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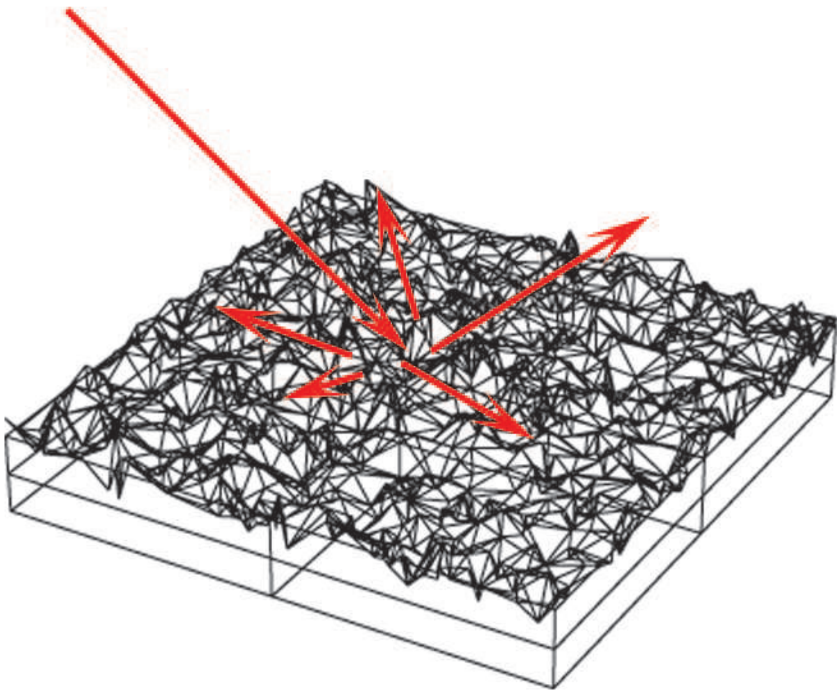

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Report title

A Review of Models for Scattering from Rough Surfaces

Abstract

The objective of this report is to present results from a literature search of models for scattering of light from random rough surfaces in the IR, visible and UV wavelengths. The literature search was performed with signature management applications in mind. We have focused on methods for calculating the *bidirectional reflectance distribution function* (BRDF), which is an important input parameter in IR signature simulation software. The influence of surface roughness on optical signatures can therefore be studied using the scattering models. We have studied four analytical methods in more detail. These methods have been found to be amongst the most common in the literature: *The Kirchhoff Approximation Method*, *The Method of Small Perturbations*, *The Integral Equation Method* (IEM) and *The Small Slope Approximation* (SSA). Of these models, the IEM and the SSA have the largest ranges of validity. However, they are more complex and therefore more difficult to implement. The literature search also shows that all models are less accurate for grazing incidence. Another limitation is that the results have been derived for rough surfaces with Gaussian height distribution. Rather few comparisons between models and measurements have been found in the literature. Such comparisons are vital for the use of rough surface scattering models in signature modelling, which is our main interest. One important conclusion of the present study is therefore that the models should be implemented, mutually compared and validated against measured data of low signature materials.

Keywords

Signature materials, mathematical modelling, rough surfaces

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Rapporttitel

En översikt av modeller för spridning mot skrovliga ytor

Sammanfattning

I denna rapport redovisas resultat från en litteraturstudie med syfte att ge en översikt över modeller för spridning av av ljus i IR, visuellt och UV mot stokastiska skrovliga ytor. Litteraturstudien har utförts för applikationer inom optiska signaturer och signaturanpassningsteknik. Litteraturstudien fokuserar på modeller för beräkning av *"bidirectional reflectance distribution function"* (BRDF), vilken är en viktig indata-parameter i t.ex. koder för simulering av IR-signaturer. Inverkan av ytstrukturens skrovlighet på optiska signaturer kan därför studeras med dessa spridningsmodeller. Vi har i huvudsak studerat fyra analytiska (approximativa) metoder för beräkning av spridning mot skrovliga ytor. Dessa metoder är bland de vanligast förekommande i litteraturen. De fyra metoderna är: *Kirchhoffs metod (approximation)*, *Perturbationsmetoder*, *Integralekvationsmetoder* (IEM) och *"The Small Slope Approximation"* (SSA). Av dessa metoder har IEM och SSA de största giltighetsområdena men de är också mer komplexa och därmed svårare att implementera. Litteraturstudien visar också att alla dessa modeller har svårighet att korrekt modellera spridning vid strykande infall. Ytterligare en begränsning är att metodernas slututtryck härletts endast för ytor med gaussisk höjdfördelning. Relativt få jämförelser mellan modeller och mätningar har hittats i litteraturen. Sådana jämförelser är av stor vikt vid användning av dessa modeller i signaturmodellering. En viktig slutsats från föreliggande studie är alltså att modellerna bör implementeras, jämföras inbördes och valideras mot mätdata av signaturmaterial.

Nyckelord

Signaturmaterial, modellering, skrovliga ytor

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Contents

1 Introduction

Models for scattering and diffraction of electromagnetic waves from random rough surfaces have developed during the last two centuries and the scientific interest in the problem remains strong today. Advances in the theory of physical optics, improved numerical methods and the availability of high performance computers have resulted in major advances in recent years. Areas of application include remote sensing, oceanography, communications, computer graphics, optics and material science. Reliable and flexible models for scattering of light in the ultraviolet (UV), visible and infrared (IR) wavelengths are useful in signature management and in development, optimization and assessment of camouflage.

An important parameter in simulation of the IR signature from objects is the angle-resolved reflectance, usually expressed in terms of the bidirectional reflectance distribution function (BRDF), for the surface materials in the object. Some infrared scene (and target) simulation software used in simulation of infrared signatures use a parametrized BRDF as input. One of the main objectives of modelling scattering of light from rough surfaces would be to calculate (predict) the BRDF from the surface hight statistics (or the topography) and the material parameters of the rough surface. These predicted BRDFs can then be used as input to IR scene/target simulation software to evaluate the effect of surface topography on the IR signature of an object. The rough surface scattering models can therefore be used to study, and therefore also in some cases improve or optimize, the influence of surface roughness on optical signatures. Although the applications we have in mind for the present survey of models are in the UV, visible and IR wavelength bands, many of the models are also applicable in for instance microwave theory (radar) and acoustics.

Models for scattering of electromagnetic waves from random rough surfaces can roughly be categorized into analytical models, numerical simulation methods and combinations of numerical and analytical methods. However, this division is by no means unambiguous since many of the analytical models require numerical calculations and since some of the numerical methods rely on analytical approximations. In this survey we will consider all three categories but our main emphasis will be on the analytical methods. We will present four analytical methods in more detail: *The Kirchhoff Approximation Method*, *The Small Perturbation Method* (SPM), *The Integral Equation Method* (IEM) and *The Small Slope Approximation* (SSA). The Kirchhoff method and the SPM represent early approaches to scattering which are still much used and the latter two represent more modern approaches which have larger domains of validity. These methods have been found to be amongst the most common in the literature and many of the other methods found in the literature are based on or have much in common with these approaches. In Section 2 we begin by giving a brief presentation of the scattering problem and introduce some concepts and results from the theory of electromagnetic fields which are often used in the scattering models. In Section 3 we give a brief presentation of the usage of the Kirchhoff's approximation in surface scattering models. In Section 4 we will present the integral equation method and in Section 5 we give a brief presentation of the small perturbation method. In Section 6 we give a brief survey of the Small Slope Approximation. In Section 7 we will list some other methods, including numerical methods, for scattering from rough surfaces, without presenting any details. In Section 7 we will also say a few words about volume scattering and scattering in multiple layer systems. Finally, in Section 8 we will attempt to draw some conclusions on how useful the different models are for applications in modelling of IR (and visible) signatures and also give some suggestions for further evaluation of the models.

2 Some concepts from the theories of electromagnetism and radiometry

In this section we will give a brief presentation of some concepts and results from the theories of electromagnetism, optics and statistical characterization (description) of surfaces which are often used in models for scattering of electromagnetic radiation from random rough surfaces. We will also try to define the problem of scattering of light from rough surfaces and give a definition of the bidirectional reflectance distribution function (BRDF) and its relation to other quantities used to describe angle resolved scattering.

2.1 Maxwell's equations and the scalar approximation

We begin by presenting Maxwell's equations and some other concepts and results from the theory of electromagnetism.

Maxwell's equations: The basic laws of classical (i.e. not quantum field theory) electromagnetism are given by Maxwell's equations [1] which for linear isotropic media can be written:

$$\nabla \times \mathbf{H} - \epsilon \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J}, \tag{1}$$

$$\nabla \times \mathbf{E} + \mu \frac{\partial \mathbf{H}}{\partial t} = 0, \tag{2}$$

$$\nabla \cdot \epsilon \mathbf{E} = \rho, \tag{3}$$

$$\nabla \cdot \mu \mathbf{H} = 0, \tag{4}$$

where $\mathbf{E}$ is the electric vector, $\mathbf{H}$ is the magnetic vector, ϵ is dielectric permittivity, μ is the magnetic permeability, $\mathbf{J}$ is the electric current density and ρ is the charge density. Maxwell's equations together with boundary conditions give a boundary value problem with a unique solution. In rough surface scattering, the surface enters into the boundary conditions (see Eqs. (10)–(13)). Furthermore, boundary conditions have to be supplied at infinity by specifying the incident field and the scattered field at infinity. The incoming field can be a plane wave or a directed beam and the scattered fields decay with increasing distance from the surface.

Helmholtz equations: From Maxwell's equations in Eqs. (1) and (2) and the constitutive relations it is easy to show that the fields $\mathbf{E}$ and $\mathbf{H}$ satisfy wave equations in the form of vector inhomogeneous Helmholtz equations. If we assume a monochromatic field such that

$\mathbf{E}(\mathbf{r},t) = \exp(-i\omega t)\mathbf{E}(\mathbf{r})$ the wave equation for the electric field vector can be written:

$$\nabla \times \nabla \times \mathbf{E} - k^2\mathbf{E} = i\omega\mu\mathbf{J} \tag{5}$$

where the squared wave number $k^2 = \omega^2\mu\epsilon$.

Plane, time-harmonic waves: Most analytical methods for scattering from random rough surfaces assumes that the electromagnetic wave incident on the rough surface is a plane wave. We consider a monochromatic, linearly polarized, plane wave with electric field:

$$\mathbf{E}^i(\mathbf{r}) = \hat{\mathbf{p}}E_0 \exp[i(\mathbf{k}^i \cdot \mathbf{r})] \tag{6}$$

where $\mathbf{k}^i = \hat{\mathbf{k}}^i k$, $k = \omega\sqrt{\epsilon\mu}$, $\hat{\mathbf{p}}$ is the unit polarization vector and E_0 is the amplitude. Furthermore, the time dependence, which is of the form $\exp(-i\omega t)$, is omitted in Eq. (6) as it will be in the following equations.

Magnetic field: The magnetic field associated with the electric field is given by:

$$\mathbf{H}^i(\mathbf{r}) = \hat{\mathbf{k}}^i \times (\mathbf{E}^i)/\eta \tag{7}$$

where $\eta = \sqrt{\mu/\epsilon}$ is the wave impedance in the medium.

Poynting vector and irradiance: The Poynting vector associated with an electromagnetic field, denoting power flow per unit area is given by:

$$\mathbf{S} = \frac{1}{2}Re(\mathbf{E} \times \mathbf{H}^*) = \frac{\mid \mathbf{E} \mid^2}{2\eta}\hat{\mathbf{k}} \tag{8}$$

Irradiance, I is defined as radiant power incident per unit area upon a surface (units Watts per square meter). The irradiance on a surface orthogonal to $\hat{\mathbf{k}}^i$ is therefore given by $I = \mid \mathbf{S} \mid = \mid \mathbf{E} \mid^2 /(2\eta)$.

Scalar approximation: Although the boundary value problem for Maxwell's equations often can be formulated quite easily, solving it (by for instance numerical methods) is in many cases extremely difficult or computationally expensive. Therefore, approximations and simplifications are often introduced. A simplification that is sometimes used in scattering problems is the use of a scalar wave-field $U(\mathbf{r}, t)$, the spatial part of which (monochromatic field is assumed) satisfies Helmholtz inhomogeneous equation [2]:

$$\nabla^2 U + k^2 U = -\rho, \tag{9}$$

The scalar wave function U which approximates the electric field defines the irradiance, $I \propto |U(\mathbf{r})|^2$. The range of validity of the scalar approximation is not obvious but, as is stated in [2], for problems with no depolarization and no multiple interactions it is possible to characterize the electric field by a scalar field. More information about the validity of the scalar approximation can be found in [2] and references therein.

2.2 Integral theorems and other results used in scattering models

We will now present some (exact) results for electromagnetic fields which are often used as a starting point in analytical models for scattering from rough surfaces. These equations are approximated and simplified using different methods and assumptions in the analytical methods for scattering from rough surfaces. We will not show how the equations in this section are derived but derivations can be found in the references.

Consider an electromagnetic field incident on a rough surface as shown in Figure 1.

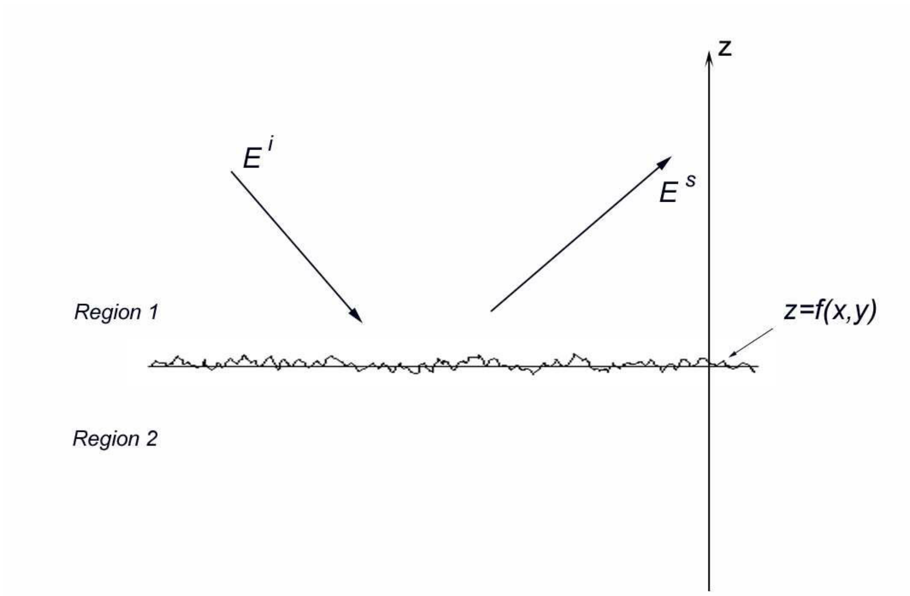

Figure 1: *Scattering of electromagnetic field on surface separating two media*

Boundary conditions: Across any surface interface, the electromagnetic field should satisfy continuity conditions given by [1]:

$$\hat{\mathbf{n}} \times (\mathbf{E} - \mathbf{E}_2) = 0 \tag{10}$$
$$\hat{\mathbf{n}} \times (\mathbf{H} - \mathbf{H}_2) = \mathbf{J}_s \tag{11}$$
$$\hat{\mathbf{n}} \cdot (\epsilon \mathbf{E} - \epsilon_2 \mathbf{E}_2) = \rho_s \tag{12}$$
$$\hat{\mathbf{n}} \cdot (\mu \mathbf{H} - \mu_2 \mathbf{H}_2) = 0 \tag{13}$$

where $\hat{\mathbf{n}}$ is the unit normal to the rough surface (pointing into Region 1). The electric surface current density, $\mathbf{J}_s$, and the charge density, ρ_s at the rough surface interface are zero unless the scattering surface (or one of the media) is a perfect conductor (see e.g. [2]).

