An integral equation method for low frequency electromagnetic fields in three-dimensional marine environments

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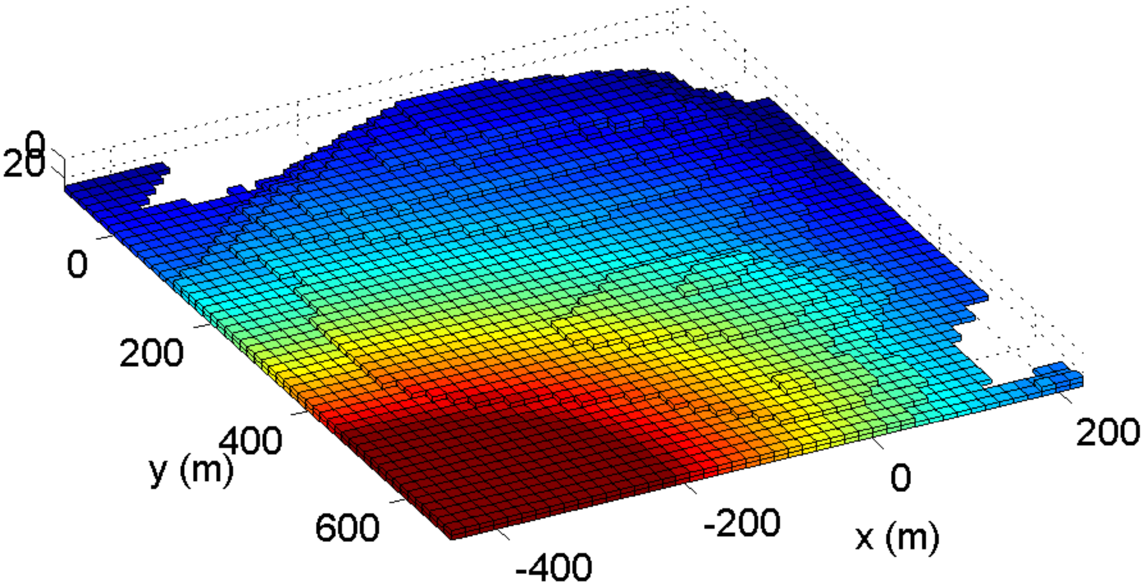

FOI-R-- 1838 --SE
ISSN 1650-1942

Systems Technology
Technical report

December 2005

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FOI-R--1838--SE
ISSN 1650-1942

Systems Technology
Technical report

December 2005

Issuing organization	Report number, ISRN	Report type
FOI – Swedish Defence Research Agency Systems Technology SE-164 90 Stockholm	FOI-R--1838--SE	Technical report
	Research area code 4. C4ISTAR	
	Month year December 2005	**Project no.** E60701
	Sub area code 43 Underwater Surveillance, Target acquistion and Reconnaissance	
	Sub area code 2	
Author/s (editor/s) Johan Mattsson	**Project manager** Robert Sigg	
	Approved by Monica Dahlén	
	Sponsoring agency Swedish Armed Forces	
	Scientifically and technically responsible Leif Abrahamsson	

Report title

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Abstract

Electromagnetic fields in three-dimensional marine environments are modelled in this paper. In particular, a volume integral equation method is used to calculate the electromagnetic fields from low frequency dipole sources in regions with varying bathymetry and sub-bottom structure. The horizontal variations are imbedded inhomogeneities in a horizontally stratified background model of the environment. A contracted (regularised) integral equation is derived and the resulting linear system of equations are preconditioned by the extended Born operator and solved by a conjugate-gradient algorithm. The solution to the extended Born approximation is chosen as the initial guess. Low frequency electromagnetic fields in conductive environments imply sparse system matrices, which makes this numerical method fast and memory efficient.

A numerical example demonstrates the convergence of the solution as a function of the cell size in the discretisation of the volume. The convergence speed of the iterative solver as well as the accuracy of the extended born approximation is shown. The example also demonstrates the effects on the electric field due to a range dependent bathymetry in a shallow marine environment.

