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Sara Linder, Otto Tronarp and
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A Kurtosis-based Approximative Method for Intersystem Interference Analysis

Abstract

The problem to determine the impact on digital communication systems due to electromagnetic interference often requires complex calculations and analyses. Therefore, there is a need for approximate methods suitable for computer-based tools for intersystem interference analyses. Here we have investigated the impact on a digital radio receiver from different kinds of pulsed interfering signals. We also estimate the kurtosis for these signals, both analytically and through simulations. Furthermore, we propose a low-complexity method that uses the kurtosis measure to estimate the impact from intersystem interference on a digital radio receiver. Such a method can for example be used in a tool for intersystem interference analyses.

Keywords

Intersystem interference, digital communication systems, AWGN-approximation, higher-order statistics, cumulants, kurtosis, pulsed signals, intersystem interference analysis tools

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En approximativ metod för telekonfliktanalys baserad på kurtosis

Sammanfattning

Att bestämma påverkan på ett digitalt radiosystem som orsakas av en elektromagnetisk interferens kräver ofta relativt komplexa beräkningar och analyser. Det finns därför ett stort behov av approximativa metoder som är lämpade för datorbaserade verktyg avsedda för telekonfliktanalyser. Vi har undersökt olika typer av pulsade signalers påverkan på en digital mottagare. Kurtosis har skattats för dessa signaler, både analytiskt och genom simuleringar. Vi föreslår även en metod, med låg komplexitet, som använder kurtosis för att skatta påverkan på en digital mottagare. En sådan metod skulle till exempel kunna användas i ett telekonfliktverktyg.

Nyckelord

Telekonflikter, digitala kommunikationssystem, AWGN-approximation, högre ordningens statistik, kumulanter, kurtosis, pulsade signaler, telekonfliktverktyg

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Contents

Chapter 1

Introduction

1.1 Background

The use of wireless communications increases rapidly both in the civilian and military society. The ongoing transformation of the Swedish Armed Forces towards a Network Based Defence (NBD) will lead to an increased amount of equipment for wireless communications and information processing on, and in the proximity of, military platforms. Combined with reduced defense budgets this transformation is anticipated to increase the use of civilian electronics in military applications. The emission specifications for civilian electronics, such as computers and printers, allows considerably higher emission levels than for military specified equipment. With this foreseen increase of radiating equipment in the vicinity of radio receivers, it is obvious that intersystem interference is destined to increase. At the same time, the need for reliable communications increases, since the distribution of information collected at different platforms to those who need the information (the right information at the right place and at the right time) is a critical part of the NBD concept. Wireless communications will continue to play an important role in future military communication networks. In many scenarios it is the only feasible communications solution, e.g. for airplanes, ships, and for the so-called last tactical mile. Hence, it is important to develop efficient tools in order to judge the effect on radio receivers due to intersystem interference. We must be able to analyze an intersystem interference situation, or to plan a co-location of different systems, so that the level of intersystem interference remains manageable.

The computations necessary to determine the performance degradation on a radio receiver, caused by intersystem interference, are often quite complicated. Thus, simplified methods, with low computational complexity, are desired in computer-based tools for intersystem interference analysis. One commonly used simplified method is to approximate the interfering signal as additive white Gaussian noise (AWGN); however, it is often difficult to evaluate the quality of the AWGN-approximation. The AWGN-approximation has previously been shown to perform fairly well for modulated interference signals, i.e. interference caused by other transmitters [6, 8]. However, it should be used with caution for pulsed interference signals since it can lead to large underestimations of the resulting bit error probabilities (BEP) in the radio receiver. Also, a higher-order statistics (HOS) measure, the kurtosis, has shown to be useful when attempting to distinguish between modulated and periodic pulsed interference [5, 6]. This opens the possibility to use HOS in computer-based tools for intersystem interference analysis.

1.2 Contributions

In this report we show that it is possible to use higher-order statistics (i.e. the kurtosis) to determine if the use of the AWGN-approximation for a pulsed interference leads to an over- or underestimation of the bit error probability (BEP). This has previously been shown for periodic pulses with fixed amplitude and arrival time [5, 6]. Here we show that this is also the case for pulses with randomly distributed amplitude and arrival time. The kurtosis for these pulsed signals is also estimated, both analytically and through simulations.

Furthermore, a low complexity method for estimating the impact of intersystem interference on digital radio systems is proposed. It is intended for use in computer-based intersystem interference analysis tools. In the method, we propose to use a kurtosis measure as a threshold to decide if the AWGN-approximation can be used or not. For kurtosis values close to zero, the AWGN-approximation can be used safely. For higher kurtosis values, two other approximate methods are used. For periodic pulses, with constant amplitude, that are longer in duration than the bit time, it is possible to estimate the resulting BEP in the receiver with use of the estimated kurtosis value. Finally, when the pulse is shorter than the bit time we also need to estimate the relationship between the pulse repetition time and the bit time in order to estimate the BEP. All the methods need to know both the average ratio between the desired signal and the noise and the average ratio between the desired signal and the interference.

1.3 Outline

The outline of the remainder of this report is given here. In Chapter 2, the effects of intersystem interference on a binary phase-shift keying (BPSK) receiver is described. Also, the validity of the additive white Gaussian noise (AWGN) approximation is discussed. Thereafter, in Chapter 3, a short introduction to the basics of higher-order statistics is given. This theory is then used, in Chapter 4, in order to derive analytical expressions fort he kurtosis for various pulsed interference sources. The kurtosis is derived for periodic pulses or for pulses with Poisson distributed arrival times, as well as for pulses with constant amplitude or with a uniformly distributed amplitude. In Chapter 5, the performance degradation for an uncoded coherent BPSK-receiver caused by pulsed interference is shown. The main result is that the AWGN-approximation can be used for practical purposes for pulsed interference with low kurtosis values. However, for pulsed interference with high kurtosis values, the AWGN-approximation may yield unacceptably large underestimations of the resulting bit error probabilities. Based on these results, we derive a low-complexity kurtosis-based intersystem interference analysis method in Chapter 6. The proposed method is of interest for use in, for instance, computer-based intersystem interference analysis tools. Finally, the main conclusions are summarized in Chapter 7.

Chapter 2

Intersystem Interference in Digital Communication Systems

In a wireless communication scenario, the antenna typically receives the desired signal as well as noise and various interfering signals. These undesired signals may originate from various wireless communication transmitters, for example from other military radio systems that are sharing the same frequency, from co-located military radio systems using adjacent frequency bands, or from out-of-band emissions from civilian wireless systems. These intersystem interference signals generally consist of modulated signals and they can sometimes be received at relatively large distances.

Other intersystem interference sources are different kinds of electronic equipment, such as computers, faxes, and micro-wave ovens. All electronic equipment unintentionally emits electromagnetic energy. These intersystem interference sources are relatively weak, but they can seriously deteriorate the performance of radio receivers if they are placed in the vicinity of the receiver. The performance degradation caused by intersystem interference can, for example, result in a decreased capacity or a reduced operating range [4].

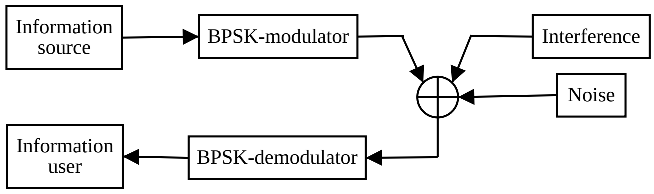

Figure 2.1: The BPSK receiver is subjected to the desired signal, interference signals, and additive white Gaussian noise.

