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Report title

On determining multiple positions and velocities from bistatic measurements -- fast bistatic association in AASR

Abstract (not more than 200 words)

This report considers a method for fast bistatic association and positioning of targets in a radar system like AASR – Associative Aperture Synthesis Radar. AASR consists of a network of distributed radar stations where the stations may be used in both monostatic and bistatic modes. The stations are not assumed to have any angular resolution, which means that the sensor information is range (both monostatic and bistatic) and the corresponding doppler information. The problem of correctly matching data from one sensor with data from another sensor, where data should be matched if they stem from the same target, is called the association problem.

In this report we describe a new method for fast association which uses bistatic measurements only. The main advantage, besides the fact that bistatic configurations are more numerous than monostatic ditto, is that bistatic measurements has a much better capacity to detect stealth targets.

Keywords

distributed radar network, bistatic measurements, association

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Bestämning av många lägen och hastigheter med bistatiska mätningar – snabb bistatisk associering för AASR

Sammanfattning (högst 200 ord)

Rapporten behandlar en metod för snabb bistatisk associering och positionering av mål i ett radarsystem à la AASR – Associativ apertursyntesradar. AASR består av ett ytdistribuerat nätverk av radarstationer där radarstationerna kan verka monostatiskt och bistatiskt. Stationerna antas inte ha någon vinkelupplösningsförmåga, vilket innebär att tillgänglig information om mål är avstånd (monostatiskt eller bistatiskt) samt motsvarande dopplerinformation. Problemet att korrekt para data från en sensor med data från en annan sensor, där data skall paras om de härrör från samma mål, kallas associeringsproblemet.

I denna rapport redovisas en ny metod för snabb associering som endast använder bistatiska mätningar. Den främsta fördelen, vid sidan av det faktum att antalet bistatiska konfigurationer är fler än de monostatiska dito, är att bistatiska mätningar har en överlägsen förmåga att upptäcka smyganpassade mål.

Nyckelord

distribuerat radarnätverk, bistatiska mätningar, associering

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Contents

1 Introduction

AASR - Associative Aperture Synthesis Radar, is a proposed system for air surveillance. It consists of a network of homogeneously distributed ground radar stations which can detect, position and track air targets. One proposed design is to have the stations distributed some 20-50 km apart and let the stations perform both monostatic and (for nearby stations) bistatic measurements. Tentative frequencies will lie in the lower UHF band, and the system will use a band width of, say, 40 MHz. By communicating and combining data from nearby stations, the system will produce a scene where air targets are located in position space and also given velocities. For a full account of the system as a whole, we refer to [1],[2].

Since any neighbouring radar facilities can form a pair for bistatic measurements, there are obviously more bistatic facilities available than monostatic. (This is provided, of course, that the separation between the radar stations is not too large.) The geometry of bistatic measurements is essentially the geometry of ellipsoids, while the geometry of monostatic measurements are connected to spheres. Since intersections of ellipsoids are described through equations of fourth order, the explicit algebra of bistatic measurements can be awkward, compared to the simpler equations related to spheres/monostatic measurements.

On the other hand, apart from being more numerous, the bistatic measurement have another advantage over the monostatic ditto. Namely, in the situation of stealth targets, it may be that the monostatic measurements are to weak to be detectable, while scattering from the target may be sufficiently strong for bistatic configurations. Therefore, efficient trilateration methods are of interest.

In this report, we study the problem of efficiently resolving (i.e., determine position and velocity) many targets over a scene, where we have bistatic measurements (only). It will turn out that the calculations can carried out easily and fast if we are given a certain redundancy. Below, we will describe the method when applied to one target, and then extend the situation to many targets. First, however, we will look at the formulation of the problem and, for future reference, look at the monostatic case.

2 The association problem

In this report, the relevant situation is the following. We imagine N_s radar stations (in $\mathbf{R}^3$). The stations are denoted by $s_j, j = 1, \dots N_s$ and their radii vectors with $\overline{r}_j, j = 1, \dots N_s$. Each radar station transmits pulses which are scattered against targets and then received by the transmitting station (monostatic measurements) but also by the neighbouring radar stations (bistatic measurements). Each measurement gives a distance and a Doppler estimate, i.e., if there is a target located at the time dependent position $\overline{\rho} = \overline{\rho}(t)$, with velocity $\overline{v} = \dot{\overline{\rho}} = \frac{d}{dt}\overline{\rho}$ and if the transmitter and receiver are located at $\overline{r}_{tx}$ and $\overline{r}_{rx}$ respectively, the measurements are