Keywords

integral equation method, marine electromagnetics, three-dimensional

Further bibliographic information	Language English
ISSN 1650-1942	**Pages** 13 p.
	Price acc. to pricelist

Utgivare	Rapportnummer, ISRN	Klassificering
FOI - Totalförsvarets forskningsinstitut Systemteknik 164 90 Stockholm	FOI-R--1838--SE	Teknisk rapport
	Forskningsområde 4. Ledning, informationsteknik och sensorer	
	Månad, år December 2005	**Projektnummer** E60701
	Delområde 43 Undervattenssensorer	
	Delområde 2	
Författare/redaktör Johan Mattsson	**Projektledare** Robert Sigg	
	Godkänd av Monica Dahlén	
	Uppdragsgivare/kundbeteckning Försvarsmakten	
	Tekniskt och/eller vetenskapligt ansvarig Leif Abrahamsson	

Rapportens titel

En integralekvationsmetod för lågfrekventa elektromagnetiska fält i tredimensionella marina miljöer

Sammanfattning

I den här rapporten modelleras elektromagnetiska fält i tredimensionella marina miljöer. En integralekvationsmetod används för att beräkna lågfrekventa elektromagnetiska fält från dipolkällor i havsområden med varierande batymetri och bottenegenskaper. Bottenvariationerna modelleras som stora inhomogeniteter i en planskiktad miljömodell. En regulariserad integralekvation härleds där det, efter diskretisering, resulterande linjära ekvationssystemet förkonditioneras med den utvidgade Born operatorn. Systemet löses sedan medelst en konjugerad gradientmetod där den utvidgade Bornlösningen används som initialgissning. Lågfrekventa elektromagnetiska fält i elektriskt ledande miljöer medför att systemmatrisen blir gles vilket gör metoden snabb och minneseffektiv.

I ett numeriskt exempel demonstreras hur lösningen konvergerar som en funktion av cellstorleken i de diskretiserade volymerna. Konvergenshastigheten med avseende på antalet iterationer samt noggrannheten på den utvidgade Bornlösningen belyses också. Exemplet visar även effekten på det elektriska fältet från en uppstickande bergsplatå i en förövrigt plan botten med ett tjockt lager av lera ovanpå berggrunden.

Nyckelord

integralekvationsmetod, marin elektromagnetik, tre dimensioner

Övriga bibliografiska uppgifter	**Språk** Engelska
ISSN 1650-1942	**Antal sidor:** 13 s.
Distribution enligt missiv	**Pris: Enligt prislista**

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1. Introduction

The need for accurate numerical modelling in marine electromagnetics is increasing. Naval applications as well as hydrocarbon prospecting, where low frequency electromagnetics fields are used, are effectively designed and analysed by numerical computations taking into account the three-dimensional (3D) behaviour of the environment. Hence, it is important to develop fast and efficient computer codes for this purpose.

Many sub-bottom and bathymetry structures in marine electromagnetics can mathematically be modelled as a set of 3D inhomogeneities embedded in a horizontally stratified medium. This type of geometry is well suited for integral equation (IE) methods where the time consuming computational part is limited to the inhomogeneities [1-6]. The IE-method is inherently divided into several independent steps, making it ideal for parallelization.

Modern computer technology with lots of fast internal memory, allows for solving large problems where the inhomogeneities are discretised in tens of thousands of cells. However, a direct inversion of the coefficient matrix for the resulting linear system of equations is in practice impossible. Therefore, the feasible way to go is to apply iterative solvers like the CGMRES algorithm [7]. The convergence properties of several iterative solvers are investigated in [4]. It is shown that the convergence speed is significantly improved if the original integral equation is rewritten to a contracted form where the Green's operator has an L_2 - norm lesser than one.

The electric Green's function for a layered marine environment behaves very much like a Dirac-δ function due to the strong attenuation of the electric field, [5]. Hence, the extended Born approximation of the integral equation provides a fast and accurate solution. This is demonstrated in [5] where the extended Born solution is used as the initial guess to an iterative conjugate gradient algorithm. The extended Born approximation is also used as a preconditioner to the original integral equation.