Huygen's principle and the extinction theorem: Using the fact that the fields satisfy Helmholtz wave equation, Eq. (5), it can be shown that the fields in region 1, $\mathbf{E}$ and $\mathbf{H}$, satisfy Huygen's principle and the extinction theorem [3]:

$$\begin{aligned} &\mathbf{E}^i(\mathbf{r}) + \int ds' \{ i\omega\mu_1 \mathcal{G}(\mathbf{r}, \mathbf{r}') \cdot [\hat{\mathbf{n}} \times \mathbf{H}(\mathbf{r}')] \\ &+ \nabla \times \mathcal{G}(\mathbf{r}, \mathbf{r}') \cdot [\hat{\mathbf{n}} \times \mathbf{E}(\mathbf{r}')] \} = \begin{cases} \mathbf{E}(\mathbf{r}), & z > f(x,y) \\ 0, & z < f(x,y) \end{cases} \end{aligned} \tag{14}$$

where $\mathcal{G}$ is the dyadic Green function (to the vector Helmholtz equation) which is represented by

$$\mathcal{G}(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi} \left(\mathbf{I} + \frac{\nabla\nabla}{k^2} \right) G(\mathbf{r}, \mathbf{r}') \tag{15}$$

Here $\mathbf{I}$ is the unit dyadic and G is the Green function to Helmholtz equation, represented by the outgoing spherical wave,

$$G(\mathbf{r}, \mathbf{r}') = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tag{16}$$

In the same way the fields in Region 2, $\mathbf{E}_2$ and $\mathbf{H}_2$, satisfy

$$\begin{aligned} &- \int ds' \{ i\omega\mu_2 \mathcal{G}_2(\mathbf{r}, \mathbf{r}') \cdot [\hat{\mathbf{n}} \times \mathbf{H_2}(\mathbf{r}')] \\ &+ \nabla \times \mathcal{G}_2(\mathbf{r}, \mathbf{r}') \cdot [\hat{\mathbf{n}} \times \mathbf{E}_2(\mathbf{r}')] \} = \begin{cases} 0, & z > f(x,y) \\ \mathbf{E}_2(\mathbf{r}), & z < f(x,y) \end{cases} \end{aligned} \tag{17}$$

Scattered field: The scattered field, E^s can be obtained from Eq. (14) [3]:

$$\begin{aligned}\mathbf{E}^s(\mathbf{r}) &= \int ds'\{i\omega\mu_1\mathcal{G}(\mathbf{r},\mathbf{r}')\cdot[\hat{\mathbf{n}}\times\mathbf{H}(\mathbf{r}')] \\ &+ \nabla\times\mathcal{G}(\mathbf{r},\mathbf{r}')\cdot[\hat{\mathbf{n}}\times\mathbf{E}(\mathbf{r}')]\}\end{aligned} \tag{18}$$

If the observation point is in the far field region, the Green function in Eq. (18) can be simplified and the scattered field can be written [4]:

$$\begin{aligned}\mathbf{E}^s(\mathbf{r}) &= \frac{ik}{4\pi r}\exp(ikr)(\mathbf{I}-\hat{\mathbf{k}}_s\hat{\mathbf{k}}_s) \\ & \int[\hat{\mathbf{k}}_s\times(\hat{\mathbf{n}}\times\mathbf{E})+\eta(\hat{\mathbf{n}}\times\mathbf{H})]\exp[-i(\mathbf{k}_s\cdot\mathbf{r})]ds\end{aligned} \tag{19}$$

where $\mathbf{k}_s = \hat{\mathbf{k}}_s k$.

Green's function on spectral form It is sometimes convenient to use a spectral (plane wave) representation of the Green function in Eq. (16) and its derivative [3, 5]:

$$\begin{aligned}G &= \frac{1}{2\pi}\int dudv\frac{i}{\sqrt{k^2-u^2-v^2}} \\ & \exp[iu(x-x')+iv(y-y')+i\sqrt{k^2-u^2-v^2}\mid z-z'\mid]\end{aligned} \tag{20}$$

$$\begin{aligned}\nabla' G &= \frac{1}{2\pi}\int dudv\frac{\hat{\mathbf{x}}u+\hat{\mathbf{y}}v\pm\hat{\mathbf{z}}\sqrt{k^2-u^2-v^2}}{\sqrt{k^2-u^2-v^2}} \\ & \exp[iu(x-x')+iv(y-y')+i\sqrt{k^2-u^2-v^2}\mid z-z'\mid]\end{aligned} \tag{21}$$

Using these equations, the dyadic greens function can also be given in a spectral representation. The dyadic spectral Green function can be found in [3].

Tangential surface fields: The tangential surface fields $\hat{\mathbf{n}}\times\mathbf{E}$ and $\hat{\mathbf{n}}\times\mathbf{H}$ can be determined by using Eqs. (14) and (17) but we will also use the following expressions given by Poggio and Miller [5, 6]:

$$\hat{\mathbf{n}}\times\mathbf{E} = 2\hat{\mathbf{n}}\times\mathbf{E}^i - \frac{2}{4\pi}\hat{\mathbf{n}}\times\int[\mathcal{E}]ds' \tag{22}$$

and

$$\hat{\mathbf{n}}\times\mathbf{H} = 2\hat{\mathbf{n}}\times\mathbf{H}^i + \frac{2}{4\pi}\hat{\mathbf{n}}\times\int[\mathcal{H}]ds' \tag{23}$$

where

$$\mathcal{E} = jk\eta(\hat{\mathbf{n}}'\times\mathbf{H}')G - (\hat{\mathbf{n}}'\times\mathbf{E}')\times\nabla' G - (\hat{\mathbf{n}}'\cdot\mathbf{E}')\nabla' G \tag{24}$$

$$\mathcal{H} = \frac{jk}{\eta}(\hat{\mathbf{n}}'\times\mathbf{E}')G + (\hat{\mathbf{n}}'\times\mathbf{H}')\times\nabla' G + (\hat{\mathbf{n}}'\cdot\mathbf{H}')\nabla' G \tag{25}$$

2.3 Rough surfaces and their characterization

Roughness is a measure of the topographic height variations on the surface [7]. The roughness can arise from for instance polishing marks, machining marks, marks left by rollers, dust or other particles. Rough surfaces can be observed with microscopes such as electron microscopes, scanning probe microscopes and confocal scanning optical microscopes (see [7] for more information).

There are basically two categories of methods which are being used to measure surface roughness. The roughness can either be determined directly from surface profile measurements, or it can be calculated using some theory which relates scattering measurements to surface roughness. Here we will not enter more deeply into the different measurement methods for determining the surface roughness but a good general overview is found in [7].

In some cases, such as comparisons with experimental measurements for fixed, real surfaces, the scattered fields from deterministic rough surfaces are useful. However, in comparison with measurements the angular resolution of the detector and the accuracy of surface topography used in the simulation should be considered (see [7, 8]).

More commonly than considering a deterministic surface description, one would like to obtain the stochastic scattering properties of an ensemble of surfaces with a given statistical distribution. A random rough surface can be characterized either by joint probability density functions (PDF) or by the statistical moments (which are associated with the PDFs), or by a combination of the two. For instance, a rough surface described by $z = f(x, y)$ with first-order PDF, $p(z)$, and second-order PDF, $p_{z_1,z_2}(z_1, z_2)$, where $z_i = z(x_i, y_i)$, $i = (1, 2)$, has a mean (first order moment) given by $m = \langle z \rangle = \int_{-\infty}^{\infty} zp(z)dz$ and an *autocovariance* function (second order moment) given by $B = \langle (z_1 - m)(z_2 - m) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (z_1 - m)(z_2 - m)p_{z_1,z_2}(z_1, z_2)dz_1dz_2$. The autocovariance function is a measure of the length scales over which the hight changes over the surface. The related *autocorrelation function* is given by $C = \langle z_1 z_2 \rangle$, i.e. the autocovariance function for zero mean. In general, to completely describe a rough surface, the PDF of all orders or the statistical moments of all orders have to be known but often the random rough surface is only characterized by a surface height probability distribution and the surface hight autocovariance function [7, 8]. For more information on random processes, see for instance [9]. The Fourier transform of the autocovariance function is called the Power Spectral Density Function (PSD) and it is often used due to its occurrence in smooth surface models for angle resolved scattering [7].

The Gaussian distribution plays a central role in models for scattering from random rough surfaces because it is encountered under a great number of different conditions and because Gaussian variates have the unique property that the random process is entirely determined by the height probability distribution and the autocorrelation. All higher order correlations can be expressed in terms of the (second order) autocorrelation function which simplifies modelling of the scattering process. A simple and often used form for the autocorrelation is a Gaussian function but other forms have also been studied [8].

We will here denote the surface root mean square (RMS) height (or the standard deviation) by σ. If the surface is homogeneous and isotropic, the autocorrelation function is of the form $C(x - x', y - y') = C(\xi)$, where $\xi = \sqrt{\|(x - x', y - y')\|}$. The roughness spectrum at n'th power of the autocorrelation function, $W^{(n)}$, which often enters in results for rough surface scattering, is given by the Fourier transform

$$W^{(n)}(u,v) = (1/2\pi) \int C^n(x',y') e^{j(ux'+vy')} dx'dy' \tag{26}$$

Some often used forms (see for instance [5]) of the autocorrelation function is the Gaussian correlation function, the exponential correlation function, combinations of the gaussian and exponential functions and the so called 1.5-power correlation function. For all these, the roughness spectrum at n'th power can be evaluated analytically (see [5]). For instance, the single parameter Gaussian correlation function (normalized to the surface height variance), $C(\xi) = \exp(-\xi^2/L^2)$, has an n'th power roughness spectrum $W^{(n)}(K) = \exp(-K^2L^2/(4n))$, where L is the *correlation length* of the surface, which monitors the typical distance between two different bumps on the surface.

One more quantity sometimes used to characterize a random rough surface is the *RMS slope*. The RMS slope is a measure of the mean slope of the "bumps" on the rough surface. It's obtained by taking the average of the gradient of the surface height function and for an isotropic surface with a stationary Gaussian height distribution it is given by [3]:

$$\sigma_s = \frac{\sigma\sqrt{2}}{L} \tag{27}$$

In Figure 2 we illustrate the influence of RMS height and correlation length on the surface profile for surfaces with Gaussian statistics.

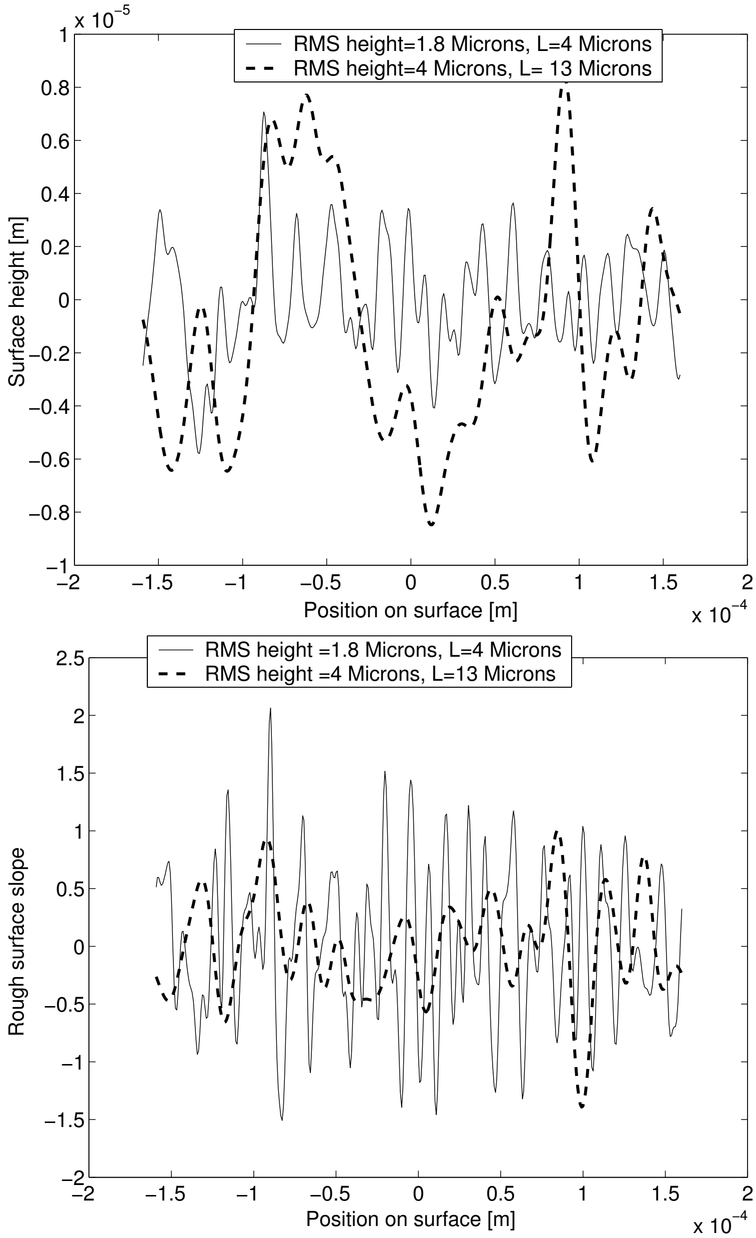

Figure 2: *Examples of two one-dimensional, computer generated rough surfaces with Gaussian statistics. The upper figure shows surface height versus position on surface and the lower figure shows the rough surface slope versus position on surface. Note that the surface with shorter correlation length, L has higher rough surface slopes. These surfaces have been generated using the freeware code in [10].*

2.4 Angle resolved scattering, BRDF and bistatic scattering coefficient

We consider a (polarized) electromagnetic wave, with electric vector $\mathbf{E}^i$, incident on a rough surface that varies about the (x, y)-plane, Π. The scattering geometry is illustrated in Figure 3. The corrugation of the rough surface is given by the equation $z = f(\mathbf{r}_\perp)$ where $\mathbf{r}_\perp = (x, y)$. We denote the electromagnetic vector of the scattered field by $\mathbf{E}^s$.

