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Survey of Positioning Algorithms for Surveillance UGVs

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FOI-R--2268--SE Scientific report
ISSN 1650-1942 April 2007

Defence and Security, Systems and Technology

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Defence and Security, Systems and Technology

Issuing organisation FOI – Swedish Defence Research Agency Defence and Security, Systems and Technology SE-164 90 STOCKHOLM	**Report number, ISRN** FOI-R--2268--SE	**Report type** Scientific report
	Research area code Strike and Protection	
	Month year April 2007	**Project no.** E28004
	Sub area code Ground Vehicles	
	Sub area code 2	
Author/s (editor/s) Ulrik Nilsson, Petter Ögren	**Project manager** Petter Ögren	
	Approved by Helena Bergman	
	Sponsoring agency FMV, 297316-LB704859	
	Scientifically and technically responsible David Anisi	
Report title Survey of Positioning Algorithms for Surveillance UGVs		

Abstract

This survey is focused on the so-called Camera Coverage Problem (CCP). The CCP is the problem of positioning a number of mobile surveillance cameras in an optimal fashion. The survey covers a number of papers in different fields of research. An overview of the field is provided, along with detailed accounts for problem formulations and proposed solutions in the different papers. The purpose of the survey is to give a comprehensive description of current research relevant to the CCP, as well as form a basis for future research.

Keywords

Autonomy, Cooperative, Surveillance, Path Planning, Task Assignment, Search, Coverage, Obstacle Avoidance, UGV

Further bibliographic information	**Language** English
ISSN ISSN-1650-1942	**Pages** 42 p.
	Price According to price list

Utgivare	Rapportnummer, ISRN	Klassificering
FOI – Totalförsvarets forskningsinstitut Försvars- och säkerhetssystem 164 90 STOCKHOLM	FOI-R--2268--SE	Vetenskaplig rapport
	Forskningsområde Bekämpning och skydd	
	Månad år April 2007	**Projektnummer** E28004
	Delområde Fordonsteknik	
	Delområde 2	
Författare/redaktör Ulrik Nilsson, Petter Ögren	**Projektledare** Petter Ögren	
	Godkänd av Helena Bergman	
	Uppdragsgivare/kundbeteckning FMV, 297316-LB704859	
	Tekniskt och/eller vetenskapligt ansvarig David Anisi	

Rapportens titel

Litteraturöversikt rörande autonom placering av spaningsrobotar

Sammanfattning

Denna rapport är en litteraturstudie kring positionering av kameror för övervakning av ett område. Studien täcker ett antal artiklar inom olika forskningsfält. En översikt över resultaten ges, samt detaljerade beskrivningar av problemställningar och föreslagna lösningar i de olika artiklarna. Litteraturstudiens syfte är att spegla forskningen i världen och att ligga till grund för framtida forskning.

Nyckelord

autonomi, samverkan, övervakning, banplanering, uppgiftstilldelning, spaning, hinderundvikande, UGV

Övriga bibliografiska uppgifter	**Språk** Engelska
ISSN ISSN-1650-1942	**Antal sidor:** 42 s.
Distribution **enligt missiv**	**Pris:** **Enligt prislista**

Contents

1 Introduction and Definition of Scope

In this chapter we will define the problem of interest and thereby the scope of the survey. We start by describing three motivating example scenarios and then formally define a problem class.

1.1 Scenario 1: Security Guard

Suddenly, the alarm goes off in a factory area that has been closed down for the night. A few minutes later a security officer arrives at the scene. The guard notices a broken window and concludes that this time the alarm was not false. Requesting backup, he activates the three UGVs at the back of his SUV and commands them to jointly survey all sides of the building complex except for the front, which is already in view from his vehicle. In less than a minute, all sides of the building complex is covered and an escaping intruder is bound to be noticed by the motion detection algorithms analyzing the image sequences from the UGVs.

1.2 Scenario 2: International Peace-Keeping and Detection of Hostile Snipers

During an international peace-keeping operation, a group of refugees must travel by foot through a part of town controlled by hostile warlords. The risk of snipers trying to take a shot at the refugees is assessed to be high. The group is escorted by international peace-keepers traveling in HMMWVs, but in order to deter any sniper attempt, the risk of detection and elimination must be substantial. The peace-keepers are equipped with surveillance UGVs that can monitor areas and buildings and alert operators in case of movements. By positioning the UGVs in key spots, right before the group passes, movements and or nozzle flares in buildings can be detected and acted upon. The geometry of both buildings and terrain must be taken into account to make sure the image sequences are useful. If good and sparse enough surveillance spots can be efficiently computed, and the UGVs are able to move faster than the group, a leap frogging movement of the UGVs can be used to keep the area around the moving refugees constantly monitored.