2.1 The Effects of Intersystem Interference on a BPSK Receiver

In Figure 2.1 a typical intersystem interference scenario can be found. The binary phase-shift keying (BPSK) receiver is subjected to the desired signal, interference signals, and additive white Gaussian noise (AWGN). To describe this scenario we use the signal-to-noise ratio, SNR, and the signal-to-interference ratio, SIR.

The BPSK receiver used in this report is a coherent receiver optimized for AWGN. Also, the information is memoryless and the decisions are made bit-by-bit. Furthermore, no error-correcting codes are used.

BPSK is a basic modulation scheme and other popular modulation schemes such as minimum-shift keying (MSK) and quadrature phase-shift keying (QPSK) are based on decision-making algorithms that can be related to the decision-making algorithms of BPSK. Finally, the performance degradation (in terms of BEP) in a BPSK-receiver, due to interference, does not differ much from other modulation schemes. Although the absolute performance in terms of BEP differ for different modulation schemes their general behavior in the presence of interference are similar. Hence, conclusions made on results for BPSK can often be extended to other related schemes.

For a digital radio receiver, the bit error probability (BEP) is a widely used measure of the receiver's performance. For AWGN interference there exist well-known equations that can be used to determine the BEP. In other cases it is necessary to calculate the BEP through a comprehensive analysis of the digital radio receiver. This can be difficult and the assumption commonly used to make these calculations easier is to model the noise, or interfering signal, as AWGN with the same mean power as the interference. This simplifies the calculations, but this approximation is not always valid. Furthermore, it is often difficult to evaluate the reliability of the AWGN-approximation.

When estimating the BEP, the SNR and SIR are important. However, the SNR and SIR are not sufficient as the waveform of the interfering signal can be of great importance for the impact on the receiver.

2.2 The AWGN-approximation

It has previously been shown, in [6], that when approximating the interfering modulated signals as AWGN, the BEP is overestimated. Furthermore, it is better to make an overestimation of the BEP, rather than underestimating the BEP, since that could lead to serious communication problems. Hence, the AWGN-approximation is valid, for practical purposes, for modulated signals.

In [8] it is shown, through a comprehensive theoretical analysis, that the AWGN-approximation is valid also for an MSK receiver when subjected to an interfering MSK signal.

However, the AWGN-approximation cannot be used safely for periodic pulsed signals, i.e. without risking an underestimation of the BEP. Previously, in [5, 6], it was shown that a higher-order statistics measure, the kurtosis, could be used to indicate the reliability of the AWGN-approximation for periodic pulsed interference with fixed amplitude. This type of pulsed interference with kurtosis values close to zero could be approximated as AWGN without seriously underestimating the BEP. However, for higher kurtosis values the AWGN-approximation could result in large underestimations of the resulting BEP [5, 6].

As man-made noise in many cases do not have fixed parameters, it is desirable to see if the kurtosis can be used also for a more general pulsed source, i.e. with randomly distributed amplitude and arrival time. In this report we continue the work in [6] by investigating pulsed interference where the arrival times are given by a Poisson process and the amplitudes are uniformly distributed.

Chapter 3

Higher-Order Statistics Theory

Higher-order statistics (HOS), also known as cumulants, are an important tool in many different fields; e.g. image reconstruction, time-delay estimation, adaptive filtration, array processing, blind equalization and modulation classification [7]. The interested reader is referred to [1, 2, 3] for comprehensive introductions to the field of HOS. The first-order cumulant is the mean and the second-order cumulant is also known as the covariance function. If a random process is symmetrically distributed its third-order cumulant equals zero [2]. Communication signals are generally symmetrically distributed; therefore, the fourth-order cumulants are used in communications applications. The kurtosis measure is an example of a forth-order cumulant. The kurtosis can be interpreted as the size of the distribution's tail. Hence, the kurtosis characterizes the shape of the distribution of the signal [7]. The impact of an interfering signal on a digital radio receiver depends on the distribution of the interfering signal. This is the motivation for examining the potential use of kurtosis-based measures in intersystem interference applications.

3.1 Cumulants

The *k*th-order cumulant can be defined in terms of a weighted sum of joint moments of orders up to *k*, see [3] for a definition of the cumulant-generating function. For a set of zero-mean, real-valued random variables $\{x_1, x_2, x_3, x_4\}$

the first-, second-, third- and fourth-order cumulants are [1]

$$\text{cum}(x_1) = E\{x_1\} = 0 \tag{3.1}$$
$$\text{cum}(x_1, x_2) = E\{x_1 x_2\} \tag{3.2}$$
$$\text{cum}(x_1, x_2, x_3) = E\{x_1 x_2 x_3\} \tag{3.3}$$
$$\begin{aligned}\text{cum}(x_1, x_2, x_3, x_4) = &E\{x_1 x_2 x_3 x_4\} \\ &- E\{x_1 x_2\}E\{x_3 x_4\} \\ &- E\{x_1 x_3\}E\{x_2 x_4\} \\ &- E\{x_1 x_4\}E\{x_2 x_3\}.\end{aligned} \tag{3.4}$$

3.2 Cumulants of Stationary Processes

The cumulants of stationary processes are most easily understood for real-valued stationary processes and these will be described herein. However, in communication applications the received data is normally complex-valued (baseband data with I- and Q-channels). Therefore, the cumulants for complex-valued stationary processes are also defined.

3.2.1 Real-Valued Stationary Processes

If $x(n)$ is a real-valued stationary random process and its moments up to order *k* exists, then the cumulants depend only of the time differences [3]

$$c_k(\tau_1, \tau_2, \ldots, \tau_{k-1}) = \text{cum}[x(n), x(n+\tau_1), \ldots, x(n+\tau_{k-1})]. \tag{3.5}$$

Hence, for stationary processes, the *k*th-order cumulants are only functions of the lags $\tau_1, \tau_2, \ldots, \tau_{k-1}$.

Furthermore, by assuming that the (real-valued) process is zero-mean and setting $\tau_1 = \tau_2 = \tau_3 = 0$ we get [3]

$$C_1 = c_1 = E\{x(n)\} = 0 \quad \text{(mean)} \tag{3.6}$$
$$C_2 = c_2(0) = E\{x^2(n)\} \quad \text{(variance)} \tag{3.7}$$
$$C_3 = c_3(0,0) = E\{x^3(n)\} \quad \text{(skewness)} \tag{3.8}$$
$$C_4 = c_4(0,0,0) = E\{x^4(n)\} - 3(C_2)^2 \quad \text{(kurtosis).} \tag{3.9}$$

In order to make the kurtosis value independent of the power of the signal the kurtosis is normalized by the variance of the signal, i.e.

$$K_4 = \frac{C_4}{(C_2)^2}. \tag{3.10}$$

3.2.2 Complex-Valued Stationary Processes

For a complex-valued zero-mean stationary random process $y(n)$ the second-order moments can be defined in two different ways depending on placement of conjugation. The variance can be defined as [1]

$$C_{20} = E\{y^2(n)\} \tag{3.11}$$

$$C_{21} = E\{|y(n)|^2\}. \tag{3.12}$$

Similarly, the kurtosis measure can be defined in three ways

$$\begin{aligned} C_{40} &= \text{cum}(y(n), y(n), y(n), y(n)) \\ &= E\{y^4(n)\} - 3E^2\{y^2(n)\} \end{aligned} \tag{3.13}$$

$$\begin{aligned} C_{41} &= \text{cum}(y(n), y(n), y(n), y^*(n)) \\ &= E\{y^3(n)y^*(n)\} - 3E\{y^2(n)\}E\{|y(n)|^2\} \end{aligned} \tag{3.14}$$

$$\begin{aligned} C_{42} &= \text{cum}(y(n), y(n), y^*(n), y^*(n)) \\ &= E\{|y(n)|^4\} - |E\{y^2(n)\}|^2 - 2E^2\{|y(n)|^2\}. \end{aligned} \tag{3.15}$$

These three different versions of the kurtosis describe the complex process in different ways. For our application C_{42} is the most interesting measure, since it can be used in order to distinguish between modulated and pulsed signals.

