Two-dimensional finite element model for simulation of the interior ballistics of guns

Mattias Unosson, Lars Olovsson,
Sten Andreasson

FOI-R-- 1690 --SE
ISSN 1650-1942

Weapons and Protection
Technical report

September 2005

FOI is an assignment-based authority under the Ministry of Defence. The core activities are research, method and technology development, as well as studies for the use of defence and security. The organization employs around 1350 people of whom around 950 are researchers. This makes FOI the largest research institute in Sweden. FOI provides its customers with leading expertise in a large number of fields such as security-policy studies and analyses in defence and security, assessment of different types of threats, systems for control and management of crises, protection against and management of hazardous substances, IT-security and the potential of new sensors.

FOI
Defence Research Agency
Weapons and Protection
SE-147 25 Tumba

Tel: +46 (0) 8 5550 3000
Fax: +46 (0) 8 5550 4143

www.foi.se

Two-dimensional finite element model for simulation of the interior ballistics of guns

FOI-R--1690--SE
ISSN 1650-1942

Weapons and Protection
Technical report

September 2005

Issuing organization	Report number, ISRN	Report type
FOI – Swedish Defence Research Agency Weapons and Protection SE-147 25 Tumba	FOI-R--1690--SE	Technical report
	Research area code 5. Strike and protection	
	Month year September 2005	**Project no.** E2005
	Sub area code 51 Weapons and Protection	
	Sub area code 2	
Author/s (editor/s) Mattias Unosson Lars Olovsson Sten Andreasson	**Project manager** Dennis Menning	
	Approved by Torgny Carlsson	
	Sponsoring agency Swedish Armed Forces	
	Scientifically and technically responsible	
Report title Two-dimensional finite element model for simulation of the interior ballistics of guns		
Abstract (not more than 200 words) A model for simulation of the interior ballistics of guns is proposed and implemented into an two-dimensional explicit finite element code. The model is applied to a 40mm calibre gun with satisfactory results.		
Keywords Interior ballistics, propellant, flame spreading, arbitrary finite element method, ALE		
Further bibliographic information	**Language** English	
ISSN 1650-1942	**Pages** 28 p.	
	Price acc. to pricelist	

Utgivare	Rapportnummer, ISRN	Klassificering
FOI - Totalförsvarets forskningsinstitut Vapen och skydd 147 25 Tumba	FOI-R--1690--SE	Teknisk rapport
	Forskningsområde 5. Bekämpning och skydd	
	Månad, år September 2005	**Projektnummer** E2005
	Delområde 51 VVS med styrda vapen	
	Delområde 2	
Författare/redaktör Mattias Unosson Lars Olovsson Sten Andreasson	**Projektledare** Dennis Menning	
	Godkänd av Torgny Carlsson	
	Uppdragsgivare/kundbeteckning Försvarsmakten	
	Tekniskt och/eller vetenskapligt ansvarig	

Rapportens titel (i översättning)

Tvådimensionell finita elementmodel för innerballistisk simulering av kanoner

Sammanfattning (högst 200 ord)

En modell för innerballistiska simuleringar av kanoner föreslås och implementeras i en tvådimensionell explicit finita elementkod. Modellen tillämpas på en 40 mm kaliber kanon med tillfredställande resultat.

Nyckelord

Innerballistik, krut, flamspridning, godtycklig finita elementmetod, ALE

Övriga bibliografiska uppgifter	Språk Engelska
ISSN 1650-1942	**Antal sidor:** 28 s.
Distribution enligt missiv	**Pris: Enligt prislista**

Utökad sammanfattning

Innerballistiska simuleringar kan göras med olika grad av idealisering, eller förenkling, såsom valet av antalet dimensioner för den rumsliga beskrivningen. Syftet här var bla. att studera flamspridning och då behöver modellen vara minst tvådimensionell. I denna rapport redovisas teorin bakom ett egenutvecklat tvådimensionellt beräkningsprogram och några inledande beräkningsresultat.

Modellen omfattar endast kammaren och kanonröret och projektilen modelleras som en massa utan ytterligare egenskaper. För att beräkningsmässigt kunna hantera problemet definieras två tvillingdomäner där den ena innehåller oförbränt krut och den andra innehåller förbränningsgaserna. Förutom de fältekvationer som styr de mekaniska och adiabatiska förloppen i krutet och gaserna var för sig, så definieras även mekaniska och termodynamiska kopplingar mellan dem.

Modellen har implementerats i ett sedan tidigare egenutvecklat beräkningsprogram som bygger på den sk. godtyckliga Lagrange-Euler finita elementmetoden (eng. Arbitrary Lagrangian-Eulerian Finite Element Method, ALE).