The extended Born approximation is also applied in [6] where 3D bathymetry variations are studied with respect to the influence on the electric field compared to a plane sea-floor. The Green's functions are efficiently computed by the Nlayer code [8] in which adaptive complex wave number integration quadratures are used for the Sommerfelt integrals. The embedded inhomogeneities are discretised in cells where the electric field is assumed constant. This yields a fast and efficient code, called EMrad, particularly suitable for inhomogeneities lying in layers with conductivity values higher than 0.1 S/m.

The EMrad code is used in [9] for numerical modelling of measured electric field data generated from a controlled dipole source in a shallow marine environment. The effects on the electric field due to the non-planar bathymetry are studied.

In this paper, the IE method in the EMrad code is extended. A further enhanced IE method is proposed by combining the contracted integral equation suggested in [4] with the regularisation method outlined in [5]. The resulting preconditioned system of equations is then solved by the iterative CGMRES algorithm. A numerical example demonstrates the convergence of the electric field solution as a function of the cell size. A comparison of this solution with the extended Born solution is also made.

2. Problem formulation

Consider the marine environment model in figure 1. It consists of a horizontally stratified background where a local three-dimensional inhomogeneity V is embedded across several layers. The layers as well as the inhomogeneity are non-magnetic and the conductivities are denoted by σ_b in the background stratification and by $\sigma = \Delta\sigma + \sigma_b$ in the inhomogeneity. The electromagnetic field at $\mathbf{r}$ is generated by a dipole source in any of the layers with harmonic time dependence $e^{-i\omega t}$, i.e. the electromagnetic fields are calculated in the frequency domain.

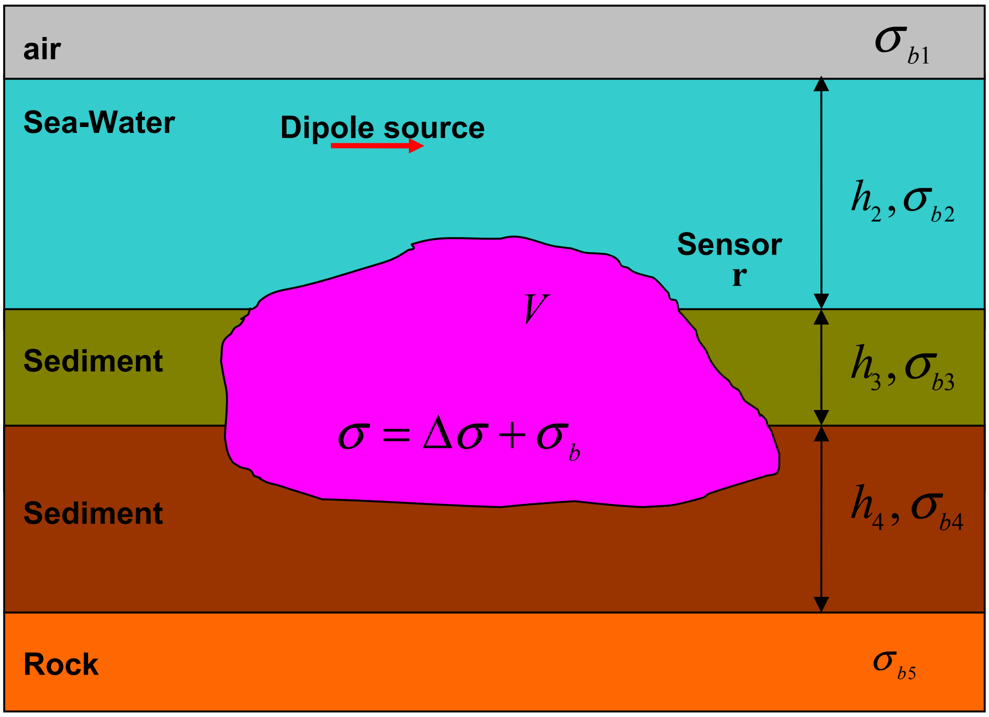

Figure 1. The three-dimensional model of the marine environment consisting of an inhomogeneity with arbitrary conductivity in a stratified background.