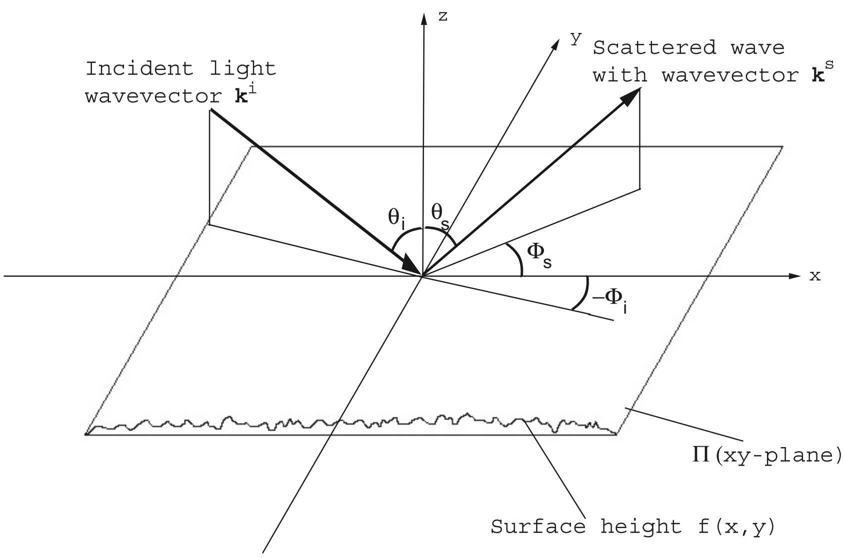

Figure 3: *Scattering geometry for an electromagnetic field incident on a rough surface.*

A special expression has become popular to present results from scattering measurements. This quantity is the Bidirectional Reflectance Distribution Function (BRDF), f^r, which has the unit [sr^{-1}]. In radiometric terms the BRDF is defined as the scattered surface radiance divided by the incident surface irradiance. The BRDF is therefore given by (see for instance [7] or [11]):

$$f^r_{qp}(\theta_i, \phi_i, \theta_s, \phi_s) = \frac{S}{I} = \frac{dP/A}{P_i/A} \cdot \frac{1}{d\Omega \cos\theta_s} = \frac{1}{P_i} \frac{dP}{d\Omega \cos\theta_s} \qquad (28)$$

In this expression, dP is the light flux scattered into a solid angle $d\Omega$, A is the illuminated area on the sample, θ_s is the scattering angle and P_i is the incident light flux on the surface. We have also added the indices p and q to include polarization effects. Here p is the incident polarization and q the scattered polarization, where p and q denote either vertical (v-pol or s-pol) or horizontal polarization (h-pol or p-pol). In Eq. (28) there is also a dependence on the wavelength, which is suppressed in the notation.

It should be noted that the BRDF in Eq. (28) is not enough to describe scattering of electromagnetic waves with arbitrary polarization. For instance it does not account for elliptic polarization. The general polarimetric case is usually treated by introducing a 4×4 matrix called the Müller matrix and 4-dimensional vector, called the Stokes vector with components having the dimension of radiance. This general polarimetric formulation is presented in for instance [5]. However, in measurements of angle resolved scattering, in the IR, visible and UV regions of the electromagnetic spectrum, it is usually the BRDF in Eq. (28) that is measured. To simulate these experiments it is therefore enough to have a model for Eq. (28).

The BRDF is usually assumed to be reciprocal for linear and isotropic media. This means that the BRDF is unchanged if the roles of scattering and incident angles, and the incident polarization and the scattered polarization, $f^r_{qp}(\theta_i, \phi_i, \theta_s, \phi_s) = f^r_{pq}(\theta_s, \phi_s, \theta_i, \phi_i)$ are interchanged. A discussion of the reciprocity of BRDF can be found in [12].

An analogous description to the BRDF for reflected light can be applied to transmitted light and in this case the commonly used quantity is called Bidirectional Transmittance Distribution Function (BTDF). To describe the combined result from angle dependent transmittance and reflectance measurements the quantity Bidirectional Scattering Distribution Function (BSDF) is often used. The BSDF is just a combination of the BRDF and the BTDF. For more information on BSDF and BTDF see [7] and references therein.

A quantity, closely related to BRDF (and BTDF and BSDF), which is often used in models and measurements of scattering in the microwave region is the *Bistatic Scattering Coefficient*, σ^0. The bistatic scattering coefficient is defined [5] as the (statistically averaged) scattered irradiance to the average incident irradiance over the surface of a sphere with radius r, where r is the distance from the scattering point to the receiver. Since the irradiance, I is related to the amplitude of the electric field by $I = \frac{1}{2}\epsilon_0 c \mid \mathbf{E} \mid^2$, the bistatic scattering coefficient can be expressed as

$$\sigma^0 = \frac{\langle \mid \mathbf{E}^s \mid^2 \rangle}{A \mid \mathbf{E}^i \mid^2 /(4\pi r^2)} \qquad (29)$$

where r is the distance to the receiver.

By comparing Eq. (28) and Eq. (29) we see that the BRDF and the bistatic sacttering coefficeint, σ^0 are related by $f^r = \sigma^0/(4\pi \cos\theta_s \cos\theta_i)$. In deriving this relation we make use of the realtions $dP = dAI^s$, $P_i = A\cos\theta_i I^i$ and $d\Omega = dA/r^2$.

An important special case of the BRDF is the case where the surface is completely smooth. In this case we only have reflection in the specular direction and the BRDF takes the form [11]:

$$f^r(\theta_i, \phi_i, \theta_s, \phi_s) = \frac{\delta(\theta_s - \theta_i)\delta(\phi_s - \phi_i)}{2\pi \sin\theta_s} \frac{1}{2}(R_h + R_v)$$

where R_h and R_v are the Fresnel reflection coefficients for horizontally and vertically polarized waves, respectively:

$$R_v = \frac{n_2 \cos\theta_i - n_1 \frac{\mu_1}{\mu 2} \cos\theta_t}{n_2 \cos\theta_i + n_1 \frac{\mu_1}{\mu 2} \cos\theta_t} \qquad (30)$$

$$R_h = \frac{n_1 \cos\theta_i - n_2 \frac{\mu_1}{\mu 2} \cos\theta_t}{n_1 \cos\theta_i + n_2 \frac{\mu_1}{\mu 2} \cos\theta_t} \qquad (31)$$

Here n_1 and n_2 are the indices of refraction in medium 1 and 2, respectively. The index of refraction is given by $n = \sqrt{(\epsilon\mu)/(\epsilon_0\mu_0)}$ where ϵ is the electric permittivity in the surface medium and the medium, μ is the magnetic permeability in the medium and ϵ_0 and μ_0 are the vacuum permittivity and permeability. Furthermore, the transmission angle, θ_t, is related to the incident angle, θ_i by Snell's law: $n_1 \sin\theta_i = n_2 \sin\theta_t$.

2.5 Relation between BRDF and emissivity

A quantity of great importance in radiometry and optical signatures is the emissivity, e_p of a surface. The emissivity of an object can be defined as the ratio of its emitted radiance to the radiance of a blackbody at the same temperature [5]. The emissivity is related to the bistatic scattering coefficient and the BRDF through (see for instance [5])

$$\begin{aligned} e_p(\theta,\phi) &= 1 - \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi/2}(\sigma^0_{pp}(\theta',\phi';\theta,\phi) \\ &+ \sigma^0_{qp}(\theta',\phi';\theta,\phi))\frac{\sin\theta'}{\cos\theta}d\theta' d\phi' = \\ &= 1 - \int_0^{2\pi}\int_0^{\pi/2}(f^r_{pp} + f^r_{qp})(\theta',\phi';\theta,\phi)\sin\theta' d\theta' d\phi' \end{aligned} \tag{32}$$

As was the case for BRDF we have suppressed the dependence on wavelength in this equation. The emissivity for unpolarized light, e is easily obtained from the polarized emissivities through $e = (e_v + e_h)/2$ (see e.g. [13]).

3 The Kirchhoff approximation

In this section we shall consider the Kirchhoff (also sometimes referred to as the tangent plane approximation) approach to light scattering by rough surfaces, which was one of the first methods applied to rough surface scattering. We will consider surfaces with random surface profiles (i.e. not periodic surfaces) and within the context of the vector theory (i.e. not the scalar approximation as in Eq. (9), we will discuss the Kirchhoff approximation. We will here consider the case of scattering from 2-dimensional dielectric surfaces. We will present results for the case of a surface which can be characterized as a Gaussian random process but we will also mention works for non-Gaussian surfaces. We will also mention some extensions of the Kirchhoff approximation and in the text give references to further reading about the Kirchhoff's approach. The reference list is by no means complete, since the literature on the Kirchhoff approximation is vast.

A more thorough introduction to Kirchoff's approximation in diffraction theory can be found in [1] and good presentations of Kirchoff's method in scattering from rough surfaces can be found in for instance [2, 3, 4, 5].

3.1 Formulation of the scattering problem

The geometry of the scattering problem we consider is shown in Figure 3. We consider a monochromatic, linearly polarized, plane wave with electric and magnetic fields:

$$\mathbf{E}^{(i)}(\mathbf{r}) = \hat{\mathbf{p}} E_0 \exp[-j(\mathbf{k}_i \cdot \mathbf{r})] \tag{33}$$

$$\mathbf{H}^{(i)}(\mathbf{r}) = \hat{\mathbf{k}}_i \times (\hat{\mathbf{p}} E^i)/\eta \tag{34}$$

where $\mathbf{k}_i = \hat{\mathbf{k}}_i k$, $k = \omega\sqrt{\epsilon\mu}$, $\hat{p}$ is the unit polarization vector, E_0 the amplitude and $\eta = \sqrt{\mu/\epsilon}$ the wave impedance in the medium above the scattering surface. Furthermore, the time dependence, which is of the form $\exp(-i\omega t)$, is omitted.

It can be shown, similarly to Eq. (19), that the far zone scattered field, E^s_{qp}, can be written in terms of the tangential surface fields, $\hat{\mathbf{n}} \times \mathbf{E}_p$ and $\hat{\mathbf{n}} \times \mathbf{H}_p$, in the medium above the scattering surface as (Stratton-Chu integrals) [2, 14]

$$\begin{aligned} E^s_{qp} &= -\frac{jk}{4\pi R}\exp(-jkR)\int[\hat{\mathbf{q}} \times \hat{\mathbf{k}}_s \cdot (\hat{\mathbf{n}} \times \mathbf{E}_p) \\ &+ \eta\hat{\mathbf{q}} \cdot (\hat{\mathbf{n}} \times \mathbf{H}_p)]\exp[j(\mathbf{k}_s \cdot \mathbf{r})]ds \end{aligned} \tag{35}$$

where

$$\begin{aligned} \mathbf{k}_s &= k\hat{\mathbf{k}}_s = \hat{\mathbf{x}}k_{xx} + \hat{\mathbf{y}}k_{yy} + \hat{\mathbf{z}}k_{zz} = \\ &= k(\hat{\mathbf{x}}\sin\sin\theta_s\cos\phi_s + \hat{\mathbf{y}}\sin\theta_s\sin\phi_s + \hat{\mathbf{z}}\cos\theta_s). \end{aligned} \tag{36}$$

The incident polarization, p can in our presentation either be horizontal, $\hat{\mathbf{h}} = \hat{\phi}$ or vertical, $\hat{\mathbf{v}} = -\hat{\theta}$. In the same way the receiving polarization, q may be horizontal, $\hat{\mathbf{h}}_s = -\hat{\phi}_s$ or vertical, $\hat{\mathbf{v}}_s = -\hat{\theta}_s$.

What needs to be calculated are the tangential surface fields in Eq. 35. In Eqs. (22)–(25) we presented integral equations for the tangential surface fields in the medium above the scattering dielectric surface. It should be noted that these expressions are exact. However, they cannot in general be solved analytically and therefore approximations have to be introduced. Below we will show that by introducing an approximation called the *tangent plane approximation* (or the *Kirchhoff approximation*), closed analytical solutions can be obtained to the scattering problem.

3.2 The tangent plane approximation and the Kirchhoff fields

In the Kirchhoff approach, the fields at any point of the surface are approximated by the fields that would be present on an infinitely extended tangent plane at that particular point on the surface. Thus, the Kirchhoff approximation amounts to assuming that the reflection at each point of the surface takes place by approximating the profile around this point by its tangent plane. The reflection is therefore considered to be locally specular. It is due to this fact that the Kirchhoff approximation is also referred to as the tangent plane approximation. The Kirchhoff approach requires a relative large radius of curvature of the surface roughness relative to the wavelength of the incident light to be valid.

The Kirchhoff approximation for the surface fields is obtained by replacing Eqs. 22 and 23 with [5]

$$\hat{\mathbf{n}} \times \mathbf{E} = (\hat{\mathbf{n}} \times \mathbf{E})_K = \hat{\mathbf{n}} \times (\mathbf{E}^i + \mathbf{E}^r) \tag{37}$$

and

$$\hat{\mathbf{n}} \times \mathbf{H} = (\hat{\mathbf{n}} \times \mathbf{H})_K = \hat{\mathbf{n}} \times (\mathbf{H}^i + \mathbf{H}^r) \tag{38}$$

Here the subscript K stands for the Kirchhoff approximation and $\mathbf{E}^r$ and $\mathbf{H}^r$ are the fields *reflected on the tangent plane* at a point on the rough surface, propagating along the reflected direction, $\hat{\mathbf{k}}_r$. For instance, if the incident electric field has vertical polarization, $\mathbf{E}^i = \hat{\mathbf{v}} E^i$, the field locally reflected on the tangent plane is given by $\mathbf{E}^r = R_v \mathbf{E}^i$, where R_v is the Fresnel reflection coefficient for vertical polarization given in Eq. (30) and the angle of incidence is the local angle of incidence with respect to the tangent plane.

The way to proceed from here is in most presentations of the Kirchhoff method to express the tangential fields under the Kirchhoff approximation in terms of the incident electric field components and the local Fresnel reflection coefficients, as was mentioned above. The local Fresnel reflection coefficients depend on the local angles of incidence. This results in expressions which can be handeled more easily. However, in these expressions there is still a dependence on the spatial variables (slope and angles), which complicates the calculation of the scattered field in Eq. (35). Various approximations have been applied in the literature to simplify this problem.

One way to remove the dependency on local variables is to expand the integrand about zero slope and then perform an integration by parts and discard the edge terms. This approach is presented for second order slope corrections in [4].

Here we will show the results presented by Fung in [5]. In [5] the Kirchhoff tangential fields are first simplified by assuming that the sum $R_h + R_v$ of the local Fresnel reflection coefficients is small (compared with other terms). For perfectly conducting surfaces, $R_h + R_v = 0$ (using the sign convention for the fields as in [5]) renders this approximation exact. The dependency on local angles in the Fresnel coefficients is removed by approximating the local incidence angle in the Fresnel reflection coefficients by the *incident angle*, θ_i, for surfaces with *small scale roughness* and by the *specular angle*, θ_{sp}, $\cos(\theta_{sp}) = -\hat{\mathbf{k}}_r \cdot \hat{\mathbf{k}}_i$, for surfaces with large scale roughness. The regions where these two choices of Fresnel reflection coefficients are valid will be discussed in the context of the Integral Equation Method (see Section 4) and the criterion $(k\sigma)(kL) < 1.2\sqrt{\epsilon}$ is suggested for using the incident angle in the Fresnel coefficients and a limit $kL > 5$ (assuming Gaussian correlation function) for using the specular angle. The final approximation which is performed by Fung is to remove the dependence on spatial variables in slope terms in the scattered field by integrating by parts and discarding the edge terms.

