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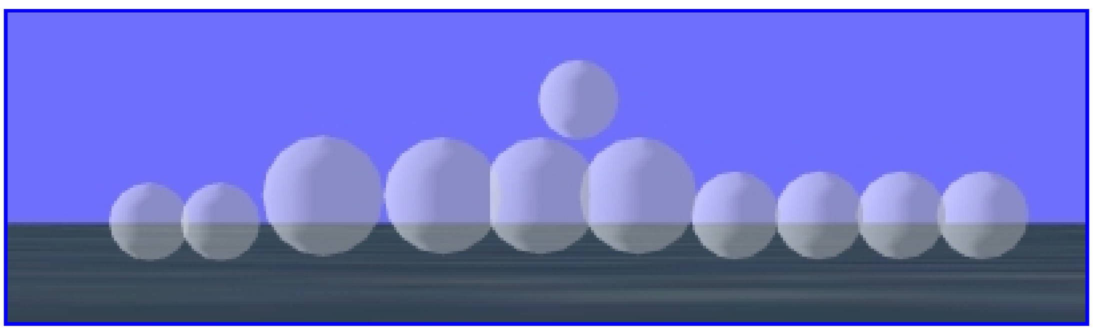
Extended Targets in a Point Source Model



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Report title

Extended Targets in a Point Source Model

Abstract (not more than 200 words)

This report describes how an IR simulation model for the duel between a ship with IR decoys and an anti-ship missile has been altered. The simulation model has been changed from using point sources to describe ship and IR decoys to handle both ship and IR decoys as extended targets. The report describes why the changes were made and how. The report also serves to document changes made in the simulation model.

Keywords

modelling, simulation, IR, extended targets, ship, decoy

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Rapportens titel (i översättning)

Utbredda mål i en punktmåls-modell

Sammanfattning (högst 200 ord)

Rapporten beskriver hur en simuleringsmodell för duellen mellan sjömålsrobot och ett fartyg med IR-skenmål har modifierats. Simuleringsmodellen har ändrats från att behandla fartyg och IR-skenmål som punktmål till att behandla dessa som utsträckta mål. Rapporten beskriver varför och hur simuleringsmodellen har ändrats. Rapporten tjänar också som dokumentation av ändringar i den använda modellen.

Nyckelord

modellering, simulering, IR, utbredda mål, fartyg, skenmål, fackla

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Index

1. Introduction

In a previous assignment FOI was asked to find an algorithm for the use of IR-decoys against an anti-ship missile. The tool to be used for this was a simulation model consisting of (at top level) ship, decoys, missiles and weather conditions. Both ship and decoys were regarded as point sources.

The ship had (and has) an IR-signature as a function of azimuth angle in the model. This was accomplished by a lookup-table where input is azimuth angle and output is radiance value. The IR-decoys was given their radiance values as a function of time, from burst (also this accomplished by look-up table) to the end of duration for the flare.

This report describes how and to some extent why the simulation model was transformed from being a point source approach to an extended object simulation model. To most problems however concerning simulations and modelling there are several ways of solving them. Some ways are better than others depending on different requirements. The problem faced within this project could also have been solved by generating one image for each time increment showing the scenario seen from the missile seeker. By using image processing the detector signal could be generated (the model works on detector signal level). The drawback with this approach for the project was that it would be too time consuming. A single simulation run took about 20 seconds, by generating images and use image processing the same simulation would take at least 80 seconds. This may not appear so dramatic, but the plan for this project included half a million of simulation runs. Apart from the extra programming the simulation time had to be held short in order to meet the projects deadline.

Chapter 2 to 4 describes step by step the order in which the simulation model was changed and why. In chapter 2 a description of how the decoys were modified concerning contact with the water is given. Chapter 3 describes how much of a decoy's energy falls upon a detector element. In chapter 4 a presentation is given of how the ship could be represented as an extended object. Chapter 5 discusses circles that overlap each other. Chapter 6 handles miss distance and hit coordinates. In chapter 7 calculations of relative wind is shown (not that it has a lot to do with extended targets, but for documentation purposes).

The calculations described in this report was first tested in Matlab, then transferred and implemented in the model.

After the described changes were made a single simulation run takes about 50% longer time then it did before, but the time cost is acceptable considering the more accurate results that are obtained.

1.1 Acknowledgements

The author of this report would like to thank the colleagues Patrik Strand and Peter Johansson (both from the department of Electronic Warfare Assessments) for their input and participation.

The project team would also like to express our gratitude to our sponsor Monica Strömstedt at FMV. Not only for a challenging, mostly fun and interesting assignment, but also for the trust shown during the project.

2. IR-decoy

An IR-decoy such as a flare could be described by intensity and area as a function of time. This is illustrated in figure below.

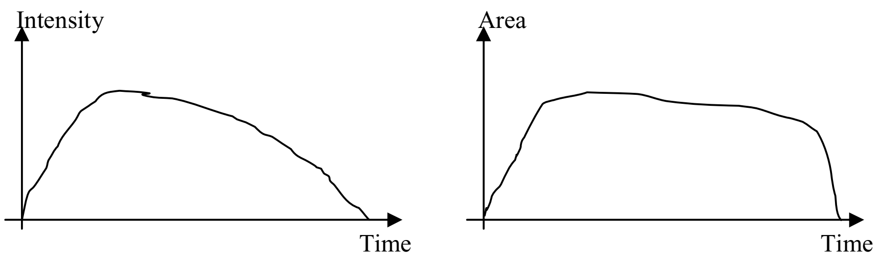

Figure 1.1 Intensity and area for a IR-decoy as function of time.

In the model that was going to be used the flare was regarded as a point source. The intensity values were stored in a look up table from which the current intensity value was calculated depending on the time. However the first problem was that the reperesentation of the flares were as point sources. One consequence of this was that when a descending flare got close to the waterline it got a digital behaviour. If the flare's point of gravity was above the water all it's intensity was used. If point of gravity was below the water surface none intensity would be taken into account.

The solution for this problem is to regard the flare as a sphere with it's projected area as a circle (from modelling point of view that is a very common way to regard flares). The projected circle is said to have the same radiance all over the circle (not quite true in reality but an acceptable simplification). By letting the flare be described as a circle with a radius R as a function of time, a better representation of the flare is achieved. See what happens to a flare when it gets close to water (see figure below).

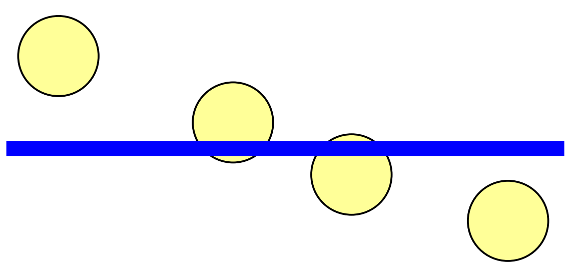

Figure 2.2 A flare's four different principle positions regarding the water surface. Completely above the water, mostly above the water, mostly below the water and finally completely below the water.

To describe the figure in a mathematical way, the height h of the flare is introduced. The height is where the point of gravity is for the flare relative to the water surface.

$h \quad R$	The flare is completely above the water surface.
$0 \quad h < R$	Most of the flare is above the water surface.
$-R \quad h < 0$	Most of the flare is below the water surface.
$h < -R$	The flare is completely below the water surface.

To calculate how much of the flare (circle) that is visible above the water surface the variables: r, d, and θ are introduced. See figure 2.3 for definitions.

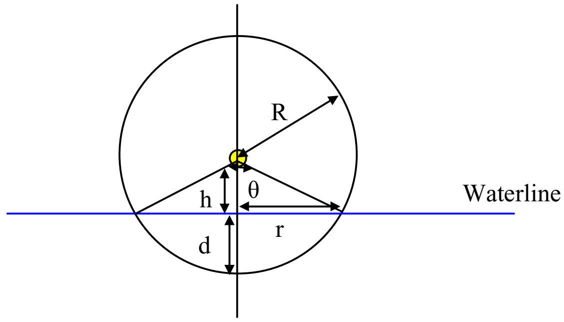

Figure 2.3 *Flare partly below water line.*

$$\theta = 2\arccos(\frac{h}{R}) \qquad \text{Eq. 2.1}$$

$$A_{circle\,sector} = \frac{1}{2}R^2\theta \qquad \text{Eq. 2.2}$$

$$r = R\sin\left(\frac{\theta}{2}\right) \qquad \text{Eq. 2.3}$$

$$A_{cone} = 2\frac{1}{2}hr = hR\sin\left(\frac{\theta}{2}\right) \qquad \text{Eq. 2.4}$$

$$A_{segment} = A_{circle\,sector} - A_{cone} \qquad \text{Eq. 2.5}$$

By using the equations above a value *G*, can be calculated to tell how much of the flare is above the water surface. *G* is within the range of 0 to 1.0. This yields:

$$\begin{array}{ll} h \geq R & G = 1.0 \\ 0 \leq h < R & G = \left(\pi R^2 - A_{segment}\right)/\pi R^2 \\ -R \leq h < 0 & G = A_{segment}/\pi R^2 \\ h < -R & G = 0.0 \end{array} \qquad \text{Eq. 2.6}$$

By multiplying the flare's intensity value with the calculated *G*-value, a more realistic value is obtained.