The electromagnetic fields in this model can be represented as the sum of the background and anomalous fields according to

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{\mathrm{b}}(\mathbf{r}) + \mathbf{E}^{\mathrm{a}}(\mathbf{r}), \quad \mathbf{B}(\mathbf{r}) = \mathbf{B}^{\mathrm{b}}(\mathbf{r}) + \mathbf{B}^{\mathrm{a}}(\mathbf{r}) \tag{1}$$

where the background fields $\mathbf{E}^{\mathrm{b}}(\mathbf{r})$ and $\mathbf{B}^{\mathrm{b}}(\mathbf{r})$ are generated by the dipole source in the layered distribution of conductivity σ_b. These fields are computed by the Nlayer code. The anomalous fields are produced by the conductivity change $\Delta\sigma$.

Assuming low frequencies, the following standard integral representations for the anomalous fields can be derived from the Maxwells equations, c.f. [10]:

$$\mathbf{E}^{a}(\mathbf{r}) = \int_{V} \Delta\sigma(\mathbf{r}')\,\mathbf{G}_{e}(\mathbf{r},\mathbf{r}')\cdot\mathbf{E}(\mathbf{r}')\,dV(\mathbf{r}') \tag{2}$$

$$\mathbf{B}^{a}(\mathbf{r}) = \mu_0 \int_{V} \Delta\sigma(\mathbf{r}')\,\mathbf{G}_{m}(\mathbf{r},\mathbf{r}')\cdot\mathbf{E}(\mathbf{r}')\,dV(\mathbf{r}')\,. \tag{3}$$

The effect from the surrounding environment on these fields is implicitly incorporated in the electric, $\mathbf{G}_e(\mathbf{r},\mathbf{r}')$, and magnetic, $\mathbf{G}_m(\mathbf{r},\mathbf{r}')$, Green's functions. They are mathematically tensors of the second order i.e. 3×3 matrices where the columns are normalised electric and magnetic fields, respectively, at the field point $\mathbf{r}$ from three mutually perpendicular electric dipoles at $\mathbf{r}'$. The first column corresponds to a dipole in the x-direction, while the second and third correspond to dipoles in the y- and z-directions, respectively. The free space magnetic permeability is denoted by μ_0.

The unknown electric field $\mathbf{E}(\mathbf{r}')$ within the volume V can be calculated from the integral equation obtained when the field point $\mathbf{r}$ is moved into the inhomogeneity V. Unfortunately, this integral equation has the L_2 - norm of its Green's operator lesser than one only for certain conductivity distributions, which means that an iterative solution of the resulting system of equations may not always converge. However, this problem is circumvented by utilising a so called contracted integral equation, which always has its norm lesser than one for any conductivity distribution. In this case, the contracted integral equation is obtained as, c.f. [4]:

$$\frac{\sqrt{\sigma_b(\mathbf{r})}}{a(\mathbf{r})}\widetilde{\mathbf{E}}(\mathbf{r}) = \sqrt{\sigma_b(\mathbf{r})}\,\mathbf{E}^{\mathrm{b}}(\mathbf{r}) + \sqrt{\sigma_b(\mathbf{r})}\int_{V}\frac{\Delta\sigma(\mathbf{r}')}{a(\mathbf{r}')}\,\mathbf{G}_{e}(\mathbf{r},\mathbf{r}')\cdot\widetilde{\mathbf{E}}(\mathbf{r}')\,dV(\mathbf{r}') \tag{4}$$

where

$$\widetilde{\mathbf{E}}(\mathbf{r}) = a(\mathbf{r})\mathbf{E}(\mathbf{r}), \quad a(\mathbf{r}) = \frac{2\sigma_b(\mathbf{r}) + \Delta\sigma(\mathbf{r})}{2\sqrt{\sigma_b(\mathbf{r})}}\,. \tag{5}$$