The result, after performing the approximations briefly mentioned above, is that the scattered field can be written as:

$$E^s_{qp} = -\frac{jk}{4\pi R}\exp(-jkR)\int f_{qp}\exp[j(\mathbf{k}_s - \mathbf{k}_i)\cdot\mathbf{r}]ds \qquad (39)$$

where the f_{qp} are independent functions of the spatial variables and for the considered polarizations given by [5]

$$\begin{aligned} f_{vv} &= \frac{2R_v}{\cos\theta_i + \cos\theta_s}[\sin\theta_i\sin\theta_s && (40)\\ &\quad -(1+\cos\theta_i\cos\theta_s)\cos(\phi_s-\phi_i)] \\ f_{hh} &= -\frac{2R_h}{\cos\theta_i + \cos\theta_s}[\sin\theta_i\sin\theta_s && (41)\\ &\quad -(1+\cos\theta_i\cos\theta_s)\cos(\phi_s-\phi_i)] \\ f_{hv} &= (R_v - R_h)\sin(\phi_s-\phi_i) && (42)\\ f_{hv} &= (R_v - R_h)\sin(\phi_i-\phi_s) && (43) \end{aligned}$$

What remains is to find expressions for the bistatic scattering coefficient (or BRDF) for a statistical ensemble of rough surfaces. This is usually made by separately considering the power that is scattered coherently (proportional to $\langle E_{qp}\rangle\langle E_{qp}\rangle^*$, where * denotes the complex conjugate) and incoherently (proportional to $\langle E_{qp}E_{qp}^*\rangle - \langle E_{qp}\rangle\langle E_{qp}\rangle^*$). The bistatic scattering coefficient can then be calculated, taking the statistical averages, using Eq. (39). Here we only present the incoherent bistatic scattering coefficient which is obtained for surfaces with small or moderate heights compared with the wavelength for rough surfaces with Gaussian statistics. In this case the bistatic scattering coefficient, σ_{qp}^{0K}, can be written as the infinite sum [5]:

$$\sigma_{qp}^{0K} = \frac{1}{2}k^2 \mid f_{qp} \mid^2 \exp[-\sigma^2(k_{sz}+k_z)^2] \sum_{n=1}^{\infty} \frac{[\sigma^2(k_{sz}+k_z)^2]^n W^{(n)}(k_{sz}-k_x, k_{sy}-k_y)}{n!} \tag{44}$$

where $W^{(n)}$ is the roughness spectrum at n'th power of the autocorrelation function (Eq. 26), $k_x = k\sin\theta\cos\phi$, $k_y = k\sin\theta\sin\phi$, $k_z = k\cos\theta$, $k_{sx} = k\sin\theta_s\cos\phi_s$, $k_{sy} = k\sin\theta_s\sin\phi_s$ and $k_{sz} = k\cos\theta_s$.

The solution in Eq. 44 is the main result for the Kirchhoff approach to scattering from random rough surfaces. This solution is also sometimes referred to as the "physical optics solution".

The coherent bistatic scattering coefficient can also be calculated. The results for the coherent field can be found in for instance [4]. The coherent field is only scattered in the specular direction.

The expansion in Eq. 44 can be evaluated for surfaces with small RMS height, σ, compared with the wavelength. For surfaces with large heights (very rough surfaces) an expression for the bistatic scattering coefficient can be derived (Eq. (5.14) in [5]) which agrees with the geometric optics solution [5].

3.3 On the range of validity of the Kirchhoff method and shadowing effects

The tangent plane approximation, which considers the scattered field as a specular reflection on the tangent plane at each point of the surface requires that the local curvature be small compared with the wavelength and therefore that the local slope cannot be large. A criterion which is often used is that the correlation length, L, is much smaller than the wavelength, λ. In [15] a criterion for the validity of the Kirchhoff method (tangent plane approximation) was given in the form:

$$kL > 6, \; R_c > \lambda \tag{45}$$

where R_c is the mean radius of curvature of the surface. For instance a surface with a Gaussian correlation function has $R_c = (L^2\sqrt{\pi}/(2\sigma\sqrt{6})$. Furthermore, as was mentioned at the end the previous subsection, the final analytical expressions for the mean scattered intensities are in the Kirchhoff method (usually) derived for either very rough surfaces (surfaces with large heights), $k\sigma$ large enough, or for slightly rough surfaces (surfaces with small heights) $k\sigma$ small enough. A rule of thumb for when a surface can be viewed as very rough or slightly rough is given in [15] as:

$$k\sigma > \frac{\sqrt{10}}{|\cos\theta_i + \cos\theta_s|} \qquad \text{(very rough)} \tag{46}$$

$$\min\sigma_s < 0.25 \qquad \text{(slightly rough)} \tag{47}$$

where $\min\sigma_s$ is the minimum of the mean surface slope (see Eq. 27) in the x and y directions on the surface. The fact that the Kirchhoff approach is progressively less accurate as the angle of incidence is increased (and at larger departures of the scattering angle from the specular direction) is incorporated in the criterion, $R_c \cos\theta_i \gg \lambda/4\pi$ given by Brekhovskikh in [16]. These criteria are also consistent with the results obtained in [17], where it was found, by comparison with "exact" numerical methods applied to 1-Dimensional Gaussian rough surfaces, that the Kirchhoff approximation is valid for $k\sigma \ll 1$ and $kL > 6$, except for grazing angles.

The Kirchhoff method briefly described in this section does not account for shadowing of the incident irradiance by the roughness of the surface. Attempts to include corrections for shadowing have been made in the literature by multiplying the incident irradiance by a shadowing function, $S(\theta_i, \sigma_s)$, where σ_s is the rough surface RMS slope. The existing shadowing functions are unfortunately only accurate under the geometric optics condition. An example of an often used shadowing function is given by Smith in [18]. The usage of a shadowing function can extend the validity of the Kirchhoff method to somewhat larger angles of incidence.

3.4 Some concluding remarks on the Kirchhoff method

As was mentioned in the previous paragraph, the Kirchhoff method does neither in itself account for shadowing and nor does it (in the form described here) account for multiple scattering on the surface. Due to the lack of multiple scattering and shadowing in the Kirchhoff method energy conservation is not satisfied in this method. The use of a shadowing function improves the accuracy of the method but does not ensure that energy is conserved [4].

For the case of scalar (see Eq. (9)) rough surface scattering using the Kirchoff method, Vernold and Harvey [19] use a re-normalization of the scattered radiance which they claim assures conservation of energy.

Some work have been done to extend the Kirchoff method to account for multiple scattering. Ishimaru et al. [20] have developed a method which accounts for double scattering and is claimed to be applicable to medium rough surfaces.

In the literature, the surface height distribution is in most cases assumed to be Gaussian. The reason for this is, as was mentioned in Section 2 that the surface roughness RMS height and the autocorrelation function entirely determine the random process, and therefore the bistatic scattering coefficient can be expressed in terms of these two quantities.

The Kirchhoff method has been applied to surfaces described by fractal geometry. As an example we can mention that in [21] a band-limited Weierstrass fractal function was used for modelling the rough surface. In combination with the Kirchhoff method an analytical solution for the bistatic scattering coefficient was obtained.

4 The integral equation method

A relatively new method for calculating scattering of electromagnetic waves from rough surfaces is the Integral Equation Method (IEM). The IEM has been used extensively in the microwave region in recent years and it has proved to provide good predictions for a wide range of surface profiles. The method can be viewed as an extension of the Kirchhoff method and the Small Perturbation Method (to be presented in Section 5) since it has been shown to reproduce results of these two methods in appropriate limits. The IEM is a relatively complicated method in its general form (including multiple scattering) and it is beyond the scope of the present overview to give a full presentation of the method. A more detailed, and easy to understand, presentation of the IEM can be found in the book by Fung [5].

4.1 A brief presentation of the theory for the IEM

In the IEM the equations for the tangential surface fields are solved iteratively. In the first iteration, the Kirchhoff approximation fields are used. The tangential fields at the surface are represented as a sum of the Kirchhoff fields plus a complementary field. Then, integral equations are derived for the complementary fields at the surface.

The starting point of the IEM is the Stratton-Chu integral for the scattered field in Eq. (35). The tangential surface fields which enters the Stratton-Chu integral are given in Eqs. (22)–(25). In the Kirchoff approach, the tangential surface fields are approximated using the tangent plane approximation, replacing the complete tangential surface fields with the Kirchhoff tangential surface fields in Eqs. 37 and 38. It is clear that the Kirchhoff tangential surface fields cannot alone provide a good estimate of the surface fields since the integral terms in Eqs. (22) and (23) are not accounted for in the Kirchhoff approach. In the IEM, a complementary term is included in Eqs. 37 and 38 to correct for this:

$$\hat{\mathbf{n}} \times \mathbf{E} = (\hat{\mathbf{n}} \times \mathbf{E})_K + (\hat{\mathbf{n}} \times \mathbf{E})_C \tag{48}$$

$$\hat{\mathbf{n}} \times \mathbf{H} = (\hat{\mathbf{n}} \times \mathbf{H})_K + (\hat{\mathbf{n}} \times \mathbf{H})_C \tag{49}$$

In these equations, the first terms on the right hand side are tangential fields under the Kirchhoff approximation (discussed in section 3) and the complementary fields are given by:

$$(\hat{\mathbf{n}} \times \mathbf{E})_C = \hat{\mathbf{n}} \times (\mathbf{E}^i - \mathbf{E}^r) - \frac{2}{4\pi}\hat{\mathbf{n}} \times \int [\mathcal{E}]ds' \tag{50}$$

$$(\hat{\mathbf{n}} \times \mathbf{H})_C = \hat{\mathbf{n}} \times (\mathbf{H}^i - \mathbf{H}^r) - \frac{2}{4\pi}\hat{\mathbf{n}} \times \int [\mathcal{H}]ds' \tag{51}$$

The Kirchhoff tangential fields are simplified in the way briefly described in Section 3. For the complementary tangential surface fields a set of integral equations can be found by combining Eqs. (22) and (23) with the corresponding expression for the surface fields in the scattering surface medium (see [5] for these expressions). The complimentary tangential surface fields are simplified, in a way similar to the Kirchhoff method, in order to remove dependencies on spatial variables. This means that the Fresnel reflection coefficients are introduced in the complementary tangential surface fields but in contrast to the case for the Kirchhoff term, the angle in the Fresnel reflection coefficients is always chosen as the incident angle (never the specular angle) in the complementary term. The resulting approximated complementary tangential surface field equations (electric and magnetic surface field equations for horizontal, vertical and cross polarization) can be found in for instance [5]. Here we only show the equation for the electric field for vertical polarization to illustrate the form of the equations

$$(\hat{\mathbf{n}} \times \mathbf{E}_v)_C \approx -\frac{1}{4\pi}\hat{\mathbf{n}} \times \int [(1 - R_v)\mathcal{E}_v + (1 + R_v)\mathcal{E}_{vt}]ds' \qquad (52)$$

where $\mathcal{E}_{vt}$ is given by an equation similar to Eq. (24) for the surface fields in the scattering surface medium, which can be expressed in the surface fields in the upper medium through the material parameters (see [5]).

The simplified tangential surface fields can then be inserted in the Stratton-Chu integral and by writing the scattered field in the far zone as $E^s_{qp} = E^K_{qp} + E^C_{qp}$ where E^K_{qp} is the scattered field from the Kirchhoff component and E^C_{qp} is the scattered field from the complementary component of the tangential surface fields given by Eq. (35). In the same way, the scattered field from the complementary component can be written as

$$\begin{aligned} E^C_{qp} &= -\frac{jk}{32\pi^3 R}\exp(-jkR)\int F_{qp}\exp[ju(x-x') \\ &\quad +jv(y-y') + (\mathbf{k}_s - \mathbf{k}_i)\cdot\mathbf{r}']dxdydx'dy'dudv \end{aligned} \qquad (53)$$

where the field coefficients F_{qp} are functions of the complementary tangential surface fields (as in Eq. (52)). To calculate the field coefficients, the Green function and its gradient in $\mathcal{E}$ and $\mathcal{H}$ (and the corresponding quantities for the transmitted fields) are taken in a spectral representation as in Eqs. (20) and (21). In [5] the $\mid z - z' \mid$ terms and the term with the $\pm$ sign is dropped in the spectral representation of the Green function and its gradient in order to simplify the calculations. However, in [22] these terms are kept in the analysis. Also, as was the case in the Kirchhoff method in Section 3, Fung [5] removes the dependence on spatial variables in slope terms in the scattered field by integrating by parts and discarding the edge terms. An analysis where the edge terms are kept (i.e. not discarded) has been performed in [22].

Once the approximate scattered fields have been obtained, it is possible to calculate the average scattered irradiance and from this the bistatic scattering coefficients for the random rough surface given as a stochastic function. Since the scattered field consists of two terms (the Kirchoff term and the complementary term), the scattered irradiance and therefore the bistatic scattering coefficient, σ_{qp}^0, will contain three terms:

$$\sigma_{qp}^0 = \sigma_{qp}^{0K} + \sigma_{qp}^{0KC} + \sigma_{qp}^{0C} \tag{54}$$

where σ_{qp}^{0K} is the contribution from the Kirchhoff term, σ_{qp}^{0C} the contribution from the complementary term and σ_{qp}^{0KC} the contribution from the cross term. The term σ_{qp}^{0K} in Eq. (54) is given by Eq. (44) in Section 3.

As was mentioned in Section 3, the Kirchoff terms only contribute to single scattering from the rough surface. The complementary term will on the other hand contribute to multiple scattering (to the second order) as was shown in [5].

In Section 3 we mentioned that the effect of shadowing must be included separately in the Kirchhoff approach. This is also true for the IEM. In [5] the shadowing functions derived by Smith in [18] are used.

The final expressions for the bistatic scattering coefficients in IEM are rather lengthy expressions, especially when multiple scattering is included. Furthermore, separate expressions are derived for surfaces with small or moderate surface heights (defined as $k\sigma < 2$ in [5]) and surfaces with large roughness heights (corresponding to the geometrical optics limit in Section 3). The complete expressions for the bistatic scattering coefficients in IEM can be found in [5] and [22]. In the multiple scattering contributions to the bistatic scattering coefficients, a double integral over pairs of surface spectral components and these integrals have to be evaluated numerically. The pair of spectral components can be seen as the interaction between two points on the surface, although in the spectral representation. Therefore the double integral is likely to at least represent double scattering.

