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Scientific report

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Report title

Initially increasing penetration resistance, friction and target size effects in connection with rigid projectile penetration and perforation of steel and metallic targets

Abstract (not more than 200 words)

A model for rigid projectile penetration and perforation of hard steel and metallic target plates is suggested. The intended application is in computer programs for assessment of effects and vulnerability.

The target material resistance to penetration is assumed to be the sum of nose resistance and friction along the part behind the nose. The nose resistance increases initially with the penetration depth to a constant value, which corresponds to lateral displacement of the material along the projectile trajectory. The friction also increases from the start and becomes constant when the rear end of the projectile passes the front face of the target. When the front end of the projectile is sufficiently close to the rear surface, then the remaining volume of target material in front of the projectile is crushed and forms fragments. At the penetration depth where this occurs the force required for crushing equals the force required for continued lateral displacement of the target material. The fragments are ejected with the same velocity as the projectile.

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Rapportens titel (i översättning)

Inverkan av initialt ökande inträngningsmotstånd, friktion samt målets storlek vid stela projektilers inträngning i och genomträngning av hårt stål och metalliska mål.

Sammanfattning (högst 200 ord)

En modell för stela projektilers penetration och perforation av målplåtar av stål och metalliskt material föreslås. Modellen är avsedd att tillämpas i datorprogram för värdering av verkan och sårbarhet..

Målmaterialets inträngningsmotstånd antas vara summan av motstånd vid nosen och friktion bakom nosen. Nosmotståndet ökar från starten med inträngningsdjupet till ett konstant värde, vilket motsvaras av att målmaterialet deplaceras väsentligen radiellt utåt för att ge plats för projektilen. Friktion ökar också från start och blir konstant när projektilens bakända passerar målets framsida. När projektilens främre ända är tillräckligt nära den bortre plåtytan, krossas det återstående materialet framför projektilen och bildar splitter. Vid det inträngningsdjup där detta inträffar är kraften som behövs för krossningen lika med kraften som skulle behövas för fortsatt sidledes förflyttning av materialet. Splittren kastas ut med samma hastighet som projektilen.

Nyckelord

Penetration perforation friktion stel projektil modell

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Notation

D	target diameter	[m]
d_P	projectile diameter	[m]
E_T	elastic modulus for target material	[Pa]
h	target thickness	[m]
h^*	transition thickness between penetration and perforation	[m]
$h^{(*)}$	thickness of target material that forms secondary fragments	[m]
L_P	projectile length	[m]
m_P	projectile mass	[kg]
m_T	mass of secondary fragments from target rear surface	[kg]
P	projectile penetration depth	[m]
R_T	axial target resistance	[Pa]
u	projectile penetration velocity	[m/s]
t	time	[s]
v_P	projectile impact velocity	[m/s]
v_{exit}	projectile velocity after perforation	[m/s]
W	projectile impact energy	[J]
W_p	minimum target perforation energy	[J]
Y_T	uniaxial yield strength of target material	[Pa]
β	deep hole target penetration resistance parameter	
β_i	initial target penetration resistance parameter	
Λ	plastic deformation diameter	[m]
γ	projectile sharpness parameter	
ρ_T	target density	[kg/m^3]
θ	half apex angle for a ogive-nose projectile	
τ	fragment acceleration time	[s]
ξ	parameter describing initial penetration resistance	
ζ	parameter describing initial penetration resistance	
μ	friction coefficient	
λ	lateral pressure coefficient	
ν_T	Poisson ratio for target material	

Introduction

A model for rigid projectile penetration and perforation of hard steel and metallic materials was recently suggested [1]. The projectile is assumed to be retarded by a constant force $R_T=\beta Y_T$, where Y_T is the uniaxial yield strength of the target material and $\beta=5$ is the penetration coefficient. This is always a simplification of reality. Real penetration resistance R_T should increase with the penetration depth P and smoothly reach the limit βY_T at some depth ζd_P. Here d_P is the projectile diameter and ζ is a non-dimensional parameter, which depends on the length and sharpness of the projectile nose. Furthermore real penetration resistance should be described with two additive components, one of which corresponds to the work that is required to displace the target material in front. This component should be proportional to the square of the diameter d_P. The other component corresponds to friction between the projectile and the target material. If it is assumed that there is contact between the target and the projectile along the entire length L_P of the latter, then the friction force should be proportional to the product $d_P L_P$. Thus rigid projectiles with the same mass m_P, nose shape, diameter d_P and impact velocity v_P but with different length L_P should penetrate to different depths P so that shorter projectiles reach larger depth.

It is also reasonable to expect the penetration coefficient β to depend on the shape of the projectile nose, so that β is smaller for a sharper shape. This is based on the theoretical difference between the pressures that are required to expand cylindrical and spherical holes in elastic-plastic materials of infinite extent.

Below the basic model in [1] is first described. Thereupon it is modified to account for (non-viscous) friction only. Next initially increasing penetration resistance without friction is accounted for. Thereupon both effects are simultaneously considered. This is followed by suggestions for how β should vary with the half apex angle θ for conical-nose projectiles in semi-infinite targets and for how large targets must be in order to be considered as semi-infinite. Finally the model is compared with some experimental results.

Rigid projectiles are assumed to make holes with the same diameter d_P. If the impact velocity is sufficiently high, then the projectile is eroding instead of rigid and makes a larger hole diameter. The corresponding penetration resistance is often assumed to be proportional to the product of the target density and the square of the penetration velocity. With such a penetration resistance model the effects of friction and initially increasing penetration resistance cannot be calculated analytically. Furthermore, the value for the penetration coefficient β would be unrealistically small.

Basic model

With the basic model in [1] the penetration depth in a semi-infinite target is

$$P = \frac{2 m_P v_P^2}{\pi d_P^2 \beta Y_T} . \tag{1}$$

A target with the thickness h will be perforated if the impact energy

$$W = \frac{1}{2} m_P v_P^2 \tag{2}$$

is greater than the minimum perforation energy W_p. For $h < h^*$

$$W_p = \frac{\pi}{8} d_P h \left(\pi h + \gamma d_P\right) Y_T \tag{3}$$

and for $h > h^*$

$$W_p = \frac{\pi}{4} \beta d_P^2 \left(h - h^*\right) Y_T + W_p^* \tag{4}$$

with

$$W_p^* = \frac{\pi}{8} d_P h^* \left(\pi h^* + \gamma d_P\right) Y_T . \tag{5}$$

Here γ is a parameter that depends on the shape of the projectile nose. For a conical nose with the half apex angle $\theta=\pi/6$ the value is $\gamma\approx1$ and for a spherical nose it is $\gamma\approx2$. The transition thickness

$$h^* = d_P \frac{2\beta - \gamma}{2\pi} \tag{6}$$

is determined be requiring that $\partial W_p / \partial h$ is continuous for $h = h^*$.