1.3 Scenario 3: Squad Under Fire

A recon squad is suddenly under fire, and is forced to take cover. In order to improve their situational awareness and reclaim the initiative, they command their two surveillance UGVs to find positions that cover as much as possible of the immediate surroundings in the direction of the threats.

1.4 Definition: The Camera Coverage Problem

The scope of this survey is defined and limited by the following problem statement:

> **The Camera Coverage Problem (CCP):** Given a map or an environment model and a user defined area of interest, composed of ground and/or buildings, the problem is to *find a number of camera locations* in such a way that the area of interest is covered in an optimal way.

We are interested in a wide class of variations of this problem in terms of camera model, environment model, and choice of objective function in the optimization. Having stated the problem we now list a number of possible variations.

1. Objective function: Optimal in what sense?
    - a) To cover as large area as possible.
    - b) To cover a given area with as few cameras as possible.
    - c) To cover a given set of walls with as few cameras as possible.
    - d) Other objective functions, such as a weighted sum. Such objective functions can be used to capture problems where some areas are more important than others, and some are good to have multiple views of.
2. Camera model: Field of view, range and image quality constraints
    - a) Omnidirectional
    - b) Range limitations
    - c) Field of view limitations
    - d) Other constraints, such as zoom, image quality or angle of incidence.
3. Environment model and types of occluding objects
    - a) 2 dimensional environments
    - b) 3 dimensional environments

The following chapter describes results found in the literature relevant to the above problem in different variations. After that, a set of reviews of the individual papers can be found in Chapter 3. Finally, conclusions are drawn in Chapter 4.

2 Survey of Literature Relevant to the Camera Coverage Problem

The base line problem studied in this survey is the Camera Coverage Problem (CCP), described in section 1.4 above.

To get a good overview of the work relevant to the CCP, the survey covers papers in many different research disciplines. Reviews of the individual papers can be found in Chapter 3, while this Chapter attempts to give a more unified view of the results found.

We divide this chapter into three sections dealing with different aspects of the CCP. First we describe the environment models used, then the different camera models, and finally we give a more detailed account for the different objective functions and the algorithms used to find good solutions.

2.1 Search Environments

In many papers, the area to be guarded has been considered to be either indoor, or outdoor in an urban environment. Common obstacles that restrict visibility in these environments are walls, pillars and other stationary objects with vertical extension. Hence, a natural simplification is to consider a two dimensional problem. However, as will be seen, there are also papers addressing the full 3 dimensional case. In this section we describe the different environment models used in more detail.

2.1.1 Polygons

One of the most straightforward search environment models used is the polygon. A polygon consist of a number of points (vertices) in a certain order together with line segments (edges) joining consecutive points. A simple polygon is a polygon that is not self-intersecting, see Figure 2.1. Thus, the inside and outside of a simple polygon is well defined. Since rooms and buildings

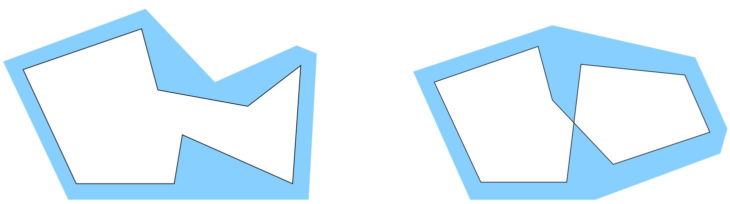

Figure 2.1: A simple and a complex polygon.

are modeled well by simple polygons, they are used in many of the surveyed papers.

2.1.2 Simple Polygons with Holes

Objects inside a polygon that limit the field of view, so-called holes, are also generally modeled as polygonal regions, [23, 18]. Polygons with holes are said to be *multiply connected* in contrast to polygons without holes that are *simply connected*, see Figure 2.2.

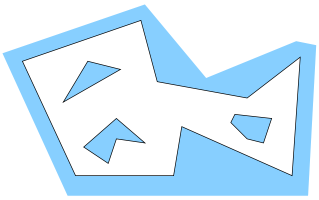

Figure 2.2: A polygon with holes, *i.e.* a multiply connected polygon.