4.2 Validity of the IEM

The main assumption (approximation) in the IEM is that the unknown tangential fields in the integral terms of the expressions for the total tangential surface fields can be approximated by the corresponding Kirchhoff tangential fields. In the formulation given in [22], the major additional simplifying approximations are that terms in the tangential field expression involving the sum of R_h and R_v are discarded. Furthermore, the dependency on local angles in the Fresnel coefficients is removed by approximating the local incidence angle in the Fresnel reflection coefficients by the *incident angle* for surfaces with *small scale roughness* and by the *specular angle* for surfaces with large scale roughness. An attempt to find a transition model between the two choices of angle in the Fresnel reflection coefficient has been made in [23]. The exact range of validity of IEM is not known. However, as a rule of thumb, supported by validation and based on the different choices of angle in the Fresnel reflection coefficients mentioned above, Fung [5] suggest the following criteria for surfaces with a Gaussian correlation function:

$$(k\sigma)(kL) < 1.2\sqrt{\epsilon_r} \qquad \text{(slightly rough surface)} \tag{55}$$

$$kL > 5 \qquad \text{(very rough)} \tag{56}$$

where Fung gives the criterion $k\sigma < 2$ for a slightly rough surface but a criterion for very rough surfaces is not given.

The validity of the IEM against measurement and/or more accurate numerical methods has been studied in for instance [5], [22], [24], [23]. Since the tangent plane approximation is used in the IEM, the method gives less accurate results for small $k\sigma$ and kL, and the method is also less accurate for grazing incidence. However the comparisons show that results calculated using IEM, in general, compare very well to experiments and numerical calculations. Furhermore, the IEM can be used in the region of moderately rough surfaces where neither the Kirchhoff method nor the small perturbation method (see Section 5) can be used. Another advantage of the IEM is that it can predict enhanced backscattering which is a consequence of multiple scattering, see [25].

5 The small perturbation method

The Small Perturbation Method (SPM) belongs to a large family of perturbation expansion solutions to the wave equation. The approach is based on formulating the scattering as a partial differential equation boundary value problem. The basic idea is to find a solution in terms of plane waves that match the surface boundary conditions, which state that the tangential component of the field must be continuous across the boundary. The surface fields are expanded in a perturbation series, $\mathbf{E} = \mathbf{E}^0 + \mathbf{E}^1 + ...$ where $\mathbf{E}^i$ depends on the n-th power of the surface elevation and its gradient. In the expansion $\mathbf{E}^0$ would be the surface field if the surface was flat. The philosophy behind this approach is that small effective surface currents on a mean surface replace the role of the small-scale roughness. So this method applies to surfaces with small surface height variation and small surface slopes independently of the radius of curvature of the surface roughness. Therefore, the surface need no longer be approximated by planes. The small-scale roughness is expanded in a Fourier series and the contributions to the field are therefore analyzed in terms of the different wavelength components.

Here we will only give a very brief presentation of the SPM in order to give some idea of its construction. A more detailed description of SPM can be found in [2] and, in particular, in [4].

5.1 A Brief Presentation of the SPM

For the present discussion of SPM we consider the case of a dielectric scattering surface. The perfect conductor case can be found in for instance [2] and [4]. As in previous sections we consider an incident plane wave given by Eq. (6). The basic equations which are used as a starting point are Eqs. (14) and (17) in Section 2. A spectral plane wave representation is used for the dyadic Green function in these equations (see Section 2 and [4]). Using the continuity conditions for the electric and magnetic fields, Eqs. (10) and (11) in Section 2 surface field unknowns, $\mathbf{a}(\mathbf{r}_\perp)$ and $\mathbf{b}(\mathbf{r}_\perp)$, can be defined as [4]:

$$ds\eta\hat{\mathbf{n}} \times \mathbf{H}(\mathbf{r}) = d\mathbf{r}_\perp \mathbf{a}(\mathbf{r}_\perp) = ds\eta\hat{\mathbf{n}} \times \mathbf{H}_2(\mathbf{r}) \quad (57)$$

$$ds\eta\hat{\mathbf{n}} \times \mathbf{E}(\mathbf{r}) = d\mathbf{r}_\perp \mathbf{b}(\mathbf{r}_\perp) = ds\eta\hat{\mathbf{n}} \times \mathbf{E}_2(\mathbf{r}) \quad (58)$$

where $\mathbf{r}_\perp = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$, i.e. a point in the "mean height plane" of the rough surface, and η is the wave impedance (see Section 2). Once $\mathbf{a}(\mathbf{r}_\perp)$ and $\mathbf{b}(\mathbf{r}_\perp)$ have been determined, the surface fields are also known through Eqs. 57 and 58. Using the spectral representation it can be shown that Eqs. (14) and (17) can be expressed as [4] (see Figure 4 for an explanation of the symbols):

$$\begin{aligned}
\mathbf{E}_i(\mathbf{r}) &= \frac{1}{8\pi^2}\int d\mathbf{k}_\perp e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp} e^{-ik_z z}\frac{k}{k_z}\int d\mathbf{r}'_\perp e^{i\mathbf{k}_\perp\cdot\mathbf{r}'_\perp} e^{-ik_z f(\mathbf{r}'_\perp)} \qquad (59)\\
&\times \{[\hat{\mathbf{h}}(-k_z)\hat{\mathbf{h}}(-k_z)+\hat{\mathbf{v}}(-k_z)\hat{\mathbf{v}}(-k_z)]\cdot\mathbf{a}(\mathbf{r}_\perp)'\\
&+ [-\hat{\mathbf{v}}(-k_z)\hat{\mathbf{h}}(-k_z)+\hat{\mathbf{h}}(-k_z)\hat{\mathbf{v}}(-k_z)]\cdot\mathbf{b}(\mathbf{r}'_\perp)\}\\
0 &= \frac{1}{8\pi^2}\int d\mathbf{k}_\perp e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp} e^{-ik_z z}\frac{k}{k_z}\int d\mathbf{r}'_\perp e^{i\mathbf{k}_\perp\cdot\mathbf{r}'_\perp} e^{-ik_z f(\mathbf{r}'_\perp)} \qquad (60)\\
&\times \{[\hat{\mathbf{h}}(-k_z)\hat{\mathbf{h}}(-k_z)+\hat{\mathbf{v}}(-k_z)\hat{\mathbf{v}}(-k_z)]\cdot\mathbf{a}(\mathbf{r}'_\perp)\\
&+ [-\hat{\mathbf{v}}(-k_z)\hat{\mathbf{h}}(-k_z)+\hat{\mathbf{h}}(-k_z)\hat{\mathbf{v}}(-k_z)]\cdot\mathbf{b}(\mathbf{r}_\perp)'\}
\end{aligned}$$

where $f(\mathbf{r}'_\perp)$ is the surface height. The definition of the horizontal unit polarization vectors is $\hat{\mathbf{h}}(\pm k_z) = (\hat{\mathbf{x}}k_y - \hat{\mathbf{y}}k_x)/k_\rho$ and $k_\rho = \sqrt{k_x^2+k_y^2}$. The vertical polarization vectors are given by $\hat{\mathbf{v}}(\pm k_z) = \mp k_z(\hat{\mathbf{x}}k_x - \hat{\mathbf{y}}k_y)/(k_\rho k) + \hat{\mathbf{z}}k_\rho/k$.

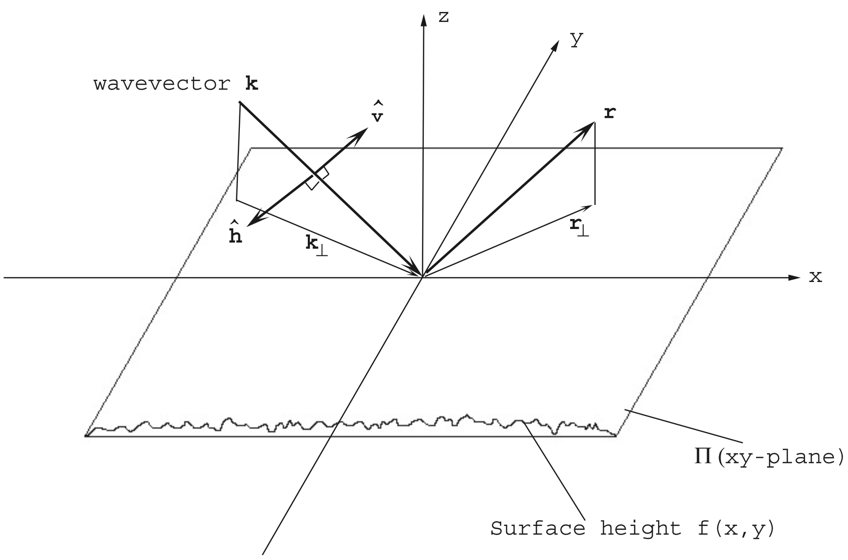

Figure 4: *Illustration of polarization vectors and wave vectors.*

In the same way, the scattered field in Eq. (18) can be written in spectral representation as:

$$\begin{aligned}
\mathbf{E}_s(\mathbf{r}) &= -\frac{1}{8\pi^2}\int d\mathbf{k}_\perp e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp} e^{ik_z z}\frac{k}{k_z}\int d\mathbf{r}'_\perp e^{-i\mathbf{k}_\perp\cdot\mathbf{r}'_\perp} e^{-ik_z f(\mathbf{r}'_\perp)} \qquad (61)\\
&\times \{[\hat{\mathbf{h}}(k_z)\hat{\mathbf{h}}(k_z)+\hat{\mathbf{v}}(k_z)\hat{\mathbf{v}}(k_z)]\cdot\mathbf{a}(\mathbf{r}'_\perp)\\
&+ [-\hat{\mathbf{v}}(k_z)\hat{\mathbf{h}}(k_z)+\hat{\mathbf{h}}(k_z)\hat{\mathbf{v}}(k_z)]\cdot\mathbf{b}(\mathbf{r}'_\perp)\}
\end{aligned}$$

It should be noted that the above equations are exact and Eqs. (59) and (60) are used to solve for the surface fields (i.e. the unknowns $\mathbf{a}$ and $\mathbf{b}$

defined in Eqs. (57) and (58)). Next, Eq. (61) is used to obtain the scattered field.

To solve for the surface fields, the SPM makes use of series expansions:

$$\mathbf{a}(\mathbf{r}'_\perp) = \sum_{m=0}^{\infty} \mathbf{a}^{(m)}(\mathbf{r}'_\perp) \tag{62}$$

$$\mathbf{b}(\mathbf{r}'_\perp) = \sum_{m=0}^{\infty} \mathbf{b}^{(m)}(\mathbf{r}'_\perp) \tag{63}$$

and

$$e^{\pm ik_z f(\mathbf{r}'_\perp)} = \sum_{m=0}^{\infty} \frac{[\pm ik_z f(\mathbf{r}'_\perp)]^m}{m!} \tag{64}$$

where m denotes the mth order solution.

By using the continuity relations in Eqs. (57) and (58) and expressing the local surface normal, $\hat{\mathbf{n}}$ in derivatives of the surface height $f(\mathbf{r}'_\perp)$, the following relations are obtained between the components of the surface field unknowns:

$$\begin{aligned}
\mathbf{a}_z^{(0)}(\mathbf{r}_\perp) &= \mathbf{b}_z^{(0)}(\mathbf{r}_\perp) = 0 \\
\mathbf{a}_z^{(m)}(\mathbf{r}_\perp) &= \left(\hat{\mathbf{x}}\frac{\partial f}{\partial x} + \hat{\mathbf{y}}\frac{\partial f}{\partial y}\right) \mathbf{a}_\perp^{(m-1)}(\mathbf{r}_\perp) \\
\mathbf{b}_z^{(m)}(\mathbf{r}_\perp) &= \left(\hat{\mathbf{x}}\frac{\partial f}{\partial x} + \hat{\mathbf{y}}\frac{\partial f}{\partial y}\right) \mathbf{b}_\perp^{(m-1)}(\mathbf{r}_\perp)
\end{aligned} \tag{65}$$

In SPM, the surface height, $f(\mathbf{r}'_\perp)$ and its derivatives are regarded as small parameters. Thus it is not only the RMS heights that are assumed to be small but also the surface slopes. The basic underlying assumptions for the SPM are therefore

$$k_z f(\mathbf{r}'_\perp), \frac{\partial f}{\partial x'}, \frac{\partial f}{\partial y'} \ll 1 \tag{66}$$

The way to proceed from the equations given above is to substitute the series expansions in Eqs. (62)–(64) into Eqs. (59) and (60). Next, by using the relations in Eq. (65), terms of the same order can be equated. In this way the surface fields to the zeroth-order, first-order, and so on can be calculated. When the surface fields have been obtained, the scattered fields of different orders can be obtained from Eq. (61). A detailed derivation of the fields up to second order can be found in [4].

The zeroth order solutions are just the reflected fields of a flat surface which can be expressed as

$$\mathbf{E}_s^0 = \{R_h[\hat{\mathbf{h}}(-k_{iz}) \cdot \hat{\mathbf{q}}]\hat{\mathbf{h}}(k_{iz}) + R_v[\hat{\mathbf{v}}(-k_{iz}) \cdot \hat{\mathbf{q}}]\hat{\mathbf{v}}(k_{iz})\}e^{i\mathbf{k}_i \cdot \mathbf{r}} \tag{67}$$

where R_h and R_v are the Fresnel reflection coefficients. The zeroth order scattered field is therefore purely coherent and reflected in the specular direction.

The first order solution gives the lowest order correction to the incoherent scattered irradiance. The lowest order incoherent bistatic scattering coefficient can therefore be calculated by taking statistical averages of the first order field solution. The lowest order incoherent bistatic scattering coefficient, assuming a gaussian surface height distribution, can be written as [4]:

$$\sigma^0_{qp} = \frac{k^2 cos^2\theta_s}{4\pi} W^{(1)}(\mathbf{k}_\perp - \mathbf{k}_{i\perp})|f^1_{qp}|^2 \tag{68}$$

where $W^{(1)}$ is the roughness spectrum to the first power given in Eq. 26. The coefficients f^1_{qp} can be found in [4] and here we only give the coefficient f^1_{hh}:

$$f^1_{hh} = \frac{(k_2^2 - k^2)2k_{iz}}{(k_z + k_{2z})(k_{iz} + k_{2zi})} \cos(\phi_s - \phi_i) \tag{69}$$

The cross polarized coefficient, f^1_{hv} and f^1_{vh} both have a factor $\sin(\phi_s - \phi_i)$. This implies that first order SPM does not predict depolarization in the plane of incidence (i.e. for $\phi_s = \phi_i(\pm\pi)$).

The second order perturbative solution and the corresponding bistatic scattering coefficients can be found in [4]. The second order solution gives the lowest order correction to the coherent reflection coefficients. Furthermore, depolarization in the plane of incidence is manifested in the second order solution.

5.2 Some remarks on the range of validity of SPM

The basic criterion for the SPM to be applicable is that the rms heights and the surface slopes are small. This means that the SPM is applicable in the small roughness regime. Since the zeroth-order field in the SPM series corresponds to the coherent field scattered from a flat surface, the accuracy of the mean irradiance calculated by means of the first terms of the perturbative series requires that the diffuse component is small. Due to the increasing difficulty in analytically calculating higher order terms in the perturbative series, the convergence and accuracy of the perurbative series is in general very difficult to assess [2]. As a rule of thumb, a surface *with Gaussian correlation function* can be considered to be in the small roughness regime (slightly rough) when [15]:

$$k\sigma < 0.3 \text{ and } kL > \sqrt{2}k\sigma/0.3 \tag{70}$$

The SPM has been compared to more accurate numerical simulations by Thorsos and Jackson in [17, 26] for one-dimensional rough surfaces with a Gaussian roughness spectrum. Under these conditions Thorsos and Jackson show that the first-order SPM give accurate results for $k\sigma \ll 1$ and $kL \approx 1$. The results obtained by Thorsos and Jackson show that for $k\sigma \ll 1$ and $kL > 6$, the sum of the first three orders of SPM is required to obtain accurate results.

