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Methodology report

Recursive Terrain Navigation

Application of the Correlation Method

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Metodrapport

Rekursiv terrängnavigering

Tillämpning av korrelationsmetoden

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Report title

Recursive Terrain Navigation, Application of the Correlation Method

Abstract (not more than 200 words)

This report describes the use of the correlation method in the recursive estimation of the position of an underwater vehicle.

It also discusses the improved robustness and accuracy achieved by the correlation method.

Further it describes a Hardware Correlator which calculates a Terrain Position in 266 ms.

Keywords

Terrain navigation, Terrain positioning, AUV

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Rekursiv Terrängnavigering, Tillämpning av korrelationsmetoden

Sammanfattning (högst 200 ord)

Denna rapport beskriver användningen av korrelationsmetoden vid rekursiv estimering av en undervattensfarkosts position.

Den diskuterar också den förbättrade robustheten och noggrannheten som erhålls vid korrelationsmetoden.

Vidare beskrivs en hårdvarukorrelator som beräknar en terrängposition på 266 ms.

Nyckelord

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Contents

1. Introduction

Determining the positioning of underwater vehicles using the Correlation method ref. [3] has been studied over the past years. The correlation method has proven to be robust, highly precise and minimally revealing. It is also easily implemented and require a minimum of underwater maps.

In navigating by this method, the vehicle moves between "navigation cells" for which seabed maps are available with help of the INS system (Inertial Navigation System), Figure 1.1. When the vehicle is within a particular navigation cell it determines its position using terrain correlation, updates the INS system, and moves on,
Figure 1.2.

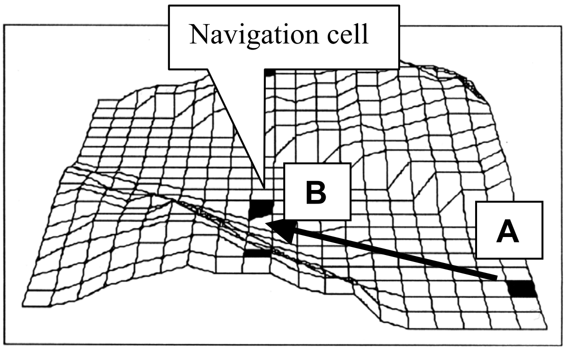

Figure 1.1. The vehicle moves between the navigation cells using the INS system. The position in the cell is determined by terrain correlation.

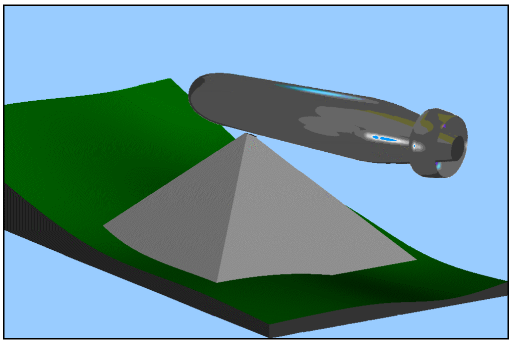

Figure 1.2. Principle of the Correlation method. The sonar normally measures the topography of an area between approx. 100 x 100 m^2 and 300 x 300 m^2. The area may have a more circular form.

There are however other principles for terrain navigation, including Recursive Bayesian Terrain Navigation. This report relates the Correlation method with that method, starting with a short presentation of the fundamentals of recursive Bayesian estimation thereafter comparing the methods. The report concludes with a description of a hardware correlator for the Correlation method.

2. The Bayesian Recursive Estimation Method

The Bayes method for recursive estimation may be suitable for non-linear problems that are not easily linearized to enable use of the Extended Kalman Filter (EKF). The method requires that the estimation problem is solved numerically and it is quite demanding of computer resources.

A basic element of the method is the conception of a likelihood function. A likelihood function is defined as

$$L(\mathbf{y} \mid \mathbf{x}) = \Pr\{\mathbf{Y} = \mathbf{y} \mid \mathbf{X} = \mathbf{x}\} \text{ for } \mathbf{x} \in \text{state space of } \mathbf{x} \qquad (2.1)$$

The notation Pr means here probability when we have discrete random variables and probability density when we have continuos variables. The likelihood function gives a measure of how likely it is to have a particular measured value of $\mathbf{y}$ at the position $\mathbf{x}$ in the state space.

Likelihood functions can be multiplied if the measurements are independent and if the process generating $\mathbf{x}$ is a Markov process.

According to Bayes theorem the PDF (Probability Density Function) for the position $\mathbf{x}$ given the measurement $\mathbf{y}$ is for the continuos case

$$p(\mathbf{x}\,|\,\mathbf{y}) = \frac{p(\mathbf{y}\,|\,\mathbf{x})p(\mathbf{x})}{p(\mathbf{y})} = \frac{L(\mathbf{y}\,|\,\mathbf{x})p(\mathbf{x})}{\mathrm{C}} \tag{2.2}$$

where C can be interpreted as a normalizing constant. The PDF $p(\mathbf{x}\,|\,\mathbf{y})$ is referred to as the posterior PDF (after the measurement), while $p(\mathbf{x})$ is referred to as the prior PDF (before the measurement).

