Polarisation Efficiency in Radiated Susceptibility Testing

Magnus Höijer

$$p = \sin^2\left(\frac{\Delta\tau_t + \Delta T_r}{2}\right) + \cos\left(\Delta\tau_t\right)\cos\left(\Delta T_r\right)\cos^2\left(\varphi_t - \Phi_r\right)$$

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Report title

Polarisation Efficiency in Radiated Susceptibility Testing

Abstract (not more than 200 words)

At Radiating Susceptibility Testing of electronic equipment against High Power Microwaves, the result depends strongly on the applied polarisation. It is not affordable to apply many different polarisations. Therefore only a few different polarisations are applied. This report quantifies the error introduced by applying only a few different polarisations.

The report does also tell which polarisations are the most optimal to use if the maximum possible error are to be minimised. Under some common conditions, described in the report, it is optimal to apply circular polarisation when only one polarisation is used. If more than one polarisation is applied consecutively, it is optimal to apply linear polarisations. At the trade-off between accuracy and affordability, we have developed a criterion showing that two or three different polarisations are optimal.

By applying unpolarised field, the error is deterministically -3 dB.

Keywords

Polarisation efficiency, Friis transmission equation, Unpolarised transmitted field, Radiated Susceptibility Testing, High Power Microwaves

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Polarisationsverkningsgrad vid RS-provning

Sammanfattning (högst 200 ord)

Vid RS-testning av elektronikutrustning mot HPM, beror resultatet starkt på det påstrålade fältets polarisation. På grund av begränsade resurser testas utrustning bara för ett fåtal olika polarisationer. Denna rapport ger kvantitativa uttryck för storleken på det fel som därmed uppkommer.

Rapporten ger även anvisningar om vilka polarisationer som är optimala för att minimera det maximalt möjliga felet. Under några allmängiltiga antaganden, beskrivna i rapporten, är det optimalt att använda cirkulär polarisation om bara en polarisation anbringas. Om mer än en polarisation anbringas konsekutivt, är linjär polarisation optimal. I kompromissen mellan noggrannhet och resursutnyttjande har vi tagit fram ett kriterium som visar att två eller tre olika polarisationer är optimalt.

Genom att anbringa opolariserat fält blir felet deterministiskt -3 dB.

Nyckelord

Polarisationsverkningsgrad, Opolariserat utsänt fält, Friis transmissionsformel, RS-provning, HPM

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Contents

Chapter 1

Radiated Susceptibility Testing - An Electromagnetic Coupling between Antennas

Figure 1.1: *A Radiated Susceptibility Test can be described by the Friis transmission equation* (1.1).

Radiated Susceptibility Testing (RST) can be seen as nothing else than an electromagnetic coupling between antennas. Fig. 1.1 shows an example. On the left hand side of Fig. 1.1, a strong source radiates electromagnetic energy. Through a transmitting antenna the electromagnetic energy is directed toward, the on the right hand side of Fig. 1.1 positioned, Equipment Under Test (EUT). The EUT acts as a receiving antenna and receives electromagnetic energy. Somewhere in the EUT there is a critical component which constitutes the load to our receiving antenna (=the EUT). The whole process can be described with the Friis transmission equation,

$$P_t(\nu)D_{t,max}(\nu) \times \frac{|\hat{\boldsymbol{e}}_t(\nu) \cdot \hat{\boldsymbol{e}}_r^*(\hat{\boldsymbol{r}}, \nu)|^2}{4\pi r^2} \times D_r(\hat{\boldsymbol{r}}, \nu)\frac{c^2}{4\pi\nu^2}\eta_r(\nu)q_r(\nu) = P_r(\hat{\boldsymbol{r}}, \nu) \ . \tag{1.1}$$

The Friis transmission equation is an expression for the power absorbed in the critical component (P_r). It assumes that the far field conditions are fulfilled, cf. [1, p. 75],

$$r \gg d_1 + d_2 \ , \tag{1.2}$$

$$r \gg \frac{2\pi}{\lambda}(d_1 + d_2)^2 \ , \tag{1.3}$$

$$r \gg \lambda \ , \tag{1.4}$$

where r is the distance between the transmitting antenna and the EUT, λ is the wavelength in use, d_1 is the maximum extent[1] of the transmitting antenna and d_2 is the maximum extent of the EUT. Even in the cases that the far field conditions are not fulfilled, the Friis transmission equation gives a qualitative understanding.

We will now go through and comment every part on the left hand side of the equal sign in (1.1). We first notice that we have explicitly indicated that some of the quantities depend on the frequency (ν) in use and the direction from which the EUT is irradiated ($\hat{\boldsymbol{r}}$). The first factor in (1.1) is the total power radiated by our source (P_t), and the second factor, the directivity of our transmitting antenna (D_t), describes how well the radiated power is focused in a specific direction compared to the average. Most often we want to focus as much power as possible onto our EUT, and therefore we assume that the EUT is positioned at boresight[2] as seen from our transmitting antenna. Hence we do not consider the precise directional dependence of the directivity of the transmitting antenna, but simply uses the maximum directivity ($D_{t,max}$). The two first factors in (1.1) completely describe the strength of our radiating source, and the product is called powermaxdirectivity.

The distance between the transmitting antenna and the EUT is r, and assuming that the far field conditions are fulfilled, the power density at the EUT is,

$$S(r,\nu) = \frac{P_t(\nu)D_{t,max}(\nu)}{4\pi r^2} \ . \tag{1.5}$$

If we are in the far field, the power density incident on the EUT does unequivocally define the stress we put onto the EUT. It follows from (1.5) that the stress we put onto our EUT can be regulated in three ways, the power radiated by our source, the maximum directivity of the transmitting antenna and the distance from the transmitting antenna to the EUT (=receiving antenna).

The last two factors on the left hand side of the equal sign in (1.1), the radiation efficiency (η_r) and the impedance mismatch factor (q_r), are essential in creating a

[1] Maximum extent is defined as the maximum distance to any point on the object from the objects origin of coordinates. The distance r is the distance from the transmitting antenna's origin of coordinates to the EUT's origin of coordinates. The choice of the position of the objects origin of coordinates will affect the exact value of d and r, but that has no physical implications, it is only a mathematical technicality. We have to remember that the exact definition of, *much greater than*, of course is arbitrarily. (However, we do assume that the origin of coordinates is within the objects.)

[2] We define boresight as the beam-maximum direction of our transmitting antenna.

good electromagnetic shielding. We will however not describe those factors here, for a further description see [2]. The factor,

$$\frac{c^2}{4\pi\nu^2} = \frac{\lambda^2}{4\pi} \ , \tag{1.6}$$

is fundamental and shows that any antenna includes a receiving property which decreases with the square of the frequency. The directivity of the EUT (D_r) varies substantially with the direction ($\hat{\boldsymbol{r}}$) as well as the frequency (ν). The interest of this topic has grown substantially during the last years, and special sessions has been addressed to this issue at conferences. The main work in this field has been done by the Swedish Defence Research Agency FOI, see e.g. [3, 4, 5, 6, 7], and by the (United States) National Institute of Standards and Technology NIST, see e.g. [8, 9, 10, 11, 12]. Reference [12] is a good review article, but does also give an overview of the mathematics involved. There are further work to be performed in this field, but in this report, we will focus on the remaining factor in (1.1), the polarisation efficiency[3],

$$p(\hat{\boldsymbol{r}},\nu) \triangleq \left|\hat{\boldsymbol{e}}_t(\nu)\cdot\hat{\boldsymbol{e}}_r^*(\hat{\boldsymbol{r}},\nu)\right|^2 \ . \tag{1.7}$$

Due to that the polarisation of the by our source radiated electromagnetic field ($\hat{\boldsymbol{e}}_t$) in the general case not is matched to the receiving polarisation of the EUT ($\hat{\boldsymbol{e}}_r$), there will be a decrease in the power which couples to the critical component in the EUT. The polarisation efficiency[4],

$$0 \le p(\hat{\boldsymbol{r}},\nu) \le 1 \ , \tag{1.8}$$

is a quantitative measure of the fraction of the electromagnetic power which couples to the critical component in the EUT, compared to the maximum electromagnetic power which would couple to the critical component in the EUT at the ideal case with a complete polarisation match.

The polarisation of the field leaving the radiating source varies with from which direction we look upon the transmitting antenna, but as we stated above, we assume radiation at boresight, and hence we suppress the directional dependence in $\hat{\boldsymbol{e}}_t(\nu)$. Actually, we have control of our source and transmitting antenna, and they may be manufactured in such a way that there is no frequency dependence either in $\hat{\boldsymbol{e}}_t(\nu)$. Consequently, we could have suppressed the frequency dependence also. However, there might be a, desired or not, frequency dependence and we do here include that possibility. When it comes to the receiving polarisation of the EUT, the situation is different, the receiving polarisation varies substantially with from which direction

[3]The complex conjugation of the receiving polarisation of the EUT comes from that we have defined the polarisation of the EUT in receiving mode and not in transmitting mode.