$$d = |\overline{\rho} - \overline{r}_{tx}|, \quad v = \frac{d}{dt}|\overline{\rho} - \overline{r}_{tx}| \quad (\text{monostatic case, } \overline{r}_{tx} = \overline{r}_{rx})$$

and

$$d = |\overline{\rho} - \overline{r}_{tx}| + |\overline{\rho} - \overline{r}_{rx}|, \quad v = \frac{d}{dt}(|\overline{\rho} - \overline{r}_{tx}| + |\overline{\rho} - \overline{r}_{rx}|) \quad (\text{bistatic case})$$

If we know the distance to a target from several (at least three) measurement facilities, then the position of the target can be determined up to possibly a trivial ambiguity. Given the Doppler information, the velocity can also be determined. In practice, however, we have several targets located at $\overline{\rho}_i$, $i = 1, 2, 3 \ldots$. This means that to a given measurement configuration, monostatic or bistatic, where we have several targets, i.e., several measurements, it is not a priori known which measurement from one measurement configuration that corresponds to a given measurement from *another* measurement configuration. The problem of correctly matching measurements from one configuration with the measurements from the other configurations is called the association problem. In this report one method for solving this problem is described; other methods are presented or investigated in [1],[2],[3],[4],[5]. The method presented here, which gives both position and velocity of the targets uses, only bistatic measurements.

2.1 Formulation of the problem

We thus have the stations $s_j, j = 1, \ldots N_s$ at $\overline{r}_j, j = 1, \ldots N_s$. We also have N_t moving targets t_i, $i = 1, \ldots N_t$ which are to be detected, and they are located at $\overline{\rho}_i = \overline{\rho}_i(t)$, $i = 1, \ldots N_t$. Each station has the capability that to each target (up to a certain maximum distance) measure distance and radial speed. Thus, station s_j, $1 \leq j \leq N_s$ will, at a certain time, register

$$d_j(k) = |\overline{\rho}_k - \overline{r}_j|, \quad k = 1, 2, \ldots N_{d_j} \leq N_t$$

$$v_j(k) = \frac{d}{dt}|\overline{\rho}_k - \overline{r}_j|, \quad k = 1, 2, \ldots N_{d_j} \leq N_t$$

For stations nearby enough, we also get bistatic information, i.e., one transmits from one station and receives at another. For the station pair (s_i, s_j), this means that one register, for $1 \leq i, j \leq N_s$,

$$\begin{array}{rcll} d_{ij}(k) & = & |\overline{\rho}_k - \overline{r}_i| + |\overline{\rho}_k - \overline{r}_j| = d_i(k) + d_j(k), & k = 1, 2, \ldots N_{d_{ij}} \leq N_t \\ v_{ij}(k) & = & \frac{d}{dt}|\overline{\rho}_k - \overline{r}_i| + \frac{d}{dt}|\overline{\rho}_k - \overline{r}_j| = v_i(k) + v_j(k), & k = 1, 2, \ldots N_{d_{ij}} \leq N_t \end{array} \quad (1)$$

Note that with these notations, $d_{ii}(k) = 2\, d_i(k), \quad v_{ii}(k) = 2\, v_i(k)$, $i = 1, 2, \dots, k = 1, 2, \dots$.

As mentioned above, it is a priori not possible to know which measurement from one sensor that is related to a measurement from another sensor, i.e., if the measurements stem from the same target. If measurements from different sensors are paired wrongly, we get false targets, or ghosts. The association problem is to match data correctly so that among all possible combinations or candidates, we discriminate between correct pairings (targets) and incorrect pairings (ghosts). The targets are to be located both with respect to position and velocity, i.e., the targets can be considered as points in the state space $S = \mathbf{R}^6$. The problem is thus the following.

Given indata of the form (1), determine targets t_i, $i = 1, 2, 3, \dots$ in S compatible with indata.

Note that monostatic measurements may be included in (1) if $i = j$.

3 Position and velocity of one target from monostatic measurements.

In this section, we will, for future reference, look at the following problem. Suppose that we are given three points $\overline{r}_1, \overline{r}_2, \overline{r}_3$ in $\mathbf{R}^3$, which we for convenience place in the xy-plane. Given three (compatible) distances, ρ_1, ρ_2, ρ_3 from these points, these distances determine, up to a trivial reflection symmetry in the xy-plane, a point $\overline{r} \in \mathbf{R}^3$. We will here show how this point can be determined. It turns out that we may as well consider more than three distances from given points, but we will at present assume that the distances are compatible, i.e., that there exits a point $\overline{r}$ giving the distances (see section 3.1 below). Note also that unless all points lie in the same plane, the reflection symmetry is lost.