In order to not slow simulations down, a function was introduced that uses the height (*z*) and the radius *R* of the flare. The input parameter to the function is the parameter *z/R*. The function G (z/R) is plotted in figure 2.4. The values in this plot were put in a look up table to be used in the model. The values that are used for the function in the look up table is given in table 2.1. Note that in model used, the coordinate system is a right-handed rectangular coordinate system with the z-axis pointing downwards. This means that the corresponding z value for the height h is equal to –h.

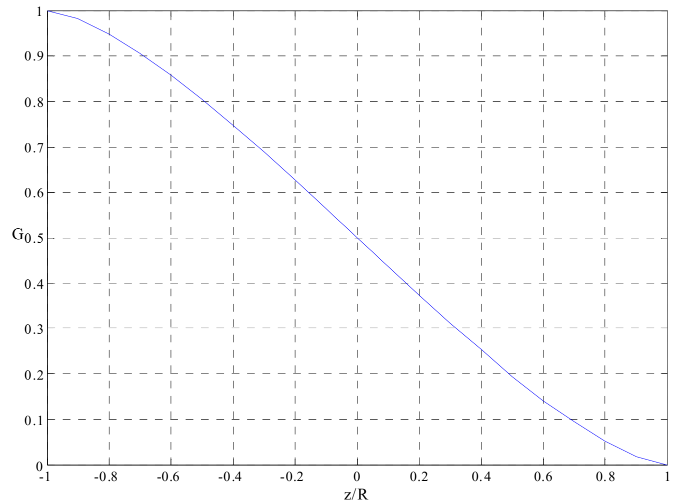

Figure 2.4 Variabel G as a function av z/R.

z/R	**G_{fcwl}**
-500.000	1.0000
-1.000	1.0000
-0.900	0.9813
-0.800	0.9480
-0.700	0.9059
-0.600	0.8576
-0.500	0.8045
-0.400	0.7477
-0.300	0.6881
-0.200	0.6265
-0.100	0.5636
-0.075	0.5477
-0.050	0.5318
-0.025	0.5159
0.000	0.5000
0.025	0.4841
0.050	0.4682
0.075	0.4523
0.100	0.4364
0.200	0.3735
0.300	0.3119
0.400	0.2523
0.500	0.1955
0.600	0.1424
0.700	0.0941
0.800	0.0520
0.900	0.0187
1.000	0.000
500.0000	0.0000

Table 2.1 G as a function of z/R, where z = -h. Z-axis pointing downwards.

So far the only thing introduced in the model is the radius for the flares and a look up table for the function G.

3. A flare's projection on a detector element

When calculating the resulting detector signals in the model the same approach are used as in the case with the flare and water surface. This means that either the flare's point of gravity was within the detector element and all the flare's intensity considered to be projected or the flare's point of gravity was outside the detector element and did not contribute anything to the detector signal. This was a model with the approach all or nothing. After the changes were made that are described in the previous chapter, it was argued that changes had to be made for the projection of a flare on a detector element.

Using the same reasoning as in previous chapter a flare can be projected as a circle. In order to solve the problem following conditions has been used:

The detector element is a rectangle with its origin in the centre of itself.

The detector element's sides are parallel with x- respectively y-axis.

The detector element has the width b and the height h.

The flare is projected on the detector plane as a circle with the radius R.

The flare's point of gravity is given by the coordinates x, y.

In order to minimize the calculations within a simulation, the most common situation has to be determined. In the model used, the most common situation is that the flare is not projected on the detector.

3.1 No projection on the detector element.

To determine if the projected circle in the detector plane will give an effect on the detector following parameters are used:

b	width of the detector
h	height of the detector
R	radius of circle
x,y	position of point of gravity for circle

The parameters are shown below in figure 3.1

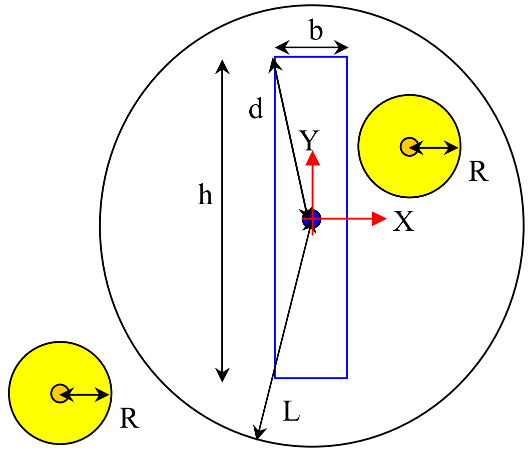

Figure 3.1 Parameters used to determine if the circle will coincide with the detector element.

In the figure, two new parameters *d* and *L* are introduced. Where

$$d=\sqrt{\left(\frac{b}{2}\right)^2+\left(\frac{d}{2}\right)^2} \qquad \textit{Eq. 3.1}$$

and

$$L=d+R \qquad \textit{Eq. 3.2}$$

If the condition in equation 3.3 is true the projected circle will not be projected on the detector at all, and can therefore be neglected.

$$\sqrt{x^2+y^2}>L \qquad \textit{Eq. 3.3}$$

In case that the condition is false the circle is not necessarily projected on the detector element. So a closer determination, to decide if the flare is projected on the detector element, has to be done.

Next step is to investigate if the center of the circle is closer to any of the sides of the detector than the radius of the circle. This is done by study if the condition in equation 3.4 is true.

$$\left(-\frac{b}{2}-R\leq x\right)AND\left(x\leq\frac{b}{2}+R\right)AND\left(-\frac{h}{2}-R\leq y\right)AND\left(y\leq\frac{h}{2}+R\right) \qquad \textit{Eq. 3.4}$$

If the statement above is false there is no need to go further. But if the statement is true, the circle is projected on the detector element. Two cases are possible, the first is that the center of the circle is within the detector element and the second is that the center of the circle is outside the detector element. The condition to be fulfilled if the center of the circle is inside the detector element is given in equation 3.5

$$\left(-\frac{b}{2} \leq x\right) AND \left(x \leq \frac{b}{2}\right) AND \left(-\frac{h}{2} \leq y\right) AND \left(y \leq \frac{h}{2}\right) \qquad \textit{Eq. 3.5}$$

3. 2 Point of gravity within the detector element.

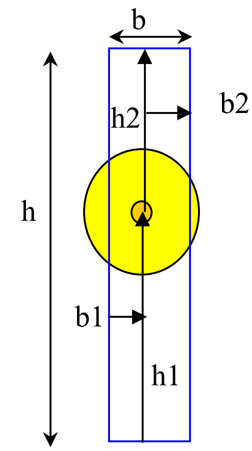

Figure 3.2 The circle's projection on the detector element. Definition of help variables.

Since the problem with calculating the area of circle, above or below a line was solved previously, the problem can be restricted to four cases. This is achieved by introducing four help variables. The variables are

b1 Distance between the left edge of detector and the centre of the circle.

b2 Distance between the right edge of detector and the centre of the circle.

h1 Distance between the lower edge of detector and the centre of the circle.

h2 Distance between the upper edge of detector and the centre of the circle.

By using the variable *b1* a normalized circle segment that is outside the left edge of the detector element is calculated and stored in the parameter A_{left}. In the corresponding way the parameters A_{right} (area segment on the right side of the detector), A_{below} (area segment on the lower side of the detector) and A_{above} (area segment on the upper side of the detector) are calculated. In the figure 3.2 A_{below} and A_{above} would be zero. When these calculations are done the final area on the detector could be calculated according to equation 3.6

$$Area = (1 - A_{left} - A_{right}) * (1 - A_{below} - A_{above}) \qquad \textit{Eq. 3.6}$$

3.3 Point of gravity outside the detector element.

If the circle's point of gravity is outside the detector element but still overlaps the detector element, there are several possible combinations of the overlap. For reason of simplicity the possible combination for the x-axis is shown in figure 3.3.

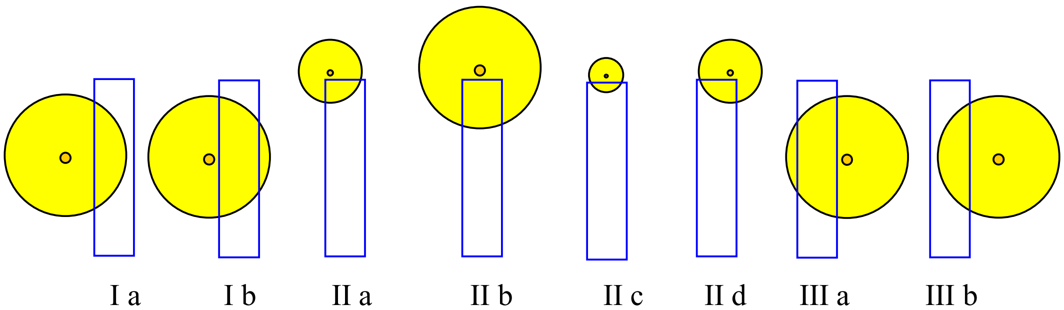

Figure 3.3 Possible combination of overlap concerning the x-axis.