The recursive procedure means that we will start from a given prior PDF that we will propagate according to the movement of the vehicle and the uncertainty of this movement. The movement can, for example, be determined from the INS system which has a specified uncertainty. We can denote the propagated, but not measurement-updated, PDF as $p^{-}(\mathbf{x})$.

The next step in the recursive procedure is to do the measurement update of $p^{-}(\mathbf{x})$ by the pointwise multiplication in state space of $p^{-}(\mathbf{x})$ and $L(\mathbf{y}\,|\,\mathbf{x})$ in order to arrive at the posterior PDF $p(\mathbf{x}\,|\,\mathbf{y})$.

The expected value and variance of the estimated position are $\hat{\mathbf{x}} = E\{\mathbf{x}\,|\,\mathbf{y}\}$ and $Var\{\hat{\mathbf{x}}\} = E\{(\mathbf{x} - \hat{\mathbf{x}})^2\}$ respectively.

For a more cogent derivation of the recursive equations see ref.[1]. A natural numeric method for the estimation is to replace the continuos PDF's with "mass points" in a grid - the mass point approach. A second numerical method is the 'Particle Filter'. Recursive Bayesian estimation is particularly easy to use when we have a low-dimensional state.

3. Recursive Terrain Navigation using the Correlation Method

3.1. Prerequisite

Ref. [2] describes a method called Bayesian Recursive Terrain Navigation, which is based on a single measurement of the height above the terrain at each sampling event. Figure 3.1 shows an underwater vehicle that instead measures the whole bottom profile at each sampling event. The underwater vehicle is moving along a mainly straight course.

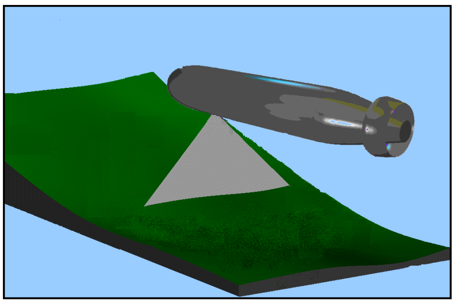

Figure 3.1. At each sampling event the vehicle measure the bottom profile.

The assumption for the movement is

$$\mathbf{x}_{t+1} = \mathbf{x}_t + \mathbf{u}_t + \mathbf{v}_t \qquad t = 1,2,3... \tag{3.1}$$

$$\mathbf{y}_t = \mathbf{h}(\mathbf{x}_t) + \mathbf{e}_t \tag{3.2}$$

where $\mathbf{x}_t$ is the position in the horizontal plane at time t and $\mathbf{u}_t$ - the distance between the positions - is provided by the INS system. The measured heights above the ground for the bottom profile are collected in the vector $\mathbf{y}$ and the vector $\mathbf{h}(\mathbf{x}_t)$ collects the depth measurement above the sea bottom according to the map if we are in position $\mathbf{x}_t$. The error in the INS system is $\mathbf{v}_t$ and $\mathbf{e}_t$ is the error in the height measurement. The number of measurement points (sonar beams) is n and we use the middle beam as a reference point if n is odd. In the case of a rectangular measurement $\mathbf{y}_t$ and $\mathbf{h}(\mathbf{x}_t)$ will be matrices but they can be vectorized if the measurements are independent. We will further assume that the errors are independent white Gaussian sequences which is not strictly necessary.

3.2. Propagation of the PDF for the vehicle position

The Equation, 3.1, describes how the position changes between two sampling events. Figure 3.2 below illustrates this.

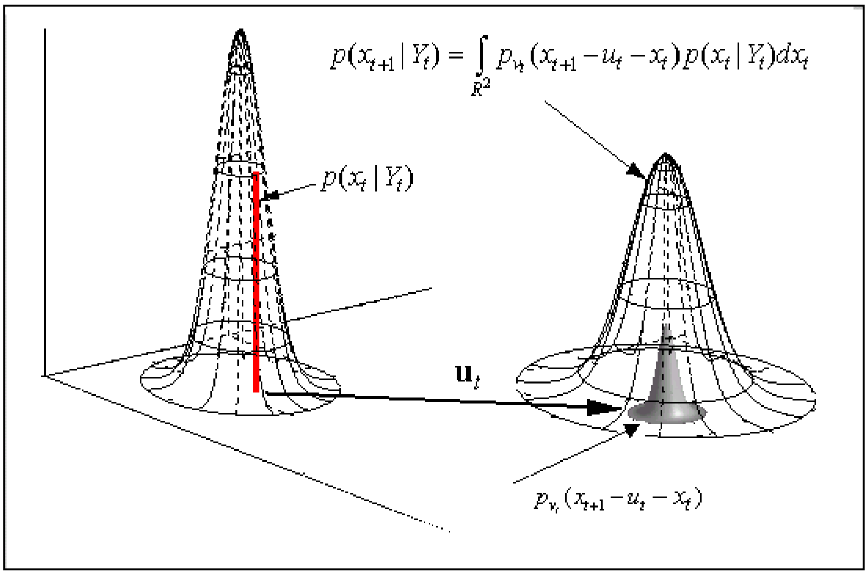

Figure 3.2. Propagation of the PDF for the vehicle position.