3.1 Position of one target from monostatic measurements

Thus we study the following problem:

Problem 1.
Suppose that we are given N points $\overline{r}_1, \overline{r}_2, \dots, \overline{r}_N$ in $\mathbf{R}^3$, $N \geq 3$. Suppose also that to a given, but unknown point $\overline{r}$, we know the distances $\rho_i = |\overline{r} - \overline{r}_i|$, $i = 1, 2, \dots N$. The problem is then, given $\overline{r}_i$ and ρ_i, $i = 1, 2, \dots N$, to recover $\overline{r}$.

One solution to this problem is to do as follows. We start by choosing the origin point so that

$$\sum_{i=1}^{N} \overline{r}_i = \overline{0}. \tag{2}$$

If we write $\overline{r} = r\widehat{r}$, $\overline{r}_i = r_i\widehat{r}_i$, etc.,. we have that

$$r^2 - 2\overline{r} \cdot \overline{r}_i + r_i^2 = \rho_i^2, \quad i = 1, 2, \dots, N \tag{3}$$

By summing over i, and using (2), we find that

$$r^2 = \frac{1}{N} \sum_{i=1}^{N} (\rho_i^2 - r_i^2) \tag{4}$$

Thus r is known. Knowing r, we can rewrite (3) as

$$\widehat{r} \cdot \overline{r}_i = \frac{1}{2}\left(r + \frac{1}{r}(r_i^2 - \rho_i^2)\right) \quad i = 1, 2, \dots, N \tag{5}$$

By putting the vectors $\overline{r}_i$ as rows in a matrix A, and by putting the numbers $1/2(r + 1/r(r_i^2 - \rho_i^2))$ in a vector $\overline{b}$, equation (5) becomes

$$A\widehat{r} = \overline{b} \tag{6}$$

where A is $N \times 3$ and $\overline{b}$ is $N \times 1$. We note that the rank of A is 3, unless all $\overline{r}_i$ lie in the same plane, in which case the rank of A is 2. We now use SVD-decomposition of A, so that $A = U\Sigma V^t$, where U and V are orthogonal matrices of order $N \times N$ and 3×3, and where Σ contains the singular values on its 'broken diagonal'. We also note that the decomposition of A into U, Σ and V is independent of the measured distances ρ_i, so that it can be done once and for all, as soon as all $\overline{r}_i$ are known. With $\overline{\beta} = U^t\overline{b}$, $\widehat{\rho} = V^t\widehat{r}$, (6) becomes

$$\Sigma\widehat{\rho} = \overline{\beta} \tag{7}$$

The structure of (7) is

$$\begin{pmatrix} \Sigma_I \\ \Sigma_{II} \end{pmatrix} \widehat{\rho} = \begin{pmatrix} \beta_I \\ \beta_{II} \end{pmatrix} \tag{8}$$

where Σ_I and β_I consist of the first three rows/elements of Σ and β respectively. Thus Σ_{II} is the zero-matrix of size $(N-3) \times 3$, and the 'equation' $\Sigma_{II}\widehat{\rho} = 0 = \overline{\beta}_{II}$ is a measurement of the compatibility of data. As for the remaining equation, $\Sigma_I\widehat{\rho} = 0 = \overline{\beta}_I$, two situations may occur. First, if Σ_I is invertible, which is the case if not all $\overline{r}_i$ lie in the same plane, we simply have

$$\widehat{\rho} = \Sigma_I^{-1}\overline{\beta}_1 \tag{9}$$

Ideally, this should give $\widehat{\rho}$ the correct length directly, but due to inconsistencies and/or measurement errors, one may in practice have to normalize $\widehat{\rho}$;

$$\widehat{\rho} = \frac{\Sigma_I^{-1}\overline{\beta}_1}{|\Sigma_I^{-1}\overline{\beta}_1|} \tag{10}$$