Den implementerade modellen har sedan tillämpats på Bofors 40 mm L/70-kanon med spårljusspränggranat. Indata redovisas här i tabellform och resultaten som kurvor och tvådimensionella figurer. Beräkningar ger att projektilen börjar accelerera efter 2 ms, att krutet brunnit upp efter 5 ms och att projektilen lämnar kanonröret efter 7 ms. Det högsta gastrycket i kammaren och kanonröret är 340 MPa, vilket uppnås efter 4 ms. Projektilens utgångshastighet blev 944 m/s , vilket stämmer bra överens med produktspecifikationer för den aktuella kanonen och ammunitionen där utgångshastigheten anges till 1005 m/s.

Contents

List of symbols

a – parameter in the thermal interaction equation (s^{-1})
b – Co-volume (m^3/kg)
c – parameter in the burning law (-)
c_1 – parameter in the solid interaction law (kg/m^3s)
c_v – heat capacity at constant volume (J/kgK)
c_p – heat capacity at constant pressure (J/kgK)
D – barrel diameter (m)
e – specific internal energy (J/m^3)
E – modulus of elasticity (Pa)
ϵ_v – volumetric strain (-)
f – form function (-)
F_f – projectile-barrel friction force (N)
$\mathbf{g}$ – solid-gas interaction force vector (N/m^3)
η – propellant volume fraction (-)
k – exponential parameter in the burning law (-)
K – bulk modulus (Pa)
m – mass (kg)
ν – Poisson's ratio (-)
p – pressure (Pa)
p_0 – parameter in the burning law (Pa)
ρ – density (kg/m^3)
r – burning rate (m/s)
s – distance of un-burnt propellant (m)
T – temperature (K)
T_i – ignition temperature (K)
u – axial displacement (m)
$\mathbf{v}$ – velocity vector (m/s)
V – volume (m^3)
x – axial distance from the breech end (m)

In addition to the symbols above, subscripts s, g and $proj$ denote solid propellant phase, combustion gas phase and projectile respectively.

1 Introduction

Interior ballistics involves the combustion of the propellant and the acceleration of the projectile by the expanding combustion gases. For simulation the interior ballistics processes can be modelled to various degree of detail depending on the purpose. Lumped parameter (or 0-dimensional or thermodynamic) models are useful for studying parameter variations of e.g. propellant properties, while one-dimensional (or gas dynamic) models can compute the distribution of e.g. pressure along the chamber and barrel and longitudinal waves. An axi-symmetric two-dimensional model gives an opportunity to simulate the flame spreading through the propellant charge during ignition to a degree that is useful in many cases.

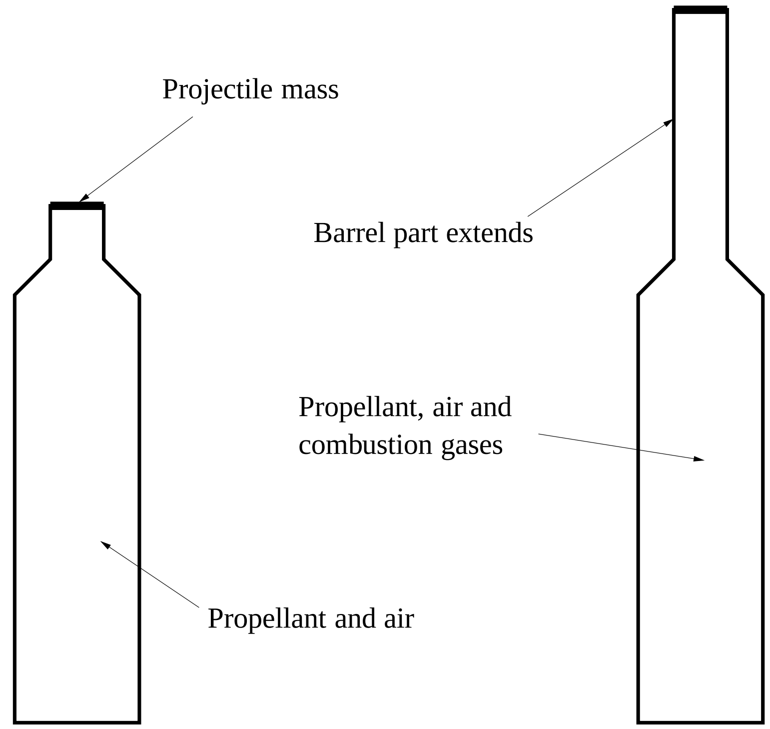
Hence, a two-dimensional model can be used to study the effects of distributing the propellant charge in different ways as well as different ignition techniques and strategies. The former property is of great importance when increasing the charge loading density and is of universal interest, while the latter is a requirement to adequately model ignition in electrothermal-chemical (ETC) guns using plasma generators, which is of particular interest at FOI.

Increasing the number of dimensions, however, increases both the computing time and the effort to set up a problem.