When $W > W_p$ the target mass m_T in front of the projectile is fragmented

$$m_T = \frac{\pi}{4} d_P^2 \, h^{(*)} \, \rho_T \tag{7}$$

where $h^{(*)}$ is the smaller of h and h^*. The fragments are assumed to emerge with the same velocity v_{exit} as the projectile

$$v_{exit} = \sqrt{\frac{2\left(W - W_p\right)}{m_P + m_T}} \, . \tag{8}$$

If $W_p - W_p^* < W < W_p$ fragmentation is initiated but is halted when the projectile stops at some depth in the interval $h - h^* < P < h$.

Friction

Friction is accounted for via a lateral pressure on the projectile that is proportional to the target yield strength Y_T with a coefficient λ, and a friction coefficient μ. For simplicity only the case with a flat-nosed projectile is considered below. Then the friction length is equal to the projectile length L_P. Friction retardation should then increase with the penetration depth until it is equal to the projectile length, whereupon it becomes constant, at least if the lateral pressure is assumed to be independent of the penetration depth. It is then easy to show that Eq. (1) is replaced by

$$P = \sqrt{\left(\frac{\beta \, d_P}{4 \mu \lambda}\right)^2 + \frac{m_P \, v_P^2}{\pi \, d_P \, \mu \lambda Y_T}} - \frac{\beta \, d_P}{4 \mu \lambda} \tag{9}$$

for $P < L_P$. Otherwise the projectile reaches the penetration depth $P = L_P$ with the velocity

$$u_{L_P} = \sqrt{v_P^2 - \frac{\pi \, d_P \, L_P \, Y_T}{2 m_P} \left(\beta \, d_P + 2 \mu \lambda L_P\right)} \tag{10}$$

and continues to the final depth

$$P = L_P + \frac{2 m_P \, u_{L_P}^2}{\pi \, d_P^2 \, \beta \, Y_T + 4 \pi \, d_P \, L_P \, \mu \lambda Y_T} \, . \tag{11}$$

Eqs. (10) and (11) yield

$$\mu\lambda = \frac{2 m_P v_P^2 - \pi \beta d_P^2 P Y_T}{2\pi d_P L_P Y_T (2P - L_P)} . \tag{12}$$

and

$$P = \frac{2 m_P v_P^2 + 2\pi \mu \lambda d_P L_P^2 Y_T}{\pi d_P Y_T (\beta d_P + 4\mu\lambda L_P)} \tag{13}$$

For projectiles with spherical, conical or ogive nose shape the projectile length is still suggested to yield realistic results with Eqs. (9) – (13). The reason is that there is higher lateral pressure along the nose where the diameter is smaller than d_P .

The perforation model in Eqs. (3) - (6) is assumed to remain valid even if there is friction retardation along the rear end of the projectile. For $0 < h - h^* < L_P$ the minimum perforation energy is

$$W_p = \frac{\pi}{4}(h - h^*) d_P^2 \beta Y_T + \frac{\pi}{2} d_P (h - h^*)^2 \mu\lambda Y_T + W_p^* \tag{14}$$

whereas $h - h^* > L_P$ corresponds to

$$W_p = \frac{\pi}{4}(h - h^*) d_P^2 \beta Y_T + \pi d_P L_P \left(h - h^* - \frac{L_P}{2} \right) \mu\lambda Y_T + W_p^* . \tag{15}$$

The notation “minimum perforation energy” for W_p is strictly not adequate when there is friction since the projectile may become stuck in the hole if the impact energy is not sufficiently much higher than W_p, as shown below in connection with Eqs. (16) - (20). Nevertheless this notation is chosen and corresponds to the energy required to reach the penetration depth $P = h - h^*$ at which fragmentation of the mass m_T is assumed to occur instantaneously. Accordingly the projectile velocity immediately after fragmentation is

$$u = \sqrt{v_P^2 - \frac{2 W_p}{m_P}} . \tag{16}$$

When the fragments are produced it is reasonable to assume that the last part h^* of the hole has an at least slightly larger diameter than the projectile so that there is no friction force on the projectile in this region. For simplicity only the case with $h - h^* > L_P$ is considered. If the velocity u is sufficiently high and it is assumed that the projectile and all fragments have the same exit velocity, then the highest possible value for this velocity is given by energy conservation

$$v_{exit} = \sqrt{\frac{m_P u^2 - \pi d_P L_P^2 \mu\lambda Y_T}{m_P + m_T}} . \tag{17}$$

The time τ during which acceleration of the fragment mass occurs is estimated via momentum conservation and the average friction force on the projectile until it leaves the target

$$\tau = \frac{2\{m_P\, u - (m_P + m_T) v_{exit}\}}{\pi\, d_P\, L_P\, \mu\, \lambda\, Y_T} \tag{18}$$

With Eqs. (17) and (18) there is no loss of energy or momentum from the fragment mass m_T via friction to the surrounding intact target material during the acceleration. Furthermore, once the fragments are produced, which required the energy W_p^*, there is no further deformation of the fragments during the acceleration.

A possibility, which cannot be neglected, is that the projectile may be stopped in the hole, whereby the fragments will emerge with a smaller velocity v_F than in Eq. (17). For simplicity only the case with $h - h^* > L_P$ and $h^* < L_P$ is considered below. At the penetration depth $P = h - h^*$ the friction length is L_P, the projectile velocity is u and the fragment acceleration starts. The velocity v_F for the projectile and the fragments is reached when the penetration depth is P_F in the interval $h - h^* < P_F < h - h^* + L_P$, whereby the rear end of the projectile, the length of which is $L_P - P_F + h - h^*$, is still in frictional contact with target material. Energy conservation yields the relation

$$m_P\, u^2 = (m_P + m_T) v_F^2 + \pi\, d_P \left(P_F - h + h^*\right)\left(2\, L_P - P_F + h - h^*\right) \mu\, \lambda\, Y_T\,. \tag{19}$$