The left PDF (prior PDF) has a bearing upon the vehicle's position at sampling time t and the red pile is the relative frequency of the number of realizations which have ended in point $\mathbf{x}_t$, i.e $p(\mathbf{x}_t | \mathbf{Y}_t)$. The notation $p(\cdot | \mathbf{Y}_t)$ indicates that the PDF is based on all measurement up to and including the measurement at time t. The realizations are moved the distance $\mathbf{u}_t$ but due to the uncertainty $\mathbf{v}_t$ we will have a spread in the new position. We multiply this new PDF - the small one in the Figure - by the relative frequency $p(\mathbf{x}_t | \mathbf{Y}_t)$ and total the contributions of all points in the left PDF by an integral so we will have the position PDF at time t+1. We can express the propagation as

$$p(\mathbf{x}_{t+1} | \mathbf{Y}_t) = \int_{R^2} p_{\mathbf{v}_t}(\mathbf{x}_{t+1} - \mathbf{u}_t - \mathbf{x}_t) p(\mathbf{x}_t | \mathbf{Y}_t) d\mathbf{x}_t \qquad (3.3)$$

3.3. The Measurement update

The measurement update is made according to equation 3.2. We assume the error in the height measurement to be Gaussian and independent. This assumption can be justified if we are close to the true position (a discussion about correlation between errors is found in ref. [3]). Therefore, according to equation 3.2 we have $\mathbf{e}_t = \mathbf{y}_t - \mathbf{h}(\mathbf{x}_t)$ and since $\mathbf{e}_t$ is Gaussian

$$L_G(\mathbf{y} \mid \mathbf{x}) = \frac{1}{\sqrt{(2\pi)^n \det(\mathbf{C}_e)}} e^{-\frac{1}{2}(\mathbf{y}-\mathbf{h}(\mathbf{x}))^T \mathbf{C}_e^{-1} (\mathbf{y}-\mathbf{h}(\mathbf{x}))} \tag{3.4}$$

The function $L_G(\mathbf{y} \mid \mathbf{x})$ is called the likelihood function since, for a given position $\mathbf{x}$, it gives the likelihood of having the measurement value $\mathbf{y}$. The assumption that the error of different beams with same measurement errors are uncorrelated makes the correlation matrix C_e diagonal and equal to $\sigma_e^2 \mathbf{I}_n$. Therefore the likelihood function at time t will be

$$L_G(\mathbf{y}_t \mid \mathbf{x}_t) = \frac{1}{\sqrt{(2\pi\sigma_e^2)^n}} e^{-\frac{1}{2\sigma_e^2}(\mathbf{y}_t-\mathbf{h}(\mathbf{x}_t))^T (\mathbf{y}_t-\mathbf{h}(\mathbf{x}_t))} = \frac{1}{\sqrt{(2\pi\sigma_e^2)^n}} e^{-\frac{1}{2\sigma_e^2}\sum_{k=1}^{n}(y_{t,k}-h(x_{t,k}))^2}$$

The index k means summation over all elements in the measurement vector/matrix $\mathbf{y}_t$. We recognize the correlation sum $T(x_t) = \sum_{k=1}^{n} (y_{t,k} - h(x_{t,k}))^2$ from the correlation method. The contour lines for the correlation sum $T(x)$ give the locus for equal likelihood.

Figures 3.3 and 3.4 below show the non-normalized likelihood function for a measurement with only one beam and a measurement with 9 x 9 beams, respectively, on a 5 km x 5 km map.

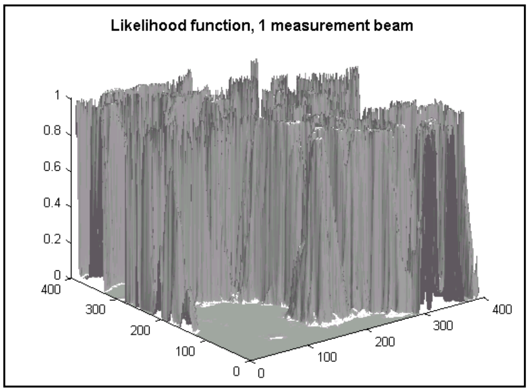

Figure 3.3. Likelihood function, 1 beam.

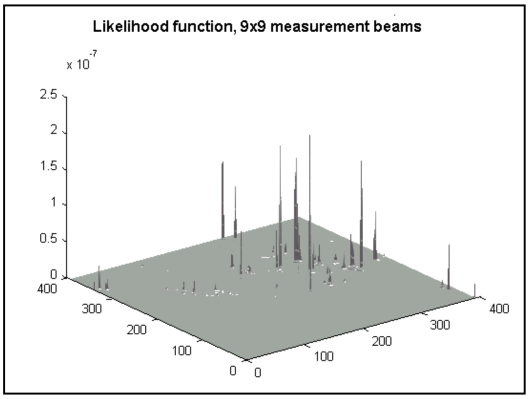

Figure 3.4. Likelihood function, 9 x 9 beams.