Well known one-dimensional codes for interior ballistics are the series of NOVA-codes, cf. for example [1], [2], [3], [4] and [5]. The latest extension of the code includes electromagnetic capacities. Other studies of different aspects of interior ballistics modelling include [6] and [7] while a historical review and outlook in the area of computational interior ballistics can be found in [8].

Recent computational studies at FOI include [9], [10] and [11]. Interior ballistic codes in use at FOI include the 0-dimensional, in-house, IB-KB [12] and the one-dimensional CELLGUN98 [13], [14]. The present two-dimensional, axi-symmetric model extends the simulation capabilities, while computing times, problem setup and handling are still reasonable.

Figure 1. Modelling approach



2 Model

The modelling approach to the problem used here is shown schematically in Figure 1. Initially, only the volume of the chamber is considered and the projectile is represented by a rigid piston with mass. As the propellant is ignited the expanding combustion gases accelerates the projectile mass and the barrel part of the geometry is extended to the projectile position. The equations that govern the process are solved with the finite element (FE) method.

2.1 Twin domains and field equations

Multi-phase problems described by field theory are difficult to handle numerically when the velocity of the individual phases differ, as in the case of a solid propellant and combustion gases. To numerically allow for different velocity fields, the idea here is to split the problem into two identical, overlapping and interacting twin domains, see Fig. 2. In the figure η is the volume fraction of solid propellant and, hence $1 - \eta$ is the volume fraction of combustion gases. Over these twin domains two separate systems of field equations are valid that represent conservation of mass, linear momentum, angular momentum and thermo-mechanical energy. The first system represents the solid propellant in vacuum and the second system represents the combustion gases with identical boundary conditions.

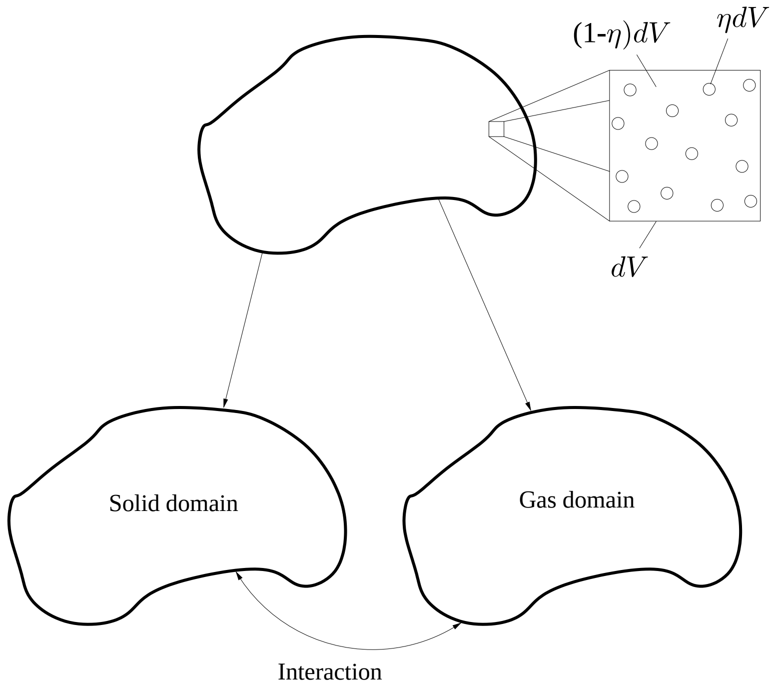
2.2 Constitutive equations

2.2.1 Mechanical

The solid propellant is modelled as an elastic fluid according to

$$p_s = -K\epsilon_v \tag{1}$$

Figure 2. Twin domains



where p_s is the pressure, K is the bulk modulus and ϵ_v is the volumetric strain. The gas domain is initially filled with an ideal gas and the combustion gas is modelled according to

$$p_g\left(1-\rho_g b\right)=\rho_g\left(c_p-c_v\right)T_g \tag{2}$$

where p_g is the pressure, b is the Co-volume, ρ_g is the density, T_g is the temperature and c_p and c_v are the specific heat capacities at constant pressure and volume respectively.

2.2.2 Thermodynamical

Constant specific heat capacities and adiabatic heating is assumed. According to [15] the energy loss due to heat transfer from the combustion gases to the gun is 5–20%, depending on the caliber.

2.3 Combustion

The propellant is assumed to ignite when

$$T_s \geq T_i \tag{3}$$

where T_s is the propellant temperature. The local rate of combustion $\dot{m}_s$ is

$$\frac{d\dot{m}_s}{dV}=\dot{\eta}\rho_s \tag{4}$$

where

$$\dot{\eta}=\eta\frac{1}{f}\frac{df}{ds}r \tag{5}$$

Here f is a propellant form function describing the propellant grain volume as a function of the burning distance s that increases with the rate r as

$$r = c\,(p_0 + p_s)^k \tag{6}$$

where c, p_0 and k are scalar valued parameters.