The second case is when Σ_I has rank two, i.e., when the singular values are $\sigma_1, \sigma_2, \sigma_3$, with $\sigma_3 = 0$. This occurs when we have only three measurements, i.e. $N = 3$, or $N \geq 4$ but all radar stations lie in the same plane, Π. This is also the case when we expect a reflection symmetry in Π for the solution. With $\widehat{\rho} = (\rho_1, \rho_2, \rho_3)^t$ and $\overline{\beta} = (\beta_1, \beta_2, \beta_3)^t$ we simply have

$$\rho_1 = \frac{\beta_1}{\sigma_1}, \quad \rho_2 = \frac{\beta_2}{\sigma_2}, \quad \rho_3 = \pm\sqrt{1 - \rho_1^2 - \rho_2^2} \tag{11}$$

The reflection symmetry shows up in the sign ambiguity of ρ_3, and we also remark that a complex ρ_3 means that we have incompatible indata.

If, in addition to range measurements, we also have (monostatic) Doppler measurements, we are of course also interested in the full three-dimensional velocity $\overline{v}$ of the target.

3.2 Velocity of one target from monostatic Doppler measurements

In this subsection we study the problem of determining the velocity $\overline{v}$, when we know the radial (Doppler) velocities of the target with respect to some radar stations are known. Thus, using the same notation as in the previous section, we also know N Doppler velocities,

$$v_i = \frac{\overline{v} \cdot (\overline{r} - \overline{r}_i)}{|\overline{r} - \overline{r}_i|}, \quad i = 1, 2, \ldots N \tag{12}$$

The values of $v_i,\ i = 1, 2, ...N$ will depend on the location $\overline{r}$ of the target. Therefore $\overline{v}$ will be expressed in terms of $\overline{r}$ which is either known or regarded as a parameter. The problem can be formulated as follows.

Problem 2.
Given the vectors $\overline{r}$ and $\overline{r}_i$, $i = 1, 2, \ldots N$, $N \geq 3$ in $\mathbf{R}^3$ and the radial velocities v_i, $i = 1, 2, \ldots N$, given by (12), determine $\overline{v}$.

With a similar approach as in the previous section, we put $(\overline{r} - \overline{r}_i)^t / |\overline{r} - \overline{r}_i|$ as rows in the $N \times 3$ matrix A. With $\overline{u} = (v_1, v_2, \ldots v_N)^t$, the equation becomes

$$A\overline{v} = \overline{u} \tag{13}$$

This time we might expect A to have rank 3, so we can either solve (13) via SVD-factorisation as in the previous section or use the normal equations

$$A^t A\overline{v} = A^t\overline{u} \tag{14}$$

4 Position and velocity of one target from bistatic measurements.

Here we will look at a similar situation to the one in section 3. The difference is that we instead of monostatic measurements will consider bistatic ditto. As mentioned earlier, intersections of ellipsoids (which are connected to bistatic measurements) are harder to handle algebraically than intersections of spheres (which are connected to monostatic measurements). This difficulty will be handled through a certain redundancy.

4.1 Notation

Consider our N radar stations at $\overline{r}_1, \overline{r}_2, \ldots \overline{r}_N$. For a while, we will assume that $N = 4$, so that we have (target at ρ) monostatic measurements

$$d_i = |\overline{\rho} - \overline{r}_i|, \quad i = 1, 2, 3, 4$$

and bistatic measurements

$$d_{ij} = |\overline{\rho} - \overline{r}_i| + |\overline{\rho} - \overline{r}_j|, \quad 1 \leq i, j \leq 4$$

so that in particular $d_{ii} = 2d_i$.

For the Doppler speed, we write

$$v_i = \dot{d}_i = \frac{d}{dt}|\overline{\rho} - \overline{r}_i| = \frac{\overline{\rho} - \overline{r}_i}{|\overline{\rho} - \overline{r}_i|} \cdot \dot{\overline{\rho}} = \frac{\overline{\rho} - \overline{r}_i}{|\overline{\rho} - \overline{r}_i|} \cdot \overline{v}, \quad i = 1, 2, 3, 4$$

where a dot indicates differentiation with respect to time. Similarly, we write

$$v_{ij} = \frac{\overline{\rho} - \overline{r}_i}{|\overline{\rho} - \overline{r}_i|} \cdot \dot{\overline{\rho}} + \frac{\overline{\rho} - \overline{r}_j}{|\overline{\rho} - \overline{r}_j|} \cdot \dot{\overline{\rho}}, \quad 1 \leq i, j \leq 4$$

so that $v_{ii} = 2v_i$. Let us also use the notation B_{ij} for the set of all bistatic measurements between station s_i and station s_j.