The next step in the recursion is to fuse the likelihood function for the measurement with the propagated PDF of the position. This is done by pointwise multiplication. The Figures 3.5 and 3.6 show the normalized measurement-updated PDF, the posterior PDF.

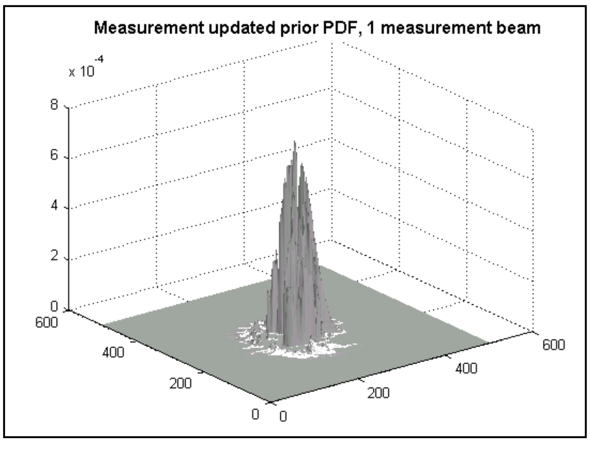

Figure 3.5. The measurement updated PDF, 1 beam.

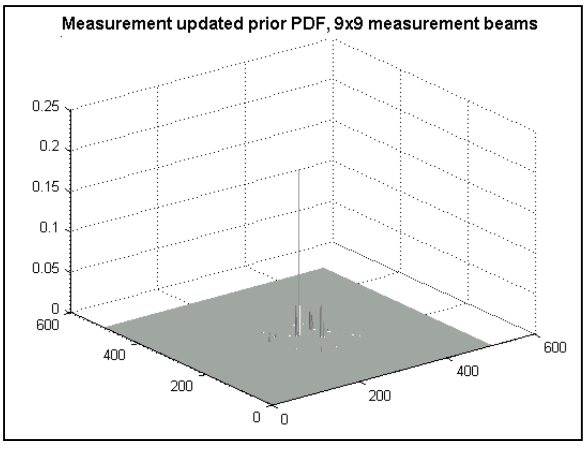

Figure 3.6. The measurement updated PDF, 9 x 9 beams.

The equation for the fusion is

$$p(\mathbf{x}_{t+1} \mid \mathbf{Y}_{t+1}) \sim L_G(\mathbf{y}_{t+1} \mid \mathbf{x}_{t+1}) p(\mathbf{x}_{t+1} \mid \mathbf{Y}_t) \qquad (3.5)$$

the notation $\sim$ means that the likelihood function does not need to be normalized.

A comparison between Figure 3.5 and 3.6 shows that the variance is much smaller when the correlation method is used. It can be shown (see Chapter 5) that the variance decreases inversely with the number of beams.

4. Mean and Variance for the Posteriori PDF of the Position.

4.1 The shape of the measurement-updated PDF

The PDF for the measurement-updated PDF can have several pronounced local maxima, especially if the measurement is made by a small number of beams. This means, from a frequentist point of view, that if repeated realizations were based on this PDF the vehicle would flip between those points without external reasons. This is not physically credible.

One interpretation of this phenomenon is that it is a consequence of our measurement method. The posterior PDF should merely be seen as a sample of an underlying PDF. We have the smoothing property $E\{E(x \mid y)\} = \int_{-\infty}^{\infty} [\int_{-\infty}^{\infty} xp(x \mid y)dx] p(y)dy = E[x]$, i.e. the underlying PDF is $p^{-}(x)$. As can be seen from Figures 3.3 and 3.4 the number of local maxima is greatly dependent on the number of measurement beams and when the number of beams $\rightarrow \infty$ the sample PDF of $p^{-}(x)$ will be unimodal if there are no terrain areas that are exactly equal, ref. [3].

4.2 Mean and variance

If the optimal estimate of the position, $\hat{\mathbf{x}}$, is given by the mean of the posterior function the variance will reach the Cramer Rao Lower Bound (CRLB). In general when we are estimating an unbiased parameter θ_i the variance is given by, ref.[4]

$$Var(\theta_i) \geq [I^{-1}(\theta)]_{ii} \text{ where } [I(\theta)]_{ij} = -E[\frac{\partial^2 \ln(p(x;\theta))}{\partial\theta_i \partial\theta_j}] \qquad (5.1)$$

where the matrix I is the Fischers Information Matrix (FIM).

Ref. [2] discusses the CRLB and shows that the CRLB is reached in the case where one measurement beam is used. In ref. [5] a proof is outlined showing that this will also be the case when the measurements are a vector. The correlation method in recursive application will thus, according to ref.[5], reach the CRLB.

Referring to Chapter 4.1 a better estimate of the position may be the maximum of the posteriori probability density function (MAP-estimate) instead of the mean estimate since the secondary peaks (bimodal) are due to similarities in the terrain and they disappear with increasing number of measurement beams. The estimate will then also be optimal in a mean square sense.