2.4 Twin domain interaction

2.4.1 Mechanical

The gas exerts drag forces on the propellant due to flow separation and frictional shearing that here are introduced as body forces

$$\mathbf{g} = \mathbf{g}_1 + \mathbf{g}_2 \tag{7}$$

where

$$\mathbf{g}_1 = \eta \nabla p_g \tag{8}$$

and

$$\mathbf{g}_2 = c_1\,(1-\eta)\,(\mathbf{v}_g - \mathbf{v}_s) \tag{9}$$

Here c_1 is a scalar valued parameter and $\mathbf{v}_g$ and $\mathbf{v}_s$ are the velocity of the gas and propellant respectively.

2.4.2 Thermodynamical

Heat is transferred from the gas, heating up the propellant according to

$$\dot{T}_s = a\,(T_g - T_s) \tag{10}$$

where T_s is the temperature of the solid propellant and a is a scalar valued parameter. When the propellant combust mass is transferred to the gas domain and at the same time more space is created allowing for the gas to expand. Taking both effects into account leads to an evolution of the gas density according to

$$\dot{\rho}_g = \frac{\dot{\eta}}{1-\eta}\,(\rho_s - \rho_g) \tag{11}$$

and the corresponding specific internal energy increase

$$\dot{e}_g = \frac{\dot{\eta}}{1-\eta} c_v\,(\rho_s T_c - \rho_g T_g) \tag{12}$$

where T_c is the combustion temperature.

2.5 Contact

The contact between projectile and barrel is modelled as a friction force $F_f(u_{proj})$, where u_{proj} is the projectile displacement.

3 Implementation

The arbitrary Lagrangian-Eulerian finite element method is used for the spatial discretization, cf. [16]. Here the formulation is based on volume weighted residuals and implemented for plane strain and rotational symmetrical problems. Temporal discretization is performed with the central difference method.

The implementation allows for up to four different gas species to be defined and the projectile is modelled as a rigid piston with a mass m_{proj} that is allowed to accelerate. A region of the mesh, defined by the user, is adapted to the motion of the projectile.

The model was implemented in the in-house code GRALE2D, cf. [17].

Table 1. Cartridge case diameter

x (mm)	37.4	45.8	57.0	304.2	346.0	363.7
D (mm)	0.0	51.6	53.3	46.0	39.0	39.0

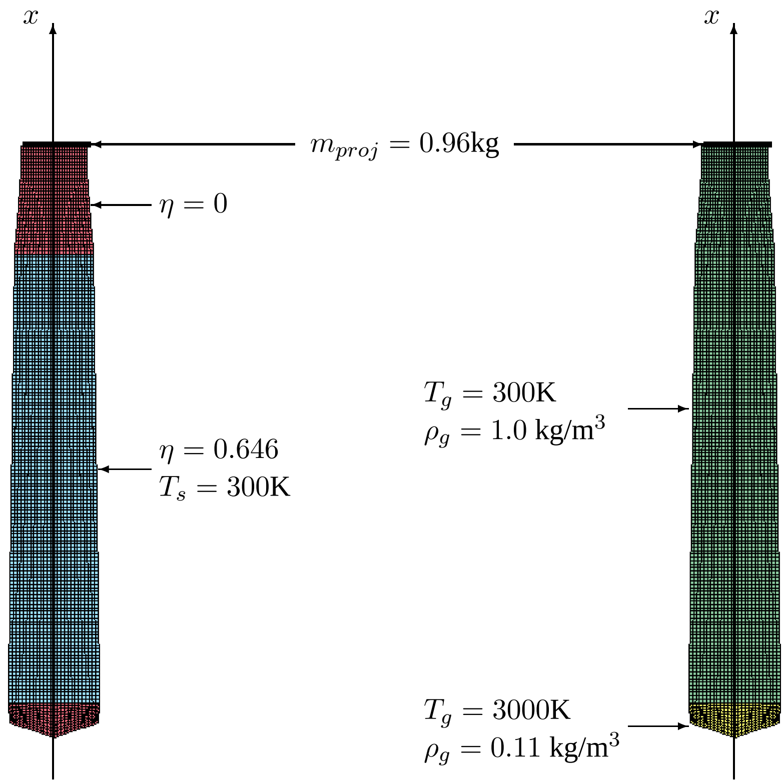


Figure 3. Discretization, boundary conditions and initial conditions

4 Application example

The model was applied to the 40 mm L/70 Bofors gun with the high-explosive tracer ammunition. Data was supplied by the manufacturer.

4.1 Input

The boundary of the cartridge case was discretized using piecewise linear segments and the cartridge case diameter, along its length from the breech end, is given in Table 1. The mesh was fixed except for the region adjacent to the projectile, ie. the barrel, where the mesh was allowed to expand to follow the projectile motion. Initial conditions and finite element discretization of the cartridge case are given in Figure 3. The total cartridge case volume was 590 cm^3 and the propellant was distributed in a volume of 480 cm^3.