4.2 Position from three bistatic measurements

In the case of three bistatic measurements (from four stations forming pairs), two different situations may occur. In the simplest case, we have the information from the three possible pairs formed by three stations, i.e., we know for instance d_{12}, d_{23} and d_{31}. The information has the structure

$$\begin{array}{rcccl} |\overline{\rho}-\overline{r}_1| & + & |\overline{\rho}-\overline{r}_2| & & & = & d_{12} \\ & & |\overline{\rho}-\overline{r}_2| & + & |\overline{\rho}-\overline{r}_3| & = & d_{23} \\ |\overline{\rho}-\overline{r}_1| & + & & & |\overline{\rho}-\overline{r}_3| & = & d_{31} \end{array} \tag{15}$$

so that we may easily find $|\overline{\rho}-\overline{r}_1|$, $|\overline{\rho}-\overline{r}_2|$ and $|\overline{\rho}-\overline{r}_3|$ explicitly and then use the methods of section 3.1.

The other, and more complicated situation is when we know for instance d_{12}, d_{23} and d_{14}. This time the information is

$$\begin{array}{rccccccl} |\overline{\rho}-\overline{r}_1| & + & |\overline{\rho}-\overline{r}_2| & & & & & = & d_{12} \\ & & |\overline{\rho}-\overline{r}_2| & + & |\overline{\rho}-\overline{r}_3| & & & = & d_{23} \\ |\overline{\rho}-\overline{r}_1| & + & & & & & |\overline{\rho}-\overline{r}_4| & = & d_{14} \end{array} \tag{16}$$

Unless all $\overline{r}_i$ lie in the same plane, there is a unique solution, which is, however, not so straightforward to find.

When we have multiple targets, there may be inconveniently many relations of the type (15), since we don't know if a suggested triple $(\rho_{12}, \rho_{23}, \rho_{31})$ corresponds to a real target or not. If, however, there are redundant measurements, it can be possible to test for consistency of data in a very fast and immediate way. It is therefore of interest to solve redundant equations of the type (16). This will be discussed in the next section.

4.3 Position from four bistatic measurements

In this section, we will look at a redundant bistatic configuration. We will assume that we from four radar stations have four bistatic measurements d_{12}, d_{34}, d_{13} and d_{24} so that we have measurements according to the following scheme:

$$\begin{array}{rcccccccl} |\overline{\rho}-\overline{r}_1| & + & |\overline{\rho}-\overline{r}_2| & & & & & = & d_{12} \\ & & & & |\overline{\rho}-\overline{r}_3| & + & |\overline{\rho}-\overline{r}_4| & = & d_{34} \\ |\overline{\rho}-\overline{r}_1| & + & & & |\overline{\rho}-\overline{r}_3| & & & = & d_{13} \\ & & |\overline{\rho}-\overline{r}_2| & + & & & |\overline{\rho}-\overline{r}_4| & = & d_{24} \end{array} \tag{17}$$

It is not possible to first solve (uniquely) for $|\overline{\rho} - \overline{r}_i|$, $i = 1, 2, 3, 4$, since in the equation

$$A \begin{pmatrix} |\overline{\rho} - \overline{r}_1| \\ |\overline{\rho} - \overline{r}_2| \\ |\overline{\rho} - \overline{r}_3| \\ |\overline{\rho} - \overline{r}_4| \end{pmatrix} = \begin{pmatrix} d_{12} \\ d_{34} \\ d_{13} \\ d_{24} \end{pmatrix}$$

the matrix A is singular. However, we still expect equation (17) to have a unique solution, since there are only three unknown variables $(x, y, z) \sim \overline{\rho}$. We now show that it is possible to solve equation (17) without invoking complicated fourth-order polynomial equations.

We first note that (17) can be solved parametrically:

$$\begin{pmatrix} |\overline{\rho} - \overline{r}_1| \\ |\overline{\rho} - \overline{r}_2| \\ |\overline{\rho} - \overline{r}_3| \\ |\overline{\rho} - \overline{r}_4| \end{pmatrix} = \begin{pmatrix} d_{12} - d_{24} \\ d_{24} \\ d_{34} \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \tag{18}$$

where we have used the fact that $d_{12} - d_{24} + d_{34} = d_{13}$. (This follows from (17).) We now choose $\overline{r}_4$ as origin point, so that $\overline{r}_4 = \overline{0}$ and $|\overline{\rho} - \overline{r}_4| = |\overline{\rho}| = \rho$. If we compare with (18), we see that $t = \rho$ and therefore that (18) can be written

$$\begin{aligned} |\overline{\rho} - \overline{r}_1| &= d_{12} - d_{24} + \rho \\ |\overline{\rho} - \overline{r}_2| &= d_{24} - \rho \\ |\overline{\rho} - \overline{r}_3| &= d_{34} - \rho \end{aligned}$$