Guarding problems on simple polygon environments with or without holes are studied in [7, 34, 23, 2, 18, 21, 37, 31, 11, 22, 25, 26]. More details on the problem statements and proposed algorithms can be found below.

2.1.3 Orthogonal Polygons with Holes

Most indoor and outdoor environments have an *orthogonal* structure with walls, pillars, display cases, and other obstacles, which motivates research on guarding *orthogonal* polygons with holes. An example can be seen in Figure 2.3.

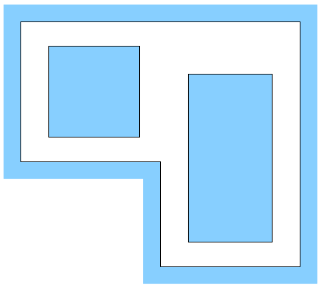

Figure 2.3: An orthogonal polygon with holes.

Guarding problems on orthogonal polygons are studied in [34, 24, 37, 31, 39].

2.1.4 Three Dimensional Terrain

A natural generalization from the 2 dimensional polygonal environment models is 3 dimensional terrain models, where a function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ represents the terrain surface $\sigma = \{(x, y, f(x, y)),\ (x, y) \in \Omega\}$ over some area of interest Ω, [8]. The standard way of storing data for these models is to use *digital elevation maps* (DEM), *i.e.* a stored set of sample points (x, y, z) on the terrain surface. If the sample points are regularly spaced and "dense enough" it is common to either represent f as piecewise constant function, or use bilinear interpolation. If on the other hand, the sample points are scattered, a triangulation of the points are often used in combination with a linear interpolation inside each triangle. Such models are called *Triangulated Irregular Networks* (TINs). The most commonly used triangulation is *Delaunay triangulation*, which is optimal in the following sense: it minimizes the maximal containing circle of any triangle, *i.e.* the created triangles are as close to equilateral as possible.

In cases where the environment can not be modeled by a level surface σ, triangulation is still used to capture the general 3D structure. In these cases however, line of sight calculations takes more time, [9].

Of the reviewed papers, guarding problems on 3 dimensional terrain are studied in [11, 37, 13, 14, 9, 8, 20, 19, 30].

2.2 Sensor Constraints

The guards in art gallery research have in many cases been considered ideal and without constraints. In this section we will discuss some natural constraint that make the camera model more realistic.

It should be noted however, that for some real world cases the unconstrained guard model works quite well, for instance for a human guard in a well illuminated gallery of moderate size. If the gallery is empty and quiet, the guard will use both vision and hearing and will perform more or less omnidirectional. Furthermore, some literature refer to *illumination problems* which are equivalent to guarding problems. In these, the guards guarding an area are replaced by light sources illuminating the area. Clearly, omnidirectional light sources are not hard to find.

2.2.1 Range Constraints

An important constraint is that of sensor range. Range constraint, both minimum and maximum, is typically used to achieve a certain quality of the acquired data, see [23]. The long range bound for a camera could be given by resolution constraints, while the close range bound could be set due to focusing limitations.

These ranges can furthermore vary to some extent for different environments. For example, the useful range of a standard camera would probably increase if the environment is well lit, while the performance of other sensors, such as an IR-camera, would be degraded by the excess of light.

Papers dealing with range constraints are: [23, 5, 20, 19].

2.2.2 Field of View

As opposed to range constraints, the field of view (FOV) of the sensor is generally well defined. Few sensors though, if any, are omnidirectional, but most papers ignore this limitation. Some exceptions are [36] were the *searchlight problem* was introduced. Here stationary flashlights with infinitesimal view angle (1-searchers), are used to search by rotating on the spot. Another example where view angle limitations were treated is [22], in which moving guards with ϕ radians view angle (ϕ-searchers) were used to clear an area. Further more, so-called Floodlight illumination problems were presented in [37]. A floodlight is a light source with limited angle of illumination and is equivalent to a ϕ-searcher in [22]. To summarize, FOV constraints are treated in: [23, 5, 37, 22, 35].

2.2.3 Image Quality, Angle of Incidence and Zoom-Cameras

In order for the images to be useful, for either a human operator or an automated detection and classification algorithm, the image quality must be good enough. One reasonable measure of this is the number of image pixels per meter of surveillance object (for 2D case). Such a constraint would imply both a maximal distance and a bound on the angle of incidence to the object, [5]. Similar constraints are used in [23], where angle of incidence bounds, together with range constraints is motivated by the use of a 3D image acquisition device. The importance of such a bound for the CCP is clear when for instance observing the wall of a building to look for possible hostile snipers. To be able to see in through the windows, a certain angle of incidence is required.