Constitutive parameters values used for the solid propellant and the combustion gases are given in Tables 2–4 and 5 respectively.

The twin domain interaction (Eqs. 7–10) was modelled with the parameter values in Table 6 and the friction between projectile and barrel is given in Table 7 as a piecewise linear curve.

Table 2. Solid propellant constitutive parameters

ρ_s	E	ν
1549 kg/m^3	10 GPa	0.3

Table 3. Solid propellant combustion parameters

T_i	e_{chem}	s_0	c	p_0	k
450 K	3.583 MJ/kg	0.43 mm	$0.513 \cdot 10^{-9}$ m^3/(kg s)	7.5 MPa	1.0

Table 4. Solid propellant form function

s/s_0	η
0.0	0.0
1.0	0.646

4.2 Results

The propellant was ignited after 0.2 ms and the projectile exited the barrel after 7.0 ms at a velocity of 944 m/s. According to [18] the muzzle velocity for this ammunition is 1005 m/s. In Figures 4–6 the time histories for relative propellant mass, the maximum gas pressure and projectile displacement are given. In Figures 7–9 the solid domain density, gas pressure and gas temperature are shown at times 1, 4 and 7 ms.

Table 5. Combustion gases constitutive parameters

b	c_v	c_p
0.001 m^3/kg	1453 J/(kg K)	1822 J/(kg K)

Table 6. Twin domain interaction parameters

a	c_1
$1.0 \cdot 10^{10}$	0.0 kg/(m^3 s)

Table 7. Friction between projectile and barrel

u_{proj} (m)	0	0.002	0.012	0.024	0.040	2.480
F_f (N)	6283	0	42724	42724	2513	2513

Figure 4. Relative propellant mass

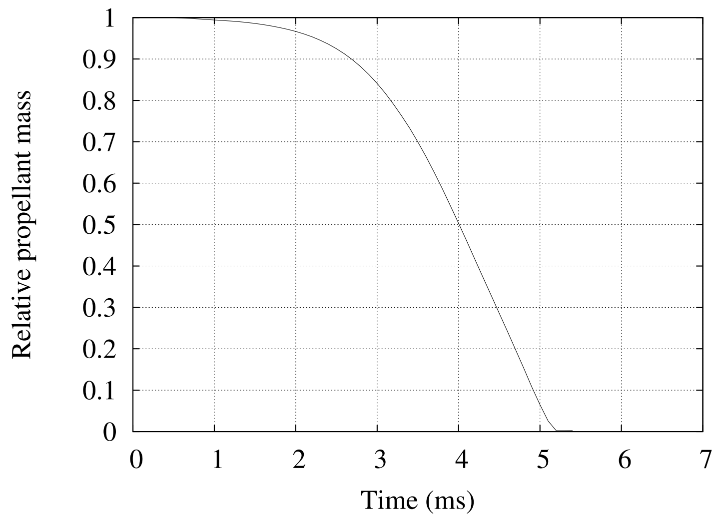
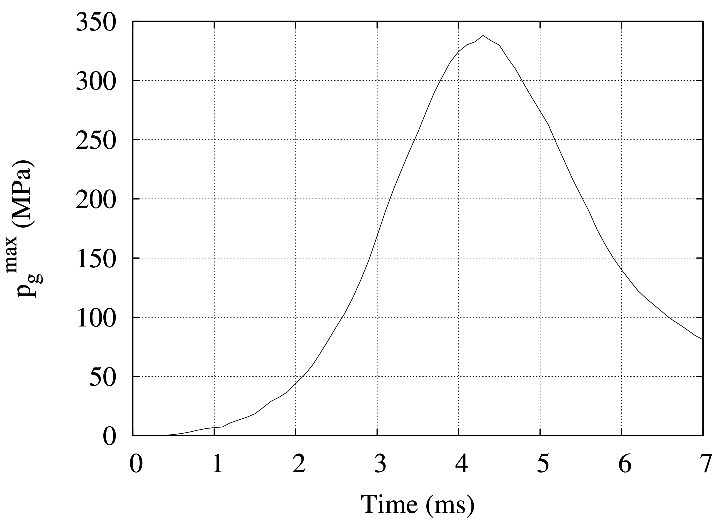