In order to determine v_F and P_F momentum conservation must also be employed. The projectile moves the distance $P_F - h + h^*$ with the initial velocity u and the final velocity v_F, so that the average velocity $(u+v_F)/2$. The ratio between the distance and the average velocity yields a reasonable estimate of the corresponding time. If the average friction force is calculated from the initial and final values, then it is $\pi\, d_P \left(2\, L_P - P_F + h - h^*\right) \mu\, \lambda\, Y_T / 2$. Strictly it is not correct to use these average values to calculate the momentum loss to the surrounding target material, but if this nevertheless is done then momentum conservation yields

$$m_P (u - v_F) = m_T\, v_F + \frac{\pi\, d_P \left(2\, L_P - P_F + h - h^*\right)\left(P_F - h + h^*\right) \mu\, \lambda\, Y_T}{u + v_F}\,. \tag{20}$$

After numerical solution of Eqs. (19) - (20) energy conservation for subsequent retardation of the projectile, which is caused by friction along the rear end that has not yet reached the depth $h - h^*$, will either yield an exit velocity that is smaller than v_F or a final penetration depth $P>P_F$, whereby the rearmost part of the projectile is stuck in the hole. It should be observed that it is possible to obtain $P>h$ so that the front part of the projectile protrudes from the rear target surface.

Initially increasing penetration resistance

In the basic model the penetration resistance is $R_T=\beta Y_T$, where the coefficient β is assumed to be constant. For small penetration depths P this assumption is not realistic. For a flat-nosed projectile the initial penetration resistance should be $\beta_i Y_T$ with $\beta_i<\beta$ but not much smaller. Furthermore the resistance should reach the limit βY_T

rather quickly and smoothly, namely at some depth ζd_P. Accordingly it may be assumed that

$$R_T = \left[\beta_i + (\beta - \beta_i) \left\{ 1 - \left(1 - \frac{P}{\zeta\, d_P} \right)^{\xi} \right\} \right] Y_T \tag{21}$$

for $P<\zeta d_P$ and

$$R_T = \beta\, Y_T \tag{22}$$

for $P>\zeta d_P$. Smooth connection for $P=\zeta d_P$ requires that $\xi>1$. The instantaneous penetration depth is given by

$$m_P \frac{d^2 P}{dt^2} = -\frac{\pi}{4} d_P^2\, R_T\,. \tag{23}$$

Integration for $P<\zeta d_P$ yields

$$\frac{dP}{dt} = \sqrt{v_P^2 - \frac{\pi\, d_P^2\, Y_T}{2\, m_P} \left[\beta\, P - (\beta - \beta_i) \frac{\zeta\, d_P}{\xi + 1} \left\{ 1 - \left(1 - \frac{P}{\zeta\, d_P} \right)^{\xi + 1} \right\} \right]}\,. \tag{24}$$

When the target is semi-infinite and the impact velocity is high enough, then Eq. (24) and $P=\zeta d_P$ yield

$$dP / dt = \sqrt{v_P^2 - \frac{\pi\, \zeta\, d_P^3 \,(\xi\, \beta + \beta_i) Y_T}{2\, m_P \,(\xi + 1)}} = u_\zeta\,, \tag{25}$$

whereupon the final penetration depth is obtained in analogy with Eq. (1)

$$P = \zeta\, d_P + \frac{2\, m_P\, u_\zeta^2}{\pi\, d_P^2\, \beta\, Y_T}\,. \tag{26}$$

Otherwise Eq. (24) and $dP/dt=0$ yield a final penetration depth P that is smaller than ζd_P.

When the target is semi-infinite and the impact velocity is high enough so that $P>\zeta d_P$ and the penetration resistance is given by Eqs. (21) and (22) with $\beta_i=0$, then Eqs. (25) and (26) yield

$$P = \frac{2\, m_P\, v_P^2}{\pi\, d_P^2\, \beta\, Y_T} + \frac{\zeta\, d_P}{\xi + 1} \tag{27}$$

Comparison between Eqs. (1) and (27) quantifies the error corresponding to use of Eq. (22) for all penetration depths.

If the target thickness is so large that $h-h^* > \zeta\, d_P$, then modification of the basic model to account for initially increasing penetration resistance is trivial. Otherwise it is suggested that the minimum perforation energy is estimated by assuming that the fragmentation energy and transition thickness still are given by Eqs. (5) and (6). Accordingly it is also assumed that the projectile reaches the penetration depth $P = h - h^*$ with the velocity u^* obtained from Eq. (24)

$$u^* = \sqrt{v_P^2 - \frac{\pi\, d_P^2\, Y_T}{2\, m_P}\left[\beta\left(h - h^*\right) - \left(\beta - \beta_i\right)\frac{\zeta\, d_P}{\xi + 1}\left\{1 - \left(1 - \frac{h - h^*}{\zeta\, d_P}\right)^{\xi + 1}\right\}\right]}\,. \quad (28)$$

The corresponding minimum perforation energy is

$$W_p = \frac{1}{2} m_P \left(v_P^2 - u^{*2}\right) + W_p^*\,. \quad (29)$$

Thus $\partial W_p / \partial h$ is not a continuous function for $h = h^*$ when $h - h^* < \zeta\, d_P$, contrary to the basic model and for $h - h^* > \zeta\, d_P$ with initially increasing penetration resistance. Nevertheless it is suggested that Eqs. (28) and (29), as well as the corresponding results below, represent realistic ways to extend the basic model in [1] to account for friction or initially increasing penetration resistance or for both these effects.

Initially increasing penetration resistance and friction

When both initially increasing penetration resistance and friction is accounted for then the projectile velocity *u* first decreases with increasing penetration depth *P* in a semi-infinite target as given by

$$u = \sqrt{v_P^2 - \frac{\pi\, d_P^2\, Y_T}{2\, m_P}\left[\beta P - \left(\beta - \beta_i\right)\frac{\zeta\, d_P}{\xi + 1}\left\{1 - \left(1 - \frac{P}{\zeta\, d_P}\right)^{\xi + 1}\right\}\right] - \frac{\pi\, d_P\, \mu\, \lambda\, P^2\, Y_T}{m_P}}\,. \quad (30)$$

Subsequently different cases must be considered. If $\zeta d_P > L_P$ then the penetration depth $P = L_P$ is reached with the velocity

$$u_{L_P} = \sqrt{v_P^2 - \frac{\pi\, d_P^2\, Y_T}{2\, m_P}\left[\beta L_P - \left(\beta - \beta_i\right)\frac{\zeta\, d_P}{\xi + 1}\left\{1 - \left(1 - \frac{L_P}{\zeta\, d_P}\right)^{\xi + 1}\right\}\right] - \frac{\pi\, d_P\, \mu\, \lambda\, L_P^2\, Y_T}{m_P}}\,. \quad (31)$$