3.3 Sample Estimates

In practice, the cumulants must be estimated from a finite set of sample data. The sample average is removed before the cumulant estimation in order to get a zero-mean process. The sample estimates of the correlations (with zero-lag) of a zero-mean complex-valued process, $y(n), n = 1, 2, \ldots, N$, can be calculated as [7]

$$\hat{C}_{20} = \frac{1}{N}\sum_{n=1}^{N} y^2(n), \tag{3.16}$$

$$\hat{C}_{21} = \frac{1}{N}\sum_{n=1}^{N} |y(n)|^2, \tag{3.17}$$

where $\hat{}$ denotes the sample average estimate. Similarly, the fourth-order cumulants can be expressed in three different ways, depending on the placement of conjugation [7]. One of the three kurtosis estimates, from (3.15), is

$$\hat{C}_{42} = \frac{1}{N}\sum_{n=1}^{N} |y(n)|^4 - |\hat{C}_{20}|^2 - 2\hat{C}_{21}^2. \tag{3.18}$$

The normalized kurtosis estimate is

$$\hat{K}_{42} = \frac{\hat{C}_{42}}{\hat{C}_{21}^2}. \tag{3.19}$$

The normalized kurtosis has a low computational complexity, of order N.

Chapter 4

Kurtosis for Pulsed Interference

In this chapter we will derive expressions for the normalized kurtosis for several different types of pulsed interference. We start by deriving a general expression for pulses with an arbitrary arrival process, an arbitrary distributed amplitude, and a uniformly distributed phase. From that we then derive expression for periodic pulses and pulses that arrive according to a Poisson process, with constant or uniform amplitude.

4.1 General Expression for the Kurtosis

Depending on the pulse length and the arrival process there can be several excited pulses at the same time. We let the discrete random variable $M(n)$ denote the number of active pulses at time n. Furthermore, we model each excited pulse, ξ, as having a random amplitude, denoted A_ξ, and a random phase, denoted Φ_ξ. Finally, we assume that the A_ξ:s are independent and identically distributed real random variables, that the Φ_ξ:s are independent and uniformly distributed on the interval $[0, 2\pi]$, and that A_ξ, Φ_ξ and $M(n)$ are independent. We can now write the resulting signal $y(n)$ as

$$y(n) = \sum_{\xi=1}^{M(n)} A_\xi e^{i\Phi_\xi}. \tag{4.1}$$

First we make two observations that will be useful:

$$E\left\{e^{i\Phi_\xi}\right\} = \frac{1}{2\pi}\int_0^{2\pi} e^{i\phi}\, d\phi = \frac{1}{2\pi}\left[\frac{e^{i\phi}}{i}\right]_0^{2\pi} = 0, \tag{4.2}$$

and

$$E\left\{e^{2i\Phi_\xi}\right\} = \frac{1}{2\pi}\int_0^{2\pi} e^{2i\phi}\, d\phi = \frac{1}{2\pi}\left[\frac{e^{2i\phi}}{2i}\right]_0^{2\pi} = 0. \tag{4.3}$$

As we recall from (3.15) C_{42} can be written as

$$C_{42} = E\{|y(n)|^4\} - |E\{y^2(n)\}|^2 - 2E^2\{|y(n)|^2\}, \tag{4.4}$$

and therefore we need expressions for these three moments.

We start with the moment $E\{y^2(n)\}$, which can be written as a sum of conditional expected values given $M(n) = m$ as

$$E\left\{y^2(n)\right\} = \sum_{m=0}^{\infty} p(m) E\left\{ y^2(n) \middle| M(n) = m\right\}, \tag{4.5}$$

where $p(m)$ is the probability that $M(n) = m$ and

$$E\left\{ y^2(n) \middle| M(n) = m\right\} = E\left\{\left(\sum_{\xi=1}^{m} A_\xi e^{i\Phi_\xi}\right)^2\right\} = \sum_{\xi=1}^{m}\sum_{\eta=1}^{m} E\left\{A_\xi e^{i\Phi_\xi} A_\eta e^{i\Phi_\eta}\right\}. \tag{4.6}$$

Since all variables are independent we can write the terms in the last sum (4.6) as

$$E\left\{A_\xi e^{i\Phi_\xi} A_\eta e^{i\Phi_\eta}\right\} = \begin{cases} E\left\{A_\xi\right\} E\left\{e^{i\Phi_\xi}\right\} E\left\{A_\eta\right\} E\left\{e^{i\Phi_\eta}\right\} & \text{for } \xi \neq \eta \\ E\left\{A_\xi^2\right\} E\left\{e^{2i\Phi_\xi}\right\} & \text{for } \xi = \eta. \end{cases} \tag{4.7}$$

With Eqs. (4.3) and (4.2) we see that all terms in the sum are zero. Hence

$$E\left\{ y^2(n) \middle| M(n) = m \right\} = 0, \tag{4.8}$$

and consequently

$$E\left\{ y^2(n) \right\} = 0. \tag{4.9}$$

We use the same method for $E\{|y(n)|^2\}$ and get

$$E\left\{ |y(n)|^2 \right\} = \sum_{m=0}^{\infty} p(m) E\left\{ |y(n)|^2 \middle| M(n) = m \right\}, \tag{4.10}$$

where $p(m)$ is the probability that $M(n) = m$ and

$$\begin{aligned} E\left\{ |y(n)|^2 \middle| M(n) = m \right\} &= E\left\{ y(n)y^*(n) \middle| M(n) = m \right\} \\ &= E\left\{ \left(\sum_{\xi=1}^{m} A_\xi e^{i\Phi_\xi} \right) \left(\sum_{\xi=1}^{m} A_\xi e^{-i\Phi_\xi} \right) \right\} \\ &= \sum_{\xi=1}^{m} \sum_{\eta=1}^{m} E\left\{ A_\xi e^{i\Phi_\xi} A_\eta e^{-i\Phi_\eta} \right\}. \end{aligned} \tag{4.11}$$