One interesting camera model, which to our knowledge is unstudied in this context, is the zoom camera. A zoom-camera can increase its range to the cost of reduced view angle. Without the image quality constraints, a zoom camera would always be used in the wide angle mode, to maximize the field of view, but with the constraint the zoom functionality becomes important.

Imagine a zoom camera of fixed position and orientation. Then due to the range/field of view tradeoff, any object facing the camera within the corridor in Figure 2.4 can be viewed with good enough image quality.

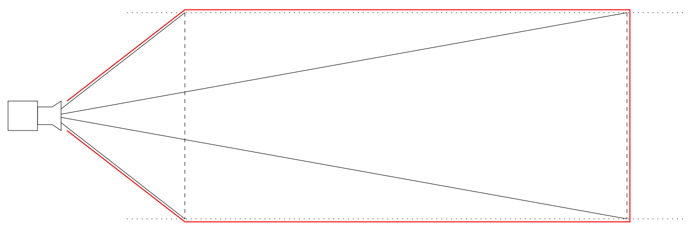

Figure 2.4: Zoom-Camera.

The exposed area in the Figure is perpendicular to the camera direction, and we say that the angle of incidence is zero. For non-zero angles the image quality will be reduced since the exposed length per camera pixel decreases with increased angle of incidence. These and similar constraints are treated in [5].

2.3 Problem Objectives

In this section we review the literature from the standpoint of what problem objective is optimized. The section is divided into four parts. After some background, the first two parts are concerned with finding the minimum number of guards needed to guard an area. In the first part, general upper bounds are presented for different problem types and sizes, and in the second part, algorithms to actually find the guard positions for a specific instance are studied. The third part concerns the reverse problem. How to cover as much area as possible with a fixed number of guards. Finally, the fourth part discusses other optimization objectives.

Area Partitionings and Guard Types

Before going into details of the different parts, we briefly discuss different partitionings of the area to be guarded, as illustrated in Figure 2.5. Polygon triangulation is a key tool in the art gallery theorem, [7] (see below) and an $\mathcal{O}(n)$ triangulation algorithm is presented in [4] and improved in [1]. One corresponding natural decomposition for orthogonal polygons is quadrilateralization [27] where the polygon is decomposed into convex 4-gons. Decomposition into star shaped regions is very useful and utilized in [24, 21]. We can see in

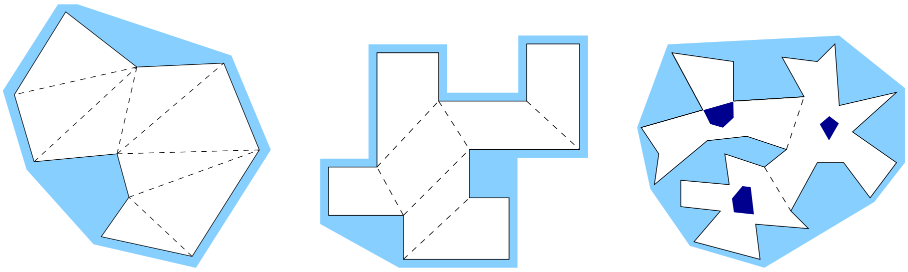

Figure 2.5: Triangulation, quadrilateralization and star decomposition (kernels in dark blue).

Figure 2.5 that none of these decompositions are unique. More on polygon decompositions can be found in [28].

Finally, we note that the literature contains a set of different guard types, namely vertex guards (guards that may only be positioned at vertices), edge guards (guards that may patrol along a single edge of the polygon) and point guards (guards that may be placed anywhere in the polygon), see Figure 2.6.

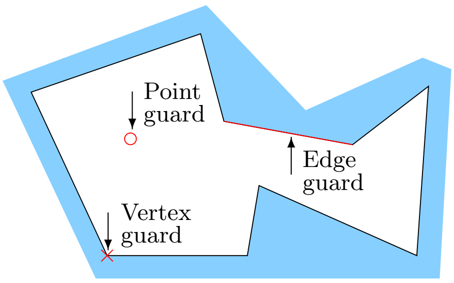

Figure 2.6: A polygon with a vertex guard, an edge guard and a point guard.

2.3.1 Finding an Upper Bound on the Number of Guards

The problems discussed in this section are all versions of the following problem.