It has been argued that the SPM does account for multiple scattering up to the order of the perturbative expansion [27]. This means that the first order perturbative solution does not account for multiple scattering but that some multiple scattering effects can be observed in the higher order solutions.

6 The small slope approximation

The small slope approximation (SSA), introduced by Voronovich in the mid-1980s [28], is a very promising method for modelling wave scattering from rough surfaces. In the general formulation of SSA, the roughness slope is the only parameter required to be small. This smallness parameter differs from those used in classical perturbation theory and in Kirchhoff approximation because it is independent of wavelength. When the first two terms of the series are retained, the theory reduces to perturbation theory, which is intrinsic to the formulation, and to the Kirchhoff approximation under appropriate conditions.

6.1 Theory in brief

The small slope approximation is a rather complex method and here we only present some of the main features of the method. In particular, we discuss the most important mathematical and physical approximations used. A more detailed presentation of SSA can be found in for instance [28] and [29].

We once more consider a plane wave incident on a dielectric rough surface. We follow the conventions and notations used in [28], where the incident plane electromagnetic wave is taken to travel in the positive z direction in dielectric Medium 1 towards the rough surface (dielectric Medium 2). The scattering geometry in cross section is illustrated in Figure 5. For

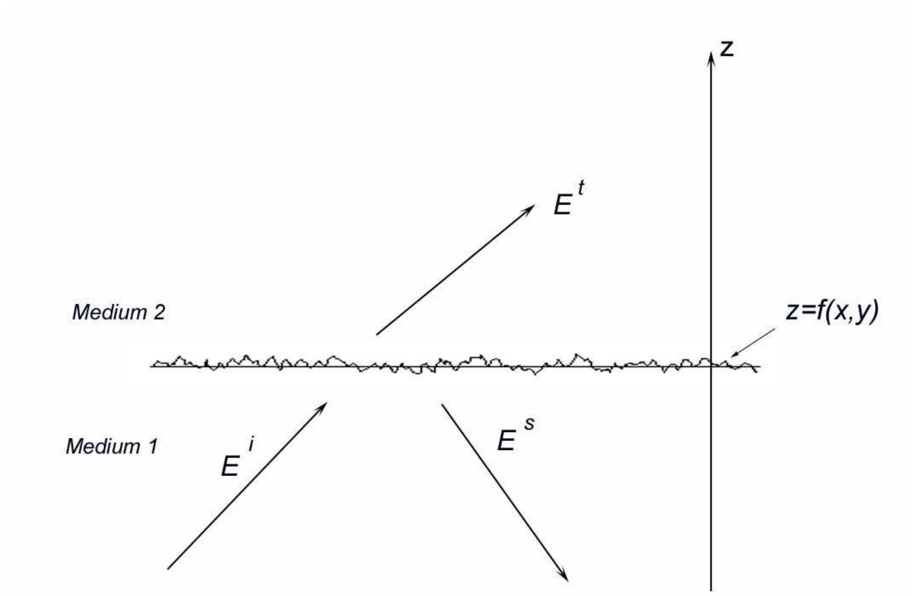

Figure 5: *Schematic illustration of scattering geometry (cross section) used in the SSA model.*

simplicity we also assume that the magnetic permeabilities of both media can be set equal to one (i.e. $\mu_1 = \mu_2 = 1$), which is almost always a valid assumption when considering scattering in the optical (IR, visible and UV) region of the electromagnetic spectrum. The incident electric field vector is then given by (compare with Eq. (6)):

$$\mathbf{E}^{(i)}(\mathbf{r}) = \hat{\mathbf{q}}^E_{\alpha_0}(k^i_z)^{-1/2}\exp[i(\mathbf{k}^i_\perp \cdot \mathbf{r}_\perp + k^i_z z)] \tag{71}$$

where $(\mathbf{k}^i_\perp, k^i_z)$ are the horizontal and vertical projections of the incident wavevector with wavenumber $k^i = \sqrt{\epsilon}\omega$, so that $k^i_z = \sqrt{(k^i)^2 - (k^i_\perp)^2}$. The factor $(k^i_z)^{-1/2}$ is introduced for normalization purposes. The subscript, $\alpha_0 = 1, 2$, in the unit polarization vectors, $\hat{\mathbf{q}}^E_{\alpha_0}$, is the index describing vertical and horizontal polarization, respectively ($\hat{\mathbf{q}}^E_1 = \hat{\mathbf{v}}$ and $\hat{\mathbf{q}}^E_2 = \hat{\mathbf{h}}$). These polarization vectors can be expressed in terms of the components of the wave vector, $\mathbf{k}^i$ in the same way as we did for Eqs. (59) and (60) in Section 5. The incident magnetic field associated with the incident electric field is then given by Eq. (7).

It follows from the linearity of Maxwell's equations, analogously to the spectral representation of Green's functions in Eq. (20), that a plane wave expansion can be used to write the scattered electromagnetic wave, $\mathbf{E}^{(s)}$ and $\mathbf{H}^{(s)}$, as [29]:

$$\mathbf{E}^{(s)} = \sum_\alpha \int \hat{\mathbf{q}}^E_\alpha k_z^{-1/2}\exp[i(\mathbf{k}_\perp \cdot \mathbf{r}_\perp - k_z z)] S^{11}_{\alpha\alpha_0}(\mathbf{k}_\perp, \mathbf{k}^i_\perp) d\mathbf{k}_\perp \tag{72}$$

where $k_z = \sqrt{(k)^2 - (k_\perp)^2}$. The value, $S^{11}_{\alpha\alpha_0}$ is one element of the so called *scattering amplitude* matrix, $S^{NN_0}_{\alpha\alpha_0}$, where N, $N_0 = 1, 2$, representing a wave incident from Medium N_0 and propagating in Medium N. The subscripts, $\alpha, \alpha_0 = 1, 2$, on the unit polarization vectors and the scattering amplitude are the indices describing vertical and horizontal polarization, respectively. Notice that the polarization vectors depend on $\mathbf{k}_\perp$. The scattering amplitude matrix components are defined by Eq. (72), Eq. (74) below and by analogous equations, to be found in [28], for the remaining combinations of incident and propagating fields.

In the same way as for the electric field, the scattered magnetic field can be expressed as:

$$\mathbf{H}^{(s)} = \sqrt{\epsilon_1}\sum_\alpha \int \hat{\mathbf{q}}^H_\alpha k_z^{-1/2}\exp[i(\mathbf{k}_\perp \cdot \mathbf{r}_\perp - k_z z)] S^{11}_{\alpha\alpha_0}(\mathbf{k}_\perp, \mathbf{k}^i_\perp) d\mathbf{k}_\perp \tag{73}$$

Similar equations can be written for the transmitted fields by introducing the elements $S^{12}_{\alpha\alpha_0}$ and $S^{21}_{\alpha\alpha_0}$ of the scattering amplitude. For the transmitted electric field, $\mathbf{E}^{(t)}$, (see Figure 5) we have [28]:

$$\mathbf{E}^{(t)} = \sum_\alpha \int \hat{\mathbf{q}}^{E(2)}_\alpha (k^{(2)}_z)^{-1/2}\exp[i(\mathbf{k}_\perp \cdot \mathbf{r}_\perp + k^{(2)}_z z)] S^{21}_{\alpha\alpha_0}(\mathbf{k}_\perp, \mathbf{k}^i_\perp) d\mathbf{k}_\perp \tag{74}$$

where we have added a superscript (2) on the unit polarization vectors and on the z-component of the transmitted wave vector to indicate that they are defined for the transmitted field in Medium 2.

The expressions in Eqs. (72)–(74) are general (exact) solutions of the Maxwell equations. To determine the scattering amplitude, one must then use the continuity conditions for the tangential fields over the rough surface boundary in Eqs. (10)–(13).

The form of the equation which serves as a staring point for the SSA relies on transformation properties of the scattering amplitude. The physics (for instance the scattered irradiance) must be invariant under a translation of the rough surface in the horizontal or vertical directions. As follows from geometrical arguments (see [29]), the scattering amplitude picks up a phase factor so that [28]:

$$f(\mathbf{r}_\perp) \to f(\mathbf{r}_\perp - \mathbf{d}) \;\Rightarrow\; S^{NN_0}_{\alpha\alpha_0}(\mathbf{k}_\perp, \mathbf{k}^i_\perp) \to S^{NN_0}_{\alpha\alpha_0}(\mathbf{k}_\perp, \mathbf{k}^i_\perp) e^{-i(\mathbf{k}_\perp - \mathbf{k}^i_\perp)\mathbf{d}} \quad (75)$$

$$f(\mathbf{r}_\perp) \to f(\mathbf{r}_\perp) + H \;\Rightarrow\; S^{NN_0}_{\alpha\alpha_0}(\mathbf{k}_\perp, \mathbf{k}^i_\perp) \to S^{NN_0}_{\alpha\alpha_0}(\mathbf{k}_\perp, \mathbf{k}^i_\perp) e^{-i(k_z - k^i_z)H} \quad (76)$$

where $\mathbf{r}_\perp = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$, $f(\mathbf{r}_\perp)$ is the rough surface height function and $\mathbf{d} = d_x\hat{\mathbf{x}} + d_y\hat{\mathbf{y}}$.

In order to fulfil the transformation properties in Eqs. (75) and (76) the scattering amplitude is in SSA written as:

$$\begin{aligned} S^{NN_0}_{\alpha\alpha_0}(\mathbf{k}_\perp, \mathbf{k}^i_\perp) &= \int \frac{d\mathbf{r}_\perp}{(2\pi)^2} \exp[-i(\mathbf{k}_\perp - \mathbf{k}^i_\perp)\cdot \mathbf{r}_\perp \\ &\quad + i(k_z + k^i_z) f(\mathbf{r}_\perp)] \Phi^{NN_0}_{\alpha\alpha_0}(\mathbf{k}_\perp, \mathbf{k}^i_\perp; \mathbf{r}_\perp; [f]) \end{aligned} \quad (77)$$

where $\Phi^{NN_0}_{\alpha\alpha_0}$ is an arbitrary functional, which for smooth functionals can be expanded in terms of an integral power series:

$$\begin{aligned} &\Phi^{NN_0}_{\alpha\alpha_0}(\mathbf{k}_\perp, \mathbf{k}^i_\perp; \mathbf{r}_\perp; [f]) = (\Phi^{NN_0}_{\alpha\alpha_0})^{(0)}(\mathbf{k}_\perp, \mathbf{k}^i_\perp) + \\ &\int (\Phi^{NN_0}_{\alpha\alpha_0})^{(1)}(\mathbf{k}_\perp, \mathbf{k}^i_\perp; \xi) f(\xi) e^{i\xi\cdot\mathbf{r}_\perp} d\xi + \\ &\int (\Phi^{NN_0}_{\alpha\alpha_0})^{(2)}(\mathbf{k}_\perp, \mathbf{k}^i_\perp; \xi) f(\xi_1) f(\xi_2) e^{i(\xi_1+\xi_2)\cdot\mathbf{r}_\perp} d\xi_1\xi_2 + \dots \end{aligned} \quad (78)$$

Notice that Eq. (78) contains an integral power series. This is in contrast to the SPM in Section 5 where the fields were expanded in an ordinary Taylor expansion.

Voronovich [28] now proceeds by inserting Eq. (78) into Eq. (77). Furthermore, by using the transformation properties of the scattering amplitude, Eqs. (75) and (76), integration by parts shows that any of the terms

$(\Phi_{\alpha\alpha_0}^{NN_0})^{(n)}$ in the expansion can be transferred to terms of order $(n-1)$ and order $(n+1)$. For details see [28, 29]. This procedure shows that the n'th term in the expansion is proportional to $|\nabla_{\mathbf{r}_\perp} f(\mathbf{r}_\perp)|^n$, which also is a consequence of the integration by parts. This means that n terms in the expansion in Eq. 78 can be chosen so that the accuracy of the scattering amplitude is of $\mathcal{O}((\nabla f)^{n+1})$.

The preceding discussion indicates that SSA can be used when ∇f is sufficiently small, which means that the local surface slopes must be small everywhere on the surface. In fact, Voronovich [29] states that, if the coefficients in an expansion of the field are known, the range of validity for SSA is:

$$\nabla f \ll 1 \tag{79}$$

This means that the slopes of the irregularities on the surface should everywhere on the surface be much smaller than one.

The coefficients, $(\Phi_{\alpha\alpha_0}^{NN_0})^{(m)}$, in Eq. (78) are still unknown and have to be determined. To arrive at expressions for the coefficients, the continuity conditions for the tangential fields over the rough surface boundary in Eqs. (10)–(13)have to be used. The unknown coefficients are in SSA determined by comparing the integral expansion in Eqs. (77) and (78) with a known expansion of the scattering amplitude. Usually the known expansion of the scattering amplitude is based on SPM (see Section 5), although other expansions could also be used [29]. It can be shown that the scattering amplitude in SPM can be written as [28, 29]:

$$\begin{aligned}
&S_{\alpha\alpha_0}^{NN_0}(\mathbf{k}_\perp, \mathbf{k}_\perp^i) = \\
&V_{\alpha\alpha_0}^{NN_0}(\mathbf{k}_\perp, \mathbf{k}_\perp^i)\delta(\mathbf{k}_\perp - \mathbf{k}_\perp^i) \\
&+2i(k_z^{(N)} k_z^{i(N_0)})^{1/2} B_{\alpha\alpha_0}^{NN_0}(\mathbf{k}_\perp, \mathbf{k}_\perp^i) f(\mathbf{k}_\perp - \mathbf{k}_\perp^i) \\
&+(k_z^{(N)} k_z^{i(N_0)})^{1/2} \sum_{m=1}^{\infty} \int (B_{\alpha\alpha_0}^{NN_0})^{(m+1)}(\mathbf{k}_\perp, \mathbf{k}_\perp^i; \xi_\mathbf{1}, ..., \xi_\mathbf{m}) \\
&\times f(\mathbf{k}_\perp - \xi_\mathbf{1}) ... f(\xi_\mathbf{m} - \mathbf{k}_\perp^i) d\xi_\mathbf{1} ... d\xi_\mathbf{m}
\end{aligned} \tag{80}$$

where $f(\mathbf{k}_\perp) = \int \exp[i(\mathbf{k}_\perp \cdot \mathbf{r}_\perp)] f(\mathbf{r}_\perp) d\mathbf{r}_\perp/(2\pi)^2$, i.e. the Fourier transform of the surface height $f(\mathbf{r}_\perp)$. The explicit expressions for the coefficients $V_{\alpha\alpha_0}^{NN_0}$ and $(B_{\alpha\alpha_0}^{NN_0})^{(m+1)}$ can be found in [28, 29].