Since all variables are independent we can write the terms in the sum as

$$E\left\{ A_\xi e^{i\Phi_\xi} A_\eta e^{i\Phi_\eta} \right\} = \begin{cases} E\left\{ A_\xi \right\} E\left\{ e^{i\Phi_\xi} \right\} E\left\{ A_\eta \right\} E\left\{ e^{-i\Phi_\eta} \right\} & \text{for } \xi \neq \eta \\ E\left\{ A_\xi^2 \right\} E\left\{ e^{i\Phi_\xi} e^{-i\Phi_\xi} \right\} & \text{for } \xi = \eta. \end{cases} \tag{4.12}$$

Equation (4.2) gives that all terms for $\xi \neq \eta$ are zero, whereas the terms for $\xi = \eta$ are non-zero and equal to $E\left\{A^2\right\}$. This results in

$$E\left\{ |y(n)|^2 \middle| M(n) = m \right\} = \sum_{\xi=1}^{m} E\left\{ A_\xi^2 \right\} = E\left\{ A^2 \right\} m \tag{4.13}$$

and by inserting into (4.10) we get

$$E\left\{ |y(n)|^2 \right\} = \sum_{m=0}^{\infty} p(m) E\left\{ A^2 \right\} m. \tag{4.14}$$

Finally, for $E\{|y(n)|^4\}$ we get

$$E\left\{|y(n)|^4\right\} = \sum_{m=0}^{\infty} p(m) E\left\{ |y(n)|^4 \middle| M(n) = m \right\}, \tag{4.15}$$

where $p(m)$ is the probability that $M(n) = m$ and

$$\begin{aligned} E\left\{ |y(n)|^4 \middle| M(n) = m \right\} &= E\left\{ (y(n)y^*(n))^2 \middle| M(n) = m \right\} \\ &= E\left\{ \left(\left(\sum_{\xi=1}^{m} A_\xi e^{i\Phi_\xi} \right) \left(\sum_{\xi=1}^{m} A_\xi e^{-i\Phi_\xi} \right) \right)^2 \right\}. \end{aligned} \tag{4.16}$$

Since $E\left\{e^{i\Phi_\xi}\right\}$ and $E\left\{e^{2i\Phi_\xi}\right\}$ are zero (see Eq. (4.2) and (4.3)) the only non-zero terms are terms with the following structures $A_k e^{i\Phi_k} A_k e^{-i\Phi_k} A_k e^{i\Phi_k} A_k e^{-i\Phi_k}$ and $A_k e^{i\Phi_k} A_k e^{-i\Phi_k} A_l e^{i\Phi_l} A_l e^{-i\Phi_l}$, which gives $E\left\{A^4\right\}$ and $\left(E\left\{A^2\right\}\right)^2$ respectively. There are m of the first type and $2m(m-1)$ of the second type. This together with Eq. (4.15) gives

$$E\left\{|y(n)|^4\right\} = \sum_{m=0}^{\infty} p(m) \left(E\left\{A^4\right\} m + \left(E\left\{A^2\right\}\right)^2 2m(m-1) \right). \tag{4.17}$$

4.2 Periodic Pulses

Periodic pulses can be characterized by a pulse time (pulse duration) T_p, a pulse repetition time T_r, and an amplitude A. The pulsed interference is compared to the communication signal in Figure 4.1, where T_b denotes the bit time for the communication system. We will consider pulses for which the pulse time is shorter than the pulse repetition time. Periodic pulses with this constraint can at most have one active pulse and the probability for a pulse is $T_\mathrm{p}/T_\mathrm{r}$. Thus, the probability density function for $M(n)$ is

$$p(m) = \begin{cases} 1 - T_\mathrm{p}/T_\mathrm{r} & m = 0 \\ T_\mathrm{p}/T_\mathrm{r} & m = 1 \\ 0 & \text{otherwise} \end{cases}. \tag{4.18}$$

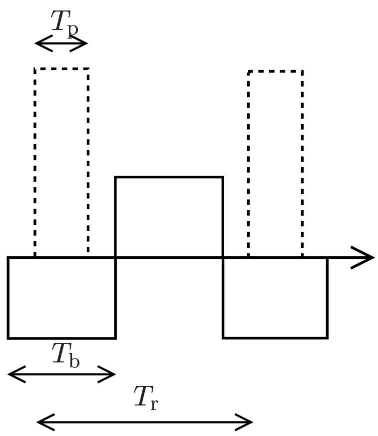

Figure 4.1: Pulse time, T_{p}, bit time, T_{b}, and the (average) repetition time, T_{r}.

With (4.18), (4.14), and (4.17) we can write the non-zero moments as

$$E\left\{|y(n)|^2\right\} = E\left\{A^2\right\}\frac{T_{\mathrm{p}}}{T_{\mathrm{r}}}, \tag{4.19}$$

and

$$E\left\{|y(n)|^4\right\} = E\left\{A^4\right\}\frac{T_{\mathrm{p}}}{T_{\mathrm{r}}}. \tag{4.20}$$

By inserting (4.19) and (4.20) into (3.15) we get C_{42} for periodic pulses as

$$C_{42} = E\left\{A^4\right\}\frac{T_{\mathrm{p}}}{T_{\mathrm{r}}} - 2\left(E\left\{A^2\right\}\frac{T_{\mathrm{p}}}{T_{\mathrm{r}}}\right)^2 \tag{4.21}$$

By dividing (4.21) with (4.19) squared we get the normalized kurtosis for periodic pulses as

$$K_{42} = \frac{E\left\{A^4\right\}}{\left(E\left\{A^2\right\}\right)^2}\cdot\frac{T_{\mathrm{r}}}{T_{\mathrm{p}}} - 2. \tag{4.22}$$

4.2.1 Constant Amplitude

If the amplitude is constant we can assume $A = 1$, since normalized kurtosis is invariant to amplitude scaling, and we get

$$E\{A^4\} = E\{A^2\} = 1. \tag{4.23}$$

Inserting (4.23) into (4.22) yields the normalized kurtosis for periodic pulses with constant amplitude as

$$K_{42} = \frac{T_\mathrm{r}}{T_\mathrm{p}} - 2. \tag{4.24}$$

4.2.2 Uniform Amplitude

If the amplitude is uniformly distributed we can assume that it is distributed on the interval $[0,1]$, since normalized kurtosis is invariant to amplitude scaling, and we get $E\{A^2\}$ as

$$\begin{aligned} E\left\{A^2\right\} &= \int_0^1 1 \cdot a^2\, da \\ &= \left[\frac{a^3}{3}\right]_0^1 \\ &= \frac{1}{3}, \end{aligned} \tag{4.25}$$

and $E\{A^4\}$ as

$$\begin{aligned} E\left\{A^4\right\} &= \int_0^1 1 \cdot a^4\, da \\ &= \left[\frac{a^5}{5}\right]_0^1 \\ &= \frac{1}{5}. \end{aligned} \tag{4.26}$$

By substituting (4.25) and (4.26) into (4.22) we get the normalized kurtosis for periodic pulses with uniform amplitude distribution as

$$K_{42} = \frac{9}{5} \cdot \frac{T_\mathrm{r}}{T_\mathrm{p}} - 2. \tag{4.27}$$

4.3 Poisson Arrivals

Now we will study a more stochastic interference where pulses arrive according to a Poisson process, with intensity λ. As before, the pulses are characterized by a pulse time T_p and an amplitude A.