5. Maximum Likelihood estimation of position and variance.

If the time interval between the measurements is large, the variance in the propagated PDF, $p^{-}(\mathbf{x})$, will increase so much that it will not improve the estimation of the position based on the likelihood function. Any gyro bias will also introduce a bias into the $\mathbf{v}_t$ - PDF. This bias can indeed be included in the estimation but it will complicate the numerical procedure and is best done separately.

This means that using only one measurement beam as in the method, described in Chapter 3, will not work. However, when the PDF $p^{-}(\mathbf{x})$ has a large variance it can be approximated by a uniform distribution without introducing larger errors. This means that the estimation method turns over to a Maximum Likelihood (ML) estimation.

ML-estimation has the asymptotic properties that it is unbiased, it reaches the CRLB and its estimate is Gaussian. Here, we have two parameters to estimate, the east and north position (the components of the position vector $\mathbf{x}_t$) and the number of measurement beams is 100 – 1600.

We can conclude that the variance in this case will reach the CRLB if the estimate is based on the mean since fusing with uniform $p^{-}(\mathbf{x})$ would not change the conclusion reached in Chapter 4. We also know that the secondary peaks in the likelihood function will disappear when the number of beams increases, i.e. the mean and maximum will coincide.

In order to have an analytic expression for the variance some, assumptions about the creation of the bottom topography have to be made. In the case where it is generated by an AR(1), ref. [3], process and the bottom profile is measured by a linear array we have

$$Var(\hat{x}) \approx \frac{1}{2}\frac{1+a}{N}\frac{\sigma_e^2}{\sigma_\varepsilon^2} \qquad (5.2)$$

where N is the number of beams and a is the pole in the AR(1) expression and $\sigma_e^2, \sigma_\varepsilon^2$ are the variances for the measurement error and the variance of the bottom topography respectively.

It is customary in the Correlation method to draw the inverse of the correlator sum $T(x)$, $\frac{1}{c_{i,j}}$, at every point of the map, instead of $\frac{1}{e^{c_{i,j}}}$ which would be correct according to the likelihood function. However, since both are monotonic function we can often use $\frac{1}{c_{i,j}}$ without introducing significant errors; we will have a sharper crest and thus somewhat smaller variance if we fuse it with the propagated PDF.

6. The Search Area for the Correlation Method

When the vehicle is moving from navigation cell A to cell B it is vital that the vehicle ends up being in cell B so it can conduct the correlation with the underwater map and determine its position to avoid losing its position. The question is thus, how large does the navigation cell B have to be?

6.1 The position error for inertial navigation system (INS)

The accuracy of the INS system has been treated extensively in the literature. The discussion below is largely taken from ref. [6] but is also found in ref. [7]. One assumption underlying the simplified treatment is that the vehicle is moving within a small area, say 50 km x 50 km at low speed as in the case here.

The errors in an INS system can roughly be categorized as bias errors, random walk errors, and other errors. The bias errors, or day-to-day errors, are stochastic errors that remain after the best nulling of the errors. The random walk errors are random errors that vary more often and are more easily to remove using the vehicle dynamics (Kalman filtering).

The day-to-day errors and the errors from initializing and setting up of the INS system will determine the size of the positioning area over which the Likelihood function should be calculated.

The following discussion assumes that the vehicle is heading north. The position errors in the northern and eastern directions can be assumed to be independent of each other. The following model, Figure 6.1, which is the same for both platform and strapped-down systems, can be drawn up. The model is valid for movements within a limited area, i.e. between navigation cells.

The navigation principle is that the accelerometers in the INS system are integrated twice in the directions pointed out by the Gyros. It is customary to examine the errors rather than the positions, so the Simulink schematic, Figure 6.1, refers to the errors.

In Figure 6.1 $dBaN$ refers to the accelerometer bias in the northern direction while $dBgE$ and $dBgD$ refer to the gyro bias in the eastern and in the down direction, respectively. The performance of the INS system is highly dependent on the size of the bias in the accelerometers and in the gyros.

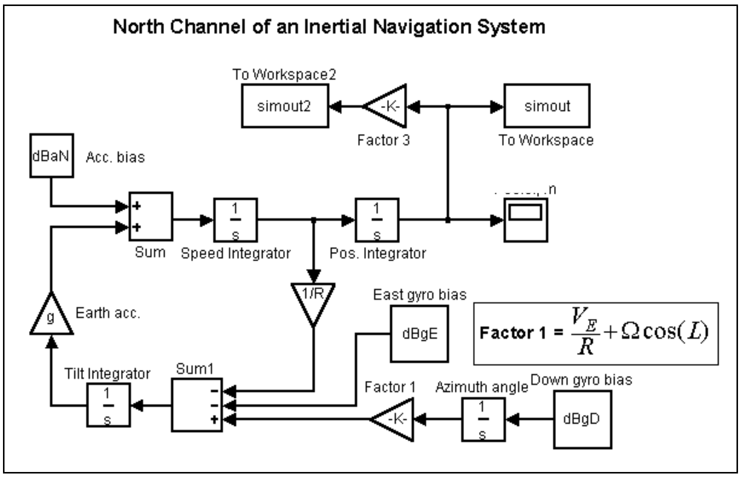

Figure 6.1. Simulink model for the north channel for studying the influence of gyro and accelerometer bias. The east channel has a similar model.