Problem 2.1 (The General Art Gallery Problem) *What is the smallest number of guards needed to cover any polygon with n vertices and h holes.*

The classical art gallery problem was stated by V. Klee in 1973, and it concerned polygons without holes, $h = 0$. In 1975 Chvátal [7] presented a bound of $\lfloor n/3 \rfloor$ on the number of vertex guards that is always sufficient and sometimes necessary. Chvátal's theorem was later called *the art gallery theorem*. This art gallery theorem formed a foundation for further research, see e.g. [31, 34, 37].

For polygons containing h holes, [31] proved that $\lfloor (n+2h)/3 \rfloor$ vertex guards suffice. This bound was however not considered tight, and a common conjecture was that $\lfloor (n + h)/3 \rfloor$ vertex guards are sufficient. The latter bound was later proved for point guards in [3, 25], and Hoffmann *et al.* furthermore showed that the bound also holds for *vertex guards* for certain types of polygons.

In [24] the guarding problem for orthogonal polygons with holes are treated. It is proved that $\lfloor n/4 \rfloor$ point guards suffice for these polygons. The example in Figure 2.3 with $n = 14$ walls is taken from [24] and require 4 vertex guards but 3 ($\lfloor 14/4 \rfloor$) point guards to be completely covered. This polygon was used to disprove the old conjecture in [31], stating that $\lfloor 3n/11 \rfloor$ vertex guards does always suffice, and propose a new conjecture of $\lfloor 2n/7 \rfloor$.

2.3.2 Minimizing the Number of Guards

Even though many of the proofs above are constructive, and can be used to place guards in specific problem instances, other algorithms tailored to solving such instances have been proposed. They will be described in this section. First however, we follow Eidenbenz, [13], to formally define the problem

Problem 2.2 (Minimum point guard) *Let A be an area. The problem minimum point guard is the problem of finding a minimum set S of points on A such that every point on A is visible from a point in S. The points in S are called guard points.*

The area A can be either a polygon of some sort, or a 3 dimensional terrain.

It has been shown that the minimum guard problem is NP-hard [29] for vertex guards, edge guards and point guards. It is further proved that finding a set of guards whose cardinality is at most $1+\epsilon$ times the optimum is also NP-hard, [12].

A straight forward way to find a near optimal solution to the minimum guard problem is to use a greedy approximation algorithm [2]. From a guard candidate set a guard is chosen, one at a time, so that the covered area increment is as large as possible.

This solution approach is very common in the literature, and can be viewed as a transcription of the *minimum point guard* problem to the *minimum set cover* problem defined below

Problem 2.3 (Minimum set cover) *Let $E = \{e_1, \ldots, e_n\}$ be a finite set of elements, and let $S = \{s_1, \ldots, s_m\}$ be a collection of subsets of E,* i.e. $s_j \subseteq E$. *The problem* minimum set cover *is the problem of finding a minimum subset $S' \subseteq S$ such that every elements $e_i \in E$ belongs to at least one subset in S'. We say that E is covered by S'.*

If the subsets $s_1, \ldots, s_m$ all represent a candidate guard positions and the elements in s_j are the areas visible from guard j, then clearly, a solution to Problem 2.3 is also a solution to Problem 2.2.

Problem 2.3 is NP-hard, but a standard greedy solution is known to achieve an approximation ratio of $\mathcal{O}(log(n+1))$. The algorithm simply adds the set containing the maximum number of yet uncovered elements. Furthermore, these approximations are *optimal up to a constant factor*, due to theoretical results on the complexity of the guarding problems, [13].

One important issue is how to find the set of "good" guard positions to chose from, *i.e.* how do we choose S. In [15] the *minimum convex cover* problem was studied. A minimum convex cover of a set is obviously a reasonable set of tentative guard positions, S.

The approach described above is also applicable to the case when A in Problem 2.2 is a 3D terrain. Some approaches, such as [19], start out with a digital elevation map (DEM) of the terrain, while others use a triangulation. In [30], a set of triangulations with different resolutions in different places were computed.

In order to find "good" guard positions to chose from, [30] use an initial vertex coloring to find a smaller subset of the vertices of the triangulation. Finally a visibility computation is done to find the sets $s_j \subseteq E$ visible from each possible guard location.