This requires that

$$v_P > \sqrt{\frac{\pi\, d_P^2\, Y_T}{2\, m_P}\left[\beta L_P - \left(\beta - \beta_i\right)\frac{\zeta\, d_P}{\xi + 1}\left\{1 - \left(1 - \frac{L_P}{\zeta\, d_P}\right)^{\xi + 1}\right\}\right] + \frac{\pi\, d_P\, \mu\, \lambda\, L_P^2\, Y_T}{m_P}}\,. \quad (32)$$

Otherwise Eq. (20) and u=0 yields the final penetration depth P.
If the penetration depth $P=\zeta d_P>L_P$ is reached then the velocity is

$$u_{\zeta d_P}=\sqrt{v_P^2-\frac{\pi\,\zeta\,(\xi\,\beta+\beta_i)d_P^3\,Y_T}{2\,m_P\,(\xi+1)}-\frac{\pi\,d_P\,\mu\,\lambda\,L_P(2\zeta\,d_P-L_P)Y_T}{m_P}}\,. \tag{33}$$

This requires that

$$v_P>\sqrt{\frac{\pi\,\zeta\,(\xi\,\beta+\beta_i)d_P^3\,Y_T}{2\,m_P\,(\xi+1)}+\frac{\pi\,d_P\,\mu\,\lambda\,L_P(2\zeta\,d_P-L_P)Y_T}{m_P}} \tag{34}$$

and then the final penetration depth is

$$P=\zeta\,d_P+\frac{2\,m_P\,u_{\zeta\,d_P}^2}{\pi\,d_P\,Y_T\,(\beta\,d_P+\mu\,\lambda\,L_P)}\,. \tag{35}$$

Otherwise the final penetration depth P is obtained from

$$v_P^2=\frac{\pi\,d_P^2\,Y_T}{2\,m_P}\left[\beta P-(\beta-\beta_i)\frac{\zeta\,d_P}{\xi+1}\left\{1-\left(1-\frac{P}{\zeta\,d_P}\right)^{\xi+1}\right\}\right]+\frac{\pi\,d_P\,\mu\,\lambda\,L_P\,(2P-L_P)Y_T}{m_P}\,. \tag{36}$$

When $\zeta d_P<L_P$ the penetration depth $P=\zeta d_P$ is reached with the velocity

$$u_{\zeta d_P}=\sqrt{v_P^2-\frac{\pi\,\zeta\,(\xi\,\beta+\beta_i)d_P^3\,Y_T}{2\,m_P\,(\xi+1)}-\frac{\pi\,d_P\,\mu\,\lambda\,\zeta^2\,d_P^2\,Y_T}{m_P}}\,. \tag{37}$$

This requires that

$$v_P>\sqrt{\frac{\pi\,\zeta\,(\xi\,\beta+\beta_i)d_P^3\,Y_T}{2\,m_P\,(\xi+1)}+\frac{\pi\,d_P\,\mu\,\lambda\,\zeta^2\,d_P^2\,Y_T}{m_P}}\,. \tag{38}$$

Otherwise Eq. (30) and u=0 yields the final penetration depth P.
If the penetration depth $P=L_P>\zeta d_P$ is reached then the velocity is

$$u_{L_P}=\sqrt{v_P^2-\frac{\pi\,\zeta\,(\xi\,\beta+\beta_i)d_P^3\,Y_T}{2\,m_P\,(\xi+1)}-\frac{\pi\,d_P^2\,(L_P-\zeta\,d_P)\beta\,Y_T}{2\,m_P}-\frac{\pi\,d_P\,\mu\,\lambda\,L_P^2\,Y_T}{m_P}}\,. \tag{39}$$

This requires that

$$v_P>\sqrt{\frac{\pi\,\zeta\,(\xi\,\beta+\beta_i)d_P^3\,Y_T}{2\,m_P\,(\xi+1)}+\frac{\pi\,d_P^2\,(L_P-\zeta\,d_P)\beta\,Y_T}{2\,m_P}+\frac{\pi\,d_P\,\mu\,\lambda\,L_P^2\,Y_T}{m_P}}\,. \tag{40}$$

and then the final penetration depth is

$$P = L_P + \frac{2\, m_P\, u_{L_P}^2}{\pi\, d_P\, Y_T\, \left(\beta\, d_P + \mu\, \lambda\, L_P\right)}. \tag{41}$$

Otherwise Eq. (39) with $u_{L_P} = 0$ and P instead of L_P yields the final penetration depth.

If the target thickness is so big that $h - h^*$ is larger than the largest of ζd_P and L_P, then modification of the basic model to account for initially increasing penetration resistance is trivial. Otherwise the velocity u^* and the minimum perforation energy W_p are determined in analogy with Eqs. (28) and (29).

Nose shape dependence of penetration resistance coefficient

Sharp-nosed projectiles should experience smaller penetration resistance than blunt-nosed projectiles, which means that β should increase with the apex angle 2θ for conical-nosed projectiles. The physical reason to expect this is that displacement of target material in front of a penetrating projectile should follow shorter and less curved trajectories with a sharp conical nose than with a blunt nose. In this connection it should be mentioned that the pressure

$$p_{sph} = \frac{2\, Y_T}{3} \left[1 + \log \left\{ \frac{E_T}{3\left(1 - \nu_T\right) Y_T} \right\} \right] \tag{42}$$

is required for expansion of a spherical hole [2, chapter V.1], whereas a somewhat smaller pressure

$$p_{cyl} = \frac{Y_T}{\sqrt{3}} \left[1 + \log \left\{ \frac{\sqrt{3}\, E_T}{\left(5 - 4\, \nu_T\right) Y_T} \right\} \right] \tag{43}$$

is required for expansion of a cylindrical hole [2, chapter V.4]. In these relations E_T and ν_T are the elastic modulus and Poisson ratio for the target material, respectively. In both cases the displaced material follows straight lines but there is more work done for a certain increase of the hole radius in the spherical case.