Figure 5. Maximum gas pressure



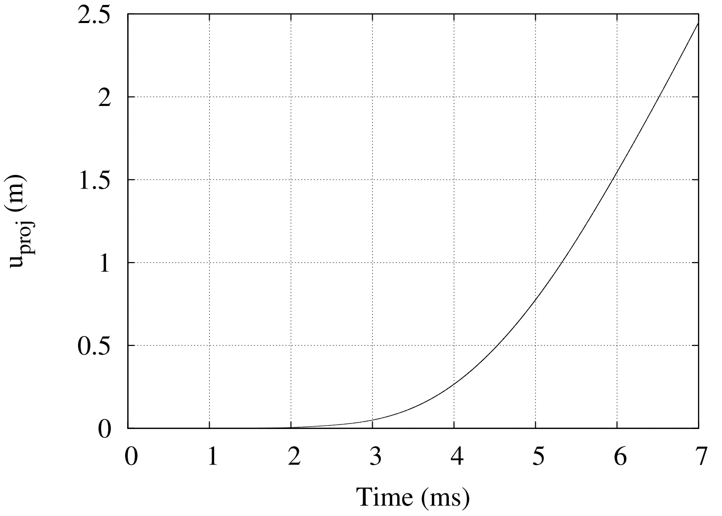


Figure 6. Projectile displacement

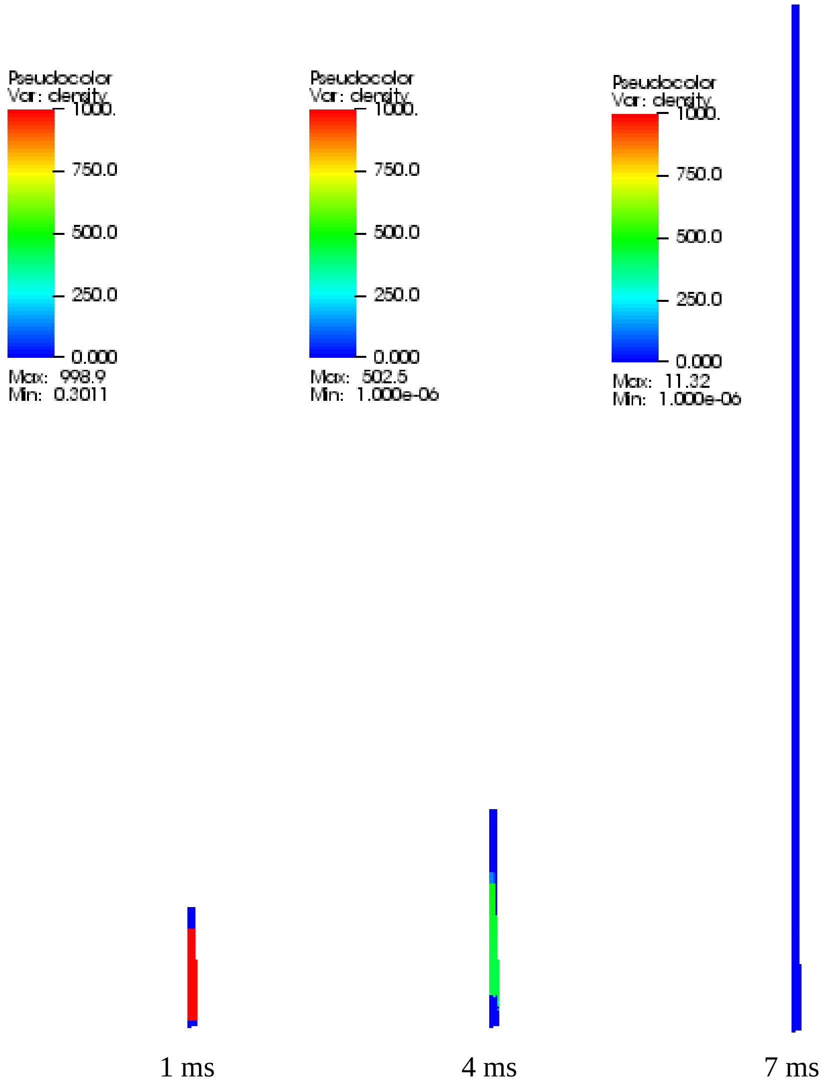


Figure 7. Solid domain density (kg/m^3) representing un-burnt propellant

Figure 8. Gas domain pressure (Pa)

