.g., by extensive simulations. Hence, a possible approach to determine the resulting BEP is then to use kurtosis-based measures to classify the modulation type and thereafter use a look-up table to determine the BEP caused by the interference.

Chapter 6

Conclusions

In this report, a new application of HOS has been investigated. We have shown that it is possible to use a higher-order statistics based measure, the kurtosis, to methodically simplify the performance analysis of a coherent digital radio receiver that is subjected to an interference signal. This measure can be used to determine if an in-band interference can be approximated as AWGN without underestimating its impact. This opens up the possibility to use the kurtosis as a quality measure in computer-based tools for intersystem interference analyses. In such tools it is favorable to use the AWGN-approximation since this gives fast calculations.

Modulated interference signals can be approximated as AWGN without underestimating the BEP. However, the AWGN-approximation is not valid for all repetitive pulsed interference signals. For kurtosis values near zero, both positive and negative, the AWGN-approximation can be used for practical purposes. Hence, this opens up the possibility to use a simple threshold, on the kurtosis value, to determine if the AWGN-approximation can be used. The kurtosis can effectively be used to create a warning for high kurtosis values saying that the estimated BEP is not reliable. Furthermore, for pulsed interference, the kurtosis can be used to quantify the impact on the communication receiver. A signal with a large kurtosis value also has large impact on the resulting BEP.

Chapter 7

Further Work

First of all, we will extend the work presented in this report by also examining pulsed interference with random amplitudes, arrival times, and pulse lengths. It is also of interest to examine how the kurtosis-based measure should be used in scenarios with more than one interference signal. Also, the kurtosis does not reveal all relevant information about an interference signal. It is possible to retain more information from different slices of the fourth-order cumulants; information which may enable more accurate predictions about the impact from interference signals on digital radio receivers.

Furthermore, the effects of noise on the cumulant estimates could be examined. In theory, the fourth-order cumulants are unaffected by Gaussian noise; however, this is only true when the number of samples are sufficiently large. In practical situations, noise can affect the variance of the kurtosis estimates.

We have earlier suggested that the kurtosis measure can be used in an intersystem interference tool in order to give a quality measure on the results from the AWGN-approximation We have also seen that the kurtosis can estimate the size of the impact, on a digital radio receiver, of pulsed interference. Hence, it might be possible to develop a method, which uses the kurtosis, to estimate the receiver performance when the AWGN-approximation is not valid. This alternative method would be very useful in a tool for intersystem interference since it could handle some of the interfering signals that the AWGN-approximation could not.

Appendix A

Numerical Calculations of the Kurtosis

In this appendix we will first show that the normalized kurtosis of an interfering signal is unaffected by a constant scaling of the interference power. Thereafter, a simple expression for the normalized kurtosis for repetitive pulsed interference, with constant power and random uniformly distributed phase, is derived. The normalized kurtosis only depends on the ratio between the number of samples the interference is active compared to the total number of samples.

A.1 Normalized Kurtosis is Unaffected by Power Scaling

Here we will show that the normalized kurtosis,

$$\tilde{C}_{42} = \frac{\hat{C}_{42}}{\hat{C}_{21}^2}, \tag{A.1}$$

is unaffected by a scaling of the amplitude of the data sequence.

In section 3.4 it was shown that the second- and fourth-order cumulants can be calculated from the following expressions

$$\hat{C}_{20} = \frac{1}{N}\sum_{n=1}^{N} y^2(n), \tag{A.2}$$

$$\hat{C}_{21} = \frac{1}{N}\sum_{n=1}^{N} |y(n)|^2, \tag{A.3}$$

$$\hat{C}_{42} = \frac{1}{N}\sum_{n=1}^{N} |y(n)|^4 - |\hat{C}_{20}|^2 - 2\hat{C}_{21}^2. \tag{A.4}$$

Hence, the normalized kurtosis can be calculated as

$$\tilde{C}_{42} = \frac{\frac{1}{N}\sum_{n=1}^{N} |y(n)|^4}{\hat{C}_{21}^2} - \frac{|\hat{C}_{20}|^2}{\hat{C}_{21}^2} - 2. \tag{A.5}$$