In [19], an approximate visibility index, *i.e.* the fraction of the area within range that can actually be seen, is first computed for each point. Then the set of tentative observers are found by choosing points with high visibility index. Some extra measure is used to prevent the spots from clustering (the neighbor of a good spot is probably also good, but sees roughly the same area). The viewshed, *i.e.* $s_j \subseteq E$ is computed for each point and a version of problem 2.3 is solved with the standard greedy algorithm.

In [13], the tentative observers are found by partitioning the whole 3D-space using a huge set of planes, where each plane is found by intersecting an edge of the triangulation with an additional vertex of the triangulation. The point of this partitioning is that in each cell, all point see the same set of triangles. This approach is similar to the one used in [22] to find areas within which the number of visible vertices do not change.

In [10], problem 2.3 is solved by a randomized search instead of the greedy approach. In this way the set of tentative guards can be very large, and the sets $s_j \subseteq E$ are only calculated when needed. It would be interesting to see how these two methods compare in terms of performance.

In [20], some different ways to calculate $s_j \subseteq E$, *i.e.* the area visible to observer j, are discussed. Tools for calculating s_j in a general triangulated 3D environment can also be found in the well written surveys [9, 8].

2.3.3 Maximizing the Coverage

A closely related problem to the minimum guard problem is to maximize the guarded polygon area or boundary using a fixed number of guards. This problem also falls within the CCP, and is perhaps even more relevant to the motivating UGV application than the previous one. However, this formulation has not received as much attention.

In [6] the near optimal position of one guard is computed based on random sampling and ϵ-approximation. The similarities to shape matching are discussed. [18] look for guard locations that maximize the value of items on the boundary of a polygon with holes, and then optimize the positions of both the guards and the valued items in a multiple knapsack fashion. A natural extension is found in [16] where the visible interior area of a polygon is to be maximized given a limited number of guards. In both these examples the positions of the guards are computed one at a time so that each guard increase the still unguarded area or boundary as much as possible. A number of heuristics for locating guards are presented in [2]. The approach is aimed at covering entire polygons, but the greedy algorithms presented can be applied to the maximum cover problem as well. The construction of candidate guard sets is clever and it would be interesting to test a randomized candidate guard set as presented in [23] on the algorithms.

2.3.4 Other Objective Functions

In this section we will describe two approaches that incorporate more than just coverage into the objective function.

The motivation of the CCP is to provide information to an operator from a set of surveillance cameras. In a paper by Vazquez *et al.*, [38], inspired by the connection between inforamtion and entropy, a measure called *viewpoint entropy* is defined and an algorithm to maximize it is proposed. Viewpoint entropy is defined as

$$I(S,p) = -\Sigma_{i=0}^{n} \frac{A_i}{A_t} \log \frac{A_i}{A_t},$$

where A_i/A_t is the fraction of the camera image covered by surface i. Thus a maximization of $I(S,p)$ gives a viewpoint such that the image contains many different surfaces covering roughly equal sized parts of the image. As an example, the maximum entropy viewpoint of a cube would be on the diagonal line connecting two opposite corners. In the paper, the optimization is carried out using exhaustive search on a set of equally spaced candidate points.

Another approach is presented in [5], where an algorithm is proposed to compute viewpoints for robot arm mounted stereo cameras, performing inspection and production tasks. The authors propose an objective function incorporating the number of viewpoints, the size of the projected objects in the image, and the satisfaction of up to nine different constraints, including field of view, view angle, range and occlusion. A heuristic genetic algorithm is used to perform the optimization.

After having categorized and presented the research litterature in terms of environment model, camera model and finally objective function, we now turn to give more detailed accounts of the different papers.

3 Short Reviews of Individual Papers

This sections contains individual reviews of most of the papers surveyed. A list of the papers, with references and corresponding review section can be found in Table 3.2. The table also contains information on what problem areas the papers address. The problem area tags correspond to the list in Chapter 1, and are summarized in Table 3.1 below.

Table 3.1: The different paper categories.

Objective function: Optimal in what sense?	
1a	To cover as large area as possible.
1b	To cover a given area with as few cameras as possible.
1c	To cover a given set of walls with as few cameras as possible.
1d	Other objective functions.
Camera model: Field of view, range and image quality constraints	
2a	Omnidirectional
2b	Range limitations
2c	Field of view limitations
2d	Other constraints.
Environment model and types of occluding objects	
3a	2 dimensional environments
3b	3 dimensional environments

Table 3.2: List and classification of papers

Paper Title and Reference	Problem Areas
-Guarding Galleries and Terrains, [10], see Section 3.13	(3a,3b)
-Approximation Algorithms for Terrain Guarding, [13], see Section 3.11	(3b)
-Higher isn't Necessarily Better: Visibility Algorithms and Experiments, [20], see Section 3.10	(3b,2b)
-System to Place Observers on a Polyhedral Terrain in Polynomial Time, [30], see Section 3.7	(3b)
-Siting Observers on Terrain, [19], see Section 3.8	(2b,3b)
Continued on Next Page. . .	