It is reasonable to assume that β increases linearly with θ for $\theta << \pi/2$ and that it levels off when $\theta = \pi/2$ is approached. A simple mathematical relation with such behaviour is

$$\beta = \beta_0\, \left\{ 1 + \alpha \sin(\theta) \right\}. \tag{44}$$

The parameters β_0 and α must be determined from experimental results. However, the hole expansion resistance in Eqs. (42) and (43) provides an idea about the magnitude of these parameters. If a hole is produced with a vanishingly small apex angle it seems reasonable to assume that the penetration coefficient should be somewhat higher than the ratio p_{cyl}/Y_T in Eq. (43) so that

$$\beta_0 = \kappa \frac{p_{cyl}}{Y_T} = \frac{\kappa}{\sqrt{3}} \left[1 + \log \left\{ \frac{\sqrt{3}\, E_T}{3\left(5 - 4\, \nu_T\right) Y_T} \right\} \right] \tag{45}$$

where κ is larger but not much larger than unity. If a hole is produced with a flat-ended projectile it seems reasonable to assume that the penetration coefficient should be higher than p_{sph}/Y_T in Eq. (42). A rather arbitrary assumption is then that the penetration resistance with the half apex angle $\theta=\pi/6$ should be higher than with a vanishingly small apex angle with the factor to p_{sph}/p_{cyl} obtained from Eqs. (42) and (43). Accordingly α is obtained from

$$\frac{2}{3}\left[1+\log\left\{\frac{E_T}{3(1-\nu_T)Y_T}\right\}\right]=\frac{1}{\sqrt{3}}\left[1+\log\left\{\frac{\sqrt{3}\,E_T}{3(5-4\nu_T)Y_T}\right\}\right]\left(1+\frac{\alpha}{2}\right). \tag{46}$$

In reality the nose of a projectile is never flat but ogival, which should correspond to an only slightly smaller penetration coefficient then for a conical nose with the same apex angle. Normally apex angles are in the interval $35^{o}<2\theta<55^{o}$. Then the model yields little relative variation of the penetration coefficient with the projectile nose shape. Furthermore there is little difference in the penetration coefficients for a material in ductile and hardened states. Accordingly the suggestion in [1] that the penetration coefficient $\beta=5$ can be used, at least in general applications, is justified.

The penetration coefficient for a projectile with a hemispherical nose must be smaller than for a flat-nosed projectile. Hence it should be equal to a conical-nosed projectile for some intermediary apex angle, presumably in the interval $90^{o}<2\theta<120^{o}$.

Target size effect

Expansion of spherical and cylindrical holes to the diameter d requires the pressures in Eqs. (42) and (43). Thereby the boundary between elastic-plastic and only-elastic deformation is at the diameter

$$\Lambda_{sph}=d\sqrt[3]{\frac{E_T}{3(1-\nu_T)Y_T}} \tag{47}$$

and

$$\Lambda_{cyl}=d\sqrt{\frac{2E_T}{(5-4\nu_T)Y_T}}\,, \tag{48}$$

respectively [2]. It should be emphasised that the pressures in Eqs. (42) and (43) are constant. With this pressure a hole with the diameter d will expand slightly so that the pressure drops. Consequently expansion stops unless the pressure medium in the hole is connected to a source that restores the pressure. Thus the rate at which the hole is expanded depends on the power supply from the pressure source.

For a spherical hole at the centre of a sphere with the external diameter D the pressure needed to produce plastic flow to the smaller diameter Λ_{sph} is given by [2]

$$p_{sph}=\frac{2Y_T}{3}\left[1-\frac{\Lambda_{sph}^3}{D^3}+3\log\left\{\frac{\Lambda_{sph}}{d}\right\}\right]. \tag{49}$$

The corresponding result for cylindrical holes is

$$p_{cyl} = \frac{Y_T}{2}\left[1 - \frac{\Lambda_{cyl}^2}{D^2} + 2\log\left\{\frac{\Lambda_{cyl}}{d}\right\}\right]. \tag{50}$$

A rough way to estimate the relative decrease of the penetration coefficient with the target diameter might be to use the ratio between the pressures in Eqs. (50) and Eq. (43)

$$\frac{\beta(D)}{\beta(\infty)} \approx \frac{1 - \dfrac{\Lambda_{cyl}^2}{D^2} + 2\log\left(\dfrac{\Lambda_{cyl}}{d}\right)}{1 + 2\log\left(\dfrac{\Lambda_{cyl}}{d}\right)}. \tag{51}$$

For hard steel $\Lambda_{cyl}/d=10$ is a representative value. Then Eq. (51) with $D/\Lambda_{cyl}=1$, 2 and 3 yields $\beta(D)/\beta(\infty)\approx 0.82$, 0.96 and 0.98, respectively.

Penetration of ductile material should be described in the same way as for hard material, but the perforation phase involves collaring around sharp-nose projectiles and plugging in front of blunt-nose projectiles instead of fragmentation. The elastic parameters E_T and ν_T are the same for hard and ductile versions of steel but the yield strength Y_T is smaller for the latter. For ductile steel with $\Lambda_{cyl}/d=14$ the corresponding results are $\beta(D)/\beta(\infty)\approx 0.87$, 0.97 and 0.98, respectively.

For steel the Poisson ratio is $\nu_T\approx 0.3$. When there is large plastic deformation it is usual to neglect compressibility, whereby $\nu_T=0.5$ is assumed instead of the real value. Then the values $\Lambda_{cyl}/d=10$ and 14 above are changed to $\Lambda_{cyl}/d\approx 11.3$ and 15.8, whereby Eq. (51) with $D/\Lambda_{cyl}=1$ yields an only slightly higher result, namely $\beta(D)/\beta(\infty)\approx 0.83$.

Eqs. (42) and (49) can be used in the same manner to estimate the smallest distance from the hole bottom to the back side of the target that should be allowed. However, since the major target deformation occurs in the vicinity of the nose of the projectile a better estimate than Eq. (51) for how the penetration coefficient should depend on the target size might be

$$\frac{\beta(D)}{\beta(\infty)} \approx \frac{1 - \dfrac{\Lambda_{sph}^3}{D^3} + 3\log\left(\dfrac{\Lambda_{sph}}{d}\right)}{1 + 3\log\left(\dfrac{\Lambda_{sph}}{d}\right)}. \tag{52}$$

For hard steel Eq. (52) with $D/\Lambda_{sph}=1$, 2 and 3 yields $\beta(D)/\beta(\infty)\approx 0.80$, 0.98 and 0.99, respectively. The differences to the results with Eq. (51) are hardly experimentally observable, especially since effects of friction and initially increasing penetration resistance are neglected in this connection.