The differential equation for the model presented in Figure 6.1 can be solved, resulting in the following analytical expression for the 1 σ-limit of the position error.

$$e(t) = \frac{gB_G t}{\omega_S^2}(1-\frac{\sin(\omega_S t)}{\omega_S t}) + B_A\frac{1-\cos(\omega_S t)}{\omega_S^2} \qquad (6.1)$$

where B_G and B_A are the RMS-values for the gyro and accelerometer biases, g is the earth acceleration and ω_S is the Schuler frequency with a period time of 84.4 minutes.

The expected variance (the RMS-value) is if B_G and B_A are assumed to be independent stochastic variables.

$$\sigma(t)=\sqrt{\left[\frac{gt}{\omega_S^2}(1-\frac{\sin(\omega_S t)}{\omega_S t})\right]^2\sigma_{B_G}^2+\left[\frac{1-\cos(\omega_S t)}{\omega_S^2}\right]^2\sigma_{B_A}^2} \qquad (6.2)$$

For a vehicle with submarine performance we will have a considerably smaller error than for a good ordinary INS system mainly due to lower gyro bias. Figures 6.2 and 6.3 show typical position errors for a good ordinary system, a so called "1 Nautical mile /h"-system and a good submarine system.

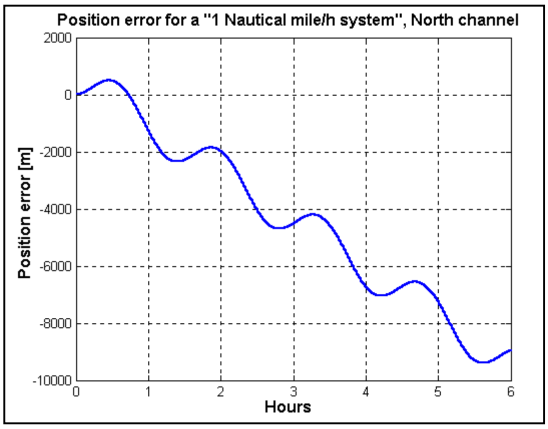

Figure 6.2. Total position error for a good INS system due to accelerometer and gyro bias, according to the model in Figure 6.1

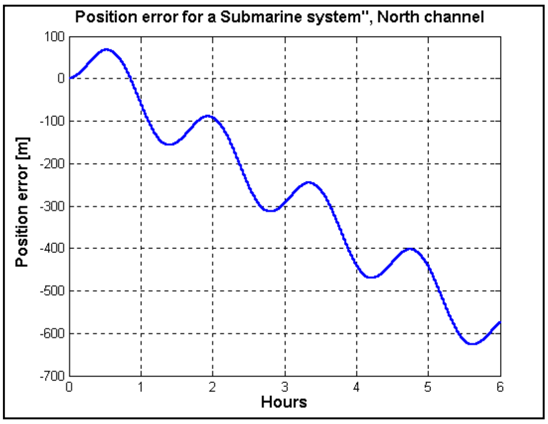

Figure 6.3. Total position error for a submarine INS system due to accelerometer and gyro bias, according to the model in Figure 6.1

7. Computing time for the correlation method

The drawback of the Correlation method in recursive calculation is the increase in computing time due to the larger number of measurement beams. Figure 7.1 shows how the computing time in Matlab for the likelihood function depends on the number of measurement beams

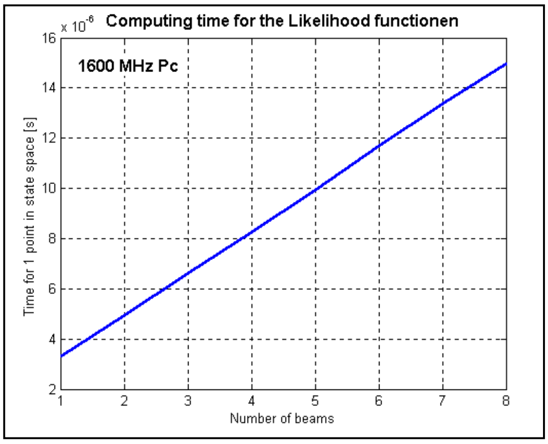

Figure 7.1. The time for computing a likelihood value in state space for various number of measurement beams.

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The computing time for calculating the likelihood function for 9 x 9 = 81 beams will be about 40 times longer compared to one beam. The total computing time will be about 20 times longer. However it turns out that the calculation of the quadratic sum is easily mechanized so the complete correlation of a whole map can be done in a few hundredths of a second with modern computer technology.

8. Hardware Correlator

8.1 The basic principle

One way to reduce the computing time is to implement the correlation algorithm in an FPGA (Field Programmable Gate Array) and a way of doing that will be described here.

Figure 8.1 shows a part of a 5 x 5 km underwater map with an inlaid sonar map. The sonar map presents gridded depth data from the sonar. In the Figure the sonar map has only 3 x 3 nodes; usually it is much larger. The sonar map is moved horizontally and vertically in one-node steps till comparison has been made with all nodes in the map. A sonar map is thus created at each sonar ping, and correlated against the larger underwater map to determine the coordinates of best fit.