Since the pulses arrive according to a Poisson process the random variable $M(n)$ is Poisson distributed with parameter λT_{p}. Hence, the probability density function for $M(n)$ is given by

$$p(m) = e^{-\lambda T_{\mathrm{p}}} \frac{(\lambda T_{\mathrm{p}})^m}{m!}. \tag{4.28}$$

By inserting (4.28) in (4.14) we can calculate $E\{|y(n)|^2\}$ as

$$\begin{aligned} E\left\{|y(n)|^2\right\} &= \sum_{m=0}^{\infty} e^{-\lambda T_{\mathrm{p}}} \frac{(\lambda T_{\mathrm{p}})^m}{m!} E\left\{A^2\right\} m \\ &= E\left\{A^2\right\} e^{-\lambda T_{\mathrm{p}}} \sum_{m=0}^{\infty} m \frac{(\lambda T_{\mathrm{p}})^m}{m!} \\ &= E\left\{A^2\right\} \lambda T_{\mathrm{p}}, \end{aligned} \tag{4.29}$$

where we use the fact that

$$\sum_{m=0}^{\infty} m \frac{(\lambda T_{\mathrm{p}})^m}{m!} = \lambda T_{\mathrm{p}} \sum_{m=1}^{\infty} \frac{(\lambda T_{\mathrm{p}})^{m-1}}{(m-1)!} = \lambda T_{\mathrm{p}} e^{\lambda T_{\mathrm{p}}}. \tag{4.30}$$

By substituting (4.28) into (4.17) and we can write $E\{|y(n)|^4\}$ as

$$\begin{aligned} E\left\{|y(n)|^4\right\} &= \sum_{m=0}^{\infty} e^{-\lambda T_{\mathrm{p}}} \frac{(\lambda T_{\mathrm{p}})^m}{m!} \Big(E\left\{A^4\right\} m + \left(E\left\{A^2\right\}\right)^2 2m(m-1)\Big) \\ &= \Big(E\left\{A^4\right\} - 2\left(E\left\{A^2\right\}\right)^2\Big) e^{-\lambda T_{\mathrm{p}}} \sum_{m=0}^{\infty} m \frac{(\lambda T_{\mathrm{p}})^m}{m!} \\ &\quad + 2\left(E\left\{A^2\right\}\right)^2 e^{-\lambda T_{\mathrm{p}}} \sum_{m=0}^{\infty} m^2 \frac{(\lambda T_{\mathrm{p}})^m}{m!} \\ &= \Big(E\left\{A^4\right\} - 2\left(E\left\{A^2\right\}\right)^2\Big) \lambda T_{\mathrm{p}} \\ &\quad + 2\left(E\left\{A^2\right\}\right)^2 \Big((\lambda T_{\mathrm{p}})^2 + \lambda T_{\mathrm{p}}\Big), \end{aligned} \tag{4.31}$$

where we use (4.30) and the fact that

$$\begin{aligned}\sum_{m=0}^{\infty} m^2 \frac{(\lambda T_\mathrm{p})^m}{m!} &= \sum_{m=0}^{\infty} m(m-1)\frac{(\lambda T_\mathrm{p})^m}{m!} + \sum_{m=0}^{\infty} m\frac{(\lambda T_\mathrm{p})^m}{m!} \\ &= (\lambda T_\mathrm{p})^2 \sum_{m=2}^{\infty} \frac{(\lambda T_\mathrm{p})^{m-2}}{(m-2)!} \\ &\quad +\lambda T_\mathrm{p} \sum_{m=1}^{\infty} \frac{(\lambda T_\mathrm{p})^{m-1}}{(m-1)!} \\ &= \Big((\lambda T_\mathrm{p})^2 + \lambda T_\mathrm{p}\Big) e^{\lambda T_\mathrm{p}}. \end{aligned} \tag{4.32}$$

Inserting (4.29) and (4.31) into (3.15) gives C_{42} for pulses that arrive according to a Poisson process as

$$C_{42} = E\left\{A^4\right\} \lambda T_\mathrm{p}. \tag{4.33}$$

By dividing (4.33) with (4.29) squared we get the normalized kurtosis for pulses that arrive according to a Poisson process as

$$K_{42} = \frac{E\left\{A^4\right\}}{\left(E\left\{A^2\right\}\right)^2} \cdot \frac{1}{\lambda T_\mathrm{p}} \tag{4.34}$$

4.3.1 Constant Amplitude

If the amplitude is constant we can, as with periodic pulses, assume that $A = 1$, thus the relevant moments are given by (4.23). Inserting (4.23) into (4.34) yields the normalized kurtosis for pulses with constant amplitude that arrive according to a Poisson process as

$$K_{42} = \frac{1}{\lambda T_\mathrm{p}}. \tag{4.35}$$

4.3.2 Uniform Amplitude

If the amplitude is uniformly distributed we can, as with periodic pulses, assume that the amplitude is distributed on the interval $[0, 1]$, thus the relevant moments are give by (4.25) and (4.26). Inserting (4.25) and (4.26) into (4.34) yields the normalized kurtosis for pulses with uniform amplitude that arrive according to a Poisson process as

$$K_{42} = \frac{9}{5} \cdot \frac{1}{\lambda T_{\mathrm{p}}} \tag{4.36}$$

Chapter 5

Interfering pulsed signals

The effect of pulsed interference on a (uncoded) coherent BPSK-modulated (binary phase-shift keying) digital communication receiver has been simulated in ACOLADE (Advanced COmmunication Link Analysis and Design Environment). The results for different kinds of pulsed interference are shown in Figures 5.1 to 5.4. In the figures the bit error probability (BEP) is shown as a function of the signal-to-interference ratio (SIR). The signal-to-noise ratio (SNR) is 10 dB. The amplitude of the interfering pulses is normalized so that the mean power of the interference is equal for all cases. We investigate four different types of pulsed interference, where we have either a periodic pulse or Poisson distributed arrival time, and where the amplitude is constant or uniformly distributed. We have investigated pulses with an average time between consecutive pulses, T_r, which are the same as the bit time, 10 times the bit time, or 50 times the bit time, i.e. $T_r = T_b$, $T_r = 10\ T_b$, or $T_r = 50\ T_b$. The pulse duration is 10% of the bit time, i.e. $T_p = 0.1T_b$ in all cases. The performance for additive white Gaussian noise (AWGN) is also shown as comparison.

In Figure 5.1, the interference consists of periodic pulses with constant amplitude. In Figure 5.2 we see the results for periodic pulses with uniformly distributed amplitude. Pulsed interference with constant amplitude and Poisson distributed arrival times is examined in Figure 5.3. Since the arrival times are random, pulses can overlap each other, which results in a larger impact on the receiver. This effect can be seen in Figure 5.3 for $T_r = T_b$ where the BEP is slightly higher for some values of SIR than the corresponding cases in Figure 5.1 and 5.2. Finally, in Figure 5.4, the interference pulses have uniformly

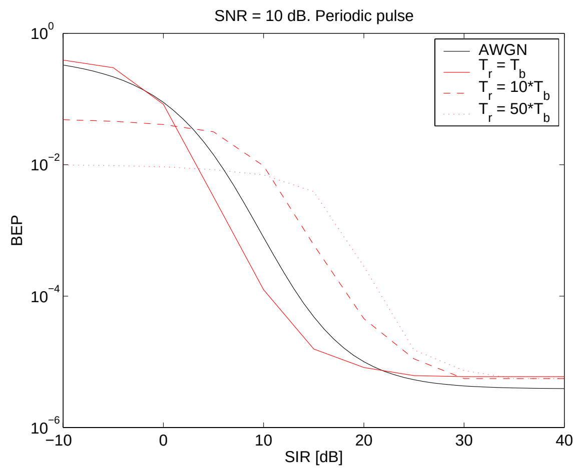

Figure 5.1: BEP for a BPSK-receiver, as a function of SIR, when subjected to periodic pulsed interference with constant amplitude.