When the target diameter is so small that Eqs. (47) and (48) yield $\Lambda>D$, then Eqs. (50) and (51) are replaced by

$$p_{sph} = \frac{2Y_T}{3}\log\left(1+\frac{D^3}{d^3}\right) \tag{53}$$

and

$$p_{cyl} = \frac{Y_T}{2}\log\left(1+\frac{D^2}{d^2}\right), \tag{54}$$

whereby it is assumed, for simplicity, that target deformation is incompressible. Thus Eq. (51) is changed to

$$\frac{\beta(D)}{\beta(\infty)} \approx \frac{\log\left(1+\frac{D^2}{d^2}\right)}{1+2\log\left(\frac{\Lambda_{cyl}}{d}\right)}, \tag{55}$$

where Λ_{cyl} is obtained from Eq. (48) with ν_T=0.5.

Experimental results for penetration

Two conical-nose projectiles of tungsten carbide have been used against targets of two steel qualities, namely SIS 1312 with the Vickers Hardness HV 1.36±0.05 GPa and SIS 2541-3 with HV 3.19±0.09 GPa. The corresponding yield strengths $Y_T=HV/3.2\approx 0.43$ and 1.00 GPa are obtained as in [3, chapter 13.4]. Projectile data are given in Table 1.

m_P, g	d_P, mm	L_P, mm	θ	v_P, km/s
3.4	4.8	≈ 19	≈18°	1.28
6.0	5.6	≈ 23	≈28°	0.98

Table 1. Projectile data

The experimental penetration depths are shown in Table 2 as the average plus/minus the maximum deviation. Every case was tested five times but sometimes the holes were not straight enough for reliable evaluation.

Target	small projectile		large projectile	
	P, mm	D/Λ_{cyl}	P, mm	D/Λ_{cyl}
SIS 1312				
D=40 mm	53.7±0.9 (2)	0.52	37.8±2.6 (5)	0.45
D=100 mm	48.2±0.9 (4)	1.30	35.5±0.3 (5)	1.12
SIS 2541-3				
D=41 mm	32.6±1.6 (5)	0.81	24.7±0.9 (5)	0.70
D=102 mm	30.8±1.8 (5)	2.02	23.0±0.1 (5)	1.73

Table 2. Experimental results for penetration depth with tungsten carbide projectiles in steel targets. Numbers of tests are given in parenthesis.

The penetration depths in the smaller targets are deeper than in the larger targets as expected. For the small projectile in the hard target Eqs. (48), (51) and (55) with

E_T/Y_T=210 and ν_T=0.5 yield $\beta(0.8\Lambda)/\beta(2.0\Lambda)\approx0.80$, which is significantly smaller than the average penetration ratio 30.8/32.6≈0.94 for practically the same values of $D/\Lambda_{\rm cyl}$. However, the experimental deviations are considerable in comparison with the difference between the average values. If half the deviation is added to the higher value and subtracted from the smaller, then the experimental ratio is 29.9/33.4≈0.90. Furthermore, friction and initially increasing penetration resistance must be accounted for. For instance, if β is determined from Eq. (27) with ζ=10 and ξ=4 the last result is changed to $\beta(D=102\text{ mm})/\beta(D=41\text{ mm})$=(29.9-9.6)/(33.4-9.6)≈0.85. With friction there is further reduction so that the model represented by Eqs. (51) and (55) seems rather realistic.

Below the penetration coefficient β is evaluated for the larger targets, even though the values $D/\Lambda_{\rm cyl}$ seem to be too small to be representative for semi-infinite targets. First the case without friction, namely Eq. (1), is considered. With the heavier projectile $\beta\approx7.8$ is obtained, whereas the result for the lighter projectile is $\beta\approx7.5$.

It is very likely that friction is important for the results of the penetration experiments mentioned above. If it is assumed that β=5 corresponds to the real penetration resistance, and that the lateral pressure is equal to the uniaxial yield strength so that λ=1, then the result for the projectile with the mass m_P=6.0 g and Eq. (12) yield the friction coefficient is $\mu\approx0.24$. In this case 65% of the impact energy is used for penetration and the remaining 35% to overcome friction. Similar evaluation of the result for the projectile with mass m_P=3.4 g yields $\mu\approx0.19$ and that 67% of the impact energy is used for penetration.

For SIS 1312 Eqs. (45) and (46) yield $\beta_0\approx3.0\kappa$ and $\alpha\approx0.80$. For the large projectile Eq. (44) and β=5 then yields $\kappa\approx1.2$. With these values Eq.(44) yields $\beta\approx4.4$ for the small projectile. Then evaluation of the friction coefficient from the result for this projectile yields the same result $\mu\approx0.24$ as for the other projectile. Consequently the shape of the projectile nose should not be neglected if very accurate results are required.

The yield strength ratio for the two target materials in Table 2 is about 2.3, whereas the corresponding ratio between the penetration depths is about $P_{1312}/P_{2341}\approx1.55$ for both projectiles in the larger targets. For the smaller projectile Eq. (13) with $\beta\approx4.4$ and $\mu\lambda\approx0.24$ yields the penetration depth ratio $P_{1312}/P_{2341}\approx1.76$. Consequently initially increasing penetration resistance must also be accounted for if very accurate results are required. Penetration depths for at least four different impact velocities are needed to determine the four quantities β, $\mu\lambda$, ζ and ξ.

Spherical-nose steel projectiles with m_P=22.8 g, d_P=7.11 mm and L_P=71.1 mm have been used against aluminium targets with the yield strength Y_T=276 MPa to obtain the results in Table 3 [4]. The projectiles are obviously not perfectly rigid. Nevertheless all these results follow a graph corresponding to Eq. (1) or Eqs. (9) – (11) reasonably close in [4, Figures 5 and 6]. For these cases the projectile length reduction is relatively small and the hole diameter is practically equal to d_P.

v_P, m/s	P_{exp}, mm	projectile hardness, R_c	projectile shortening, mm
496	37.6	39.5	0.8
572	48.1	39.5	1.2
720	67.8	36.6	3.3
781	72.7	39.5	4.6
806	74.7	36.6	6.6

821	84.3	39.5	4.0
841	91.4	39.5	2.5
892	84.1	36.6	9.4
932	96.5	39.5	6.6
909	109.6	46.2	0.5
1086	126.3	46.2	6.4

Table 3. Experimental results from [4] for three different values of the projectile hardness. For higher velocities than the underlined cases there is an almost stepwise reduction of penetration depth to about half of the previous value, accompanied by severe breaking or erosion of the projectile and hole diameter that is considerably greater than the projectile diameter.