The SPM scattering amplitude in Eq. (80) is an expansion in powers of f. As was mentioned in Section 5, SPM requires $k|f| \ll 1$. If we by the procedure mentioned above exclude the coefficient $(\Phi_{\alpha\alpha_0}^{NN_0})^{(1)}$ in Eq. (77), the accuracy of the scattering amplitude is of order $\mathcal{O}((\nabla f)^2)$. For small f, the exponential in Eq. (77) can be expanded in powers of f and we can identify terms of different orders of f in the SPM and SSA expansions of the scattering amplitude. For instance, if we look at terms up to the first order in f we find [28]:

$$(\Phi^{NN_0}_{\alpha\alpha_0})^{(0)}(\mathbf{k}_\perp, \mathbf{k}^i_\perp) = -\frac{2(k_z^{(N)} k_z^{i(N_0)})^{1/2} B^{NN_0}_{\alpha\alpha_0}(\mathbf{k}_\perp, \mathbf{k}^i_\perp)}{(-1)^N k_z^{(N)} + (-1) N_0 k_z^{i(N_0)}} \tag{81}$$

In [28] the coefficients are also determined up to second order in f for surfaces with Gaussian height statistics. For this case Voronovich gives the following estimate for the range of validity of SSA:

$$\nabla f \ll \frac{k_z}{k_\perp} \tag{82}$$

This means that the slopes of the irregularities should be small enough and at least smaller than the grazing angles of the incident and the scattered waves.

For rough surfaces with gaussian statistics, the statistical moments of the scattering amplitude can be calculated [28] and from these the bistatic scattering coefficient can be determined. In [28], the bistatic scattering coefficients up to second order are determined. The form of the bistatic scattering coefficient that corresponds to the first order of SSA is given by:

$$\begin{aligned} \sigma_{qp} &= (2k_z k_z^i)^2 \mid B_{qp}(\mathbf{k}_\perp, \mathbf{k}^i_\perp) \mid^2 \\ & \int_{A_0} e^{-i(\mathbf{k}_\perp - \mathbf{k}^i_\perp)\cdot \mathbf{r}_\perp} e^{-Q^2\sigma^2} \left(\frac{\exp(Q^2\sigma^2\rho(\mathbf{r}_\perp)) - 1}{Q^2} \right) \frac{d\mathbf{r}_\perp}{(2\pi)^2 A_0} \end{aligned} \tag{83}$$

where σ is the surface height RMS value, W is the correlation function and $Q = -k_z - k_z^i$. The integral in Eq. (83) can be calculated numerically.

6.2 Other methods and measurements related to SSA

The region of validity of SSA for surfaces with Gaussian height statistics and coefficients determined up to second order in height is given by Eq. (82). Only a few works on validating the SSA have been found in the literature. In [30] a comparison of SSA with BRDF measurements performed on an anodized Aluminium surface at wavelengths 10.6 μm and 3.39 μm has been performed using the first order SSA (with coefficients determined using SPM). The results show good agreement for scattering angles less than $\pm 45^o$ which is consistent with the validity range for low order SSA above. In [31], a comparison between the lowest order SSA and an more exact numerical method is performed for backscattering in the microwave region from ocean-like surfaces. This comparison also show that the SSA follow the more exact numerical results quite well but at gracing angles of incidence the error of the SSA predictions increase. A comparison between the SSA model, to first and second order, and an maybe exact numerical method (Method of Moments) has also been performed for the backscattering case in [32]. It is again found that the lowest order SSA model gives good predictions for low incident angles but that the error increase for larger angles of incidence. For the second order SSA model the prediction is found to be good also for higher angles of incidence.

The SSA method with coefficients determined by (low order) SPM does not account for multiple scattering (see e.g. [33]). A method which is related to the SSA method is the Non Local Small Slope Approximation (NLSSA) [33, 34]. This method takes multiple scattering into account, but requires an extra integration in the expression for the scattering amplitude.

7 Other surface scattering methods and volume scattering

In this section we will very briefly present some methods and models for scattering from rough surfaces which have not been mentioned in previous sections. We only present some examples of models and we have made no attempt to do a complete survey. These methods include the numerical methods, some of which are numerically exact, facet models in geometrical optics and empirical parameter models. All these models and the models presented in previous sections are models for scattering from a two-dimensional rough surfaces but in this section we will mention a few references to models for scattering from multiple layers and models for scattering in the bulk (volume scattering).

For further details of the models in this section and the reader is referred to the references for more details.

7.1 Numerical methods

We mentioned already in the introduction (Section 1) that methods for calculating scattering of electromagnetic fields from rough surfaces can broadly be categorized into approximate, analytical methods and numerical simulation methods. Numerical simulation methods for rough surface scattering have become increasingly popular in the last few decades. Early works on numerical simulation of rough surface scattering were mostly focused on one-dimensional rough surfaces but over the previous decade, as computational resources have increased, attention in the numerical simulation has turned more to simulation of scattering from two-dimensional rough surfaces. The analytical theories, some of which were presented in Sections 3 to 6), are often quite accurate but a problem is the lack of a precise characterization of their domain of validity. The numerical simulation methods can therefore be used to validate the approximate scattering models. The numerical simulation methods sometimes require long computation times and large computer memories. However, some of the more advanced approximate methods also require heavy computation to calculate high order expansion terms. In such cases the numerical simulation methods could be a better choice.

Many numerical simulation methods for rough surface scattering, and applications of these, can be found in the literature. Up-to-date reviews on numerical simulation methods can be found in for instance [8] and [35]. An example of a book that presents some of the most common numerical simulation methods for rough surface scattering in more detail is the book by Tsang, Kong, Ding and Ao [36].

Numerical methods for scattering from rough surfaces can roughly be classified into three categories: (a) those based on approximations; (b) integral equation methods; and (c) differential equation methods. The first type of methods solve some approximation of Maxwell's equations, whereas the other two groups of methods solve Maxwell's equations in differential form or in an equivalent integral representation. Below we briefly discuss two numerical methods based on approximations, *the geometrical optics approximation with ray tracing* and *the physical optics method*, one example of an integral equation method, *the Method of Moments (MoM)*, and one example of a differential equation method, *the Finite-Difference Time-Domain method (FDTD)*. As was mentioned in Section 3, the analytical Kirchhoff method is sometimes called the physical optics method, and a geometrical optics solution can also be obtained from the analytical Kirchhoff method in the very rough surface limit or by using a statistical facet model as will be described briefly in Section 7.2. The difference between these analytical results and the corresponding numerical geometrical optics and physical optics methods lays mainly in the manner the rough surface is described. In the analytical methods the random rough surfaces are treated statistically, and in the corresponding numerical methods a deterministic (geometrical) description of an individual rough surface is needed.

Most numerical methods for scattering from rough surfaces make use of the Monte Carlo method to average the deterministic scattered fields due to individual surfaces over an ensemble of realizations [37]. The scattered fields are calculated for each individual surface. Then the average scattered radiance or amplitude is calculated.

Geometrical optics - Ray tracing

The geometrical optics, or ray optics approximation, can be used with very rough surfaces whose typical length scales, as the RMS height, are larger than the wavelength. The numerical technique of ray tracing is implemented by launching a large number of incident rays on the surface and each ray is traced through its reflections on the surface until it finally escapes. The energy scattered in a given direction is calculated by counting the number of rays that leave the surface in that direction. For a presentation of the ray tracing technique in rough surface scattering, see for instance [38].

We would also like to mention that the ray tracing technique is a method used in some computer graphics and computer scene simulation software. An example of such a software for simulation of scenes (terrain, sky and objects) in the infrared and visible regions of the spectrum is CameoSim^{TM} [39, 40]. Codes like CameoSim^{TM} also makes it possible to apply parametrized BRDF functions on surfaces in the scene. In order to combine the angle dependent reflectance described by the BRDF with the ray tracing technique, a Monte Carlo technique is usually used [41].

Physical optics - Kirchoff approximation

The physical optics approach is based on numerical computation of the scattered fields from the Kirchhoff approximation (see Section 3) for the surface current on the surface, which only requires a direct integration over the surface. The analytical statistical averaging used in the analytical Kirchhoff method is therefore not performed in the numerical physical optics method. For a presentation of the physical optics method, see for instance [42]. The physical optics method has the same limitations (region of validity) as the analytical Kirchhoff approach. It should also be mentioned that physical optics is a traditional method for computing radar cross sections.

Method of moments The Method of Moments (MoM) is a numerical method that solves Maxwell's equations in an integral form (see also Section 2). In the case of a dielectric surface, a coupled pair of surface integral equations are used. Since MoM solves Maxwell's equations, the errors produced by this method comes from numerical errors (or from errors in the characterization of the surface, including material parameters). The MoM [36, 43, 44] is a procedure for discretizing an integral equation by expanding the unknown surface currents in a linear combination of a set of basis functions having support on the surface. A set of test functions is also introduced for the fields received from the radiating surface basis functions. This produces a finite dimensional linear equation system, the solution of which can be used to calculate the scattered field. The rough surface is generally discretized in a surface mesh (a facetted surface) and the average mesh length, compared with the wavelength, determines the accuracy of the method. For surfaces with many facets, i.e. small mesh length, the method can be expensive on memory and computing time.

Since the MoM can calculate the scattered fields with high accuracy, it is sometimes used to validate approximate analytical models (see for instance [32]). The MoM is commonly used for scattering problems in the microwave region.

Finite-difference time-domain method
The Finite-Difference Time-Domain method (FDTD) is a method which solves Maxwell's equations in differential form. It is based on approximating the derivative operators in Maxwell's equations by using the values of the electromagnetic fields on the nodes on a grid. The discretized equations are used to calculate the values of the electromagnetic fields. The fields at different times are calculated incrementally by calculating the field values at a given time from the field values obtained in previous time steps. For details about FDTD see [36, 45] and references therein. An advantage of FDTD, being a time-domain method and not a frequency-domain method, is that broadband (an interval in wavelength) information about the scattering can be obtained directly. As for MoM, FDTD can be expensive on memory and computing times if scattering from rough surfaces which are not translationally invariant in one direction is considered.

Since FDTD solves Maxwell's equations it can provide very accurate results. It is therefore sometimes used for validation of approximate analytical models. An example of this is given in [46]. FDTD is a very general method, and is therefore applied to a wide range of problems in the optical as well as microwave and radio frequency regions.

7.2 Facet and empirical models for BRDF

The methods for calculating scattering from random rough surfaces which we briefly presented in Sections 3 to 6 are all models based on reasonably well understood approximations of Maxwell's equations. These models, at least to some extent, accounts for the wave nature of electromagnetic fields. In the literature one can also find parametric (empirical) models and statistical (facet) models based on geometrical optics for describing BRDFs.

The empirical parametric models can in some cases be pure mathematical functions with free parameters which are curve fitted to measured BRDF data. Other parametric models are to some extent based on physical principles or the knowledge of the form of the solution to an analytic model like for instance the Kirchhoff approach or the SPM. An example of a "parametric type" model is the *OPTASM BRDF model* [47], in which the angular characteristics are specified by a number of Lorenzian shaped peaks. The "parametric" models for BRDF are often used in computer codes for three-dimensional graphics (visualization) and scene simulations to account for angle dependent reflection and emissivity.

The statistical (facet) models, based on geometrical optics, are of course applicable for very rough surfaces whose typical length scales, in particular, the RMS height, are greater than the wavelength. In these models the rough surface is usually considered to be composed of facets with a known distribution. Light is reflected specularly from these facets and sometimes, like in the model by Torrance and Sparrow [48], a diffuse component is also added to account for multiple scattering and/or internal scattering. A further development of the model by Torrance and Sparrow, where the effect of roughness on the diffuse component is included was given by Ginneken et al. in [49]. These models are also popular in computer graphics applications. An example of a statistical model, based on geometric optics, which accounts for multiple scattering and shadowing effects is [50].

7.3 Other analytical methods

In Sections 3 to 6 we gave brief introductions to four approximate, analytical methods for calculating the scattering of electromagnetic waves from rough surfaces. Of these methods, the ordinary Kirchhoff approach and the method of small perturbations (at low order) can be viewed as early (classical) approaches to surface scattering. The integral equation methods and the small slope approximations are more recent methods, developed as attempts to extend the theories to larger domains of validity than the Kirchhoff approach and the method of small perturbations.

When there are two relevant scales in the surface roughness spectrum (one large and one small) a method or procedure called the *Two-Scale (Composite or Hybrid) Model* [51, 52, 53] is sometimes used. In the literature, the two-scale model has primarily been used to calculate scattering from sea surfaces in the microwave region. Two different models are used for the small scale and the large scale roughness. The results from the small scale roughness model model can be used by incoherently averaging the effective incidence and scattering angles with the large scale slopes. This is usually done in four steps:

1. The surface height fluctuation is divided into a large scale and a small scale fluctuation, each with its own spectrum. The total spectrum is the sum of the two spectra.

2. The small scale spectrum is used to compute the bistatic scattering coefficient, $\sigma_{pq}^{(SPM)}(\theta, \theta_s, \phi_s)$, using the method of small perturbations (see for instance Eq. (68) in Section 5).

3. By using the large scale statistics tilting angle statistics (Ω), through the probability distribution function $P(\Omega)$, the SPM bistatic scattering coefficient is averaged,
$\overline{\sigma}_{pq}^{(SPM)}(\theta, \theta_s, \phi_s) \equiv \int d\Omega\, P(\Omega)\, \sigma_{pq}^{(SPM)}(R^{\Omega}[\theta, \theta_s], \phi_s)$, where
$R^{\Omega}[\theta, \theta_s]$ stands for the local incident and scattering angles after rotation to the locally tilted reference frame.

4. Finally,using the large scale portion of the spectrum the Kirchhoff approximation bistatic scattering coefficient, $\sigma_{pq}^{(k)}$, (see Section 3) is added to the SPM bistatic scattering coefficient to account for the specular part missing in the SPM: $\sigma_{pq}^{(total)} = \overline{\sigma}_{pq}^{(SPM)} + \sigma_{pq}^{(k)}$.

One of the problems with the two-scale model approach is that the real surface is not correctly described by two scales, and some ad hoc method must be used to divide the spectrum. Furthermore, some more recent methods, like IEM (see Section 4), can in some cases predict scattering from two-parameter, Gaussian correlated surfaces better than the two-scale model [5].

Although the methods presented in Sections 3 to 6 seem to be amongst the most common (and popular) in the literature, a number of other analytical methods for calculation of scattering from rough surfaces can also be found in the literature. Here we essentially limit ourselves to providing the names of a few methods and give one reference to each of these. These methods, like the SSA method and the IEM, are attempts to extend the region of validity of the ordinary Kirchhoff approach and the SPM.

Examples of perturbation theory extensions are the *phase perturbation technique* [54] and *the unified perturbation method* [55]. The *smoothing method* [56] is also an extension of the small perturbation method but in contrast to the previously mentioned methods it is an operator based approach. In operator based approaches the field quantities are taken to be random variables before application of the integral scattering operators. A method which results in an expansion of the scattering amplitude in powers of quasi-slopes is the *quasi-small-slope approximation* [57]. In this respect the quasi-small-slope approximation method resembles the SSA method. This method has been shown to provide a continuous transition from the results of SPM to the results of the Kirchhoff method by continuously changing the wavelength. Examples of other analytical theories for scattering from rough surfaces found in the literature are *the heuristic scattering model* [58], *mean field theory approaches* [59], *the tilt-invariant scattering theory* [60] and *local spectral expansions* [61].