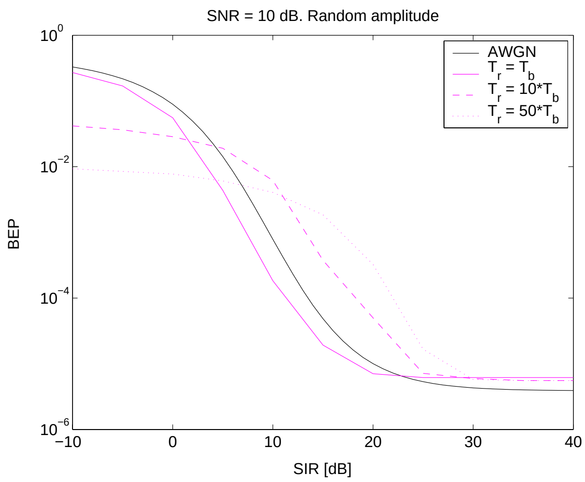

Figure 5.2: Simulated BEP for a BPSK-receiver, for periodic pulsed interference with random amplitudes.

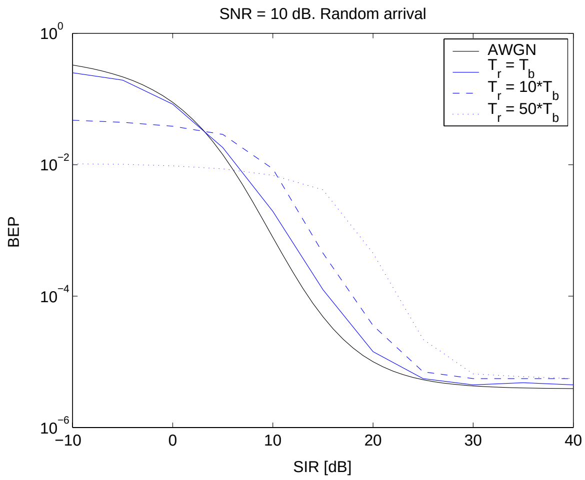

Figure 5.3: Simulated BEP for a BPSK-receiver, for pulsed interference with random arrival times and constant amplitudes.

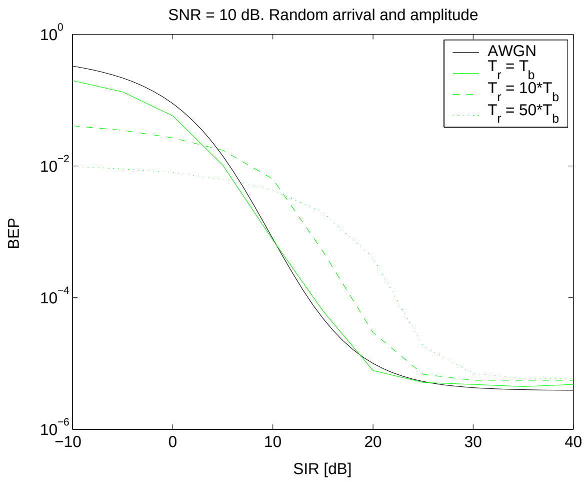

Figure 5.4: Simulated BEP for a BPSK-receiver, for pulsed interference with random amplitudes and arrival times.

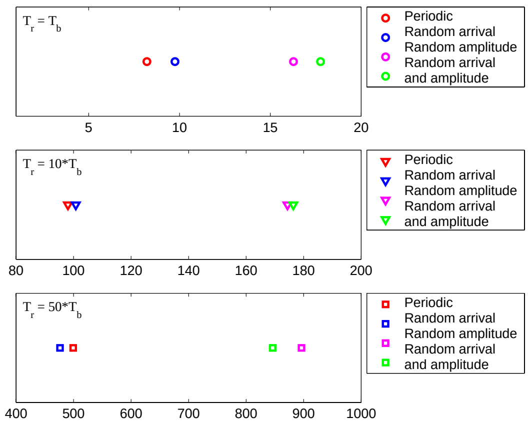

Figure 5.5: The estimated normalized kurtosis, $\hat{K}_{42}$, for pulsed interference, with different average times between pulses.

distributed amplitudes and Poisson distributed arrival times.

For all the different pulsed signals the BEP reaches a maximum value for low values of SIR, governed by the ratio T_b/T_r, since the interferece is so strong that the BEP in the disturbed bits are 0.5. For high SIR values the BEP reaches a minimum value, which depends on the SNR at hand.

From Figure 5.1 to 5.4 we can see that the AWGN-approximation cannot be used for these types of pulsed interference when the average time between consecutive pulses, T_r, is significantly larger than the bit time. Depending on the SIR and the ratio T_b/T_r, the BEP can be largely over- or underestimated. For low values of SIR the AWGN-approximation yields an overestimation. However, for moderate to high SIR it results in an underestimation of the BEP, which can have serious consequences for the communication system.

The sample estimate of the normalized kurtosis, for various types of pulsed interference, is shown in Figure 5.5, for different average times between pulses, T_r. The pulse length is 10% of the bit duration. The normalized kurtosis increases when the average time between consecutive pulses, T_r, increases. From Figures 5.1, 5.2, 5.3, and 5.4 it is clear that interference with large T_r/T_b differs

Pulse type	$T_r = T_b$			$T_r = 10T_b$			$T_r = 50T_b$		
	K_{42}	$\hat{K}_{42}$	$\Delta\epsilon$	K_{42}	$\hat{K}_{42}$	$\Delta\epsilon$	K_{42}	$\hat{K}_{42}$	$\Delta\epsilon$
Periodic	8.00	8.20	2.50	98.00	98.05	0.05	498.00	499.25	0.25
U. amp.	16.00	16.28	1.72	178.00	174.40	2.02	898.00	896.34	0.19
Po. ar.	10.00	9.75	2.48	100.00	100.76	0.76	500.00	476.46	4.71
Po. ar. U. amp.	18.00	17.77	1.29	180.00	176.41	1.99	900.00	846.19	5.98

Table 5.1: Normalized kurtosis (K_{42}), estimated normalized kurtosis ($\hat{K}_{42}$) and relative error in percent ($\Delta\epsilon$) for periodic pulses with constant amplitude (Periodic), periodic pulses with uniformly distributed amplitude (U. amp.), pulses with constant amplitude that arrive according to a Poisson process (Po. ar.), and pulses with uniformly distributed amplitude that arrive according to a Poisson process (Po. ar. U. amp.).

from the AWGN-approximation concerning the BEP performance. Hence, the normalized kurtosis value can be used to estimate the receiver performance, i.e. a large kurtosis value means a large impact on the receiver, in the form of an increased BEP. The kurtosis value of AWGN is zero, as well as all other HOS-measures of order higher than two. Hence, HOS is blind to Gaussian noise. The kurtosis also gives an indication on the size of the errors in the AWGN-approximation.