By squaring the equations and noting that the terms $\overline{\rho} \cdot \overline{\rho} = \rho^2$ cancels, we get

$$\begin{aligned} -2\overline{\rho} \cdot \overline{r}_1 + r_1^2 &= (d_{12} - d_{24})^2 + 2\rho(d_{12} - d_{24}) \\ -2\overline{\rho} \cdot \overline{r}_2 + r_2^2 &= d_{24}^2 - 2\rho d_{24} \\ -2\overline{\rho} \cdot \overline{r}_3 + r_3^2 &= d_{34}^2 - 2\rho d_{34} \end{aligned}$$

or

$$\begin{aligned} \overline{\rho} \cdot \overline{r}_1 &= -\rho(d_{12} - d_{24}) + \tfrac{r_1^2 - (d_{12} - d_{24})^2}{2} \\ \overline{\rho} \cdot \overline{r}_2 &= +\rho d_{24} + \tfrac{r_2^2 - d_{24}^2}{2} \\ \overline{\rho} \cdot \overline{r}_3 &= +\rho d_{34} + \tfrac{r_3^2 - d_{34}^2}{2} \end{aligned} \tag{19}$$

In order to (partially) eliminate ρ in the left hand side of (19) we note that if we choose α and β so that $d_{12} - d_{24} - \alpha d_{24} = 0$ and $d_{12} - d_{24} - \beta d_{34} = 0$, then

$$\begin{aligned} \overline{\rho} \cdot (\overline{r}_1 + \alpha \overline{r}_2) &= \tfrac{r_1^2 - (d_{12} - d_{24})^2}{2} + \alpha \tfrac{r_2^2 - d_{24}^2}{2} \\ \overline{\rho} \cdot (\overline{r}_1 + \beta \overline{r}_3) &= \tfrac{r_1^2 - (d_{12} - d_{24})^2}{2} + \beta \tfrac{r_3^2 - d_{34}^2}{2} \end{aligned} \tag{20}$$

Therefore, with the ansatz

$$\overline{\rho} = \gamma(\overline{r}_1 + \alpha\overline{r}_2) + \delta(\overline{r}_1 + \beta\overline{r}_3) + \varepsilon(\overline{r}_1 + \alpha\overline{r}_2) \times (\overline{r}_1 + \beta\overline{r}_3)$$

(20) gives us equations for γ and δ. Knowing these, we get ε from (19).

An alternative way to solve (19) is to create an orthogonal set of vectors from $\overline{r}_1, \overline{r}_2$ and $\overline{r}_3$. This is always possible in theory unless $\overline{r}_1, \overline{r}_2, \overline{r}_3$ and $\overline{r}_4$ all lie in the same plane (since we have chosen $\overline{r}_4 = \overline{0}$). If this is not the case, we can replace $(\overline{r}_1, \overline{r}_2, \overline{r}_3)$ by an orthonormal set $(\widehat{x}, \widehat{y}, \widehat{z})$ so that the equation (19) takes the form

$$\begin{aligned} \overline{\rho} \cdot \widehat{x} &= a\rho + \alpha \\ \overline{\rho} \cdot \widehat{y} &= b\rho + \beta \\ \overline{\rho} \cdot \widehat{z} &= c\rho + \gamma \end{aligned} \tag{21}$$

By squaring and adding these equations, we get an quadratic equation for ρ. Knowing ρ, we insert ρ in the right hand side of (21) and can thus determine $\overline{\rho} = x\widehat{x} + y\widehat{y} + z\widehat{z}$ completely. If $\overline{r}_1, \overline{r}_2, \overline{r}_3$ and $\overline{r}_4$ all happen to lie in the same plan, equation (21) becomes

$$\begin{aligned} \overline{\rho} \cdot \widehat{x} &= a\rho + \alpha \\ \overline{\rho} \cdot \widehat{y} &= b\rho + \beta \\ 0 &= c\rho + \gamma \end{aligned} \tag{22}$$

so that we know ρ immediately. This will then give us x and y and finally z up to sign.