[4]The polarisation efficiency is also called polarisation mismatch factor. That expression is however infelicitous, because the polarisation mismatch is not a measure of the polarisation mismatch, but a measure of the polarisation match. We therefore prefer to use the expression polarisation efficiency.

our EUT is irradiated ($\hat{\boldsymbol{r}}$), as well as the frequency in use (ν) [7]. However, the most fundamental difference to the radiating source, is that we cannot control and do not know the directional and frequencies dependence in the receiving polarisation of the EUT[5]. A useful way to tackle this situation is to use stochastic models of the receiving polarisation of the EUT. Outgoing from that stochastic model of the receiving polarisation of the EUT, we can calculate a stochastic model of the polarisation efficiency, because the polarisation of the field coming from the radiating source is known.

When we perform a Radiated Susceptibility Test we are most often interested in the worst case. We want to test our EUT with the polarisation of the incident field at which it is as most susceptible, or differently stated, the polarisation of the incident field which gives a polarisation efficiency of 100%. However, as a consequence of the discussion above it is not doable to know the polarisation of the incident field which corresponds to a polarisation efficiency of 100%. We are left with statistical models, and this report describes how we in a Radiated Susceptibility Test, by applying different kinds of polarisations of the field coming from the irradiating source, may achieve a prescribed probability of the polarisation efficiency being within a certain interval.

One final remark: The Friis transmission equation is a useful tool, but it is to be mentioned that, the Friis transmission equation is a description in the frequency domain, time aspects is not explicitly included in it.

[5]We do not know the directional and frequency dependencies even after we have performed the Radiated Susceptibility Test, because we have only performed a test and not a true measurement.

Chapter 2

Polarisation Efficiencies

2.1 Linear Receiving Polarisation

2.1.1 Linear Polarisation of the Source Field

2.1.1.1 Polarisation Efficiency

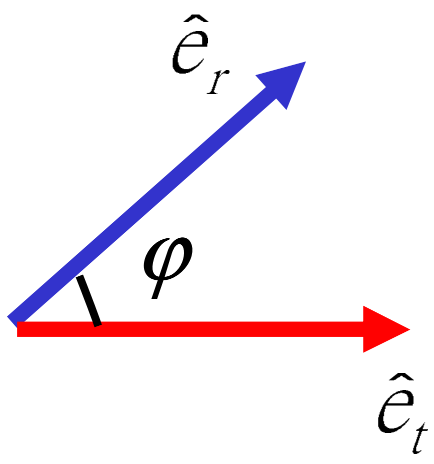

Figure 2.1: *The polarisation efficiency is given by the angle φ between the polarisation of the field transmitted from the radiating source ($\hat{\boldsymbol{e}}_t$) and the receiving polarisation of the Equipment Under Test ($\hat{\boldsymbol{e}}_r$).*

In a first attempt to find a stochastic model of the polarisation efficiency, we start with some very simple assumptions. We assume that the receiving polarisation of the EUT is linear. We also prescribe the polarisation of the field transmitted from the radiating sources to be linear. The direction of the linear polarisation of the field transmitted from the radiating source we probably have full control of, we might e.g.

choose it to be horizontal, but the receiving polarisation of the EUT is unknown. The polarisation efficiency can be written as,

$$p = \cos^2(\varphi) \ . \tag{2.1}$$

where φ is the angle between the two linear polarisations, see Fig. 2.1. Assuming that we have no knowledge of the direction of the receiving polarisation of the EUT and that any direction is equally likely, the angle will be a random variable, which is uniformly distributed on the interval 0 to 2π,

$$\Phi \in U[0, 2\pi] \ , \tag{2.2}$$

and from (2.1) it follows that the polarisation efficiency is also a random variable,

$$P = \cos^2(\Phi) \ . \tag{2.3}$$

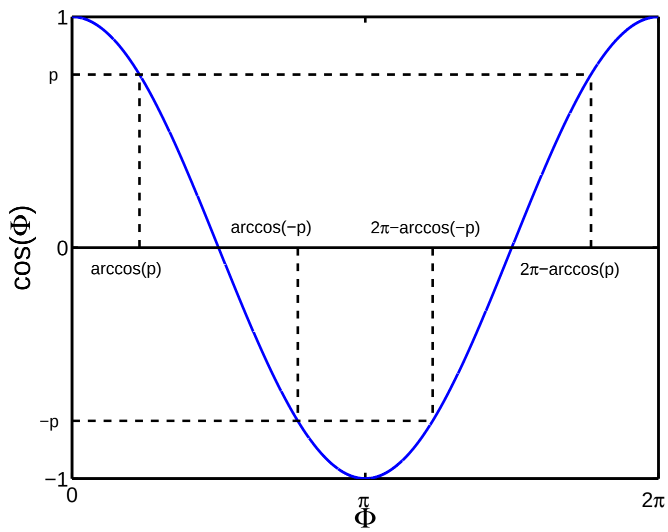

Figure 2.2: *This figure of the cosine function does hopefully help in understanding the steps taken in the derivation of (2.4).*

Outgoing from (2.2) and (2.3), the cumulative distribution function (cdf) for the polarisation efficiency can be calculated[1],

[1]The expression $\mathbb{P}(P \leq p)$ signifies: *the probability* $(\mathbb{P})$ *that the random variable* P *is less than or equal the number* p.

$$
\begin{aligned}
F_P(p) &\equiv \mathbb{P}(P \le p) = \mathbb{P}(\cos^2(\Phi) \le p) = \mathbb{P}\left(-\sqrt{p} \le \cos(\Phi) \le \sqrt{p}\right) \\
&= \mathbb{P}\left(\arccos\sqrt{p} \le \Phi \le \arccos(-\sqrt{p})\right) \\
&+ \mathbb{P}\left(2\pi - \arccos(-\sqrt{p}) \le \Phi \le 2\pi - \arccos\sqrt{p}\right) \\
&= 4\mathbb{P}\left(\arccos\sqrt{p} \le \Phi \le \frac{\pi}{2})\right) = 4\frac{\frac{\pi}{2} - \arccos\sqrt{p}}{2\pi} \\
&= \frac{2}{\pi}\arcsin\sqrt{p}\ ,
\end{aligned}
\tag{2.4}
$$

where some of the steps taken in (2.4) is hopefully more easily understood with help of Fig. 2.2 and by remembering that the value domain of the arcus cosine function is the interval $[0, \pi]$. If we take into account that the polarisation efficiency is limited to the interval $[0, 1]$ we get that,

$$
F_P(p) = \begin{cases} 0, & p < 0 \\ \frac{2}{\pi}\arcsin\sqrt{p}, & 0 \le p \le 1 \\ 1, & p > 1 \end{cases} . \tag{2.5}
$$

The probability density function (pdf) is easily calculated to,

$$
f_P(p) \equiv \frac{\mathrm{d}F_P(p)}{\mathrm{d}p} = \begin{cases} 0, & p < 0 \\ \frac{1}{\pi\sqrt{p(1-p)}}, & 0 \le p \le 1 \\ 0, & p > 1 \end{cases} . \tag{2.6}
$$

In Fig. 2.3, we have plotted both (2.5) and (2.6); on the left hand side with the polarisations efficiency in a linear scale, and on the right hand side with the polarisations efficiency in a dB-scale.

2.1.1.2 Compound Polarisation Efficiency

As discussed in chapter 1, we want the polarisation efficiency to be as close as possible to its maximum value when we perform a Radiated Susceptibility Test. We cannot simply rely on luck and hope that we got the maximum polarisation efficiency. We want to know how close we are to the maximum value 1, or more specific, what is the risk that we are outside an interval close to one.

If we look upon (2.6) or Fig. 2.3, we see that the polarisation efficiency may actually for some incident polarisation be zero, or differently stated, the polarisation efficiency is infinitely dB lower than its maximum value. The probability density function does actually also equals infinity at 0. However, it is not a delta pulse, so the probability of getting a polarisation efficiency of $-\infty$ dB is still zero, but, as can be

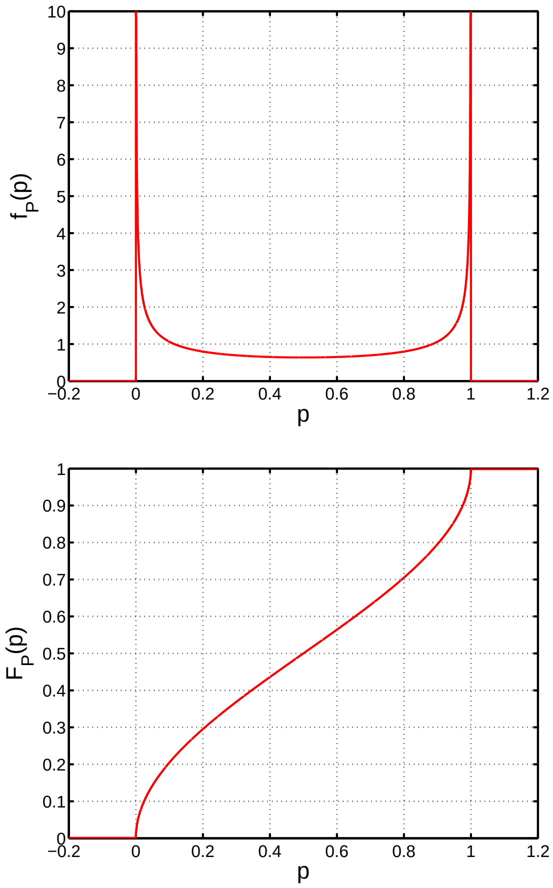

Figure 2.3: *Probability density function (upper figures) and cumulative density function (lover figures) of the polarisations efficiency. In the left hand side figures, the polarisations efficiency is in a linear scale and in the right hand side figures, the polarisations efficiency is in a dB-scale.*

seen in Fig. 2.3 with a good eye, there is a 50% risk of a polarisation efficiency lower than −3 dB, a 20% risk of a polarisation efficiency lower than −10 dB and actually a 2.0% risk of a polarisation efficiency as low as −30 dB or lower.