Three representative cases in Table 3 are used in Table 4 for comparison with the model above, despite the fact that the projectiles are not perfectly rigid. If friction is neglected, $\mu\lambda$=0, then. Eq. (1) and the result for the highest impact velocity yields $\beta\approx10$, which is unrealistically high. With friction Eq. (12) and the results for the two smaller velocities yield two equations, from which $\beta\approx5.94$ and $\mu\lambda\approx0.134$ are obtained. Thereupon Eq. (13) and the impact velocity for the third case yields P=126 mm in agreement with the experimental result. These calculated values are shown in the fourth column in Table 4, where the fifth column shows the corresponding ratios between friction energy and impact energy.

v_P, m/s	P_{exp}, mm	P_{cal}, mm β=9.74 $\mu\lambda$=0	P_{cal}, mm β=5.94 $\mu\lambda$=0.134	friction vs impact energy	projectile hardness, R_c
780	73	65	(73)	32%	39.5
932	97	93	(97)	36%	39.5
1086	126	(126)	126	39%	46.2

Table 4. Experimental and calculated results for penetration depth with spherical-nose projectiles as function of impact velocity. Values in parenthesis are used to determine the corresponding quantities β in the third column, and β and μλ in the fourth.

Penetration depth in the same target material for five impact velocities for ogive-nose projectiles with the same diameter and length but slightly smaller mass m_P=20.4 g is shown in the two columns to the left in Table 5 [5]. For these experiments there is no reported projectile length reduction before “breaking or erosion” occurs. Also these results closely follow graphs corresponding to Eq. (1) or Eqs. (9) – (11) in [5, Figures 5 and 6].

v_P, m/s	P_{exp}, mm	projectile hardness, R_c
569	58	38.2
570	55	38.1
679	72	38.1
794	103	52.9
821	102	38.9
966	140	38.0
1076	160	53.1
1147	190	38.3

1237	224	38.0
1255	229	53.1
1348	254	53.2
1365	249(252)	39.2
1396	249(267)	38.0
1493	277(303)	38.0
1538	332	53.8
1654	389	53.4
1786	452	53.0
1817	462	53.3

Table 5. Experimental results from [5] for two different values of the projectile hardness. Values in parenthesis are measured along curved trajectories. For highter velocities than the underlined cases there is an almost stepwise reduction of penetration depth to about half of the previous value, accompanied by severe breaking or erosion of the projectile and hole diameter that is considerably larger than the projectile diameter.

Five representative cases in Table 5 are used in Table 6 for comparison with the model above, The calculated results for β=5.00 and $\mu\lambda$=0.043 agree very well with the experiments.

v_P, m/s	P_{exp}, mm	P_{cal}, mm β=6.57 $\mu\lambda$=0	P_{cal}, mm β=5.00 $\mu\lambda$=0.043	P_{cal}, mm β=1.84 $\mu\lambda$=0.134	projectile hardness, R_c
679	72	65	73	87	38
966	140	132	(140)	146	38
1237	224	217	221	(224)	38
1538	332	335	337	332	52
1786	452	(452)	(452)	438	52

Table 6. Comparison between experimental and calculated results for penetration depth with ogive-nose projectiles as function of impact velocity.

Discussion

Comparison with experimental results for the penetration depth with appropriate variation of the projectile mass, length, diameter and velocity should yield the model penetration coefficient β and the product $\mu\lambda$. Expansion of a cylindrical hole in an infinite target requires a pressure of about $3Y_T$. Such lateral pressure is certainly representative for the real pressure around the projectile nose, but along the rear end of the projectile the pressure should mainly be determined by the elastic reaction that follows the elastic-plastic expansion by the nose. Accordingly the lateral pressure coefficient λ should probably be considerable smaller than 3. A reasonable guess may be $\lambda\approx1$. Direct experimental measurement of λ seems to be quite difficult. Furthermore, a reasonable guess for the friction coefficient should probably be in the interval $0.1<\mu<0.3$.

The assumptions above about λ and μ are in reasonable agreement with experimental results in both steel and aluminium. The penetration depths decrease with increasing hardness of the steel in Table 2. For instance, the penetration depth for the smaller projectile is reduced with the factor 30.8/48.2≈0.64 when the hardness is increased with the factor 3.19/1.36≈2.35. Since the inverse $2.35^{-1}\approx0.42$ of the latter is significantly smaller than the former it is concluded that both friction and initially increasing penetration resistance must be accounted for in order to obtain calculated results in close agreement with experiments. Thereby the experiments must be carried out with targets of sufficiently large dimensions so that the influence of free surfaces can be neglected. The lateral distance from the hole to the closest surface should be at least about $2\Lambda_{cyl}$ from Eq. (48). Similarly the distance from the hole bottom to the back side of the target should be at least $2\Lambda_{sph}$ from Eq. (47).

It should be mentioned that the tungsten carbide projectiles always are stuck in the holes in SIS 1312, but sometimes have bounced slightly away from the hole bottom, whereas they bounce out of the holes in about a third of the cases with SIS 2541-3 (the firing direction is horizontal). If the projectile can bounce out there should be negligible friction when this occurs. This indicates that hole expansion at a certain depth is not finalized when the nose has just passed but continues via elastic-plastic waves in the target material. Consequently it is understandable that there can be friction during penetration but that friction is almost negligible afterwards, at least in the harder target material.

It might be argued that the friction causes melting of the contact area of either the target or the projectile or both. In any case there will then be a thin liquid layer between the rapidly moving projectile material and the stationary target material. The liquid is characterized by an unknown (but certainly non-vanishing) viscosity. Since the liquid layer thickness is unknown the velocity gradient is also unknown. Thus a retarding viscous force on the projectile cannot be estimated. Accordingly it appears better to account for friction retardation as suggested above. However, with very carefully designed and evaluated experiments it might be possible to distinguish between velocity-and-length dependent (viscous) retardation on one hand and only-length dependent retardation (as above) on the other.