As can be noticed, there are three major cases. The first is that the centre of the circle is on the left side of the detector (marked in the figure with I). The second case is that the circle's centre is inside the x-values determining the sides of the detector (marked as II in the figure). The third case is when the circle's centre is on the right side of the detector (marked as III in the figure).

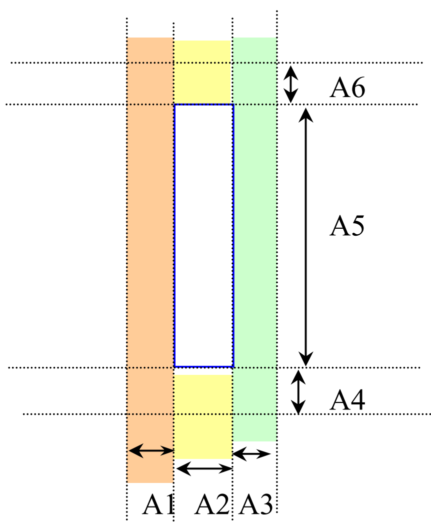

Figure 3.4 Name of variables connected to the area for calculation.

If the circle's centre is on the left side of the detector (case I), then area right of the detector's left side is calculated and stored in variable *A2*. The value of the area is normalized with the total area of the circle. If the circle reaches outside the right side of the detector, the normalized area *A3* is calculated. The part of the circle that is overlapping the detector element can now be calculated as:

$$Ax = A2 - A3 \qquad \text{Eq. 3.7}$$

The calculated area is normalized and tells how much of the circle that overlaps the detector. If there is an overlap as illustrated in the figure 3.3 case I a, then *A3* would equal zero.

If the circle's center is on the right side of the detector (case III), then area left of the detector's right side is calculated and stored in variable *A2*. If the circle reaches outside the left side of the detector, the normalized area *A1* is calculated. The solution in this case can be calculated as:

$Ax = A2 - A1$ *Eq. 3.8*

The last combination is case II, circle's centre inside the x values determining the sides of the detector. If the circle has any part outside the left side of the detector that area will be calculated and saved in variable *A1*. In case of that the circle has any part outside the right side of the detector, calculate the segment *A3*. The final area will be calculated as:

$Ax = 1 - A1 - A3$ *Eq. 3.9*

Note that if overlap as illustrated in the figure 3.3 case II c, then both *A1* and *A3* would equal zero.

In the corresponding way, the area for the y-axis is calculated. To get the final result for the area overlapping the detector, multiply the calculated areas *Ax* and *Ay* with each other (eq. 3.10).

$A = Ax * Ay$ *Eq 3.10*

3. 4 Estimation of error in calculations.

In order to determine how large error the described method will give the result following method has been applied. A detector element with the size 2 times 7, represented as 200 x 700 points (squares). Each of these points is tested to see whether it lies within the radius for a circle or outside. The circle's position is altered in steps of 0.5 along both x- and y-axis.

	Error in % of circle's area		Error in % of detector's area	
Radius	Maximal error	Average error	Maximal error	Average error
1.75	2.73 %	0.44 %	1.87 %	0.30 %
3.0	2.11 %	0.71 %	4.26 %	1.43 %
7.0	2.38 %	0.61 %	26.16 %	6.70 %

Table 3.1 Error in % as a function of radius when calculating the area from a circle projected on a detector element.

The largest error will occur when the circle is placed as in figure 3.3 case II b. The size of the errors indicates that they could be neglected considering the other presumptions made (circularity of flares and that the flare would have the same intensity within the whole area). Some of the values for the radius 7 do not look very good, but this corresponds in the used model for very close ranges, so it is actually quite acceptable.

4. Model of the ship

In the point source model the ship was defined as one point, with different intensities depending on the aspect angle. The intensity values were given in a table as a function of the azimuth angle. A total number of 72 values were used, which means a step of five degrees between each value. This is illustrated in the table below, only 5 values presented as an example.

Azimuth	Intensity
0	I_0
45	I_{45}
90	I_{90}
135	I_{135}
180	I_{180}

Table 4.1 Intensity as function of azimuth.

After the changes concerning the flares as described in previous chapters, it did not seem like a very bright idea to continue to treat the ship as a point source. The first approach was to describe the ship by using seven point sources. This was accomplished by dividing the ship into seven volume segements. Each segment is represented by a projected area, depending of the aspect angle. The seven segments are shown in figure 4.1.

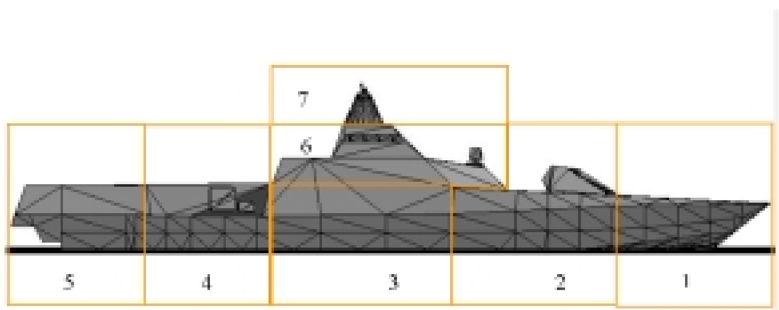

Figure 4.1.a The seven segment volumes that defines the ship.

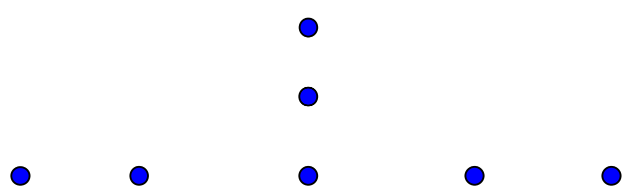

Figure 4.1.b The seven corresponding points that defines the ship.

As a result of using seven point sources, each point would have to be described with its intensity as a function of the azimuth angle.

Azimuth	P1	P2	P3	P4	P5	P6	P7	Total Intensity
0	I_{A0}	I_{B0}	I_{C0}	I_{D0}	I_{E0}	I_{F0}	I_{G0}	I_0
45	I_{A45}	I_{B45}	I_{C45}	I_{D45}	I_{E45}	I_{F45}	I_{G45}	I_{45}
90	I_{A90}	I_{B90}	I_{C90}	I_{D90}	I_{E90}	I_{F90}	I_{G90}	I_{90}
135	I_{A135}	I_{B135}	I_{C135}	I_{D135}	I_{E135}	I_{F135}	I_{G135}	I_{135}
180	I_{A180}	I_{B180}	I_{C180}	I_{D180}	I_{E180}	I_{F180}	I_{G180}	I_{180}

Table 4.2 Intensity for each point as a function of azimuth, in the seven point model.

Simulations showed that the seven point model was not ideal for the purpose. The points regarded from side aspect, where to far apart. During the discussions an idea was formulated. Why not describe the ship by using spheres, and use the calculation method described earlier. After some work, including test and simulation it was decided to describe the ship with 14 spheres. This is illustrated below by figure 4.3.

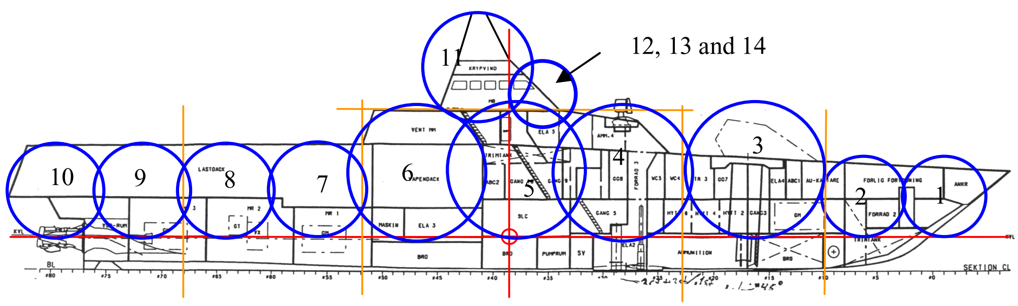

Figure 4.3 Ship seen from starboard view. Note that the three last spheres are laid side by side and therefore only one can be seen in this view.

Note that depending on the aspect angle, the size of the spheres will vary and in some cases even disappear when they are not seen from a certain view. The method used to get the tables to the simulation model can be described as follows.

First a CAD model is used to generate a 3-dimensionell IR CAD model of the ship. At FOI, a software program called RadThermIR is used for this purpose. This model is complemented with input from analysis of the material properties for the ship's different parts. If possible IR-measurements is used to validate (or modify) the 3D IR CAD model. The program calculates an image of the ship in the desired aspect and the grey level in the image corresponds to the radiance for each picture element. Next step is to generate images for the desired wavelength band to study or use in the simulation model. These generated images shows different aspects in azimuth, for this job the step selected was 22.5 degrees which gives 16 images. By using drawings of the ship, (side, front, aft and top view) the number of sub areas to use is determined. For each of the images the sub areas are identified and their corresponding projected area and radiance is determined. Finally the determined values describing the sub areas are stored in tables to be used in the simulation model. The method is also shown below as a flowchart.