However, by using two consecutive orthogonal (linear) polarisations ($\hat{\boldsymbol{e}}_H$ and $\hat{\boldsymbol{e}}_V$) of the incident electromagnetic field, the situation is drastically changed. The two polarisation efficiencies, which the two orthogonal polarisations give rise to, we denote p_H and p_V. One might think of the two polarisation efficiencies as corresponding to a horizontal and vertical polarisation, respectively, of the incident field, but the orientation of the two incident orthogonal linear polarisations is arbitrarily. The two polarisation efficiencies (p_H and p_V) will, of course, in the general case differ. One is larger than the other, and hence is the most interesting one because it corresponds to the more severe Radiated Susceptibility Test. We therefore introduce the compound polarisation efficiency,

$$r \triangleq \max\{p_H, p_V\} \ . \tag{2.7}$$

Two polarisations, which are not linearly dependent, is a complete set so that any polarisation can be built up of theses two polarisations. In our case, we assumed that the two polarisations of the incident field, $\hat{\boldsymbol{e}}_H$ and $\hat{\boldsymbol{e}}_V$, giving rise to the polarisation efficiencies p_H and p_V, are orthogonal. That, together with that all polarisations are normalised to one, gives us that,

$$p_H + p_V = 1 \ . \tag{2.8}$$

With (2.8) plugged into (2.7), we get,

$$r = \max\{p_H, 1 - p_H\} \ . \tag{2.9}$$

The polarisation efficiency p_H is a random variable described by (2.5) and (2.6), and hence the compound polarisation efficiency is also a random variable,

$$R = \max\{P, 1 - P\} \ . \tag{2.10}$$

Due to that,

$$0 \leq P \leq 1 \ , \tag{2.11}$$

it follows from (2.10) that,

$$0.5 \leq R \leq 1 \ , \tag{2.12}$$

which tells us that the compound polarisation efficiency can never be smaller than -3 dB. The cumulative density function for the compound polarisation efficiency is,

$$\begin{aligned} F_R(r) &\equiv \mathbb{P}(P \leq r) = \mathbb{P}(\max\{P, 1-P\} \leq r) = \mathbb{P}(0.5 \leq P \leq r) \\ &+ \mathbb{P}(0.5 \leq 1 - P \leq r) = \mathbb{P}(0.5 \leq P \leq r) + \mathbb{P}(1 - r \leq P \leq 0.5) \\ &= \mathbb{P}(1 - r \leq P \leq r) = \int_{1-r}^{r} f_P(p) \mathrm{d}p = F_P(r) - F_P(1-r) \\ &= \frac{2}{\pi}\left(\arcsin\sqrt{r} - \arcsin\sqrt{1-r}\right) = \frac{2}{\pi}\left(\arcsin\sqrt{r} - \arccos\sqrt{r}\right) \\ &= \frac{4}{\pi}\arcsin\sqrt{r} - 1 \ , \end{aligned} \tag{2.13}$$

or with (2.12) explicitly included,

$$F_R(r) = \begin{cases} 0, & r < 0.5 \\ \frac{4}{\pi}\arcsin\sqrt{r} - 1, & 0.5 \leq r \leq 1 \\ 1, & r > 1 \end{cases} . \tag{2.14}$$

The probability density function (pdf) is, in similarity to (2.15), easily calculated to,

$$f_R(r) \equiv \frac{\mathrm{d}F_R(r)}{\mathrm{d}r} = \begin{cases} 0, & r < 0.5 \\ \frac{2}{\pi\sqrt{p(1-p)}}, & 0.5 \leq r \leq 1 \\ 0, & r > 1 \end{cases} . \tag{2.15}$$

In Fig. 2.4, we have plotted both (2.14) and (2.15); on the left hand side with the compound polarisations efficiency in a linear scale, and on the right hand side with the compound polarisations efficiency in a dB-scale.

2.1.1.3 Higher Order Polarisation Efficiencies

In section 2.1.1.2, we saw that by using two consecutive orthogonal polarisations, the polarisation efficiency in Radiated Susceptibility Testing can be limited to the interval $[-3\text{ dB}, 0\text{ dB}]$. The interval can be further limited by using more than two polarisations, e.g. three consecutive linear polarisations with an intermediate angle of 60°. In the general case n consecutive linear polarisations with an intermediate angle of $180°/n$, will cause a n:th order polarisation efficiency,

$$p(n) \triangleq \max\{p_i\}_1^n , \tag{2.16}$$

where p_i is the polarisation efficiency received when the i:th polarisation from the radiating source is applied. The special cases, the polarisation efficiency, described in section 2.1.1.1, and the compound polarisation efficiency, described in section 2.1.1.2, are then the first and second order polarisation efficiencies, respectively.

The n:th order polarisation efficiency is a random variable,

$$P(n) = \max\{P_i\}_1^n , \tag{2.17}$$

limited to the interval,

$$\cos^2\left(\frac{\pi}{2n}\right) \leq P(n) \leq 1 = \cos^2(0) , \tag{2.18}$$

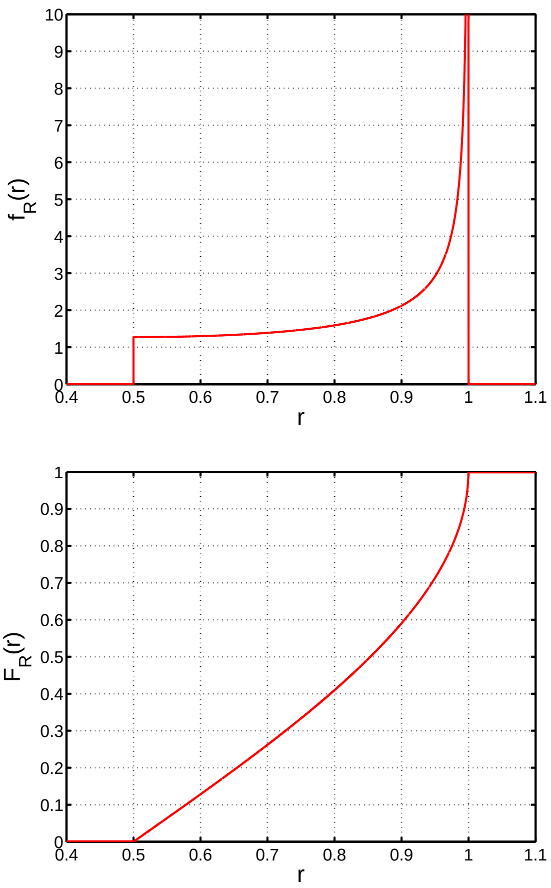

Figure 2.4: *Probability density function (upper figures) and cumulative density function (lover figures) of the compound polarisations efficiency. In the left hand side figures, the compound polarisations efficiency is in a linear scale and in the right hand side figures, the polarisations efficiency is in a dB-scale.*

because if the orientation of the incident polarisation corresponding to the maximum polarisation efficiency is changed to be outside this interval, the nearby polarisation will cause a polarisation efficiency within the interval in (2.18). In Fig. 2.5 the special case with four applied polarisations is shown.