Table 3.2 – Continued

Paper Title and Reference	Problem Areas
-A Randomized Art-Gallery Algorithm for Sensor Placement, [23], see Section 3.23	(1c,2b,2c,2d,3a)
-Locating Guards for Visibility Coverage of Polygons, [2], see Section 3.21	(1b,2a,3a)
-Maximizing the Guarded Interior of an Art Gallery, [16], see Section 3.20	(1a,1d,2a,3a)
-On Finding a Guard That Sees Most and a Shop That Sells Most, [6], see Section 3.22	(1b,2a,3a)
-Approximation Algorithms for Two Optimal Location Problems in Sensor Networks, [11], see Section 3.26	(1b,1d,2a,3a)
-Distributed Deployment of Asynchronous Guards in Art Galleries, [21], see Section 3.4	(3a)
-An Approximation Algorithm for Minimum Convex Cover with Logarithmic Performance Guarantee, [15], see Section 3.12	(1b,1a)
-Visibility-based Pursuit-evasion with Limited Field of View, [22], see Section 3.5	(2c,3a)
-Distributed Surveillance and Reconnaissance Using Multiple Autonomous ATVs: CyberScout, [32], see Section 3.6	(2c)
-Probabilistic Strategies for Pursuit in Cluttered Environments with Multiple Robots, [26], see Section 3.3	(2a,3a),
-Viewpoint selection using viewpoint entropy, [38], see Section 3.1	(1d, 2c,3b)
-Automatic sensor placement for model-based robot vision, [5], see Section 3.2	(1d,2c,2d,2b,3b)
-Inapproximability Results for Guarding Polygons without Holes, [12], see Section 3.25	(1c,2a)
-How to Place Efficiently Guards and Paintings in an Art Gallery, [18], see Section 3.19	(1c,1d,2a,3a)
-A Combinatorial Theorem in Plane Geometry, [7], see section 3.17	(1b,2a,3a)
-The Art Gallery Problem for Rectilinear Polygons With Holes, [24], see Section 3.24	(1b,2a,3a)
-Computational Complexity of Art Gallery Problems, [29], see Section 3.28	(1b,1c,2a)
-Orthogonal Art Galleries With Holes: A Coloring Proof of Aggarwal's Theorem, [39], see Section 3.29	(1b,3a)
-The Art Gallery Theorem for Polygons With Holes, [25], see Section 3.15	(3a)
-Allocating Vertex p-Guards in Simple Polygons via Pseudo-Triangulations, [35], see Section 3.16	(1b, 2c, 3a)
-Art Gallery Theorems and Algorithms, [31], see Section 3.30	(1b,2a,3a,3a)

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Table 3.2 – Continued

Paper Title and Reference	Problem Areas
-Art Gallery and Illumination Problems, [37], see Section 3.27	(1b,1c,2a,2c,3a,3b)
-Recent Results in Art Galleries, [34], see Section 3.18	(1b,1c,2a,3a)
-A Multidisciplinary Survey of Visibility, [9], see Section 3.14	(3b)
-Applications of Computational Geometry to Geographic Information Systems, [8], see Section 3.9	(3b)

3.1 Viewpoint Selection using Viewpoint Entropy, by Vazquez, Feixas, Sbert and Heidrich

This paper can be found in reference [38].

3.1.1 Problem Formulation

The problem studied is the one of automatically selecting the best viewpoint, given a geometric scene description. Furthermore, the problem of finding N different viewpoints that jointly cover a scene is studied.

3.1.2 Relation to the Camera Coverage Problem

Selecting viewpoints is the main theme of the CCP.

3.1.3 Proposed Solution Method and Mathematical Tools Used

The authors notes that in many cases, the best viewpoint of a scene is the one that gives the most information. They then go on to define *viewpoint entropy*, a measure of how much information is available at a given viewpoint, and finally choose a viewpoint that maximizes that measure.