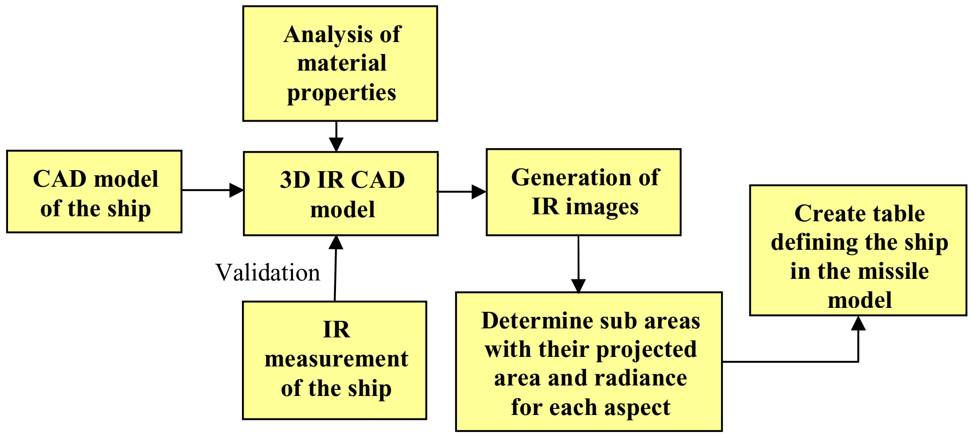

Figure 4.4 Flowchart over the method or process of defining the tables describing the sub areas of the ship (i.e spheres).

The area of each sub area is presented in table 4.3 below.

Az	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	Total area
0	A_{Aa}	A_{Ba}	A_{Ca}	A_{Da}	A_{Ea}	A_{Fa}	A_{Ga}	A_{Ha}	A_{Aa}	A_{Ja}	A_{Ka}	A_{La}	A_{Ma}	A_{Na}	A_0
45	A_{Ab}	A_{Bb}	A_{Cb}	A_{Db}	A_{Eb}	A_{Fb}	A_{Gb}	A_{Hb}	A_{Ab}	A_{Jb}	A_{Kb}	A_{Lb}	A_{Mb}	A_{Nb}	A_{45}
90	A_{Ac}	A_{Bc}	A_{Cc}	A_{Dc}	A_{Ec}	A_{Fc}	A_{Gc}	A_{Hc}	A_{Ac}	A_{Jc}	A_{Kc}	A_{Lc}	A_{Mc}	A_{Nc}	A_{90}
135	A_{Ad}	A_{Bd}	A_{Cd}	A_{Dd}	A_{Ed}	A_{Fd}	A_{Gd}	A_{Hd}	A_{Ad}	A_{Jd}	A_{Kd}	A_{Ld}	A_{Md}	A_{Nd}	A_{135}
180	A_{Ae}	A_{Be}	A_{Ce}	A_{De}	A_{Ee}	A_{Fe}	A_{Ge}	A_{He}	A_{Ae}	A_{Je}	A_{Ke}	A_{Le}	A_{Me}	A_{Ne}	A_{180}

Table 4.3 The areas of each of the surfaces as a function of the azimuth angles.

Each area could easily be translated to a radius using the formula for a circle's area, which yields following table of the radius of each sub area (sphere) as function of the azimuth angles.

Az	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14
0	r_{Aa}	r_{Ba}	r_{Ca}	r_{Da}	r_{Ea}	r_{Fa}	r_{Ga}	r_{Ha}	r_{ra}	r_{Ja}	r_{Ka}	r_{La}	r_{Ma}	r_{Na}
45	r_{Ab}	r_{Bb}	r_{Cb}	r_{Db}	r_{Eb}	r_{Fb}	r_{Gb}	r_{Hb}	r_{rb}	r_{Jb}	r_{Kb}	r_{Lb}	r_{Mb}	r_{Nb}
90	r_{Ac}	r_{Bc}	r_{Cc}	r_{Dc}	r_{Ec}	r_{Fc}	r_{Gc}	r_{Hc}	r_{rc}	r_{Jc}	r_{Kc}	r_{Lc}	r_{Mc}	r_{Nc}
135	r_{Ad}	r_{Bd}	r_{Cd}	r_{Dd}	r_{Ed}	r_{Fd}	r_{Gd}	r_{Hd}	r_{rd}	r_{Jd}	r_{Kd}	r_{Ld}	r_{Md}	r_{Nd}
180	r_{Ae}	r_{Be}	r_{Ce}	r_{De}	r_{Ee}	r_{Fe}	r_{Ge}	r_{He}	r_{re}	r_{Je}	r_{Ke}	r_{Le}	r_{Me}	r_{Ne}

Table 4.4 The radius for each of the surfaces as a function of the azimuth angle.

Finally the intensity for each sub area is defined also by using the images generated by RadThermIR.

Az	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	Total intensity
0	I_{Aa}	I_{Ba}	I_{Ca}	I_{Da}	I_{Ea}	I_{Fa}	I_{Ga}	I_{Ha}	I_{Ia}	I_{Ja}	I_{Ka}	I_{La}	I_{Ma}	I_{Na}	I_0
45	I_{Ab}	I_{Bb}	I_{Cb}	I_{Db}	I_{Eb}	I_{Fb}	I_{Gb}	I_{Hb}	I_{Ib}	I_{Jb}	I_{Kb}	I_{Lb}	I_{Mb}	I_{Nb}	I_{45}
90	I_{Ac}	I_{Bc}	I_{Cc}	I_{Dc}	I_{Ec}	I_{Fc}	I_{Gc}	I_{Hc}	I_{Ic}	I_{Jc}	I_{Kc}	I_{Lc}	I_{Mc}	I_{Nc}	I_{90}
135	I_{Ad}	I_{Bd}	I_{Cd}	I_{Dd}	I_{Ed}	I_{Fd}	I_{Gd}	I_{Hd}	I_{Id}	I_{Jd}	I_{Kd}	I_{Ld}	I_{Md}	I_{Nd}	I_{135}
180	I_{Ae}	I_{Be}	I_{Ce}	I_{De}	I_{Ee}	I_{Fe}	I_{Ge}	I_{He}	I_{Ie}	I_{Je}	I_{Ke}	I_{Le}	I_{Me}	I_{Ne}	I_{180}

Table 4.5 The intensity for each of the surfaces, as a function of the azimuth angle.

The values in the tables 4.4 and 4.5 are used in the simulation model. Before using the calculated values a check is performed to control that both the total area and the total intensity for each aspect is correct when using the sphere model. The representations with spheres are presented in figure 4.5 to 4.9.

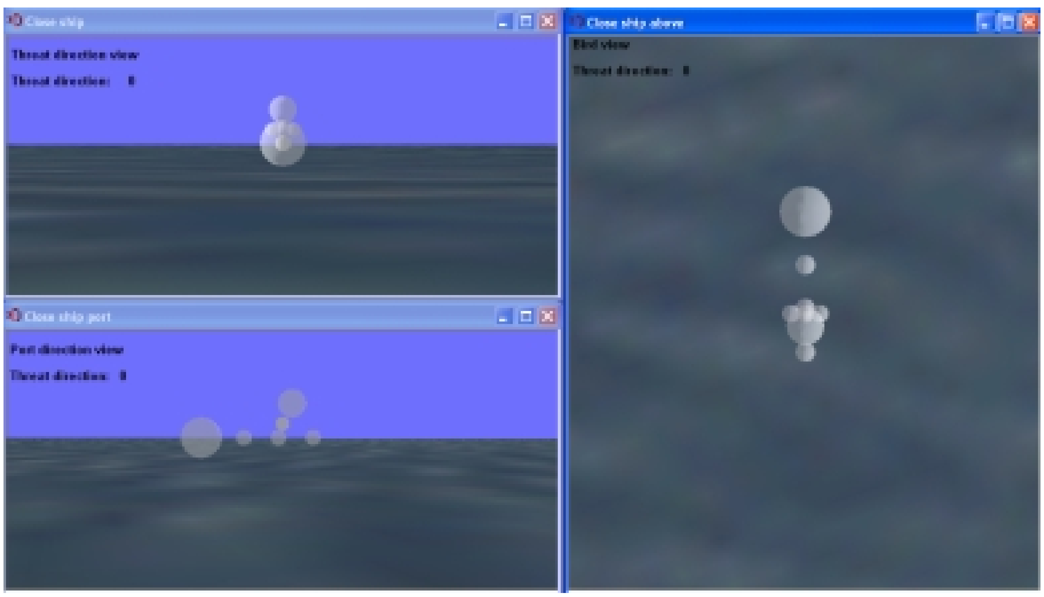

Figure 4.5 Front aspect seen from missile (above to the left). Port view and bird view shows the spheres distribution.