The n:th order polarisation efficiency random variable may be written,

$$P(n) = \cos^2(\Phi(n)) \ , \tag{2.19}$$

and we assume that the random variable $\Phi(n)$ is uniformly distributed on the interval 0 to $\frac{\pi}{2n}$,

$$\Phi \in U\left[0, \frac{\pi}{2n}\right] \ . \tag{2.20}$$

Outgoing from (2.19) and (2.20), the cumulative distribution function (cdf) for the polarisation efficiency can be calculated as,

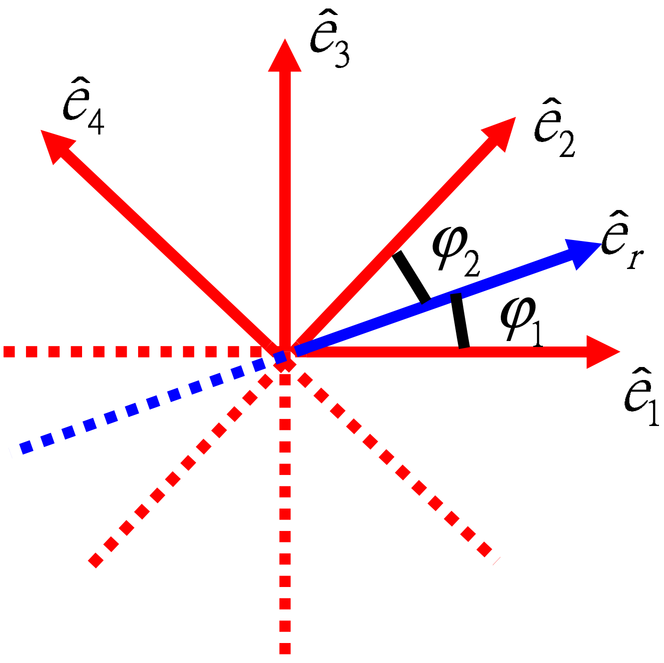

Figure 2.5: *For the forth order polarisation efficiency, the angle between the receiving polarisation of the Equipment Under Test* ($\hat{\mathbf{e}}_r$) *and the applied polarisation from the radiation source which corresponds to the maximum polarisation efficiency can never be larger than* $\frac{\pi}{8}$.

$$\begin{aligned} F_{P(n)}(p) &\equiv \mathbb{P}(P(n) \leq p) = \mathbb{P}(\cos^2(\Phi(n)) \leq p) = \mathbb{P}\left(\cos\left(\frac{\pi}{2n}\right) \leq \cos\left(\Phi(n)\right) \leq \sqrt{p}\right) \\ &= \mathbb{P}\left(\arccos\sqrt{p} \leq \Phi(n) \leq \frac{\pi}{2n}\right) = \frac{\frac{\pi}{2n} - \arccos\sqrt{p}}{\frac{\pi}{2n}} \\ &= 1 - \frac{2n}{\pi}\arccos\sqrt{p} \ , \end{aligned} \quad (2.21)$$

and if we take explicitly include that the n:th order polarisation efficiency is limited to the interval $[\cos^2\left(\frac{\pi}{2n}\right), 1]$, we get that,

$$F_{P(n)}(p) = \begin{cases} 0, & p < \cos^2\left(\frac{\pi}{2n}\right) \\ 1 - \frac{2n}{\pi}\arccos\sqrt{p}, & \cos^2\left(\frac{\pi}{2n}\right) \leq p \leq 1 \\ 1, & p > 1 \end{cases} . \quad (2.22)$$

The probability density function (pdf) is easily calculated to,

$$f_{P(n)}(p) \equiv \frac{\mathrm{d}F_P(p)}{\mathrm{d}p} = \begin{cases} 0, & p < \cos^2\left(\frac{\pi}{2n}\right) \\ \frac{n}{\pi\sqrt{p(1-p)}}, & \cos^2\left(\frac{\pi}{2n}\right) \le p \le 1 \\ 0, & p > 1 \end{cases} . \tag{2.23}$$

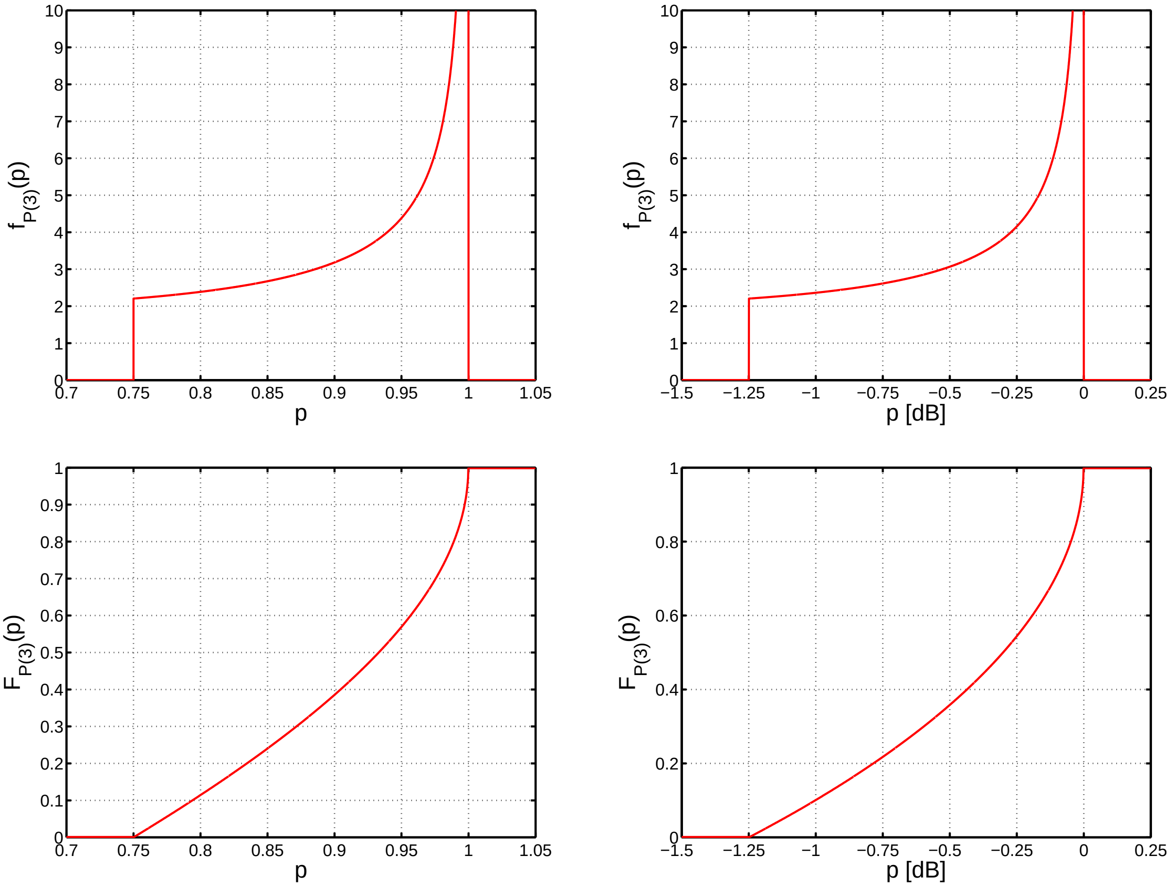

Figure 2.6: *Probability density function (upper figures) and cumulative density function (lover figures) of the third order polarisations efficiency. In the left hand side figures, the third order polarisations efficiency is in a linear scale and in the right hand side figures, the third polarisations efficiency is in a dB-scale.*

In Fig. 2.6, we have plotted both (2.22) and (2.23) for $n = 3$; on the left hand side with the third order polarisations efficiency in a linear scale, and on the right hand side with the third order polarisations efficiency in a dB-scale. We do here also give an expression for the expectation value of the n:th order polarisation efficiency,

$$\mathbb{E}\left\{P(n)\right\} = \frac{1}{2}\left[1 + \frac{n}{\pi}\sin\left(\frac{\pi}{n}\right)\right] . \tag{2.24}$$

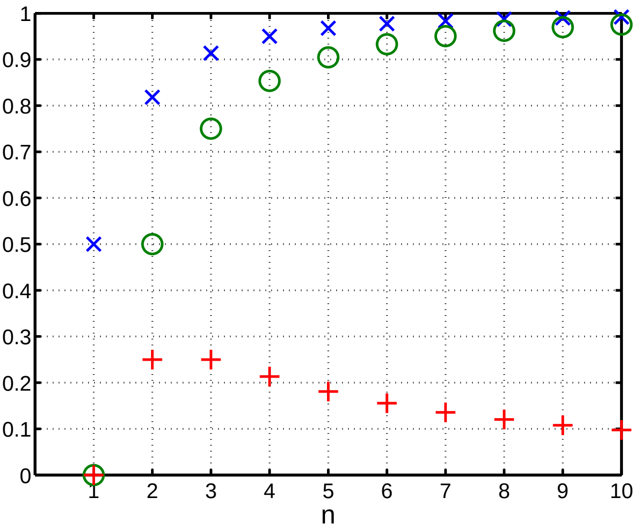

Figure 2.7: *The blue crosses are the expectation value of the n:th order polarisation efficiency plotted as function of the integer n. The green circles are the minimum value of the n:th order polarisation efficiency plotted as function of the integer n. The reed plus signs are the green circle values divided by n.*

In Fig. 2.7, we have plotted the expectation and minimum values of the n:th order polarisation efficiency as function of the integer n. Already the fifth order polarisation efficiency has a value which is always larger than 90%. One conceivable criterion for how many linear polarisations which are optimal to use, is to normalise the minimum value of the n:th order polarisation efficiency by dividing it by n, and to chose the n which corresponds to the maximum value of this quotient. In Fig. 2.7, the quotient is plotted as function of the integer n. The maximum value of the quotient (0.25) is reached for both $n = 2$ and $n = 3$. So using this criterion two orthogonal linear polarisations or three linear polarisations with an intermediate angle of 60°, are to be applied.