It is reasonable to expect a somewhat smaller value for the penetration coefficient β for the ogive-nose projectiles in Table 6 than for the spherical-nose projectiles in Table 4. However, it is also reasonable to expect the friction coefficients to be about equal. Accordingly the two cases β=5.94, $\mu\lambda$=0.134 for spherical-nose projectiles and β=5.00, $\mu\lambda$=0.043 for ogive-nose projectiles appear to show that the lateral pressure coefficient λ is about three times higher with a spherical nose than with the actual ogive-nose projectile. It seems reasonable to expect such difference since the latter expands the hole over a longer distance and with a slower rate, which very well can correspond to a smaller elastic reaction than for the former.

Numerical simulation of rigid projectile penetration is reported in [6], whereby friction is not accounted for. In [6, Figure 5] there is separation between the projectile and target material at a distance about $2d_P$ from the front end of the projectile with the length $L_P=5d_P$. This result provides a reason to neglect friction, as suggested for the basic model. On the other hand it happens that real projectiles can be very firmly stuck in a target with the front end protruding from the rear surface. This indicates that there is significant friction.

The assumption that the projectile and all fragments have the same exit velocity, as in Eq. (8), is too simple. Some fragments may have velocities that are close to the velocity u in Eq. (16), since this is the initial projectile velocity when acceleration of the fragments starts.

For the projectile in [6], with the shape of an ovoid of Rankine, the numerically calculated initial penetration resistance is reasonably well approximated by Eq. (21) with $\beta_i = 0$, $\zeta \approx 8$ and $\xi \approx 4$. This nose shape is similar to but somewhat “sharper” than a spherical nose. Thus the initial penetration resistance for a projectile with a spherical nose should also be well described by Eq. (21) but with somewhat different parameter values, particularly a slightly smaller value of ζ .

For projectiles with spherical or conical or ogive nose shape the initial penetration coefficient β_i should vanish. Furthermore the values of the penetration parameters ζ and ξ should be different. However, the coefficient β must also be allowed to vary somewhat with the nose shape, presumably so that $\beta_{flat}>\beta_{blunt\ cone}>\beta_{sphere}>\beta_{sharp\ cone}$, where the spherical nose might be assumed to be equivalent to a cone with a half apex angle somewhere in the interval $\pi/4<\theta<\pi/3$. Similarly the penetration coefficient for an ogival nose shape should be expected to be somewhat smaller than for a conical shape with the same apex angle. At the present stage it is assumed that the effect of the nose shape can be reasonably well described via appropriate variation of the coefficient β so that the values $\beta_i = 0$, $\zeta \approx 8$ and $\xi \approx 4$ above can be used for all nose shapes.

It is desirable to carry out additional experiments, in which the nose resistance and friction components of the projectile retardation can be studied in more detail. For instance, spherical projectile penetration should yield results whereby friction can be neglected. If these spherical projectiles are made of tungsten carbide, then they can have the same mass and diameter as spherical-nose projectiles of steel with cylindrical rear parts along which friction should occur. If the rear end of a spherical-nose projectile is hollow instead of solid, then the length of the rear end is greater without change of the projectile mass. Furthermore, spherical and spherical-nose projectiles of titanium, with and without hollow ends, can be used so that the mass is smaller without change of diameter.

When the instantaneous projectile velocity is sufficiently high, then projectiles either break or erode whereby the hole diameter in the target necessarily must be larger than the projectile diameter in order to make room for the projectile material. This effect is clearly demonstrated by the experiments in [4, 5] for higher impact velocities than the underlined cases in Tables 3 and 5. Making larger hole diameter requires more energy, which means that the penetration depth must be smaller than when the projectile is rigid. The penetration depth is also reduced since some of the projectile's kinetic energy is used for the breaking erosion of the projectile itself. The traditional way to account for this effect is to include an additional retardation force to the right in Eq. (23). This additional force is normally assumed to be proportional to $\rho_T(dP/dt)^2$, for instance in the well-known model of Tate-and Alekseevskii [7, Chapter 5.1.5]. A disturbing feature with this and similar models is that they do not account for conservation of either target-and-projectile mass or energy. Furthermore, if the rigid part of the model of Tate and Alekseevskii, namely

$$P = \frac{4\, m_P}{\pi\, d_P^2\, \rho_T} \log\left(1 + \frac{\rho_T\, v_P^2}{2\,\beta\, Y_T}\right) \tag{56}$$

is used to determine the penetration coefficient β for the highest impact velocity v_P=1817 m/s in Table 5, then the result is $\beta\approx1.6$. This is an unrealistically small value since representative values from Eqs. (42) and (43) are $p_{sph}/Y_T\approx4$ and $p_{cyl}/Y_T\approx3$, respectively. Moreover, for say v_P=966 m/s the calculated penetration depth is $P\approx26$ cm or almost twice the experimental penetration depth in Table 5.

Accordingly the author has suggested an alternative model for eroding projectile penetration [8]. In this model the specific energy for hole production is constant and, in principle, given by that required for quasi-static hole expansion [2]. Furthermore, the hole diameter is proportional to the instantaneous projectile velocity *dP/dt* as long as this is higher than a critical velocity that depends on relevant material parameters for both the projectile and the target. With this dependence the projectile retardation force is proportional to $(dP/dt)^2$, as it must be in order to yield realistic results.

The model suggested in [8] satisfies target-and-projectile mass conservation, energy conservation and momentum conservation for the instantaneous projectile. Moreover, it yields a natural criterion for the maximum impact velocity that a projectile can have if it shall penetrate as a rigid body. This theoretical limit velocity is in reasonable quantitative agreement with the experimental results in [4, 5].

There is no experimental evidence, at least to the knowledge of the author, that rigid projectiles can produce holes in hard (or ductile) steel and metallic targets with (significantly) larger diameter than the projectile diameter. Nevertheless, if ogive-nose projectiles could stay rigid for significantly higher impact velocities against the actual target material than the maximum value 1.8 km/s in Table 5, then it is possible that the retardation force may have to include a "traditional" dynamic component, as in the models of Tate and Alekseevskii and others, in order to describe experimental results. However, presently there is no reason to introduce such velocity dependent retardation forces on projectiles that are strong enough to penetrate a target as rigid bodies, at least not when simpler models such as that in [1], and the extension thereof above, yield results that are sufficiently accurate for the intended applications.

Conclusion

The suggested basic model for rigid projectile penetration into and perforation of steel and metallic targets seems to be reasonably realistic for the intended applications of the model. If very accurate results are required then initially increasing penetration resistance and friction due to lateral pressure from the target material behind the projectile nose must be accounted for. Such lateral pressure depends on the nose shape of the projectile.

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