In environments where the conductivities σ_b are relatively high, the electric Green's function behaves very much like a Dirac-δ function due to the strong attenuation of the electric field. The fact that low frequencies are considered makes the electric Green's function even more like a Dirac-δ function. Hence, the integral equation (4) can be approximated as

$$\left[\frac{\mathbf{I}}{a(\mathbf{r})} + \mathbf{A}(\mathbf{r})\right]\widetilde{\mathbf{E}}(\mathbf{r}) = \mathbf{E}^{b}(\mathbf{r}) \tag{6}$$

where

$$\mathbf{A}(\mathbf{r}) = -\int_{V}\frac{\Delta\sigma(\mathbf{r}')}{a(\mathbf{r}')}\mathbf{G}_{e}(\mathbf{r},\mathbf{r}')\,dV(\mathbf{r}') \tag{7}$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$

Equation (6) is the extended Born approximation of the integral equation (4). It is easily solved as a 3×3 linear system for any field point $\mathbf{r}$ in the volume V. The solution is formally written as

$$\widetilde{\mathbf{E}}(\mathbf{r}) = \mathbf{P}(\mathbf{r}) \cdot \mathbf{E}^{\mathbf{b}}(\mathbf{r}), \quad \mathbf{P}(\mathbf{r}) = \left[\frac{\mathbf{I}}{a(\mathbf{r})} + \mathbf{A}(\mathbf{r}) \right]^{-1} . \tag{9}$$

The extended Born solution of the electric field in any point outside V is obtained from (1)-(3) by using the solution in (9). A more accurate solution can be computed by discretising (4) and solving the resulting linear system of equations iteratively.

3. Numerical solution of the integral equation

In order to solve the integral equation (4) numerically, the volume V is discretised into N rectangular cells with horizontal side length a and vertical side length c. The electric field and the conductivity in each individual cell are assumed constant. Their values are taken at the grid point at the centre of the cell. Hence, the following linear system of equations, with $3N$ unknown electric field values at the grid points, is obtained:

$$\sum_{j=1}^{N} \left(\frac{\sqrt{\sigma_b(\mathbf{r}_l^g)}}{a(\mathbf{r}_l^g)} \mathbf{I}\delta_{lj} - \frac{\sqrt{\sigma_b(\mathbf{r}_l^g)}\,\Delta\sigma(\mathbf{r}_j^g)}{a(\mathbf{r}_j^g)} \mathbf{G}(\mathbf{r}_l^g, \mathbf{r}_j^g) \right) \cdot \widetilde{\mathbf{E}}(\mathbf{r}_j^g) = \sqrt{\sigma_b(\mathbf{r}_l^g)}\mathbf{E}^b(\mathbf{r}_l^g) \tag{10}$$

where

$$\begin{aligned} &\mathbf{G}(\mathbf{r}_l^g, \mathbf{r}_j^g) = \begin{cases} \mathbf{G}_e(\mathbf{r}_l^g, \mathbf{r}_j^g) a^2 c, & l \neq j \\ \int_{V_j} \mathbf{G}_e(\mathbf{r}_j^g, \mathbf{r}') dV(\mathbf{r}'), & l = j \end{cases} \\ &\delta_{lj} = \begin{cases} 0, & l \neq j \\ 1, & l = j \end{cases} \qquad . \\ &l = 1, \ldots, N \end{aligned} \tag{11}$$

The integral of the electric Green's function over each cell is approximated as the value of the Green's function at the grid point times the volume of the cell when $l \neq j$. In the case when the source and the field points of the Green's function coincide, a principal value integral over the sub-volume V_j has to be performed due to the singular behaviour of the Green's function when the integration point $\mathbf{r}'$ approaches the grid point $\mathbf{r}_j^g$. Taking into account the symmetry and anti-symmetry of the integrand with respect to the grid point, an adaptive Gauss-Legendre quadrature can be used for a fast numerical convergence.