For each node in the larger map that can be reached by the sonar-maps reference point we calculate

$$V(i,j) = \sum_{\substack{\text{nodes of the} \\ \text{sonarmap}}} \left| d_{k,ij} - d_{m,rs} \right| \qquad (8.1)$$

where *d* is the mean compensated depth value in the node. Thus, V(i,j) is the sum of the absolute differences of the mean compensated depth values in the terrain-map and the sonar map and (i,j) are the coordinates of the reference point.

Differences from the mean depth are examined because of the problem of establishing a common reference altitude for both the terrain map and the sonar map. We are using the absolute values (MAD - Mean Absolute Distance) since these are more practical to use than the squares of the depth errors.

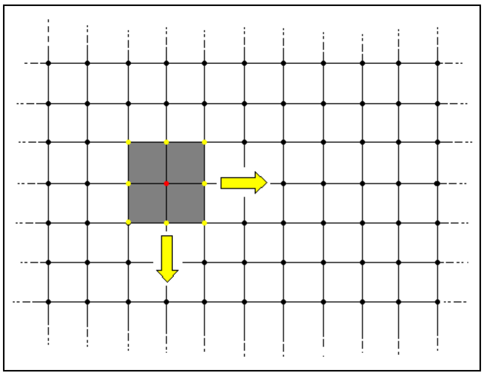

Figure. 8.1. Map grid with inlaid sonar-map.
The arrows indicate the movement of the sonar map.

In normal cases the terrain map will have 501 x 501 nodes and the sonar map 10 x 10 nodes. The distance between the nodes is 10 meters.

8.2 Principles of implementation

A principle for a correlator is

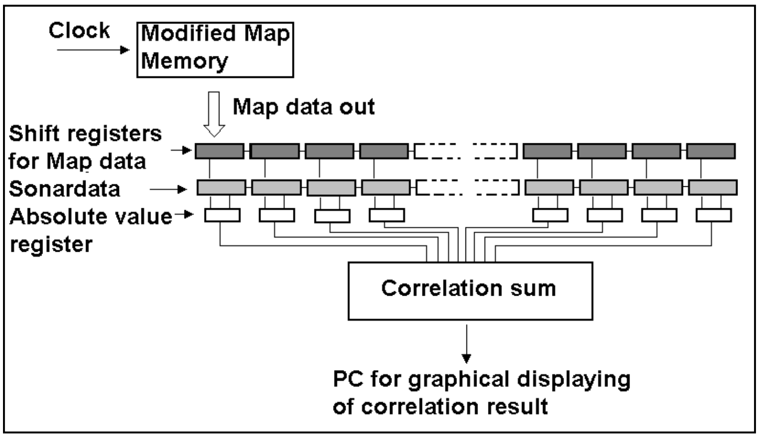

Figure 8.2. One principle for a correlator

The correlator consists of two lines of shift registers, one is for the terrain map data and the other is for the sonar-map data. There are 100 shift registers in the lines if the size of the sonar-map is 10 x 10 nodes. The length of the shift registers is 16 bits.

Mean value compensated gridded depth data from the sonar are shifted in from the left in the lower line and will be fixed there during the whole correlation process.

The modified map memory has mean value compensated depth data organized so that the correct data will always be shifted out at each clock pulse.

It turns out that the memory access time will always cause a bottleneck. A recursive solution can therefore be used without hampering the performance of the correlator. Such a solution will greatly reduce the number of gates needed in the implementation.

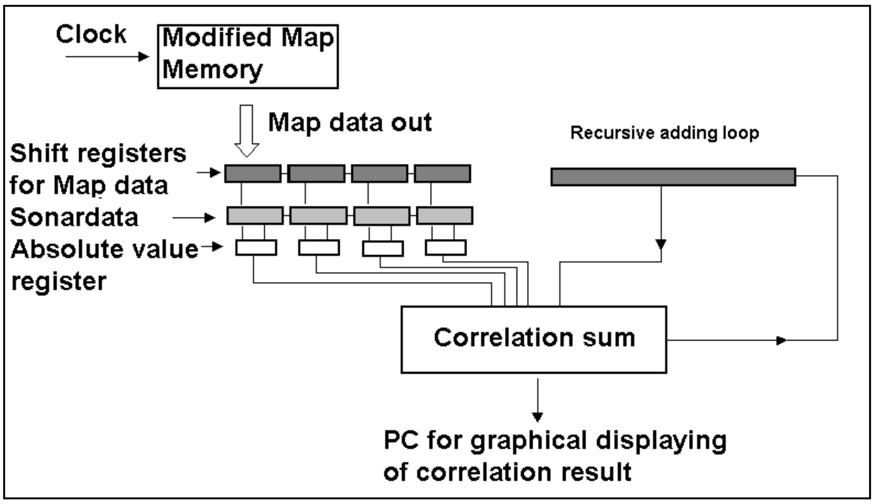

Fig. 8.3. Recursive addition instead of shift registers.