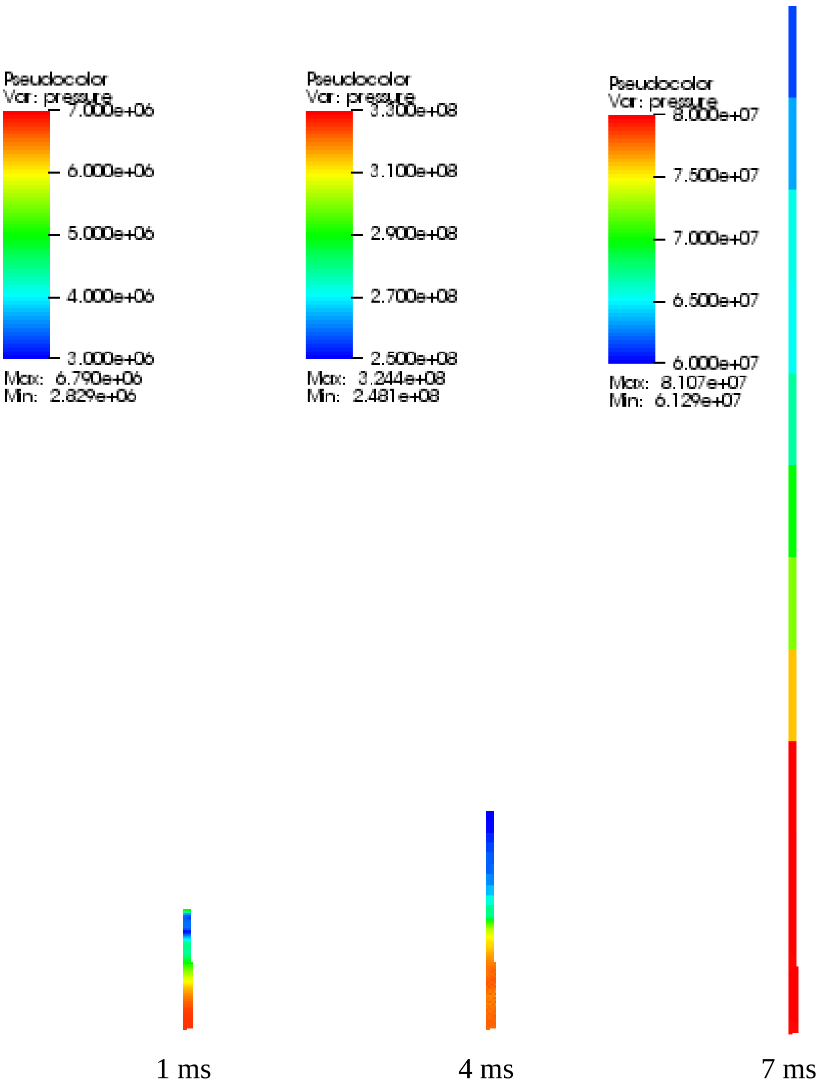
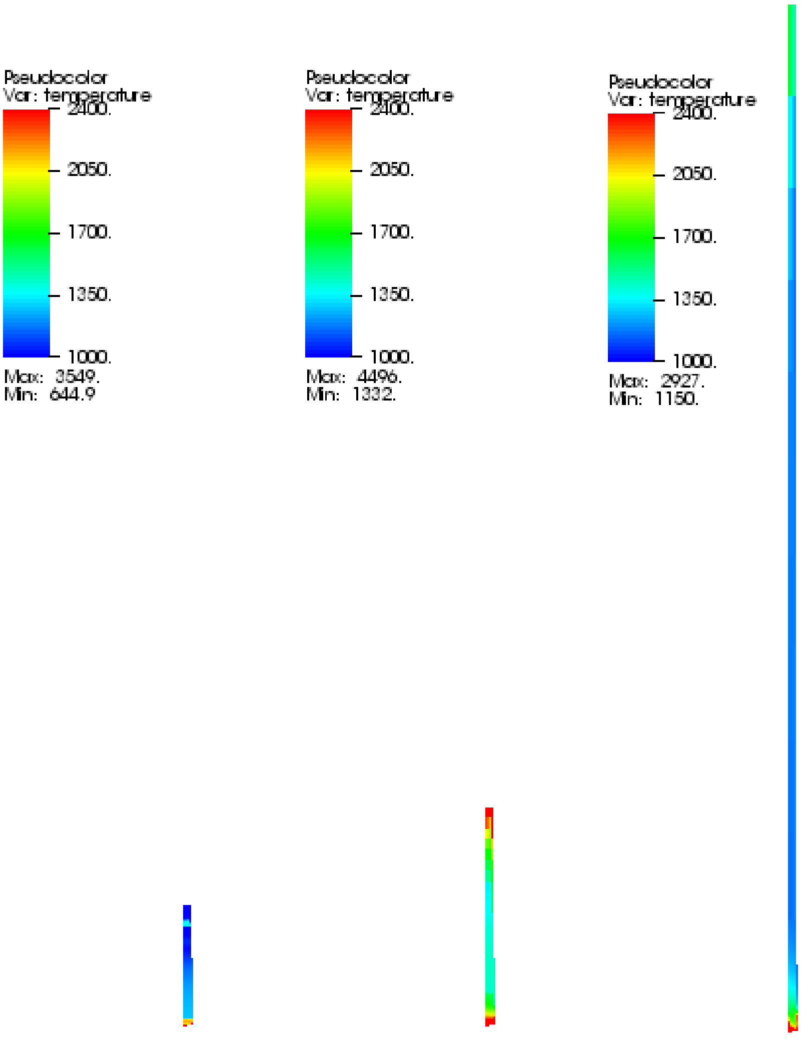


Figure 9. Gas domain temperature (K)



1 ms 4 ms 7 ms

5 Discussion

A model for internal ballistic analyses has been developed and implemented into a two-dimensional explicit finite element code. The model was applied to a 40 mm caliber gun and the results seem satisfactory.