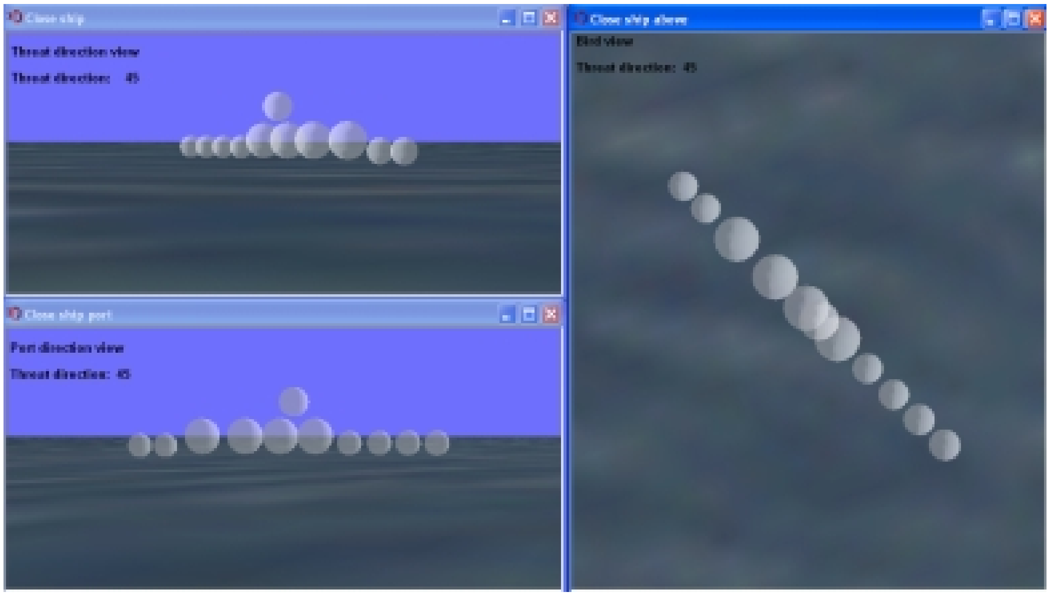

Figure 4.6 Aspect in 45 degrees, seen from missile (above to the left). Port view and bird view shows the spheres distribution.

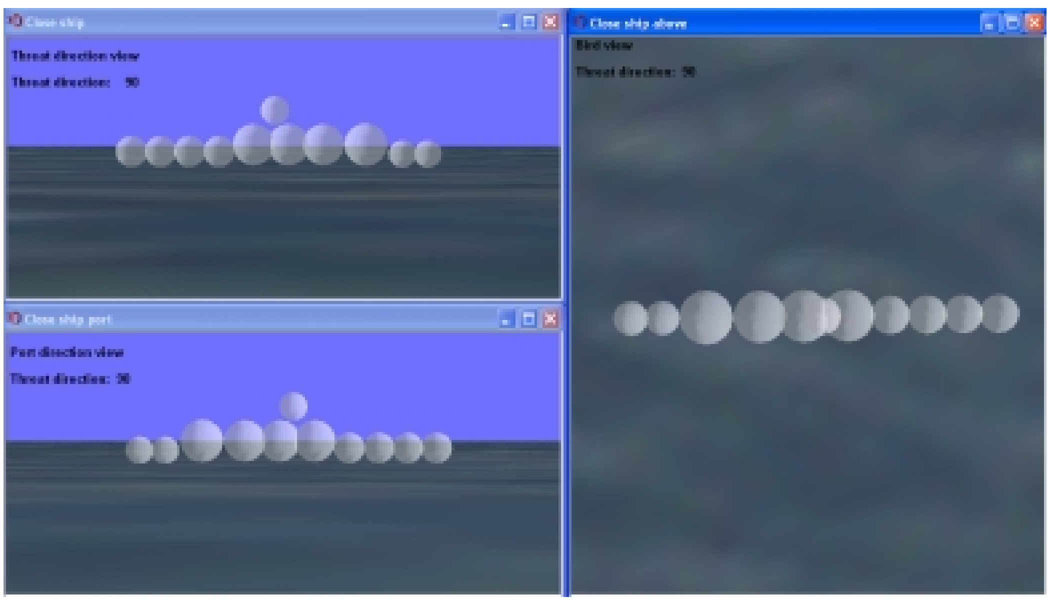

Figure 4.7 Aspect in 90 degrees, seen from missile (above to the left). Port view and bird view shows the spheres distribution.

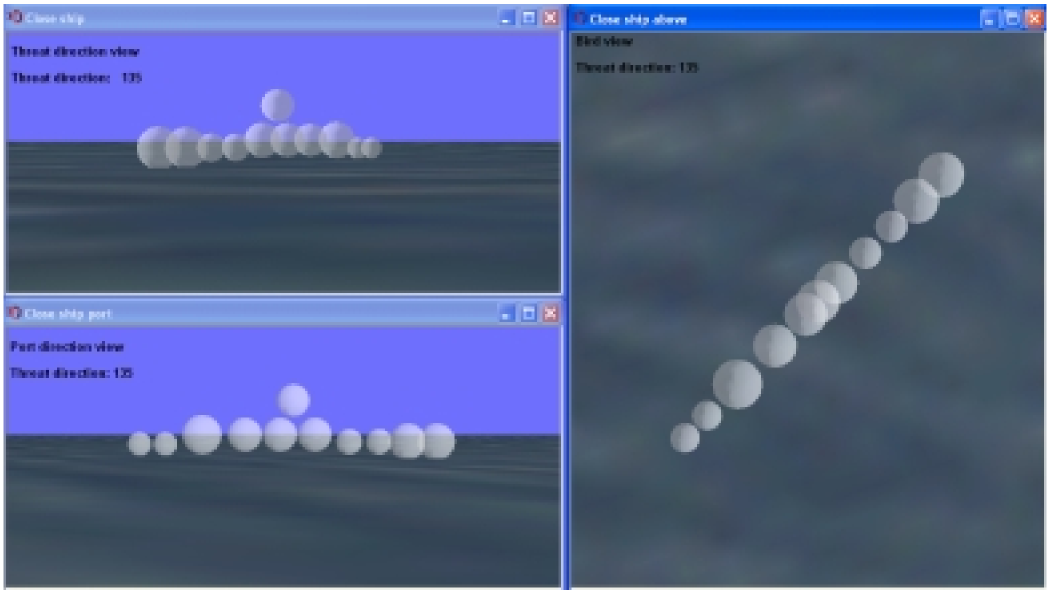

Figure 4.8 *Aspect in 135 degrees, seen from missile (above to the left). Port view and bird view shows the spheres distribution.*

Figure 4.9 *Aspect in 180 degrees, seen from missile (above to the left). Port view and bird view shows the spheres distribution.*

5. Calculation of a circle's area when overlapped by another circle.

A problem that have been solved but not implemented in the model is when there are two circles and one is overlapping the other. Again several combinations can be found. In order to simplify the description of the cases the lower circle (marked red in figure 5.1) is partly or completely covered by the upper circle (marked blue in figure 5.1). This also means that when calculating the projection of a sphere on the detector plane a third coordinate is needed to determine the order which of the circles is the upper respectively the lower circle.

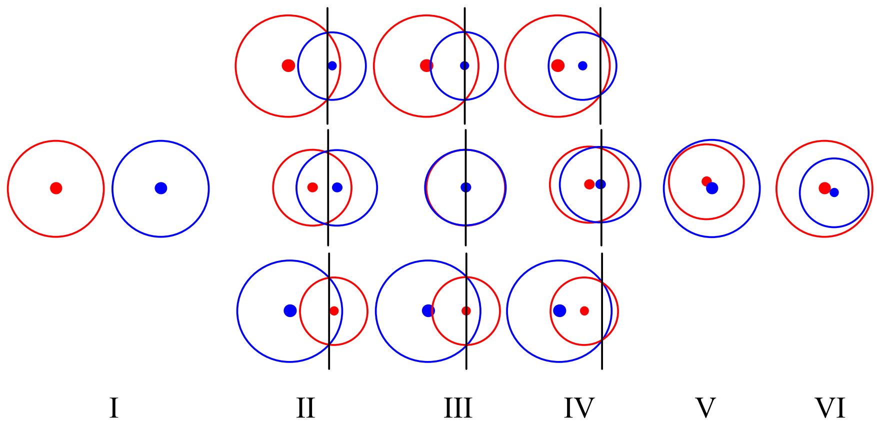

Figure 5.1 *Possible combinations of two circles in regards to overlap, where blue circle is in front of red circle.*

- *I.* *The two circles do not overlap each other.*
- *II* *Overlap, the centres of the circles are on each side of the intersection line (marked black in figure 5.1)*
- *III* *Overlap, the centres of one of the circles is on the intersection line.*
- *IV* *Overlap, the centres of the circles are on the same side of the intersection line.*
- *V* *Overlap, the upper circle is completely covering the lower circle.*
- *VI* *Overlap, the upper circle covering the lower circle but lower circle is larger then the upper circle.*

The circles are described with their coordinates, and their radius. In order to determine if they are overlapping, the distance between the circles origin can be calculated as indicated in equation 5.1.