The methods presented in Sections 3–6 and the methods mentioned above calculate the scattered field from the incident field, the surface topography (using a statistical characterization) and material parameters (dielectric permittivity and magnetic permeability). However, methods have also been developed for solving the inverse problem, which amounts to calculating the surface characteristics (topography) from known scattering properties. Examples of such methods can be found in for instance [62, 63].

7.4 Scattering in multi-layer systems and volume scattering

In the present overview of scattering models we have focused on the problem of scattering of electromagnetic waves from a random rough surface interface (boundary) between two media. However, in some situations there could be a cascade of rough surface layers on which the electromagnetic fields are scattered, as is schematically illustrated in Figure 6. Materials designed for low signature purposes in the infrared region are sometimes built up from more than one layer. This could for instance be a paint on a metal surface or several layers of paint on a surface. This situation would involve calculating the scattered radiance from a cascade of layers with rough surface interfaces. We will not describe the different models and theories for treating multi-layer systems but many of these models are based on the knowledge of a solution for scattering from a single rough surface through for instance one of the methods mentioned in previous chapters. A few approaches to scattering in multi-layer systems can be found in [4, 5, 64] and the references therein.

Another important case for scattering of electromagnetic waves, often encountered in real cases is scattering in the bulk, or volume, of the material. This could be some kind of particles enclosed in a homogenous medium on which the electromagnetic radiation is scattered. Paints for instance often consists of color pigments (flakes or particles) enclosed in some binding (matrix) material. We could also have the case that we have

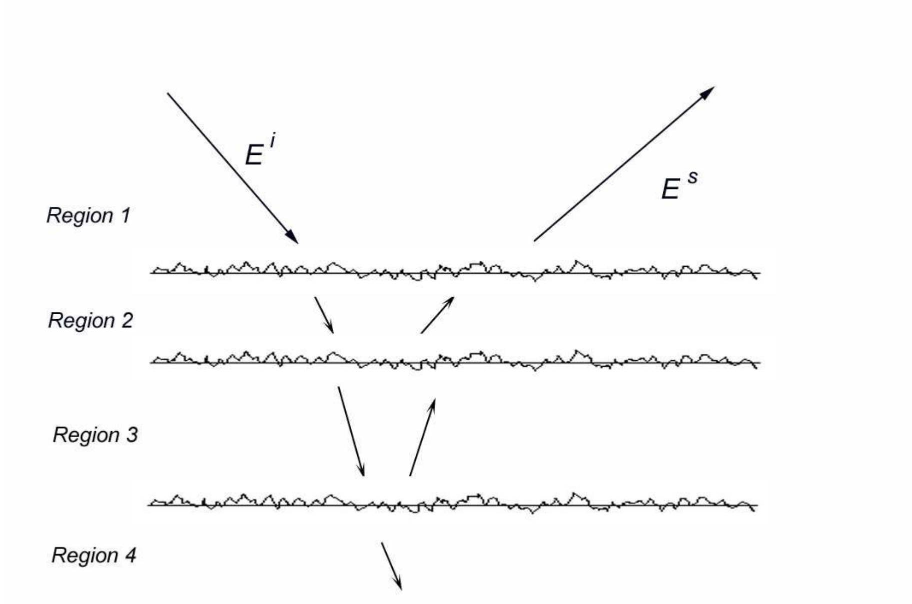

Figure 6: *Schematic illustration of scattering from three-layered rough surfaces.*

a random medium for which the scattering originates from permittivity (or refractive index) fluctuations which can not be described as particles. The classical example of this is the scattering of light in a turbulent medium where the permittivity fluctuations originates from the variation in density, temperature and so forth in a turbulent medium. Although modelling of volume scattering is important for many practical applications we will not present these theories here. Some of the more popular methods for volume scattering of electromagnetic waves are presented in [4] and the references therein.

It should also be mentioned that it is in some cases possible to treat volume scattering by homogenization of the heterogeneous medium [65, 66]. Examples of such methods are *effective medium theories*, *mixing formulas* and *mathematical homogenization*. A common restriction of such methods is that the particles should be much smaller than the wavelength, although there are exceptions to this limitation.

8 Summary and conclusions

We have presented the results from a literature search of models for scattering of electromagnetic waves from random rough surfaces. In particular, we have focused on the calculation of the *bidirectional reflectance distribution function* (BRDF) (or the related quantity *bistatic scattering coefficient*). We have remarked that the BRDF is the quantity usually used in IR and visible signature prediction (IR scene simulation) software to account for scattering of IR light from a rough surface. Therefore, methods for calculating the BRDF from surface topography and material parameters can be of great use in IR signature modelling and assessment. Models for rough surface scattering can be used, in combination with signature simulation software, to study, predict and optimize the influence of rough surface characteristics on optical signatures.

The present literature search has shown that several analytical models for scattering from random rough surfaces can be used for calculation and prediction of BRDF in the optical (IR, visible and UV) region of the electromagnetic spectrum. Much of the work on application and validation of these methods has been performed in the microwave region but the models themselves are not limited to the microwave region. The ranges of validity of the models are determined by the wavelength compared with parameters describing the surface roughness topography. For rough surfaces which are described statistically, examples of such parameters are the root mean square surface height, the correlation length and the root mean square surface slope.

The methods for calculating scattering of electromagnetic fields from rough surfaces which have been found in the literature can broadly be categorized into approximate, analytical methods, and numerical simulation methods. In this report we have mainly focused on four different analytical methods which have been found to be amongst the most studied and common in the literature. These four methods (or actually classes of methods) are: *The Kirchhoff Approximation*, *The Method of Small Perturbation*, *The Integral Equation Method* and *The Small Slope Approximation*. Of these, the first two, are amongst the early approaches to scattering from rough surfaces which however are still much used in applications. The latter two methods are examples of more recent approaches which have been developed as attempts to extend the validity of the former methods. In the following we will very briefly summarize some of the main features of these four methods.

The methods which have been studied have all a region of validity in which they can be considered valid. These regions of validity can in generally not be determined exactly. However, a number of criteria for each method can be found in the literature and therefore some rules of thumb can be used to determine if the model is applicable for a particular case. In Table 1 we have summarized some of the criteria (rules of thumb) for the different models and for the case of Gaussian height statistics and Gaussian correlation function. These criteria have been obtained for rough surfaces with Gaussian height distribution and Gaussian correlation function. The criteria in Table 1 should not be taken as unconditionally true. There may be situations where these criteria can be exceeded and the methods still give satisfactory results. Also, even though the criteria are satisfied, it does not always ensure sufficient accurate results. In the same way, the criteria for different models cannot be compared unconditionally since they do not tell the whole story about the validity and since they have been obtained in different ways. However, it is clear, from the criteria in the table, from comparisons with measurements and from the theory of the methods, that the IEM and the SSA in general have larger ranges of validity than the Kirchoff method and (low order) SPM. Many real surfaces have surface roughness statistics and optical constants which lie within the region of validity of one or several methods but in the end it is the validation of the model against measured data that determines the range of validity. Although some comparisons with measured data can be found in the literature, it is of interest to implement the models and compare them with measurements. It is of course also of interest to mutually compare results obtained with the various methods and to compare the analytical models to more exact numerical methods.

Of the four models that have been studied in detail, the IEM and the SSA have the largest ranges of validity. In fact their ranges of validity can be said (with some minor exceptions) to cover the ranges of validity of the Kirchhoff approach and the SPM. However, the SSA and IEM are more complex methods which are more difficult to implement and therefore the other two methods are still of value. Although the methods themselves do not assume a particular surface height statistics, the analytical expressions which have been derived for the mean scattered intensity and the bistatic scattering coefficient (or BRDF) are for rough surfaces with Gaussian height distribution. Works on other types of surface height statistics can also be found in the literature but in those cases a numerical method is most often used (see e.g. [25]). Another common limitation in the considered methods is that they all are less accurate for grazing incidence.

The Kirchhoff approximation

In the Kirchhoff approximation approach to scattering from rough surfaces the electromagnetic fields at any point of the surface are approximated by the fields that would be present on an infinitely extended tangent plane at that particular point on the surface. The Kirchhoff approximation (also called the tangent plane approximation) therefore amounts to assuming that the reflection at each point of the surface takes place by approximating the profile around this point by its tangent plane. The reflection is therefore said to be locally specular. This means that the assumption for the Kirchhoff approximation is that the incident and scattered fields from an infinitely large surface are related (linearly) through the Fresnel reflection coefficient, which is a function of the local incidence angle and polarization. In this way the integral equations from which the scattered field can be calculated are reduced to integration of the incident field over the surface. For rough surfaces with gaussian height statistics, analytical expressions for the mean scattered field and the BRDF can be obtained for the cases of either very rough surfaces or slightly rough surfaces. Due to the tangent plane approximation, the basic limitation of the Kirchhoff method is that the local curvature (and therefore the local slope) on the surface must be small compared with the wavelength (see also Table 1 at the end of this section). The ordinary Kirchhoff method does not by itself account for multiple scattering or shadowing effects on the surface. Although shadowing can partially be accounted for by introducing a shadowing function, the Kirchhoff method is increasingly less accurate for increasing angles of incidence.

The small perturbation method (SPM)

The Small Perturbation Method (SPM) belongs to a family of perturbation expansion solutions to the wave equation. The basic idea is to find a solution in terms of plane waves that match the surface boundary conditions. The surface fields are expanded in a perturbation series, $\mathbf{E} = \mathbf{E}^0 + \mathbf{E}^1 + ...$ where $\mathbf{E}^i$ depends on the n-th power of the surface elevation and its gradient. The underlying assumption of the SPM is therefore that the surface heights and the surface slopes are small compared with the wavelength. In the expansion $\mathbf{E}^0$ would be the surface field if the surface was flat. So this method applies simultaneously to surfaces with small surface height variations and small surface slopes, independently of the radius of curvature. Therefore, the surface need no longer be approximated by planes as in the Kirchhoff approach. Using this method, the mean scattered field and the BRDF can be obtained, at a given order of the perturbative expansion, for rough surfaces with gaussian height statistics. The complexity of deriving expressions for the BRDF increases with increasing order in the perturbative expansion. In practice only the fist few orders of the expansion can be calculated.

The integral equation method (IEM)

In the Kirchhoff approximation described earlier, the tangential fields on the rough surface are expressed in terms of the incident field by using the tangent plane approximation. In the integral equation method it is noted that the exact tangential fields can be written as a sum of the tangential field obtained in the Kirchhoff approximation plus a complementary term, which is an integral over the surface of the fields and their derivatives. This gives a set of integral equations for the surface fields. To be able to calculate these integrals a number of approximations and simplifications are made. It turns out that the contribution from the complementary terms to the scattered field can be expressed in terms of the Fresnel reflection coefficients and normal and tangential components of the surface fields. These normal and tangential surface fields can then be expressed in terms of the incident field by using the Kirchhoff approximation. This makes the integral equation method an iterative method: The tangential surface fields are expressed as a sum of the Kirchhoff fields plus a complementary term and the complementary term can be calculated by using the Kirchhoff approximation on tangential field components. The basic, underlying, assumption of the integral equation method is therefore that the tangential surface fields can be approximated by the tangent plane approximation. However, it should be noted that it does not give the same result as the Kirchhoff approximation since it is an iterative method where the tangent plane approximation is used in the iteration. Since the Kirchhoff approximation is used in IEM, shadowing effects have to be accounted for by separately introducing a shadowing function.

For surfaces with gaussian statistics, expressions can be derived for the mean scattered field and for the BRDF. Separate expressions are derived for very rough surfaces, and for slightly or moderately rough surfaces. IEM has a range of validity which cover, and exceeds, the ranges of validity of both the ordinary Kirchhoff method and the (low order) SPM (see also Table 1). Furthermore, IEM does account for multiple scattering which the ordinary Kirchhoff method does not do.

The small slope approximation (SSA)

In the SSA, the scattered field is expressed in terms of a plane wave expansion where the coefficient function in the expansion is called the scattering amplitude. Once the scattering amplitude has been determined, the scattered field can be calculated. The scattering amplitude is written in a form which is consistent with known transformation properties of the scattering amplitude. It turns out that the scattering amplitude, or rather coefficient functions entering the expression for the scattering amplitude, can be written as an expansion in the surface slope. In the general formulation of SSA, the roughness slope is the only parameter required to be small. The coefficient functions in the expansion can be determined by identifying terms of different order in surface height in the SSA with terms of different orders of the surface height in the small perturbation method (SPM). It should be noted that the SSA expansion is in terms of the surface slope and not the surface height which is used to determine the unknown coefficients in the expansion in slope. For the first order SSA, the criterion for SSA can be stated as (see Table 1): The slopes of the irregularities should be small enough and at least smaller than the grazing angles of the incident and the scattered waves. This means that the SSA becomes less and less accurate for increasing incident and scattering angles. This has been verified by comparison with measured data and numerical methods.

The smallness parameter in the SSA, the surface slope, differs from those used in classical perturbation theory and Kirchhoff approximation since it is independent of wavelength. The SSA theory gives a reduction to perturbation theory which is intrinsic to the formulation, and it reduces to the Kirchhoff approximation under the appropriate conditions when the

Method	Slightly rough surfaces	Rougher surfaces	Comments
Kirchhoff	$kL > 6$, $kL > 4.17\sqrt{k\sigma}$ and $\sigma\sqrt{2} < 0.25L$ (k=wavevector, L=correlation length, σ=RMS height)	$kL > 6$, $kL > 4.17\sqrt{k\sigma}$ and $k\sigma > \frac{\sqrt{10}}{\vert\cos\theta^i + \cos\theta^s\vert}$ (θ^i =incident angle, θ^s = scattering angle)	The Kirchhoff method becomes less accurate for grazing incidence due to shadowing.
SPM	$k\sigma < 0,3$ and $0.3kL > \sqrt{2}k\sigma$ (first order SPM)		Underlying criterion: $k\vert f(x,y)\vert \ll 1$, $\vert\nabla f\vert \ll 1$
IEM	$(kL)(k\sigma) < 1.2\sqrt{\epsilon_r}$, $k\sigma < 2$ (ϵ_r =relative dielectric permittivity)	$kL > 5$ (probably not a complete criterion)	The method becomes less accurate for grazing incidence due to shadowing.
SSA	$kL \gg \sqrt{2}\sigma/\cos\theta$ (θ is the incident or scattering angle)	$kL \gg \sqrt{2}\sigma/\cos\theta$	Underlying criterion is $\vert\nabla f\vert \ll k_z/k$. Meaning: Slopes of irregularities should at least be smaller than the grazing incidence and the scattering angles.

Table 1: *A list of "rule of thumb" criteria found in the literature for the Kirchhoff method, SPM, IEM, and SSA.*

first two terms of the series are retained. The SSA extends the range of validity of the SPM to rough surfaces with larger heights (i.e. rougher surfaces).

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