However, the model needs to be further developed and tested against experimental results. Developments should include additional burning laws and equations of state. An improved description of the drag forces, propellant-gas heat exchange and fluid-solid interaction to allow for stress analyses of the barrel would be valuable extensions.

References

[1] P. S. Gough, The flow of a compressible gas through an aggregate of mobile, reacting particles, Ph.D. thesis, McGill University, Montreal, Canada (1974).

[2] P. S. Gough, Two dimensional convective flamespreading in packed beds of granular propellant, Contract report ARBRL-CR-00404, Army Ballistic Research Laboratory, Aberdeen (MD), USA (jul 1990).

[3] P. S. Gough, The NOVA code: a user's manual, Tech. Rep. IHCR 80-8, Naval Ordnance Station, Indian Head (MD), USA (1980).

[4] P. S. Gough, The XNOVAKTC code, Tech. Rep. BRL-CR-627, Army Ballistic Research Laboratory, Aberdeen (MD), USA (feb 1990).

[5] P. S. Gough, Interior ballistics modeling: extensions to the one-dimensional XKTC code and analytical studies of pressure gradient for lumped parameter codes, Tech. Rep. ARL-CR-460, Army Research Laboratory, Aberdeen (MD), USA (feb 1990).

[6] K. K. Kuo, K. C. Hsieh, M. M. Athavale, Modeling of combustion processes of stick propellants via combined eulerian-lagrangian approach, in: 8th International Symposium on Ballistics, Vol. I, The American Defense Preparedness Association, Orlando, Florida, 1984, pp. 55–68.

[7] P. S. du Toit, A two-dimensional internal ballistics model for for granular charges with special emphasis on modeling the propellant movement, in: 17th International Symposium on Ballistics, Vol. 1, Midrand, South Africa, 1998, pp. 424–431.

[8] G. P. Wren, S. M. Dash, T. E. Tezduyar, New directions in computational interior ballistics, in: M. Mayseless, S. R. Bodner (Eds.), 15th International Symposium on Ballistics, Vol. 3, Armament Development Authority and Israel Institute of Technology, Jerusalem, Israel, 1995, pp. 11–18.

[9] M. Berglund, On the modeling of reacting gas-solid flows – a survey of the literature, Scientific report FOA-R--99--01289-310--SE, Swedish Defence Research Establishment, Stockholm, Sweden (dec 2000).

[10] M. Berglund, C. Fureby, Large eddy simulation of the flow in a solid rocket motor, in: 39th AIAA Aerospace sciences meeting & exhibit, no. AIAA 2001-0895, American Institute of Aeronautics and Astronautics, Reno (NV), USA, 2001, p. 16.

[11] M. Berglund, A. Larsson, Initial numerical simulations of the flow in a closed electrothermal-chemical bomb, Base data report FOI-R--0580--SE, Swedish Defence Research Agency, Stockholm, Sweden (2001).

[12] S.-E. Flygar, IB-KB, ett datorprogram för beräkning av det innerballistiska förloppet i kanoner och bakblåsare, Tech. Rep. FOA C 20566-D2, Försvarets forskningsanstalt, Stockholm (1985).

[13] M. Carlson, CELLGUN98: En 1-dimensionell gasdynamisk modell för rekylerande kanoner. version 2.1, Internrapport FOA-D–98-00395-310–SE, Försvarets Forskningsanstalt, Stockholm (sep 1998).

[14] J. T. Bédard, Internal ballistic computer programs - Description and method of use, Tech. Rep. M-2272, Defence Research Establishment Valcartier, DREV, Quebec, Canada (1973).

[15] Greenwood (Ed.), Textbook of ballistics and gunnery, Vol. 1, Her Majesty's Stationery Office, London, Great Britain, 1987.

[16] L. Olovsson, On the arbitrary lagrangian-eulerian finite element method, Ph.D. thesis, Linköpings universitet, Linköping, dissertation No. 635 (2000).

[17] L. Olovsson, A. Helte, GRALE2D - an explicit finite element code for two-dimensional plane and axi-symmetric multi-material ALE simulations, in: V. Sanchez-Galvez, C. A. Brebbia, A. A. Motta, C. E. Anderson (Eds.), Computational Ballistics II, Vol. 40 of WIT Transactions on Engineering Sciences, Wessex Institute of Technology, UK, WIT Press, Cordoba, Spain, 2005, pp. 137–145.

[18] C. Q. Cutshaw, L. Ness (Eds.), Jane's ammunition handbook 2004-2005, Jane's information group limited, Coulsdon, U.K., 2004.