2.1.2 Circular Polarisation of the Source Field

In section 2.1.1 we prescribed the polarisation of the field transmitted from the radiating sources to be linear. An alternative to that is to let the polarisation be circular. If we then still assume that the receiving polarisation of the EUT is linear, we have a situation schematically shown in Fig. 2.8. The polarisation efficiency is here no longer a random variable, but is exactly the deterministic value $\frac{1}{2} = -3$ dB. That is often a expedient property, and if constructing a radiating source for Radiated Susceptibility Testing (RST) having the feasibility to transmit only one polarisation, it should be considered to construct it in such a way that it irradiates an electromagnetic field with circular polarisation.

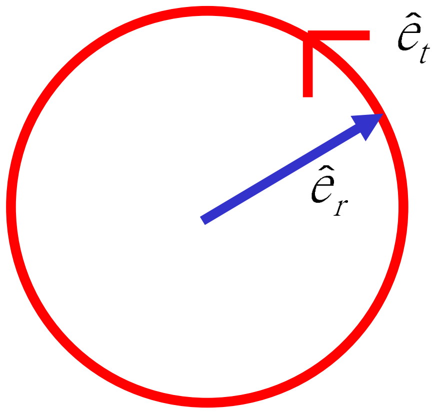

Figure 2.8: *The polarisation efficiency between a linear polarisation and circular polarisation is exactly* $\frac{1}{2} = -3$ dB.

2.2 Circular Receiving Polarisation

In section 2.1.1 and 2.1.2 we assumed that the receiving polarisation of the Equipment Under Test is linear. That assumption is probably a good one[2]. The reason being that, the incident electromagnetic energy is in the EUT at the end typically received in wires and via them lead to the critical component in the EUT, and the receiving polarisation of wires is linear. If we lock upon front-door coupling there are of course receiving antennas which are constructed to have a circular, or generally an elliptical, receiving polarisation.

If we assume that we know that the receiving polarisation of the front-door antenna is circular, we may transmit one left circular polarisation and one right circular polarisation from our radiating source. One of the two polarisations will cause a polarisation efficiency of 0%, but the other will give a polarisation efficiency of 100%. If we know whether the receiving polarisation is left or right circular polarisation we do, of course, only need to send the one which causes a polarisation efficiency of 100%. Alternatively we may transmit a linear polarisation and we get the complementary situation to section 2.1.2 and hence the polarisation efficiency is exactly the deterministic value $\frac{1}{2} = -3$ dB.

However, and this is important, the receiving polarisation of the front-door antenna is circular for the signal it is constructed. It is doubtful if the receiving polarisation remains circular for the high-level electromagnetic pulses which we apply from our irradiating source, which are also possibly at a different frequency than the one, for which the receiving antenna is constructed. It is possible, or perhaps even likely, that the receiving polarisation for high-level electromagnetic pulses is again

[2]We are however not completely certain about this statement. In complex structures, the receiving polarisation may be non-linear. That is one of the reasons why we in chapter 3 investigate the general case, where we assume that we know nothing about the receiving polarisation of the Equipment Under Test.

linear. The effect is probably more likely to occur in destructive Radiated Susceptibility Testing, than in disturbance Radiated Susceptibility Testing. The topic is to be further investigated.

It is again to be stressed, that we, without knowing for certain, assume that it often is a good assumption to assume that the receiving polarisation of the Equipment Under Test is linear for high-level electromagnetic pulses. However, we will in the next chapter investigate the general case where we assume that we have no knowledge at all about the receiving polarisation of our EUT.

Chapter 3

Polarisation Efficiency: General Case

In this chapter we will assume that we know nothing about the receiving polarisation of the Equipment Under Test (EUT). We will also assume that we have complete freedom in the choice of the polarisation coming from the radiating source. It is to be stressed that in finding a proper distribution function for the polarisation efficiency it is important to use the knowledge we have. If we e.g. have good reasons to assume a linear polarisation, as we argued for in section 2.2, we are to use that information in constructing a relevant random variable of the polarisation efficiency. However, there might be cases, where no knowledge of the receiving polarisation of the EUT is a better assumption. We will also gain understanding from studying the general case. Furthermore we will find that the cases studied in chapter 2 can easily be found as special cases of the general case. In assuming complete freedom in the choice of the polarisation coming from the radiating source, we will also be able to calculate the optimal choice of polarisation coming from the radiating source in some special cases, including the interesting case where the receiving polarisation of the EUT is linear.

The general case is however so complex that we have not yet had enough resources to develop the general distribution function for the polarisation efficiency. That is, though not easy, probably doable, and remains to be done.

3.1 Polarisation Ellipse

The electric field of a monochromatic plane wave can generally be described by four parameters. The electric field lies in the plane perpendicular to the propagation direction of the plane wave. A vector in a plane is completely described by two parameters, e.g. the vectors two components in an orthogonal coordinate system. To know the evolution of the electric field, we also need to know the phase of both components in time-space. That gives us in total four components. The polarisation of the plane wave ($\hat{\boldsymbol{e}}$) is the electric field vector ($\boldsymbol{E}$) divided by its norm ($||\hat{\boldsymbol{E}}||$),

$$\hat{\boldsymbol{e}} = \frac{\boldsymbol{E}}{||\boldsymbol{E}||} \ . \tag{3.1}$$

The norm[1] is,

$$||\boldsymbol{E}|| = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{E}\cdot\boldsymbol{E}\mathrm{d}t} \ . \tag{3.2}$$

where T is the period of the monochromatic wave. Hence, the norm of the polarisation is always one and the number of parameters necessary to describe the polarisation is one less than the number of parameters necessary to describe the electric field, i.e. three.

We choose to write the polarisation on the form,

$$\hat{\boldsymbol{e}} = \cos(\omega t+\tau)[\cos(\varphi)\hat{\boldsymbol{e}}_x+\sin(\varphi)\hat{\boldsymbol{e}}_y]+\cos(\omega t+\tau+\Delta\tau)[-\sin(\varphi)\hat{\boldsymbol{e}}_x+\cos(\varphi)\hat{\boldsymbol{e}}_y] \ . \tag{3.3}$$

Equation (3.3) might look complex, but once understood natural and useful, and we will now try to explain it. First we can notice, by plugging (3.3) into (3.2), that the condition, that the norm is one, is fulfilled. The parameter ω is the angular frequency of the monochromatic wave and t is the time. They are not parameters describing the polarisation. We have assumed a cartesian coordinate system with the electromagnetic wave propagating in the z-direction. In the polarisation plane, perpendicular to the z-direction, $\hat{\boldsymbol{e}}_x$ and $\hat{\boldsymbol{e}}_y$, are the unit vectors in the x- and y-directions, respectively. The three remaining parameters, τ, $\Delta\tau$ and φ, do completely describe the polarisation. By introducing two new orthogonal directions, see Fig. 3.1,

$$\begin{cases} \hat{\boldsymbol{e}}_x' = \cos(\varphi)\hat{\boldsymbol{e}}_x + \sin(\varphi)\hat{\boldsymbol{e}}_y \\ \hat{\boldsymbol{e}}_y' = -\sin(\varphi)\hat{\boldsymbol{e}}_x + \cos(\varphi)\hat{\boldsymbol{e}}_y \end{cases} , \tag{3.4}$$

(3.3) can be rewritten as,

$$\hat{\boldsymbol{e}} = \cos(\omega t + \tau)\hat{\boldsymbol{e}}_x' + \cos(\omega t + \tau + \Delta\tau)\hat{\boldsymbol{e}}_y' \ . \tag{3.5}$$

By looking upon Fig. 3.1 and after some thinking, we understand that by varying the parameter $\Delta\tau$ in the interval,

$$\Delta\tau \in [-\frac{\pi}{2}, \frac{\pi}{2}] \ , \tag{3.6}$$

we can completely describe the ellipticity of the polarisation. At the lower end of the interval ($\Delta\tau = -\frac{\pi}{2}$), the polarisation is left circular polarised. By going into the

[1]The norm of the electric field vector can be seen as an effective electric field value.