To improve the efficiency of solving (10) by the iterative CGMRES algorithm, the system of linear equations is multiplied by the extended Born matrix $\mathbf{P}(\mathbf{r}_l^g)$ from the left hand side. This operation reduces the condition number of the coefficient matrix. It also almost results in the solution of the unknown $\widetilde{\mathbf{E}}(\mathbf{r}_j^g)$ values when the extended Born solution is a good approximation of the true solution. Hence, the regularised version of (10) becomes:

$$\sum_{j=1}^{N} \mathbf{Q}(\mathbf{r}_l^g, \mathbf{r}_j^g) \cdot \widetilde{\mathbf{E}}(\mathbf{r}_j^g) = \mathbf{R}(\mathbf{r}_l^g), \quad l = 1, \dots, N \tag{12}$$

where

$$\mathbf{Q}(\mathbf{r}_l^g, \mathbf{r}_j^g) = \frac{\sqrt{\sigma_b(\mathbf{r}_l^g)}}{a(\mathbf{r}_l^g)} \mathbf{P}(\mathbf{r}_l^g) \cdot \mathbf{I}\delta_{lj} - \frac{\sqrt{\sigma_b(\mathbf{r}_l^g)}\, \Delta\sigma(\mathbf{r}_j^g)}{a(\mathbf{r}_j^g)} \mathbf{P}(\mathbf{r}_l^g) \cdot \mathbf{G}(\mathbf{r}_l^g, \mathbf{r}_j^g)$$

$$\mathbf{R}(\mathbf{r}_l^g) = \sqrt{\sigma_b(\mathbf{r}_l^g)}\mathbf{P}(\mathbf{r}_l^g) \cdot \mathbf{E}^b(\mathbf{r}_l^g)$$

The volume integrals $\mathbf{A}(\mathbf{r}_l^g)$ in $\mathbf{P}(\mathbf{r}_l^g)$ are discretised according to

$$\mathbf{A}(\mathbf{r}_l^g) = -\sum_{k=1}^{N} \frac{\Delta\sigma(\mathbf{r}_k^g)}{a(\mathbf{r}_k^g)} \mathbf{G}(\mathbf{r}_l^g, \mathbf{r}_k^g). \tag{13}$$

Some of the properties of the present IE-method are demonstrated below in the next section.

4. A numerical example

The environmental model for the numerical example is shown in figure 2. A $300 \times 300 \times 20$ m rectangular block of rock is standing on an infinite half-space of rock with a conductivity of 0.001 S/m. The block goes through a sediment layer of clay with a conductivity of 0.3 S/m and a thickness of 10 m. On top of that, there is a 40 m thick sea-water layer having a conductivity of 0.7 S/m. A horizontal electric dipole source in the x-direction is centred above the block at a height of 1000 km above the sea surface to model a vertically propagating plane wave of 3 Hz coming in from the ionosphere. The resulting electric field is calculated at points along a line parallel to the x-axis and starting from the centre of the block 23 m below the sea-surface. Hence, the incident field is polarised in the x-direction along this line.