4.4 Position and velocity from bistatic measurements

In this subsection we will see how also radial Doppler estimates of the target determines its three-dimensional velocity. We start by putting

$$\widehat{u}_i = \frac{\overline{\rho} - \overline{r}_i}{|\overline{\rho} - \overline{r}_i|}, \quad \overline{v} = \dot{\overline{\rho}}$$

If the measurement is of the type (15), the equation for $\overline{v}$ is

$$\begin{aligned} \widehat{u}_1 \cdot \overline{v} + \widehat{u}_2 \cdot \overline{v} &= v_{12} \\ \widehat{u}_2 \cdot \overline{v} + \widehat{u}_3 \cdot \overline{v} &= v_{23} \\ \widehat{u}_1 \cdot \overline{v} + \widehat{u}_3 \cdot \overline{v} &= v_{31} \end{aligned} \tag{23}$$

where everything is known except $\overline{v}$. Again the system is explicitly solvable and the problem is reduced to the problem described in section 3.2.

With a measurement of type (17), we must solve

$$\begin{aligned} \widehat{u}_1 \cdot \overline{v} + \widehat{u}_2 \cdot \overline{v} &= v_{12} \\ \widehat{u}_3 \cdot \overline{v} + \widehat{u}_4 \cdot \overline{v} &= v_{34} \\ \widehat{u}_1 \cdot \overline{v} + \widehat{u}_3 \cdot \overline{v} &= v_{13} \\ \widehat{u}_2 \cdot \overline{v} + \widehat{u}_4 \cdot \overline{v} &= v_{24} \end{aligned} \tag{24}$$

The immediate parameter solution is

$$\begin{pmatrix} \widehat{u}_1 \cdot \overline{v} \\ \widehat{u}_2 \cdot \overline{v} \\ \widehat{u}_3 \cdot \overline{v} \\ \widehat{u}_4 \cdot \overline{v} \end{pmatrix} = \begin{pmatrix} v_{12} - v_{24} \\ v_{24} \\ v_{34} \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \tag{25}$$

We see that $t = \widehat{u}_4 \cdot \overline{v}$ and thus

$$\begin{pmatrix} (\widehat{u}_1 - \widehat{u}_4) \cdot \overline{v} \\ (\widehat{u}_2 + \widehat{u}_4) \cdot \overline{v} \\ (\widehat{u}_3 + \widehat{u}_4) \cdot \overline{v} \end{pmatrix} = \begin{pmatrix} v_{12} - v_{24} \\ v_{24} \\ v_{34} \end{pmatrix} \tag{26}$$

This is (at least generically) an invertible system of equations, which gives $\overline{v}$. We note that in order for (24) to be solvable, the radial Doppler measurements must fulfill the consistency relations (cf. remark under equation (18))

$$v_{12} + v_{34} = v_{13} + v_{24} \tag{27}$$

This will be used in section 5.

5 Multiple positions and velocities from four bistatic measurements

As mentioned in the introduction, and also described elsewhere, there are reasons for considering redundant measurements of several targets. These measurements can be monostatic or bistatic, and in general we have a mixture of both types. The overall problem is to have an region with several targets and recover these (i.e., their position and velocity) from data.

Depending of the type of data and the degree of redundancy, several processing methods are possible. Here we will look at the following situation:

Problem 3

Suppose that we have a scene containing N targets, and four radar stations at $\overline{r}_1, \overline{r}_2, \overline{r}_3, \overline{r}_4$ monitoring the scene. Determine the positions and velocities of the targets given the bistatic measurements B_{12}, B_{34}, B_{13} and B_{24}.

Note that we could have six bistatic measurements (and four monostatic) , but that we use only four, matching the situation in section 4.3. The remaining sets B_{14} and B_{23} can be used to give extra redundancy or allow for certain data losses. All targets need not be seen by by all measurements, but for simplicity, we assume, for the moment, that all targets are seen by all measurements. Thus B_{ij} registers

$$d_{ij}(k) \text{ and } v_{ij}(k), 1 \le k \le N, \quad ij = 12, 34, 13, 24$$

In principle, we could use three measurements, B_{12}, B_{34} and B_{13} to form (up to) N^3 candidates with respect to position and velocity and then check against B_{24}. As mentioned earlier, these calculations, apart from being N^3 in number, are somewhat involved. By incorporating B_{24} directly, we can instead proceed as follows:

Step 1 Each target seen by B_{12} can be associated with a target seen by B_{34}, so that these two measurements together give N^2 candidates. Form these N^2 candidates, and sort them after their added distances, i.e., after the possible values of

$$d_{12}(k) + d_{34}(l), \quad 1 \le k, l \le N$$

We denote these distances by

$$d_{12+34}(m), \quad 1 \le m \le N^2$$

and note that the sorting can be done on $O(N^2 \log N)$ operations.