Furthermore, since $C_{20} = 0$ for the complex-valued constellations under consideration [7], the normalized kurtosis reduces to

$$\tilde{C}_{42} = \frac{\frac{1}{N}\sum_{n=1}^{N} |y(n)|^4}{(\frac{1}{N}\sum_{n=1}^{N} |y(n)|^2)^2} - 2. \tag{A.6}$$

Now, in order to examine the effects of an amplitude scaling we apply a constant scale factor c to the sequence $y(n)$,

$$\tilde{C}_{42} = \frac{\frac{1}{N}\sum_{n=1}^{N} |cy(n)|^4}{(\frac{1}{N}\sum_{n=1}^{N} |cy(n)|^2)^2} - 2 = \frac{c^4\frac{1}{N}\sum_{n=1}^{N} |y(n)|^4}{c^4(\frac{1}{N}\sum_{n=1}^{N} |y(n)|^2)^2} - 2. \tag{A.7}$$

Hence, it is clear that the normalized kurtosis $\tilde{C}_{42}$ is unaffected by an amplitude scaling of the data.

A.2 Kurtosis for Pulsed Interference

Here we examine the kurtosis of repetitive pulsed interference with constant amplitude and random uniformly distributed phase. The interference is only active during a part of the duration of the information bit T_b, and we denote this active time as T_p, see figure A.1. Furthermore, the repetitive pulsed interference is only active during some of the information bits, and we define this normalized pulse repetition frequency as $f_r = T_b/T_r$. Hence, the ratio of the number of samples the interference is active compared to the total number of samples is $p = T_p/T_r$.

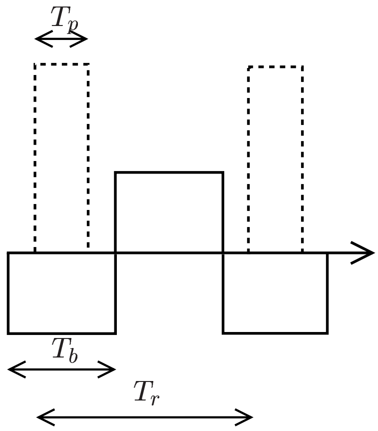

Figure A.1: Pulse time, T_p, bit time, T_b, and pulse repetition time, T_r.

The pulsed interference has, when it is active, constant amplitude and random uniformly distributed phase. In the calculation of the normalized kurtosis, see equation A.6, only the non-zero samples will contribute to the summation. The number of non-zero samples in the interfering signal $y(n)$, which is of length N, are pN. Thus, the normalized kurtosis can be calculated as,

$$\tilde{C}_{42} = \frac{\frac{1}{N}\sum_{n=1}^{N}|y(n)|^4}{(\frac{1}{N}\sum_{n=1}^{N}|y(n)|^2)^2} - 2 = \frac{\frac{1}{N}\sum_{n=1}^{pN}|y'(n)|^4}{(\frac{1}{N}\sum_{n=1}^{pN}|y'(n)|^2)^2} - 2 \qquad \text{(A.8)}$$

where $y'(n)$ contains the non-zero samples of $y(n)$.

We know that an amplitude scaling of $y(n)$ does not affect the normalized kurtosis. Therefore, we can scale the interfering signal so that $|y'(n)| = 1$, for $n = 1, 2, ..., pN$. Hence, for repetitive pulsed interference with constant amplitude and random uniformly distributed phase, the normalized kurtosis can be calculated as,

$$\tilde{C}_{42} = \frac{\frac{1}{N}\sum_{n=1}^{pN}|y'(n)|^4}{(\frac{1}{N}\sum_{n=1}^{pN}|y'(n)|^2)^2} - 2 = \frac{\frac{1}{N}pN}{(\frac{1}{N}pN)^2} - 2 = \frac{1}{p} - 2, \qquad \text{(A.9)}$$

which also can be written as,

$$\tilde{C}_{42} = \frac{1}{f_r}\frac{T_b}{T_p} - 2 = \frac{T_r}{T_p} - 2. \qquad \text{(A.10)}$$

This means that the normalized kurtosis only depends on the ratio of the number of samples the interference is active compared to the total number of samples.

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