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad \textit{Eq.5.1}$$

A situation where there is an overlap is shown in figure 5.2.

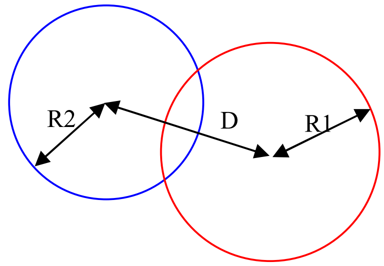

Figure 5.2 Overlap and definition of distance D.

If the circles do not overlap the condition in eq. 5.2 is fulfilled.

$R1+R2 \quad D$ *Eq. 5.2*

If the condition in eq. 5.2 is not met the circles are overlapping somehow. Next step will be to determine if one of the circles is completely inside the other circle (case V or VI in figure 5.1). This can be done by checking the conditions given in equations 5.3 and 5.4.

$D + R2 \quad R1$ *Eq. 5.3*

If true the lower circle is completely covered by the upper circle, which yields that the contribution from the lower circle will be zero.

$D + R1 \quad R2$ *Eq. 5.4*

If true the upper circle is completely inside the lower circle, which yields that the contribution from the lower circle will be:

$$f = \frac{\pi(R_2^2 - R_1^2)}{\pi R_2^2}$$ *Eq. 5.5*

In equation 5.5 the variable *f* is the factor which tells how much of the lower circle that is visible. If none of the three conditions above is true the two circles is overlapping and there are two points where the circles intersects. In order to be able to calculate the visible area of the lower circle, the intersection points have to be calculated. This is done by using the equation for a circle as described below.

$$(x - x_1)^2 + (y - y_1)^2 = R_1^2$$ *Eq. 5.6*

$$(x - x_2)^2 + (y - y_2)^2 = R_2^2$$ *Eq. 5.7*

By using equation 5.6 to express the variable *x*, and use that in equation 5.7 it is possible to calculate the y-values. After some formula exercise a number of equations are obtained in order to calculate the final y-values.

$$k_1 = 2(x_1 - x_2) \qquad \textit{Eq. 5.8}$$

$$k_2 = R_2^2 - R_1^2 - (x_1 - x_2)^2 + y_1^{\ 2} - y_2^{\ 2} \qquad \textit{Eq. 5.9}$$

$$k_3 = 2(y_1 - y_2) \qquad \textit{Eq. 5.10}$$

$$k_4 = -\frac{2(k_1^{\ 2}\ y_1 + k_2 k_3)}{(k_1^{\ 2} + k_3^{\ 2})} \qquad \textit{Eq. 5.11}$$

If the circles have the same origin the equation 5.11 would be division by zero, however that case was handled with the previous conditions.

$$k_5 = \frac{k_1^{\ 2}\ y_1^{\ 2} - k_1^{\ 2}\ R_1^2 + k_2^{\ 2}}{(k_1^{\ 2} + k_3^{\ 2})} \qquad \textit{Eq. 5.12}$$

$$y = -\frac{k_4}{2} \pm \sqrt{\left(\frac{k_4}{2}\right)^2 - k_5} \qquad \textit{Eq. 5.13}$$

If y in equation 5.13 is a double root, the solution for x is given by the equations 5.6 and 5.7.

If two y-values are obtained from the use of equation 5.13, then use equation 5.14. Note that index n in equation 5.14 refers to circle 1 and 2.

$$x = x_n \pm \sqrt{R_n^2 - (y - y_n)^2} \qquad \textit{Ekv. 5.14}$$

When this equation is used a total number of eight values for x is given, four for each y-value.

These four x-values for each y-value are compared with each other and the two solutions that have the same value is the true value. When this is done the intersection points are calculated, and there coordinates are given by(x3, y3) and (x4, y4).

The coordinates for the two intersection points are used to calculate the length of the line L described in figure 5.3.

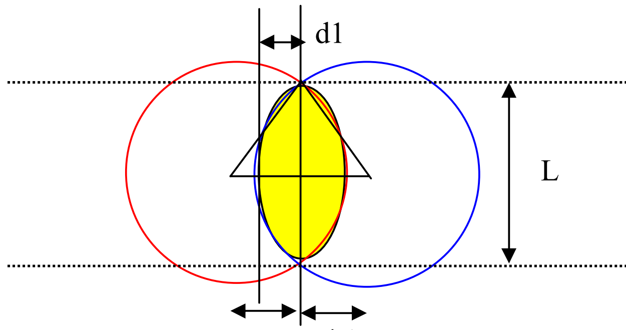

Figure 5.3 Area to be calculated is marked in yellow.

$$L = \sqrt{(x_3 - x_4)^2 + (y_3 - y_4)^2} \qquad \text{Eq. 5.15}$$

The distances between the centre of each circle and the line L are calculated.

$$h_1 = \sqrt{R_1^2 - \left(\frac{L^2}{4}\right)} \qquad \text{Eq. 5.16}$$

$$h_2 = \sqrt{R_2^2 - \left(\frac{L^2}{4}\right)} \qquad \text{Eq. 5.17}$$

Then very similar to chapter 1, the following equations are used to calculate the covered area of the lower circle. Note the index n in the equations below, the index indicates which of the circle's segment that is calculated.

$$\frac{L}{2} = R_n \sin\left(\frac{\theta_n}{2}\right) \qquad \text{Eq. 5.18}$$

$$\theta_n = 2\arcsin\left(\frac{L}{2R_n}\right) \qquad \text{Eq. 5.19}$$

$$A_{circlesector_n} = \frac{1}{2} R_n{}^2 \theta_n \qquad \text{Eq. 5.20}$$

$$r_n = R_n \sin\left(\frac{\theta_n}{2}\right) \qquad \text{Eq. 5.21}$$

$$A_{cone\,n} = 2\frac{1}{2} h_n \frac{L}{2} = \frac{1}{2} h_n L \qquad \text{Eq. 5.22}$$

$$A_{segment_n} = A_{circle\sec tor_n} - A_{cone\,n} \qquad \textit{Eq. 5.23}$$

In order to find out which of the cases II, III and IV in figure 5.1 is the actual case, an intersection line L (figure 5.3) is used. To determine on which side of the intersection line the two circle's centre's are the equation 5.24 can be used.

$$(x_4 - x_3)(y - y_3) - (y_4 - y_3)(x - x_3) = 0 \qquad \textit{Eq. 5.24}$$

By using the coordinates for each circle's centre in equation 5.24 (left side) and check the sign of the result for each of the two circles then the proper case can be determined. If the signs are different, this means that they are on the opposite side of the intersection line and thus can the hidden area can be calculated as:

$$A_{hidden} = A_{segment1} + A_{segment2} \qquad \textit{Eq. 5.25}$$

Otherwise they have their centre on the same side. If the radius of the upper circle is greater or equal to lower then the radius (radius R1> radius R2) then the hidden area is calculated as:

$$A_{hidden} = A_{segment1} + A_{circle2} - A_{segment2} \qquad \textit{Eq. 5.26}$$

If the radius of the upper circle is smaller than the lower circles radius (radius R1 < radius R2), use equation 5.27 instead.

$$A_{hidden} = A_{segment2} - A_{segment1} \qquad \textit{Eq. 5.27}$$

Finally the calculated hidden area has to be normalized.

6. Calculation of missdistance or impact point

In the simulations the criteria for when to stop the simulation is by looking at the distance between the missile and the ship. That distance will decrease during the simulation, time step by time step, until the missile will has passed the ships point of gravity. When the missile has passed the ship the distance will start to increase. By using the positions (in the ship’s coordinate system) of the missile from the two latest time steps a line equation for the path of the missile in the form given in equation 6.1 can be expressed. The closest relative distance between the missile path and the ship is calculated.

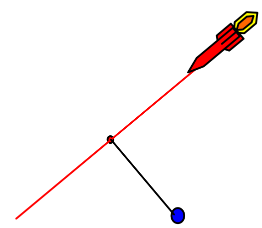

Figure 6.1 Missile passage of ship.

In the simulation model an algorithm also calculated if the ship was hit or not. If the ship is hit the coordnates has to be determined. In case of a hit, a box is used to represent the hull of the ship. The miss distance was representing the distance between point of gravity and missile passage point as mentioned before. In order to interpret the missdistance the first approach for was to translate the miss distance into some kind of safety measurement for the ship. For this purpose different radius where used. This is illustrated in figure 6.2 below.

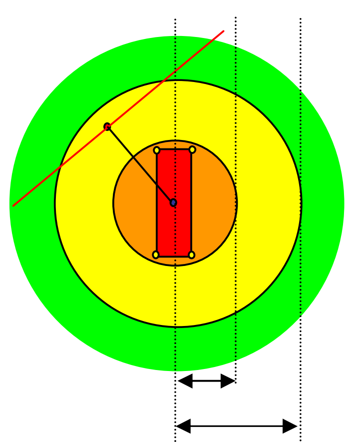

Figure 6.2 Missile passage of ship. Different safety zones indicated by color how dire the result can be. The inner circle (surrounding the the orange area) has a radius of half the length of the ship and some meteres added. The outer circle has the radius of the ship length and some metres added.