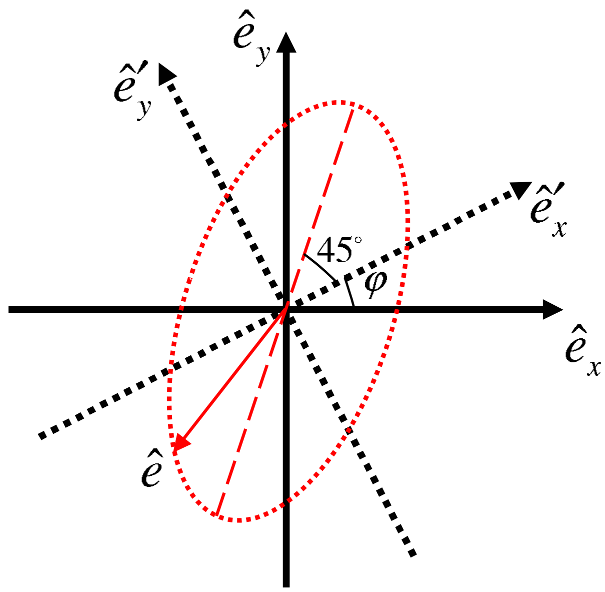

Figure 3.1: *The general polarisation can be described as an ellipse. Left and right circular polarisation are extreme cases of the elliptic polarisation with linear polarisation in the middle. The orientation of the ellipse is described by the angle φ.*

interval we get a left elliptic polarisation, and as we increase $\Delta\tau$, the elipticity of the elliptic polarisation is increased until we get a linear polarisation for $\Delta\tau = 0$. For positive $\Delta\tau$ we get right elliptic polarisation, and by increasing $\Delta\tau$, the elipticity of the elliptic polarisation is decreased and at the upper end of the interval ($\Delta\tau = \frac{\pi}{2}$), the polarisation is right circular polarised. The parameter $\Delta\tau$ does hence completely describe the type of polarisation. The major and minor axes of the polarisation ellipse are always in the same directions at 45° angle to the primed coordinate axes as shown in Fig. 3.1. A proof thereof can be found in appendix A. By looking upon Fig. 3.1, we do also see that by varying the parameter φ in the interval,

$$\varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}] \;, \tag{3.7}$$

we do completely describe the orientation of the polarisation. Hence, we have hopefully motivated that by writing the polarisation on the form in (3.3), we have a useful tool. The parameter τ is just an overall phase parameter and does only give information of when we turned on the source in the time scale at which we measures the electric field. It is often not included in the concept polarisation, but we will keep it for completeness, and, as we will see in section 3.2, when we calculate the polarisation efficiency, it will actually influence the result.

3.2 Polarisation Efficiency

In chapter 2 we used an intuitive physical understanding to calculate the polarisation efficiency. Here we will use a more strict definition of the polarisation efficiency. We calculate the polarisation efficiency[2] as the square of the time average of the scalar product of the polarisation of the source field ($\hat{\boldsymbol{e}}_t(t)$) and the receiving polarisation of the EUT ($\hat{\boldsymbol{e}}_r(t)$),

$$p = \left(\frac{1}{T} \int_0^T \hat{\boldsymbol{e}}_t(t) \cdot \hat{\boldsymbol{e}}_r(t) \mathrm{d}t \right)^2 . \tag{3.8}$$

Both the polarisation of the source field ($\hat{\boldsymbol{e}}_t(t)$) and the receiving polarisation of the EUT ($\hat{\boldsymbol{e}}_r(t)$), we express on the form proposed in (3.3)[3],

$$\hat{\boldsymbol{e}}_t(t) = \cos(\omega t + \tau_t)[\cos(\varphi_t)\hat{\boldsymbol{e}}_x + \sin(\varphi_t)\hat{\boldsymbol{e}}_y] + \cos(\omega t + \tau_t + \Delta\tau_t)[-\sin(\varphi_t)\hat{\boldsymbol{e}}_x + \cos(\varphi_t)\hat{\boldsymbol{e}}_y] , \tag{3.9}$$

and,

$$\hat{\boldsymbol{e}}_r(t) = \cos(\omega t + \tau_r)[\cos(\Phi_r)\hat{\boldsymbol{e}}_x + \sin(\Phi_r)\hat{\boldsymbol{e}}_y] + \cos(\omega t + \tau_r + \Delta T_r)[-\sin(\Phi_r)\hat{\boldsymbol{e}}_x + \cos(\Phi_r)\hat{\boldsymbol{e}}_y] . \tag{3.10}$$

The parameters $\Delta\tau_t$ and φ_t describe the field coming from the radiating source. We assume that we have deterministic control of the source, and hence may chose any values from the intervals,

$$\Delta\tau_t \in [-\frac{\pi}{2}, \frac{\pi}{2}] , \tag{3.11}$$

and,

$$\varphi_t \in [-\frac{\pi}{2}, \frac{\pi}{2}] . \tag{3.12}$$

The parameters ΔT_r and Φ_r, are parameters of the Equipment Under Test (EUT), and as we do not know their values, we treat them as random variables. To indicate that they are random variables, we have written them with capital letters. Here we assume that we have no knowledge of the receiving polarisation of the EUT, and consequently it is reasonable to assume that the two parameters are uniformly distributed on there domains,

$$\Delta\ T_r \in U[-\frac{\pi}{2}, \frac{\pi}{2}] , \tag{3.13}$$

[2]It can be noticed that the polarisation efficiency is a power quantity, but the polarisation is a field quantity.

[3]It is to be remembered that our treatment presupposes monochromatic waves. The polarisation efficiency defined in (3.8) is only well defined for a monochromatic wave, or in practice almost monochromatic wave, and is a function of frequency.

and,

$$\Phi_r \in U[-\frac{\pi}{2}, \frac{\pi}{2}] \ . \tag{3.14}$$

We will soon comment the parameters τ_t and τ_r, but first we will state that it is often common to write the polarisation in form of phasors[4] [13, pp. 294],

$$\hat{\boldsymbol{e}}_t(\omega) = \frac{1}{\sqrt{2}} \left\{ e^{i\tau_t}[\cos(\varphi_t)\hat{\boldsymbol{e}}_x + \sin(\varphi_t)\hat{\boldsymbol{e}}_y] + e^{i(\tau_t+\Delta\tau_t)}[-\sin(\varphi_t)\hat{\boldsymbol{e}}_x + \cos(\varphi_t)\hat{\boldsymbol{e}}_y] \right\} \ , \tag{3.15}$$

and,

$$\hat{\boldsymbol{e}}_r(\omega) = \frac{1}{\sqrt{2}} \left\{ e^{i\tau_r}[\cos(\Phi_r)\hat{\boldsymbol{e}}_x + \sin(\Phi_r)\hat{\boldsymbol{e}}_y] + e^{i(\tau_r+\Delta\tau_r)}[-\sin(\Phi_r)\hat{\boldsymbol{e}}_x + \cos(\Phi_r)\hat{\boldsymbol{e}}_y] \right\} \ . \tag{3.16}$$

The relation between the time notation and the phasor notation is simply,

$$\hat{\boldsymbol{e}}_t(t) = \sqrt{2}\mathrm{Re}\left\{ \hat{\boldsymbol{e}}_t(\omega)e^{i\omega t} \right\} \ , \tag{3.17}$$

$$\hat{\boldsymbol{e}}_r(t) = \sqrt{2}\mathrm{Re}\left\{ \hat{\boldsymbol{e}}_r(\omega)e^{i\omega t} \right\} \ , \tag{3.18}$$

and it is rather straight forward calculus to show that (3.8) can be calculated as[5],

$$p = \left(\mathrm{Re}\left\{ \hat{\boldsymbol{e}}_t(\omega) \cdot \hat{\boldsymbol{e}}_r^*(\omega) \right\}\right)^2 \ , \tag{3.19}$$

and by plugging (3.15) and (3.16) into (3.19), we get,

$$\begin{aligned} p &= \frac{1}{4} \Big(\mathrm{Re}\Big\{ e^{i(\tau_t-\tau_r)} \big[\cos(\varphi_t - \Phi_r) - e^{i\Delta\tau_t}\sin(\varphi_t - \Phi_r) \\ &+ e^{-i\Delta T_r}\sin(\varphi_t - \Phi_r) + e^{i(\Delta\tau_t - \Delta T_r)}\cos(\varphi_t - \Phi_r)\big] \Big\}\Big)^2 \ . \end{aligned} \tag{3.20}$$

Equation (3.20) tells us that the phase difference $\tau_t - \tau_r$ affects the result, and not both τ_t and τ_r independently of each other. That is natural, because it is only the relative phase between the two polarisations which affects the result. The overall phase will not affect the result. E.g. if we measure the polarisation efficiency now, or e.g. a tenth of a period later, does not affect the result as long as the phase difference $\tau_t - \tau_r$ remains constant.

[4]The factor of $\frac{1}{\sqrt{2}}$ is not always included in the phasor, but here we include because it implies that the inner product (scalar product) of the polarisation phasor with itself becomes one.

[5]In chapter 2 we denoted the polarisation efficiency with a capital P to emphasise that it is a random variable. Here, we do not do that, but as thew polarisation efficiency is a function of random variables, it is also itself a random variable.