The normalized kurtosis values in Figure 5.5 were estimated on the same interference signals as the BEP were simulated. In the case with $T_r = 50T_b$, the kurtosis is estimated on a sequence that is only 400 T_r long, and the kurtosis estimates, when having random time and/or amplitude, will vary for different realizations. In Table 5.1 we have tabulated the normalized kurtosis, estimated normalized kurtosis and the relative error for the different pulses. There we see that pulses with random arrival time and/or amplitude have a larger relative error than the more deterministic pulses. However, it is worth noting that, in order to give a quality measure on the AWGN-approximation, the order of magnitude of the kurtosis value is fully sufficient.

Chapter 6

Low-Complexity Intersystem Interference Method

As discussed earlier, the ability to judge the effects from intersystem interference is becoming more and more important as the amount of wireless equipment increases. Unfortunately, the computations necessary to determine the performance degradation on a radio receiver, caused by intersystem interference, are often quite complicated. Thus, simplified methods, with low computational complexity, are desired in computer-based tools for intersystem interference analysis. One commonly used simplified method is to approximate the interfering signal as additive white Gaussian noise (AWGN); however, the AWGN-approximation cannot be used for all kinds of signals. It has previously been shown to perform fairly well for modulated interference signals, i.e. interference caused by other transmitters [6, 8]. However, it should not be used for pulsed interference signals since it can lead to large underestimations of the resulting bit error probabilities (BEP) in the radio receiver. Also, the kurtosis, has shown to be useful when attempting to distinguish between modulated and periodic pulsed interference [5, 6].

The goal of the low-complexity methods, for high kurtosis values, is to give a better estimate of the resulting bit error probability than the AWGN-approximation. We know that the BEP can be underestimated several magnitudes with the use of the AWGN-approximation. The proposed method isolates some of these cases and also yields a better estimation of the BEP. An estimation error within one order of magnitude is often deemed as acceptable when using

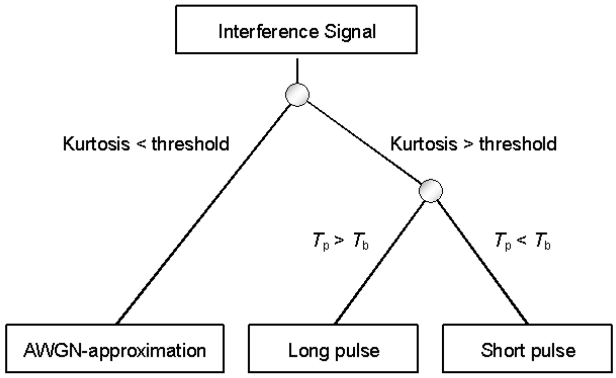

Figure 6.1: Schematic of the proposed method.

an approximate method in an intersystem interference tool. If a higher degree of accuracy is desired it might be necessary to analyze the interference signal more thoroughly.

In Figure 6.1 a schematic of the low-complexity method is shown. We propose to use the kurtosis as a threshold to decide if the AWGN-approximation can be used or not. For kurtosis values close to zero, the AWGN-approximation can be used safely. For higher kurtosis values, two other approximate methods are used. For periodic pulses, with constant amplitude, that are longer in duration than the bit time, it is possible to estimate the resulting bit error probability (BEP) in the receiver with use of the estimated kurtosis value. However, when the pulse is shorter than the bit time we need to estimate the relationship between the pulse repetition time and the bit time in order to estimate the BEP. Apart from the kurtosis, the mean SIR and SNR must be estimated for all three methods.

We derive the methods for a pulsed interference for a periodic pulsed signal, but the method could be used even for other sorts of pulsed interference, but with a larger uncertainty in the estimated BEP. However, the proposed method yields a better estimate of the BEP for these signals than the AWGN-approximation does, which might be the only practical alternative if it is not possible to derive the BEP analytically.

6.1 AWGN-approximation

For the cases where the AWGN-approximation can be used the interfering signal is treated as AWGN with spectral density N_I. For a BPSK receiver the bit error probability is

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0 + N_\mathrm{I}}}\right) = Q\left(\sqrt{\frac{2 \cdot SNR \cdot SIR}{SNR + SIR}}\right), \tag{6.1}$$

where E_b is the energy per bit and N_0 is the spectral density of the noise. The signal-to-noise ratio is $SNR = E_b/N_0$ and the signal-to-interference ratio is $SIR = E_b/N_\mathrm{I}$.

6.2 Long pulse

When the interfering signal is pulsed we can examine the bit error probability for the disturbed information bits and the BEP for the non-disturbed bits separately. We assume that when a pulse is present it disturbs the entire bits. The wanted BEP is then the weighted sum of the BEP for the disturbed and the non-disturbed bits. For a pulsed interference with a pulse time, T_p, larger than the bit time, T_b, the fraction of disturbed bits is $T_\mathrm{p}/T_\mathrm{r}$, where T_r is the pulse repetition time. The signal-to-interference ratio (SIR) in a disturbed bit is $T_\mathrm{p}/T_\mathrm{r}$ of the average SIR, i.e. the spectral density of the pulse in a disturbed bit is $T_\mathrm{r}/T_\mathrm{p}$ higher than its average spectral density. The BEP can be calculated as

$$\begin{aligned} P_b &= \frac{T_\mathrm{p}}{T_\mathrm{r}} \cdot Q\left(\sqrt{\frac{2 \cdot SNR \cdot SIR \cdot T_\mathrm{p}/T_\mathrm{r}}{SNR + SIR \cdot T_\mathrm{p}/T_\mathrm{r}}}\right) + \left(1 - \frac{T_\mathrm{p}}{T_\mathrm{r}}\right) \cdot Q\left(\sqrt{2 \cdot SNR}\right) \\ &= \frac{1}{\hat{K}_{42} + 2} \cdot Q\left(\sqrt{\frac{2 \cdot SNR \cdot SIR/(\hat{K}_{42} + 2)}{SNR + SIR/(\hat{K}_{42} + 2)}}\right) \\ &+ \left(1 - \frac{1}{\hat{K}_{42} + 2}\right) \cdot Q\left(\sqrt{2 \cdot SNR}\right). \end{aligned} \tag{6.2}$$

For a periodic pulse with fixed amplitude (4.24) we have that $T_\mathrm{p}/T_\mathrm{r} = 1/(\hat{K}_{42} + 2)$.

6.3 Short pulse

When the pulse time is short compared to the bit time the BEP can be derived in a similar way. Here we assume that a pulse only disturbs one bit. The fraction of disturbed bits is $T_\mathrm{b}/T_\mathrm{r}$ and the signal-to-interference ratio in a disturbed bit is $T_\mathrm{b}/T_\mathrm{r}$ of the average SIR. The bit error probability is the weighted sum of the BEP for the disturbed bits and the BEP for the non-disturbed bits, i.e.

$$P_b = \frac{T_\mathrm{b}}{T_\mathrm{r}} \cdot Q\left(\sqrt{\frac{2 \cdot SNR \cdot SIR \cdot T_\mathrm{b}/T_\mathrm{r}}{SNR + SIR \cdot T_\mathrm{b}/T_\mathrm{r}}}\right) + \left(1 - \frac{T_\mathrm{b}}{T_\mathrm{r}}\right) \cdot Q\left(\sqrt{2 \cdot SNR}\right) \tag{6.3}$$

In this method we also need to estimate the ratio $T_\mathrm{b}/T_\mathrm{r}$.