As can be seen in the above figure, the distance between the hull and the missile path will vary even if the radius is the same. So there was a need for a more correct calculation of the miss distance.

6.1 Calculation of miss distance

The selected solution for how to solve the problem with miss distance is illustrated in figure 6.3. The situation can be regarded as a two dimensional problem in the x and y plane. The slant rate for the missile in z is very low for anti ship missiles.

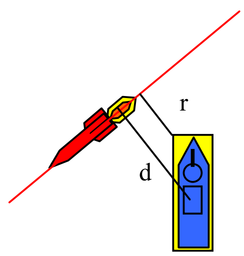

Figure 6.3 Missile passage of ship. Variable d is the closest distance between the missle's path and the point of gravity of the ship. Variable r is a more realistic miss distance.

In order to calculate the real miss distance or the hit coordinates the first step is to represent the ship with a box (as indicated in figure 6.3).

Definition of parameters:

φ	missile's course relative the ship, in the ship's coordnate system or rather the missile's velocity vector represented as an angle.
x_m, y_m, z_m	position of missile in the ship coordinate system at the closest point of the ship's point of gravity
x_i, y_i	corner coordinates of box, i = 1,2,3 and 4

The equation for a straight line is defined by either of the equation 6.1 or 6.2.

$$y = kx + l \qquad \textit{Eq. 6.1}$$

$$ax + by + c = 0 \qquad \textit{Eq. 6.2}$$

Since the missile's velocity vectors are given(i.e the course in the xy-plane), the parameter *k* (in eq. 6.1.) is easy to express.

$$k = \frac{\sin(\varphi)}{\cos(\varphi)} \qquad \textit{Eq. 6.3}$$

Using equation 6.1 and 6.3 in the equation 6.2 will give the equation 6.4.

$$-x_m \sin(\varphi) + y_m \cos(\varphi) - l\cos(\varphi) = 0 \qquad \text{Eq 6.4}$$

Identifiyng the parameters in equation 6.2 with those in equation 6.5 will give:

$$a = -\sin(\varphi)$$
$$b = \cos(\varphi)$$
$$c = -l\cos(\varphi) = y_m \cos(\varphi) - x_m \sin(\varphi)$$

In order to determine if the ship is hit or not, the equation for a line that describes the missile path can be used. The line equation 6.2 can be used. Insert the coordinate pairs for the box in the equation 6.2 and four values are obtained. If the sign of these four values are the same, the ship is not hit by the missile. In other case the ship is hit by the missile. In the case that the ship was not hit the shortest distance between the missile's path and the ship can be determined by using the coordinates for the point defining the box.

The distance between a point (given by the coordinates m and n) and a straight line expressed as in equation 6.2 can be calculated according to equation 6.5.

$$r = \frac{|am + bn + c|}{\sqrt{a^2 + b^2}} \qquad \text{Eq. 6.5}$$

With the definition from previous page for parameter *a* and *b* in equation 6.5 will yield equation 6.6.

$$r = \frac{|am + bn + c|}{\sqrt{a^2 + b^2}} = \frac{|am + bn + c|}{\cos^2\varphi + \sin^2\varphi} = |am + bn + c| \qquad \text{Eq. 6.6}$$

By inserting the corner coordinates x_i, y_i in the equation 6.6, the result with the smallest value is the miss distance. No consideration has been taken to the z value, the reason for this is that it will only have a minmal effect on the resulat and thus can be ignored.

6.2 Calculation of hit coordinates

If the ship is hit the following method can be used to determine the impact point.

The reversed direction of the missile's course relative to the ship can be exressed as:

$$\phi = \varphi + \pi \qquad \text{Eq. 6.7}$$

Where φ is the missile course. By looking at the sign for the two functions cos(ϕ) and sin(ϕ) a total number of eight combinations are possible. The sign function has three possible output, they are -1, 0 or +1. In the figure 6.4, the signs are indicated as functions of the threat direction. In table 6.1 the different possible combinations are listed with the corresponding lines that can be intersected.

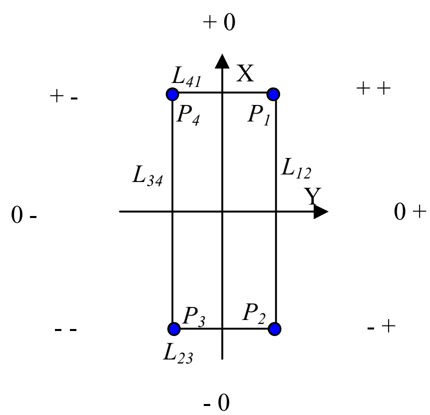

Figure 6.4 Signs of threat direction in the ship's coordinate system.

Possible solutions according to sign of threat direction							
+ 0	+ +	0 +	- +	- 0	- -	0 -	+ -
L_{41}	L_{41}	L_{12}	L_{12}	L_{23}	L_{23}	L_{34}	L_{34}
x	L_{12}	x	L_{23}	x	L_{34}	x	L_{41}

Table 6.1 Possible intersections between the missile path and the lines describing the box round the ship.

In order to select the line or the two lines that might be possible a matrix is used and all that has to be done is to calculate the index. By using a base of three, the sign can be calculated as:

$$Sign\_index = 3(sign(\cos(\phi)) + 1) + sign(\sin(\phi))+2 \qquad \text{Eq. 6.8}$$

The calculated index will be a number within the range from 1 to 9. However the index 5 is an impossible combination (because the function cos and sin can not both be zero for the same angle) and has to be eliminated. This can be done by checking if index is larger then 4 and if so subtract 1 from the index.

Possible solutions according to sign of threat direction							
1	2	3	4	5	6	7	8
- -	- 0	- +	0 -	0 +	+ -	+ 0	+ +
L_{23}	L_{23}	L_{12}	L_{34}	L_{12}	L_{34}	L_{41}	L_{12}
L_{34}	x	L_{23}	x	x	L_{41}	x	L_{41}

Table 6.2 Possible intersections between the missile path and the lines describing the box round the ship.In this table ordered by the index.

If the missile's path (a straight line) cuts the box describing the ship, there are two possibilities. The possibilities are that two of the lines L_{12}, L_{23}, L_{34} and L_{41} are crossed or the path cuts through one of the corner points P_1, P_2, P_3, and P_4. The next step is then to determine which of the lines that are crossed. This is done by using the equation 6.9 or 6.10.

$$y_{ij} = \frac{x_i \sin(\varphi) - c}{\cos(\varphi)} \qquad \textit{Eq 6.9}$$

$$x_{ij} = \frac{y_i \cos(\varphi) + c}{\sin(\varphi)} \qquad \textit{Eq.6.10}$$

Four calculations have to be performed. The calculations will be yielding the following coordinate pairs.

x_{12}, y_1
x_3, y_{23}
x_{34}, y_3
x_1, y_{41}

At least one of the above relations is going to have the value true, most likely two of them. By checking if the calculated values (two digit index) are within the border a corresponding flag (hit flag) can be set to the value true otherwise to false.

If the table 6.2 contains the line index for the lines of the box, then the sign_index can be used to find the first line index indicating which hit flag to test. If true the solution is found otherwise use the second line index given by the sign index.

If the method described above is used and the calculations show that the ship is hit, there may be a need for confirming that the ship really is hit. In figure 6.5 four different cases are shown, but all of them will indicate that the ship is hit. Case 1 is a clear hit. Case 2 will give wrong imact point. Case 3 and 4 should give a miss distance, not a hit.

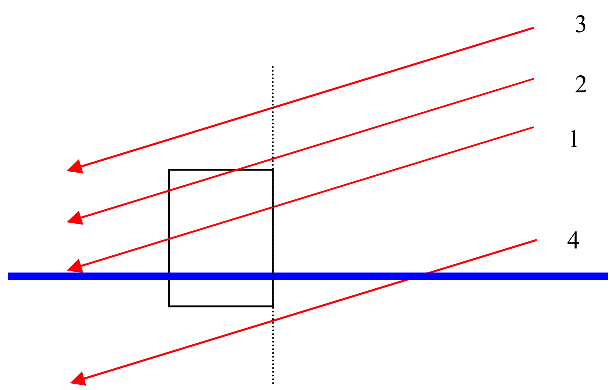

Figure 6.5 Illustrations of missile paths that may or may not be a hit.

Since the hit coordinates in x and y are given, the z value can be calculated by using equation 6.11. Note that v is the velocity vector.

$$x_h = x_m + v_x t$$
$$y_h = y_m + v_y t \qquad \textit{Eq. 6.11}$$
$$z_h = z_m + v_z t$$

Use either the equation for x or y to calculate the value for parameter *t*. Use the calculated value *t* in the equation to calculate the hit coordinate for z. Check if the calculated z-value is within the limits. If so, case 1 is solved.