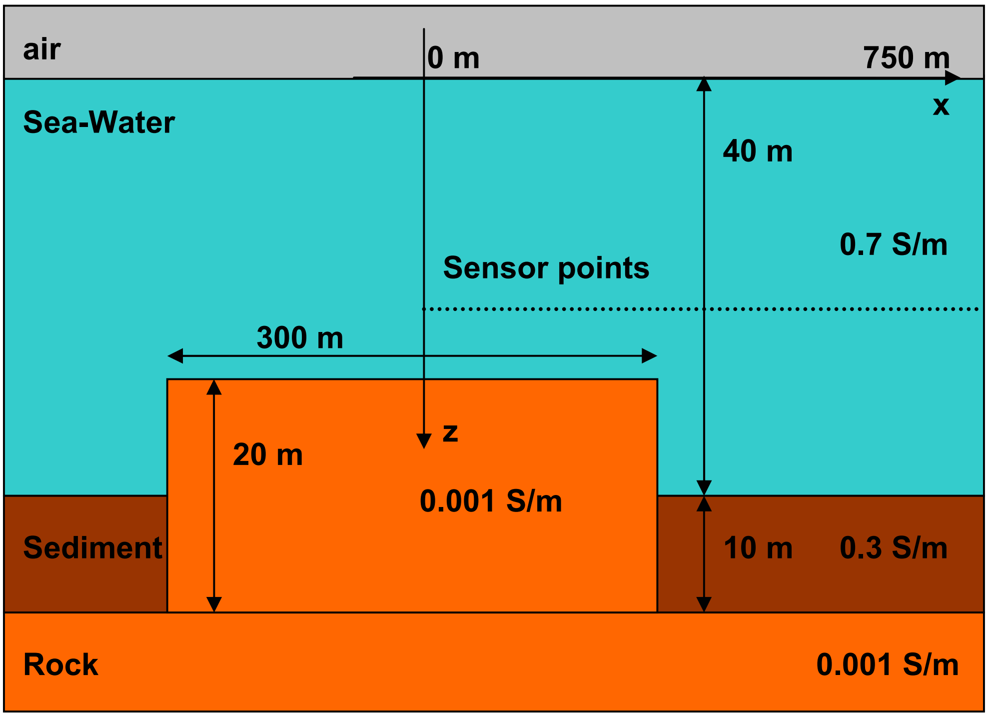

Figure 2. The geometry of the shallow water marine environment model.

The effect on the horizontal electric field component due to the 3D bathymetry and sub-bottom structure is shown in figures 3 and 4. The real and imaginary parts have been calculated with various discretisations of the inhomogeneity. As a reference,

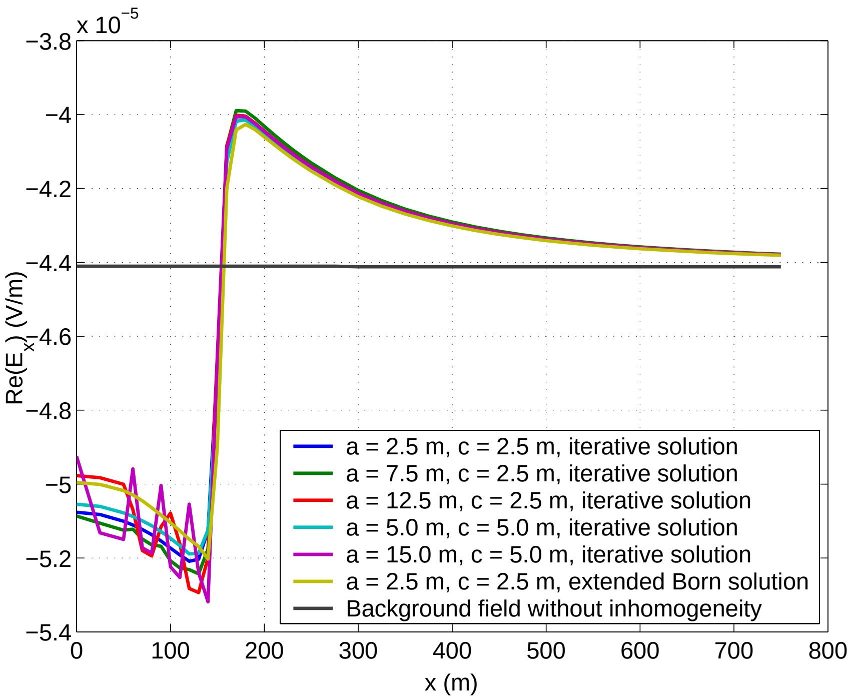

Figure 3. The real part of the horizontal electric field component along the sensor line.

the horizontal background electric field $\mathbf{E}^{b}(\mathbf{r})$ is plotted. The total field approached the values of this field in the limit where the field points go toward infinity along the sensor line.

It is clearly seen that the anomalous field converges slower with respect to the cell size when the field points are right above the inhomogeneity. The side lengths a and c of the cell need to be as short as 5 m in order to get a satisfactory solution at the field points having the x-coordinate between 0 and 150 m. Note that the distance to inhomogeneity is only 7 m in this region. Further out, i.e. when x>150 m, a good convergence is obtained with a coarser discretisation because of the rapidly increasing distance between the field points and the anomaly.