8.2 The practical implementation in an FPGA

A hardware correlator has been constructed according to the recursive addition principle presented in ref. [8]. A block schematic of the solution can be seen in Figure 8.4 and the completed correlator appears in Figure 8.5.

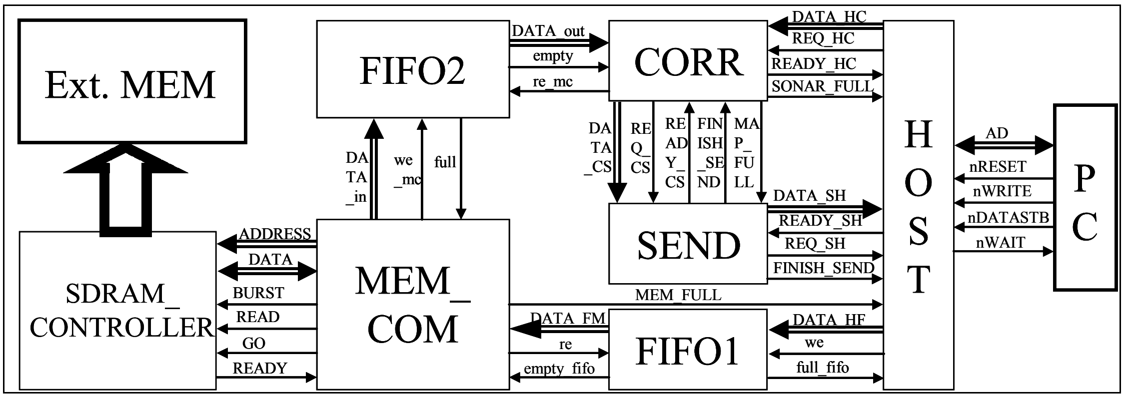

Figure 8.4. Block schematic of the implemented terrain correlator.

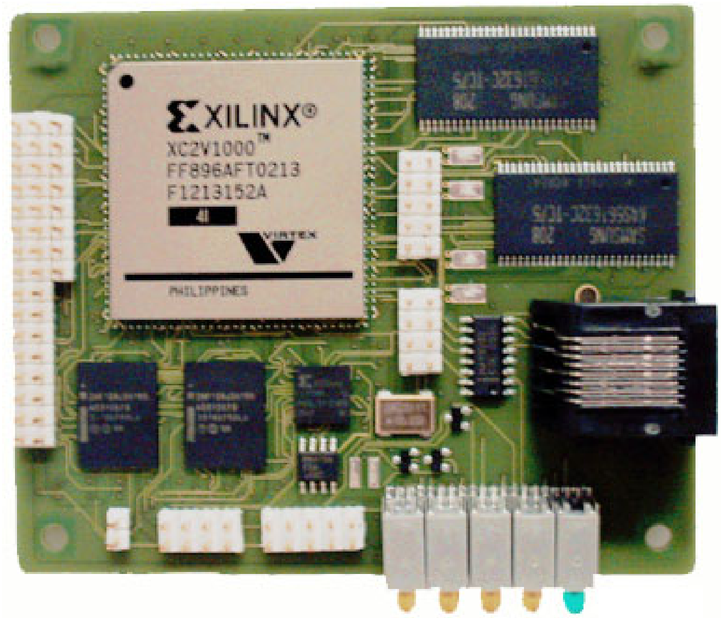

Figure 8.5 Photo of the completed correlator. Natural size.

The computing time for a map of 501 x 501 nodes and a sonar map of 10 x 10 nodes is 266 ms, but this does not include the time for transmitting the result. If the bandwidth permits, the result can be transmitted in parallel with the computation. The correlator is of a modular design and several computing machines can be hosted in the same chip and/or more than one chip can be used. The computing time will be reduced accordingly. For detailed information see ref [8].

9. References.

[1] Torsten Söderström: Discrete-Time Stochastic Systems, Estimation & Control. Prentice Hall International. 1994.

[2] Niclas Bergman: Recursive Bayesian Estimation. Navigation and Tracking Applications. Dissertation No. 579. Department of Electrical Engineering. Linköping University, Sweden.

[3] Ingemar Nygren: A Method for Terrain Navigation of an AUV, Conference Proceedings, MTS/IEEE Oceans 2001.

[4] Steven M. Kay: Fundamentals of Statistical Signal Processing: Estimation Theory. Prentice Hall International, Inc. 1993.

[5] Charles W. Therrien: Discrete Random Signals and Statistical Signal Processing. Prentice Hall Signal Processing Series. 1992.

[6] D.H. Titterton and J.L. Weston: Strapdown inertial navigation technology. IEEE Radar, Sonar and Avionic series 5. Peter Peregrinus Ltd. 1997.

[7] R.G. Brown, P.Y.C. Hwang: Introduction to Random Signals and Applied Kalman Filtering. John Wiley & Sons, Inc. 1997.

[8] Kim Andersson, Filip Traugott: Implementation of a terrain correlator for an underwater vehicle in a FPGA. Masters Thesis. Department of Electronics. Mälardalen University, 2002.