However, the phase difference $\tau_t - \tau_r$, is not arbitrarily either, which we can understand from physical insight. The EUT is a passive component and the receiving polarisation will phase lock on the incoming electric field in such a way that the polarisation efficiency is maximised. That is mathematically described by taking the absolute value instead of the real value in (3.19) and (3.20). Hence,

$$p = |\hat{\boldsymbol{e}}_t(\omega) \cdot \hat{\boldsymbol{e}}_r^*(\omega)| \ , \tag{3.21}$$

and explicitly,

$$\begin{aligned} p &= \frac{1}{4}\big|\cos(\varphi_t - \Phi_r) - e^{i\Delta\tau_t}\sin(\varphi_t - \Phi_r) \\ &+ e^{-i\Delta T_r}\sin(\varphi_t - \Phi_r) + e^{i(\Delta\tau_t - \Delta T_r)}\cos(\varphi_t - \Phi_r)\big|^2 \ . \end{aligned} \tag{3.22}$$

This is not a sort of maximum value of the polarisation efficiency, but the polarisation efficiency value we get. To get an understanding of this, assume e.g. a simple *passive* linear dipole; The phase of the receiving polarisation of the dipole cannot be phase mismatched with the incident field in such a way that no electromagnetic energy is absorbed. That is not physical. In the most extreme case of a complete mismatch, we should actually get some sort of stimulated emission from the passive dipole, which is a contradiction, and hence wrong. Actually, this knowledge is already included in the Friis transmission equation, and (3.21) is nothing else than (1.7). That does also show that the polarisation expressions used in the Friis transmission equation are phasors, which is often not pointed out. Finally we rewrite (3.22) on a more convenient form[6],

$$\begin{aligned} p &= \frac{1}{2} - \frac{1}{2}\cos(\Delta\tau_t + \Delta T_r) + \cos(\Delta\tau_t)\cos(\Delta T_r)\cos^2(\varphi_t - \Phi_r) \\ &= \sin^2(\frac{\Delta\tau_t + \Delta T_r}{2}) + \cos(\Delta\tau_t)\cos(\Delta T_r)\cos^2(\varphi_t - \Phi_r) \ . \end{aligned} \tag{3.23}$$

With help of 3.23 we will now verify the assumptions in the special cases studied in chapter 2.

3.2.1 Linear Receiving Polarisation

The special case of linear receiving polarisation we get by putting $\Delta T_r = 0$ into (3.23)[7],

$$p_{?l} = \sin^2(\frac{\Delta\tau_t}{2}) + \cos(\Delta\tau_t)\cos^2(\varphi_t - \Phi_r) \ . \tag{3.24}$$

[6] It actually takes rather tedious calculations to go from (3.22) to (3.23)

[7] The index l denotes that the receiving polarisation is linear, and the index ? denotes that we have no restrictions on the transmitted polarisation

If we also transmit linear polarised field from our radiating source ($\Delta\tau_t = 0$) we get,

$$p_{ll} = \cos^2(\varphi_t - \Phi_r) \ , \tag{3.25}$$

which is the random variable proposed in (2.3). If we instead transmit circular polarised field ($\Delta\tau_t = \pm\frac{\pi}{2}$) we get,

$$p_{cl} = \frac{1}{2} \ , \tag{3.26}$$

which is the deterministic value given in section 2.1.2.

3.2.2 Circular Receiving Polarisation

The special case of circular receiving polarisation we get by putting $\Delta T_r = \pm\frac{\pi}{2}$,

$$p_{?c} = \sin^2(\frac{\Delta\tau_t \pm \frac{\pi}{2}}{2}) \ , \tag{3.27}$$

where the minus sign is used for left circular polarisation and the plus sign is used for right circular polarisation. If we transmit linear polarised field from our radiating source ($\Delta\tau_t = 0$) we get,

$$p_{lc} = \frac{1}{2} \ , \tag{3.28}$$

which is the deterministic value given in section 2.2. If we instead transmit circular polarised field ($\Delta\tau_t = \pm\frac{\pi}{2}$) we get,

$$p_{cc} = (p_t \equiv p_r) \ , \tag{3.29}$$

i.e. if the transmitting and receiving circular polarisations are the same, the polarisation efficiency is one. If they differ, the polarisation efficiency is zero. Also that statement was given in section 2.2.

3.3 Optimal Polarisation - Random Receiving Polarisation

A key question is, of course, which is the optimal polarisation to chose for the transmitted field from our source. To be able to answer that question, we need an optimisation criterion. We choose to optimise the worst case, i.e. if we are unlucky so that the receiving polarisation are such as that we get a minimum value of the polarisation efficiency; how do we chose the transmitted polarisation in such a way that this minimum value is maximised? We will know show that this optimisation criterion in a few simple cases is unequivocally defined, and we will give the results. Actually, we implicitly used this criterion already in section 2.1.1, though we there found it so natural that we did not mention it.

3.3.1 One Transmitted Polarisation

If we look upon the second expression in (3.23), we see that all terms are positive. Hence, the closer to zero they are, the more unlucky we are. If we transmit only one polarisation from our radiating source we may be unlucky to get,

$$|\varphi_t - \Phi_r| = \frac{\pi}{2} \ . \tag{3.30}$$

which is equally likely to happen independent of the choice of the transmitted polarisation from our radiating source. If (3.30) is "fulfilled",

$$p\Big|\Big[|\varphi_t - \Phi_r| = \frac{\pi}{2}\Big] = \sin^2(\frac{\Delta\tau_t + \Delta\tau_r}{2}) \ . \tag{3.31}$$

The minimum value of (3.31) is 0, and can happen for any choice of $\Delta\tau_t$, but is less likely to happen if $\Delta\tau_t = \pm\frac{\pi}{2}$. So if we do not know anything about the receiving polarisation of our EUT and we transmit only one polarisation, the optimal polarisation to chose is, given our chosen optimisation criterion, circular polarisation. The choice of left or right circular polarisation is arbitrarily.

3.3.2 Two Transmitted Polarisations

If we transmit two polarisations with different φ_t, we may choose this two φ_t orthogonal so that even if we are at most unlucky, we get,

$$|\varphi_t - \Phi_r| = \frac{\pi}{4} \ , \tag{3.32}$$

and the corresponding polarisation efficiency to (3.31) is,

$$\begin{aligned} p\Big|\Big[|\varphi_t - \Phi_r| = \frac{\pi}{4}\Big] &= \frac{1}{2} - \frac{1}{2}\cos(\Delta\tau_t + \Delta\tau_r) + \frac{1}{2}\cos(\Delta\tau_t)\cos(\Delta\tau_r) \\ &= \frac{1}{2} + \frac{1}{2}\sin(\Delta\tau_t)\sin(\Delta\tau_r) \ . \end{aligned} \tag{3.33}$$

The minimum value of (3.33), we maximise to $\frac{1}{2}$, by choosing $\Delta\tau_t = 0$, i.e. we transmit linear polarisation.

3.4 Optimal Polarisation - Linear Receiving Polarisation

As we in section 2.2 argued for that the assumption of linear receiving polarisation is relevant, we will derive the optimal transmitted polarisation for that case. Linear receiving polarisation implies that $\Delta T_r = 0$, and plugging that into the first expression for the polarisation efficiency in (3.23), we get,

$$p_{?l} = \frac{1}{2} + \cos(\Delta\tau_t) g(\varphi_t - \Phi_r) \ , \tag{3.34}$$

where,

$$g(\varphi_t - \Phi_r) = \cos^2(\varphi_t - \Phi_r) - \frac{1}{2} \ . \tag{3.35}$$

3.4.1 One Transmitted Polarisation

If we transmit only one polarisation from our radiating source, $g(\varphi_t - \Phi_r)$ may be smaller than 0, and hence the minimum possible value of $p_{?l}$ is maximised by letting $\Delta\tau_t = \pm\frac{\pi}{2}$. Hence the optimal choice is to transmit left or right circular polarised field. The value of the polarisation efficiency is deterministic $\frac{1}{2}$. The choice $\Delta\tau_t = \pm\frac{\pi}{2}$ is optimal, so there is no meaning in sending more polarisations and only change $\Delta\tau_t$.

3.4.2 Arbitrarily Number of Transmitted Polarisations

If we transmit $n \geq 2$ consecutive polarisations with n different values of φ_t,

$$\varphi_t \in \left\{ i\frac{\pi}{n} \right\}_1^n \ , \tag{3.36}$$

the minimum value of g is,

$$g_{min} = \cos^2(\frac{\pi}{2n}) - \frac{1}{2} \geq 0, \qquad n \geq 2 \ , \tag{3.37}$$

and it follows that the minimum possible value of $p_{?l}$ is maximised by letting $\Delta\tau_t = 0$, i.e it is optimal to send linear polarisations. The choice $\Delta\tau_t = 0$ is optimal, so there is no meaning in sending any more polarisation with a different $\Delta\tau_t$. Actually, we have now turned into the same case as the one already described in section 2.1.1.3.

To summarise: If the receiving polarisation of the Equipment Under Test is linear, and we want to maximise the polarisation efficiency for the worst case, we shall use circular polarisation, if we transmit only one polarisation, and we shall use linear polarisations with equidistant intermediate angles, if we transmit more than one consecutive polarisation.