Step 2 In an identical way we form and sort the sum of the distances d_{13} and d_{24}. These gives us

$$d_{13+24}(m), \quad 1 \le m \le N^2$$

where each $d_{13+24}(m) = d_{13}(k) + d_{24}(l)$ for some k and l.

Step 3 If there is a target at $\overline{\rho}$, then there must exist a m and a m' so that

$$d_{12+34}(m) = |\overline{\rho} - \overline{r}_1| + |\overline{\rho} - \overline{r}_2| + |\overline{\rho} - \overline{r}_3| + |\overline{\rho} - \overline{r}_4| = d_{13+24}(m') \qquad (28)$$

Therefore, we start by comparing the ordered sets

$$\{d_{12+34}(m),\ 1 \le m \le N^2\} \text{ and } \{d_{13+24}(m),\ 1 \le m \le N^2\}$$

and get candidates for those m and m' where (28) holds within a given tolerance. In most situations, the number of targets is considerably less then the number of range bins, but to get pessimistic estimates, we assume that these numbers are comparable. Suppose, therefore, that we have $\sim N$ range bins so that each range bin contains $\sim N$ candidates. In this case, several $d_{12+34}(m)$ and $d_{13+24}(m')$ may be equal within the precision of the range bins so that no effective reduction of false candidates, i.e., ghosts occur. We therefore note that in addition to the equality (28), for a target, we must also have

$$v_{12}(k) + v_{24}(l) = v_{13}(k') + v_{24}(l') \qquad (29)$$

for those k, l, k', l' where

$$d_{12}(k) + d_{24}(l) = d_{12+34}(m) = d_{13+24}(m') = d_{13}(k') + d_{24}(l') \qquad (30)$$

Thus, in step 3, we compare all d_{12+34} and d_{13+24} in a given range bin and among these keep only the pairs for which (29) and (30) hold within the choosen tolerence. (If there are several d_{12+34} and d_{13+24} in the same range bin, we need not check all pairs, since we can, within each range bin, sort both d_{12+34} and d_{13+24} with respect to the velocities $v_{12}(k)+v_{24}(l)$ and $v_{13}(k')+v_{24}(l')$ respectively.)

Step 4 By the preceding steps, we will remove most, but not all ghosts. The remaining ghosts are located at points (and given velocities) which are compatible with B_{12}, B_{34}, B_{13} and B_{24}. Therefore, no further reduction can be made without incorporating further measurements. Thus step 4 consists of further comparison with additional measurements. What extra information is available depends, of course, of the situation.

Even if this method leaves some ghosts (due to insufficient data) there is a point, namely that the described procedure reduces the number of ghosts so that further investigations can be performed with a low effort.

5.1 A comments on ghosts and processing complexity

We saw in the description of step 1 and step 2 that these steps could be carried out in $O(N^2 \log N)$ operations, where N is the number of targets. If we, for each given distance ρ_{12+34}, i.e., each range bin corresponding to such a distance also sort data after velocity as mentioned in step 3, it is clear that also step 3 is performed in $O(N^2 \log N)$ operations. Since step 4 can be performed simultaneously with step 3, the overall number of operations to run through steps 1 to 4 above is still $O(N^2 \log N)$.

To get an estimate of the number of ghosts remaining, we will make the following (pessimistic) estimate. Using the rough figures N targets, N range bins and N velocity bins, step 1 distributes N^2 candidates in N^2 velocity-range bins, and the same holds for step 2. In step 3, candidates from step 1 and step 2 will be more or less paired with each other. Therefore, the number of candidates will be of the order N^2 (or less), but we note that in step 3 there will also be a effective reduction of ghosts if the number of targets is significantly less then N (N still being the number of range bins).

To summarize, we have presented a method which, in $O(N^2 \log N)$ operations, reduces N^3 potential candidates to (less that) N^2 candidates, each of which can be assigned a three-dimensional position and velocity by the methods described in section 4.3. In combination with additional measurements, this gives a $O(N^2 \log N)$ method for complete determination of targets with respect to both position and velocity.

6 References

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