If the calculated z value is outside limits and and has a value indicating that it is below the ship (case 4). The miss distance can then be calculated as

$$r_{miss} = (z_{calc} - z_{ship})\cos(\alpha) \qquad \textit{Eq. 6.12}$$

where α is

$$\alpha = \arctan\left(\frac{v_z}{\sqrt{v_x^2 + v_y^2}}\right) \qquad \textit{Eq. 6.13}$$

If the calculated z value is outside and above the ship (case 2 and 3 in figure 6.5) further calculations have to be done. This can be done be using the method described previously in this chapter (eq. 6.7 to 6.10) with the change by using φ instead of ϕ, in order to determine the exit point for the missile's velocity vector.

By checking the new calculated z value, it is either above the ship (case 3) and miss distance can be calculated by using equations 6.12 and 6.13 or it is within the range of the ship (case 3). In order to determine the hit coordinates in case 2 the equation 6.11 can be used. The z value for the height of the box is used in order to determine the variable *t*. Use variable *t* to calculate x and y coordinate.

7. Relative wind

In the work with a countermeasure algoritm it is import to be able to calculate the separation between ship and IR-decoy. The IR-decoy is expected to move with the wind, which means that the wind speed and the wind course are parameters to be taking into account. The separation is also effected by the ship's own course and speed. The interesting view of the separation is the one seen from the missile's seeker.

7.1 Calculation of relative wind and speed

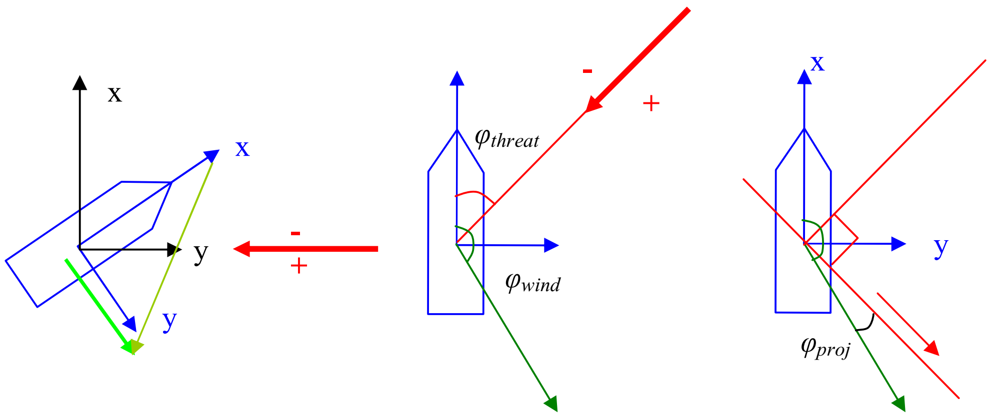

Figure 7.1 Definition of coordinate systems and angles.
Black (earth fixed coordinate system)
Blue (ship coordinate system)
Light green (wind course in earth coordinate system)
Dark green (relative wind in ship coordinate system)
Read (threat)

The following definitions will be used in this chapter. Φ represents angles in earth fixed coordinate system. φ represents the angles in the ship's coordinate system. The variable *v* is used to represent the velocity.

The relation between the wind course and the ship in the earth fixed coordinate system is given by the equationen 7.1. Corresponding for the threat direction is given by the equationen 7.2.

$$\varphi_{wind} = \Phi_{wind} - \Phi_{ship} \qquad \text{Eq. 7.1}$$

$$\varphi_{threat} + \Phi_{ship} = \Phi_{threat} \qquad \text{Eq. 7.2}$$

In order to get a value for the relative wind speed in the earth fixed coordinate system the relative wind vector has to be calculated.

$$\begin{cases} x_{rel} = v_{wind}\cos(\Phi_{wind}) - v_{ship}\cos(\Phi_{ship}) \\ y_{rel} = v_{wind}\sin(\Phi_{wind}) - v_{ship}\sin(\Phi_{ship}) \end{cases}$$ *Eq. 7.3*

The calculated vector is used to calculate the relative wind.

$$v_{rel} = \sqrt{x_{rel}^2 + y_{rel}^2}$$ *Ekv. 7.4*

The relative wind course (ship's coordinate system) is calculated by using equation 7.5 and 7.6.

$$\cos(\Phi_{rel}) = \frac{x_{rel}}{v_{rel}}$$ *Eq. 7.5*

$$\Phi_{rel} = \arccos(\frac{x_{rel}}{v_{rel}})$$ *Eq. 7.6*

In order to get the true sign for the calculated angle a test of the sign of x_{rel} and y_{rel} has to be done.

7.2 Calculation of projected wind speed

As mentioned earlier, the relativ wind speed ”seen” from the missile (variable v_{threat_rel}) is what in the end will effect the miss distance. In order to calculate the projected wind speed a unit vector is used according to:

$$\Phi_{proj} = \Phi_{threat} + \frac{\pi}{2}$$ *Eq. 7.7*

With the projected angle calculated the unit vector's composants can be calculated.

$$\begin{cases} x_{proj} = \cos(\Phi_{proj}) \\ y_{proj} = \sin(\Phi_{proj}) \end{cases}$$ *Eq. 7.8*

By using equation 7.9 and identify the parameters the projected relative wind can be determined.

$$|A||B|\cos(\alpha) = AB$$ *Eq. 7.9*

where $0 \leq \alpha \leq \pi$.

Vector A is the relative wind vector and is given by x_{rel} and y_{rel}. Vector B is the unit vector and is given by x_{proj} and y_{proj}. The projected relative wind vector is given by:

$$|A|\cos(\alpha) = \frac{AB}{|B|} \qquad \textit{Eq. 7.10}$$

Since B is a unit vector then the lenght of the vector is equal to one, as shown in equation 7.11.

$$|B| = \sqrt{\sin^2 \Phi_{proj} + \cos^2 \Phi_{proj}} = 1.0 \qquad \textit{Eq. 7.11}$$

This will give that the sought projected wind speed is calculated as:

$$v_{threat\_rel} = x_{rel} x_{proj} + y_{rel} y_{proj} \qquad \textit{Eq. 7.12}$$

7.3 Derivate of projected wind velocity

As described previously the projected wind speed is an important variable to consider.

In order to derivate the projected wind velocity, the variables are expressed in the earth fixed coordinate system.

By using equation 7.12 together with equation 7.3, 7.9 and 7.7 the expression for the relative projected wind speed is obtained.

$$v_{threat\_rel} = v_{wind}\cos(\Phi_{wind})\cos(\Phi_{threat} + \frac{\pi}{2}) - v_{ship}\cos(\Phi_{ship})\cos(\Phi_{threat} + \frac{\pi}{2}) + v_{wind}\sin(\Phi_{wind})\sin(\Phi_{threat} + \frac{\pi}{2}) - v_{ship}\sin(\Phi_{ship})\sin(\Phi_{threat} + \frac{\pi}{2}) \qquad \textit{Eq. 7.13}$$

If the equation 7.13 is derivated considering the ship's course only the second and fourth term has to be considered. The result will be:.

$$\frac{v_{threat\_rel}}{d\phi_{ftg}} = -v_{ship}\cos(\Phi_{ship} - \Phi_{threat}) \qquad \textit{Eq. 7.14}$$

If the equation 7.13 instead is derivated considering the ship's speed again only the second and fourth term has to be considered. The result will be:.

$$\frac{v_{threat\_rel}}{d v_{ftg}} = \sin(\Phi_{ship} - \Phi_{threat}) \qquad \textit{Eq. 7.15}$$

One onclusion from the expressions in the equations 7.14 and 7.15 is that the only two variables that have effect on the relative projected wind are the ship's course and speed. That may not come as a surprise, since wind and threat direction may be very hard to alter.

8. Final remarks

The method for the work done described in this report has followed in principally the same way. First there were discussions about something in the model that was not up to the standard of what the project team wanted. During those discussions a solution was brought up. After that, the solution was analysed and written down. The solution was then tested in Matlab and changed if needed (– almost every time), and after that imported in to the simulation model. The simulation results after a change was compared with prevoius results from the model, in order to check that no mistakes were made.

After the described changes were made a single simulation run takes about 50% longer time then it did before, from 20 seconds to 30 seconds. The time for a single simulation is of course depending on the processor capacity of the computer used. However, the loss of time is acceptable considering the more accurate results that are obtained.

Images in chapter 4 (figure 4.5 to 4.9) where generated by the program AFE, which is a software package made at FOI for visalization of simulations.

The big changes in the model are that the number of tables describing the target and IR decoys has increased. For instance, the IR decoys do not only have a table describing the intensity as a function of time but also a table for radius as a function of time.

The order in which the changes in the model were done is descibed by the order of the chapters in this report. Chapter 5 is not implemented in the simulation model. The reason for that is that there has been enough need for implementing the algorithm in the model.

The results of the changes are that the simulation model today better represents extended targets and as a result will give a more thrustworthy outcome.

9. References

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