Chapter 4

Unpolarised Transmitted Field

Until now we have treated the parameter Φ_r of the receiving polarisation of the EUT as a random variable[1]. In the general case, which we studied in chapter 3, we did also treat the parameter T_r of the receiving polarisation of the EUT as a random variable. The reason that we treated those parameters as random variables, is that we do not know their specific values and the best we can do is to treat them as random variables. We have also assumed that the polarisation of our radiating source is fix, and that we have total control over the parameters τ_t and φ_t. In this chapter we will do the other way around. The parameters of the receiving polarisation of the EUT are not fundamental quantum mechanical random variables, on the contrary, they do have specific values, though we do not know them. Without loss of generality we just denote them as τ_r and φ_r. We do also assume that the radiating source transmits unpolarised field, with its parameters being stochastic variables uniformly distributed on their domains,

$$\Delta T_t \in U[-\frac{\pi}{2}, \frac{\pi}{2}] \ , \tag{4.1}$$

and,

$$\Phi_t \in U[-\frac{\pi}{2}, \frac{\pi}{2}] \ . \tag{4.2}$$

Assuming ergodicity, so that the time average equals the ensemble average, we get with help of (3.23), that the polarisation efficiency is,

[1]Please notice that we uses large letters for random variables and small letters for deterministic variables.

$$
\begin{aligned}
p &= \mathbb{E}\left\{\frac{1}{2} - \frac{1}{2}\cos(\Delta T_t + \Delta\tau_r) + \cos(\Delta T_t)\cos(\Delta\tau_r)\cos^2(\Phi_t - \varphi_r)\right\} \\
&= \mathbb{E}\left\{\frac{1}{2} + \frac{1}{2}\sin(\Delta T_t)\sin(\Delta\tau_r) + \frac{1}{2}\cos(\Delta T_t)\cos(\Delta\tau_r)\cos(2\Phi_t - 2\varphi_r)\right\} \\
&= \frac{1}{2} + \frac{1}{2}\mathbb{E}\left\{\sin(\Delta T_t)\right\}\sin(\Delta\tau_r) + \frac{1}{2}\mathbb{E}\left\{\cos(\Delta T_t)\right\}\cos(\Delta\tau_r)\mathbb{E}\left\{\cos(2\Phi_t - 2\varphi_r)\right\} \\
&= \frac{1}{2}\,.
\end{aligned}
\tag{4.3}
$$

This is an interesting result, obvious for some: Transmit unpolarised field from your radiating source and we get the deterministic value one half for the polarisation efficiency. We get the same polarisation efficiency value independent of the receiving polarisation of the Equipment Under Test. That implies that, when we perform a Radiated Susceptibility Test with unpolarised transmitted field, we have deterministically undertested the Equipment Under Test with a factor of 2 compared to the polarisation which causes the largest stress onto our EUT[2].

In trying to use this favourable properties we may speculate in how to create a source which radiates unpolarised field. First we may notice that many naturally generated fields are unpolarised, e.g. sunlight. A conceivable way to generate unpolarised field is to have two incoherent sources which generate non-parallel fields. There are probably other solutions too, e.g. solutions including the use of nonlinear materials, but we will also need physical understanding and knowledge of how concepts like polarisation, coherence and bandwidth depends on each other.

[2]The factor is $\sqrt{2}$, if we talk field values, and not power values.

Chapter 5

Summary

We have developed a model describing the polarisation efficiency between the Equipment Under Test (EUT) and the electromagnetic field incident on it. Equation (3.23) is the key expression describing the polarisation efficiency. We have assumed that we have control of the field transmitted onto the EUT, and the parameters of the receiving polarisation of the EUT, ΔT_r and Φ_r in (3.23), we treat as random variables. We have so far not managed to develop a general distribution for the radiation efficiency in (3.23), but in chapter 2, we show the result for the special cases that we know that the transmitted polarisation and the receiving polarisation of the EUT, are linear or circular.

We do also show that, if the receiving polarisation of the EUT is linear, it is optimal to transmit circular polarisation if we transmit one polarisation, and optimal to transmit linear polarisations if we transmit two or more consecutive polarisations. Fig. 2.7 gives one possible answer to how many linear polarisations that is optimal to transmit. It shows that, at the trade-off between accuracy and affordability, two or three consecutive linear polarisations is optimal.

A new interesting idea is to not transmit a specific polarisation, but to transmit unpolarised field. That gives a deterministic value to the polarisation efficiency of one half.

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Appendix A

Polarisation Ellipse - The Orientation of the Major Axis and the Minor Axis

Equation (3.5) tells us that, the polarisation ellipse in Fig. 3.1, in the primed coordinate systems can be described as,

$$\hat{\boldsymbol{e}} = x'(\alpha)\hat{\boldsymbol{e}}_x^{'} + y'(\alpha, \Delta\tau)\hat{\boldsymbol{e}}_y^{'} \ , \tag{A.1}$$

where,

$$\begin{cases} x' = \cos(\alpha) \\ y' = \cos(\alpha + \Delta\tau) \end{cases} , \tag{A.2}$$

and,

$$\alpha = \omega t + \tau \ . \tag{A.3}$$

The absolute value of the polarisation,

$$|\hat{\boldsymbol{e}}| = \sqrt{x'^2 + y'^2} = \sqrt{\cos^2(\alpha) + \cos^2(\alpha + \Delta\tau)} \ , \tag{A.4}$$

is a function of α and to find its extreme values, we calculate the extreme values of,

$$f(\alpha) = \cos^2(\alpha) + \cos^2(\alpha + \Delta\tau) \ . \tag{A.5}$$

The derivatives of $f(\alpha)$ are,

$$f'(\alpha) = -\sin(2\alpha) - \sin(2\alpha + 2\Delta\tau) \ . \tag{A.6}$$

and,

$$f''(\alpha) = -2\left[\cos(2\alpha) + \cos(2\alpha + 2\Delta\tau)\right] \ . \tag{A.7}$$

The extreme values correspond to $f'(\alpha) = 0$, and there are four physically different solutions to that equation.

1 Left Circular Polarisation

The first solution is,

$$
\begin{aligned}
2\alpha + 2\Delta\tau &= 2\alpha - \pi \ , && \text{(A.8)}\\
\Delta\tau &= -\frac{\pi}{2} \ , && \text{(A.9)}
\end{aligned}
$$

which gives that,

$$\hat{\boldsymbol{e}} = \cos(\omega t + \tau)\hat{\boldsymbol{e}}_x^{'} + \sin(\omega t + \tau)\hat{\boldsymbol{e}}_y^{'} \ . \qquad \text{(A.10)}$$

The polarisation in (A.10) is left circular polarisation. The absolute value of circular polarisation is independent of the direction and hence is it is logical that the solution in (A.9) is independent of α.

2 Right Circular Polarisation

The second solution is,

$$
\begin{aligned}
2\alpha + 2\Delta\tau &= 2\alpha + \pi \ , && \text{(A.11)}\\
\Delta\tau &= \frac{\pi}{2} \ , && \text{(A.12)}
\end{aligned}
$$

which gives right circular polarisation,

$$\hat{\boldsymbol{e}} = \cos(\omega t + \tau)\hat{\boldsymbol{e}}_x^{'} - \sin(\omega t + \tau)\hat{\boldsymbol{e}}_y^{'} \ . \qquad \text{(A.13)}$$

3 Minor Axis

The third solution is,

$$
\begin{aligned}
2\alpha + 2\Delta\tau &= 2\pi - 2\alpha \ , && \text{(A.14)}\\
\alpha &= \frac{\pi}{2} - \frac{\Delta\tau}{2} \ , && \text{(A.15)}
\end{aligned}
$$

which gives that,

$$\hat{\boldsymbol{e}} = \sin(\frac{\Delta\tau}{2})\hat{\boldsymbol{e}}_x^{'} - \sin(\frac{\Delta\tau}{2})\hat{\boldsymbol{e}}_y^{'} \ , \qquad \text{(A.16)}$$

and hence for every $\Delta\tau$,

$$y' = -x' \ . \tag{A.17}$$

By plugging the solution (A.15) into (A.7), and taking (3.6) into account, we get that,

$$f''(\frac{\pi}{2} - \frac{\Delta\tau}{2}) = 4\cos(\Delta\tau) \geq 0 \ , \tag{A.18}$$

and hence we can conclude that (A.17) is the direction of the minor axis, and that it is independent of $\Delta\tau$. (The special cases that $\Delta\tau = \pm\frac{\pi}{2}$, we have already investigated above.)

4 Major Axis

The forth solution is,

$$2\alpha + 2\Delta\tau = -2\alpha \ , \tag{A.19}$$

$$\alpha = -\frac{\Delta\tau}{2} \ , \tag{A.20}$$

which gives that,

$$\hat{\boldsymbol{e}} = \cos(\frac{\Delta\tau}{2})\hat{\boldsymbol{e}}_x' - \cos(\frac{\Delta\tau}{2})\hat{\boldsymbol{e}}_y' \ , \tag{A.21}$$

and for every $\Delta\tau$,

$$y' = x' \ . \tag{A.22}$$

By plugging the solution (A.15) into (A.7), and taking (3.6) into account, we get,

$$f''(-\frac{\Delta\tau}{2}) = -4\cos(\Delta\tau) \leq 0 \ , \tag{A.23}$$

and hence we can conclude that (A.22) is the direction of the major axis, and that it is independent of $\Delta\tau$. (The special cases that $\Delta\tau = \pm\frac{\pi}{2}$, we have already investigated above.)

The result of all this four cases can be summarised in that the major and minor axes are always in the same direction at 45° angle to the primed coordinate axes of Fig. 3.1. That is true even for the degenerate case of circular polarisation, even though, the choice of directions there, of course, is arbitrarily.

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