In this example, the finest discretisation has side lengths of 2.5 m. This yields 345600 unknown electric field components to solve for. A threshold value of 0.005 is used, which means that only those elements in the coefficient matrix exceeding values higher than 0.5 % of the highest diagonal value will be retained. The highest value in the coefficient matrix always occurs in the diagonal. Hence, the number of coefficient elements used in the system of linear equations is reduced to 22136924 in this computational example. In fact, the reduction makes it possible to run the EMrad code on an ordinary PC and still be able to solve such large problems.

For all the discretisations, the number of iterations in the CGMRES algorithm does not exceed 10. It goes from 7 in the coarsest discretisation up to 10 in the finest. The fast convergence is partly due to the fact that the extended Born solution is really good as seen in the figures 3 and 4. The L_2 - norm property of the contracted integral is also a necessary condition for this fast convergence.

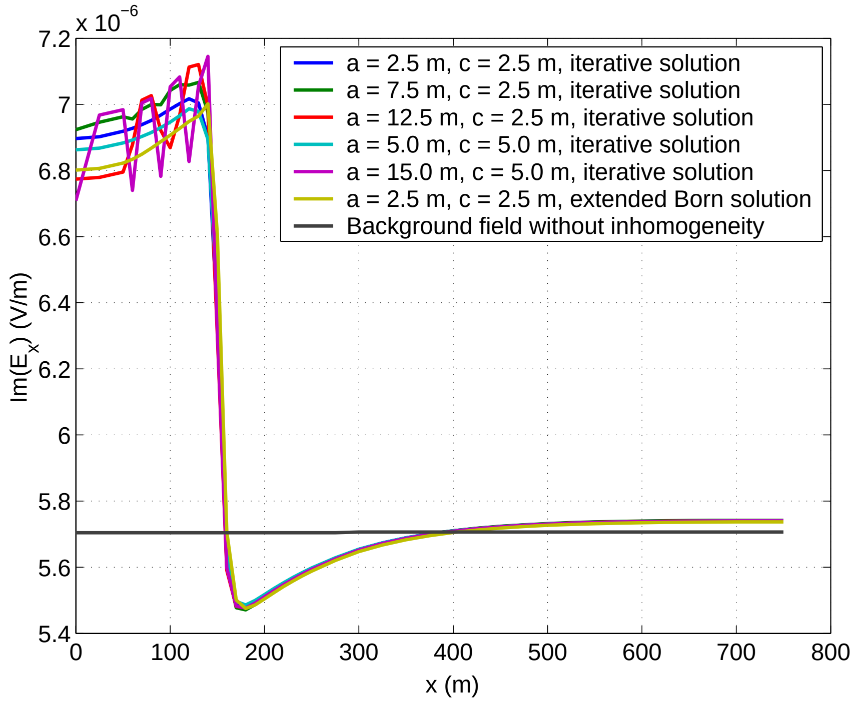

Figure 4. The imaginary part of the horizontal electric field component along the sensor line.

5. Conclusions

A fast regularised IE-method based on a contracted integral equation is developed in this paper. The extended Born approximation is used for the regularisation and its solution serves as the initial guess in a CGMRES algorithm. It is demonstrated in a numerical example that the anomaly electric field solution converges rapidly with respect to both the number of CGMRES iterations and the size of the cells in the discretisation. The distance between a field point and the inhomogeneity is critical for the appropriate choice of the cell size. It can be concluded that the side lengths a and c, should not be longer than this distance. A non-uniform discretisation, i.e. different cell sizes, is suitable for efficient and memory saving calculations. The implementation in the new EMrad code is adapted for that.

Finally, this new EMrad code using the above IE-method is currently being validated against the COMMEMI test examples reported on in [11]. The numerical example in [12] will also be used for validation in order to compare the EMrad code with the presented finite difference code. The validation results will be reported on next year.

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