6.4 Results

The BEP for a pulse with pulse duration longer than a bit time is shown in Figure 6.2. The BEP is calculated according to Equation (6.2), and the AWGN-approximation is also shown and is calculated with (6.1).

The BEP for a pulse with pulse duration shorter than a bit time is shown in Figure 6.3 and 6.4. The BEP is calculated according to (6.3), and the AWGN-approximation is also shown as a reference. In Figure 6.3 the pulse duration is 10% of the bit time and in Figure 6.4 it is 40%. From Figure 6.3 and 6.4 we can see that even when the kurtosis value is the same the BEP is different. Hence, we have to estimate the ratio between the pulse repetition time and the bit time, i.e. $T_\mathrm{r}/T_\mathrm{b}$. In Figure 6.3 the BEP from simulations with a periodic pulse with fixed amplitude is compared to the BEP calculated with the proposed method. As can be seen in the figure, there is a good agreement between the simulated and calculated BEP.

For small values of kurtosis the AWGN-approximation is applicable even for pulsed interference, which can be seen in Figure 6.2 - 6.4. However, if a too large kurtosis value is used as a threshold, then the AWGN-approximation might be used in situations where it yields an estimation error above the desired. Hence, it is important to choose the kurtosis threshold value carefully.

The methods for pulsed interference are derived for a periodic pulsed signal, but the BEP shows the same behavior for different kinds of pulsed signals, as

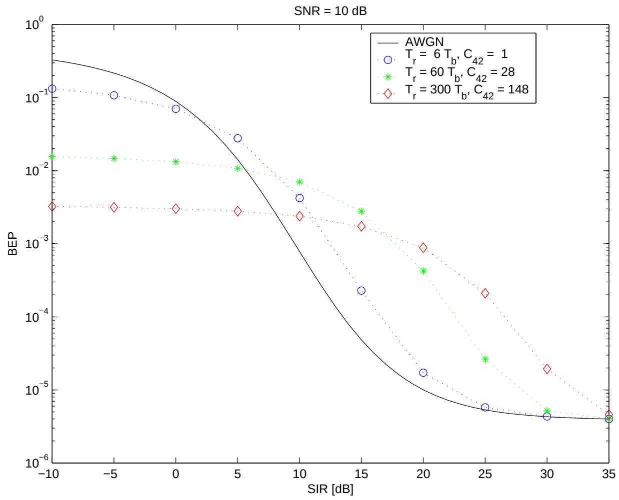

Figure 6.2: Calculated BEP as a function of SIR for a pulse with duration longer than the bit time, $T_{\mathrm{p}} = 2T_{\mathrm{b}}$.

can be seen in Figures 5.1 to 5.4. Hence, the proposed methods can be used for other sorts of pulsed interference, but with a larger uncertainty in the estimated BEP. However, the proposed method still yields a better estimate of the BEP for these signals than the AWGN-approximation does, which might be the only practical alternative if it is not possible to derive the BEP analytically.

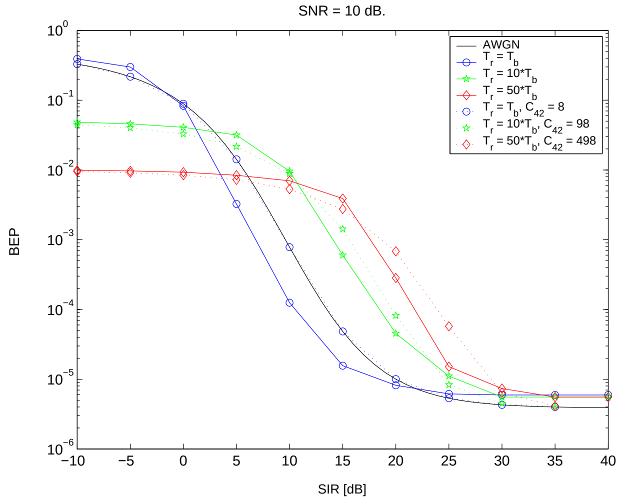

Figure 6.3: BEP as a function of SIR for a pulse with duration shorter than the bit time, $T_\mathrm{p} = 0.1T_\mathrm{b}$. Solid lines represents simulations with a periodic pulse with fixed amplitude, while dotted lines are calculated with the proposed method.

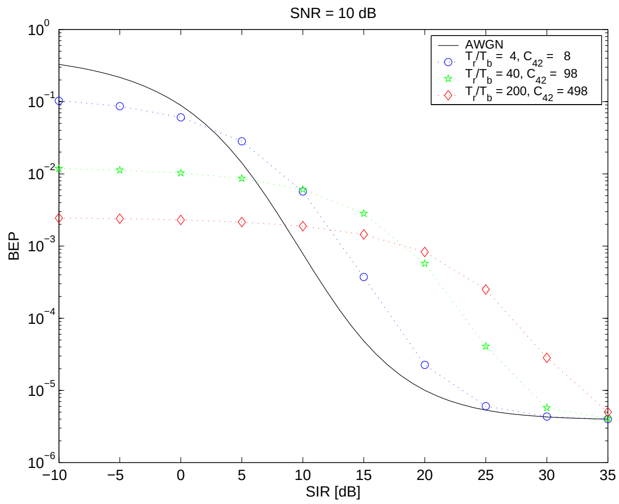

Figure 6.4: Calculated BEP as a function of SIR for a pulse with duration shorter than the bit time, $T_\mathrm{p} = 0.4T_\mathrm{b}$.

Chapter 7

Conclusions

In this report we have studied the effects on a coherent uncoded BPSK receiver from intersystem interference. We have shown that, for pulsed interference it is possible to use a higher-order statistics based measure, namely the kurtosis, to quantify the impact on the communication receiver. The kurtosis value is estimated for different kinds of pulsed signals, both analytically and through simulations. The agreement between simulated and analytical values is good.

Furthermore, we use the kurtosis measure to create a method where the BEP for different kinds of interference is estimated. The method is designed to handle both modulated interfering signals as well as pulsed interference. With use of the kurtosis value for the interfering signal the method can decide whether the interfering signal can be approximated as AWGN without largely underestimating the BEP or not. Typically, for small values on kurtosis the AWGN-approximation can be used to estimate the resulting BEP, without making a large underestimation of the BEP. An underestimation of the BEP may have serious consequences for a communication system. For large values of kurtosis the AWGN-approximation may not be used without risking underestimating the BEP. The interfering signal is typically a pulsed signal when we have a large kurtosis value. For pulsed interference two methods to calculate the BEP are proposed, depending on if the pulse duration is longer or shorter than the bit time. All the methods need the kurtosis value and the estimated SNR and SIR to calculate the BEP. However, when the pulse is short compared to a bit we must also estimate the ratio between the pulse repetition time and the bit time.

In summary, the proposed simple low-complexity method combines the kurtosis measure and the AWGN-approximation in order to obtain a better estimation of the BEP in a digital radio receiver. The comparison of BEP from simulations and calculated with the proposed method, shows a good agreement. Hence, the presented approximative method can for example be used in computer-based intersystem interference analysis tools where low-complexity methods are needed.

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