The measure of a scene S and a viewpoint p is proposed as

$$I(S,p) = -\Sigma_{i=0}^{n} \frac{A_i}{A_t} \log \frac{A_i}{A_t},$$

where A_i is the projected image area of a surface and A_t is the total area of the projection sphere.

The name viewpoint entropy stems from the similarity to *Shannon entropy*, which is defined as follows.

The Shannon entropy of a discrete random variable X with values in a set $\{a_1, \ldots, a_n\}$ is $H(X) = -\Sigma_{i=1}^{n} p_i \log p_i$, where $p_i = \Pr(X = a_i)$. $H(X)$ thus represents the uncertainty of a random variable.

The formulas are identical if we interpret p_i as the probability that a random pixel in the image belongs to surface i, *i.e.* $p_i = \frac{A_i}{A_t}$. Furthermore, the summation from $i = 0$ is to account for the background.

The authors propose to use open GL graphics hardware to compute $I(S,p)$ by simply rendering each scene, with a color coding of each surface, and then counting the pixels of each color in the image. With this approach, a speed of 17-18fps is achieved.

Given the above objective function, the authors use exhaustive search to find the best viewpoint from a number of equally spaced candidates.

The problem of finding a set of N viewpoints is then studied. Using the same objective function, all candidate points are first analyzed in terms of viewpoint entropy as well as which faces they cover. An algorithm is proposed where viewpoints are added in the order of entropy, until a threshold, of say 90 %, of the faces are covered.

3.1.4 Personal Comments, Pros and Cons, assessment of paper quality

It's a well written paper presenting a novel approach to the viewpoint selection problem.

3.2 Automatic sensor placement for model-based robot vision, by Chen and Li

This paper can be found in reference [5].

3.2.1 Problem Formulation

The paper describes a sensor placement algorithm for automated assembly or product inspection. A set of sensor positions are computed and a shortest path through these is proposed.

3.2.2 Relation to the Camera Coverage Problem

Although the application area is automated production, the problem statement is very similar to the CCP.

3.2.3 Proposed Solution Method and Mathematical Tools Used

The proposed solution uses a two step method. First a set of viewpoints is computed, then a shortest path between then is found.

The objective function for the viewpoints is

$$f(G) = -aN - \frac{b}{\Sigma \frac{\omega_j}{l_j}} - \Sigma \delta_i \phi_i,$$

where $a, b, \delta_i \in \mathbb{R}$ are weights, N is the number of viewpoints, ω_j / l_j is the projected length divided by the real length of an object and $\phi_i \in \{0, 1\}$ is a binary variable indicating which constraints are satisfied.

The constraints accounted for are visibility, viewing angle, field of view, resolution constraint, viewing distance, overlap, occlusion, image contrast, kinematic reachability of sensor pose. Here, overlap concerns desired image overlap for computer vision applications and image contrast concerns focal length, pupil diameter etc.

The objective function above is minimized using a genetic algorithm, and shortest path through these points is found with an approximation algorithm developed by Christofides.

3.2.4 Personal Comments, Pros and Cons, assessment of paper quality

It's a well written paper that is easy to read.

An impressive number of constraints are treated, including some interesting camera model details.

3.3 Probabilistic Strategies for Pursuit in Cluttered Environments with Multiple Robots, by Hollinger, Kehagias and Singh

This paper can be found in reference [26].

3.3.1 Problem Formulation

The paper addresses pursuit-evasion problems in large indoor environments with multiple pursuers. The aim is not to guarantee capture, but to minimize the expected time to capture.

3.3.2 Relation to the Camera Coverage Problem

Although this paper focuses on moving guards, the modeling of the pursuers movements can be interesting.

3.3.3 Proposed Solution Method and Mathematical Tools Used

The search area is first *manually* discretized into disjoint convex regions, converting the problem to a graph search. The motion of the evader is then modeled using a Markov chain type of model, where the evader state $p(t)$ is a set of probabilities for being in each node (including a node for "already captured"). The time evolution is modeled as follows:

$$p(t+1) = p(t)P\Pi_i C_{X_i^P(t)},$$

where $p(t) = \{p_0, p_1, \ldots, p_n\}$ is the probability of the evader being in the different cells, P is the dispersion matrix reflecting the probability of moving from cell i to j (e.g. a normalized adjacency matrix), and $C_{X_i^P(t)}$ is the capture matrix, "moving" probability from the cell of a pursuer to the captured state. The motion of the pursuer is proposed to be greedy over a short (1-5 timesteps) time horizon. A series of candidate cost functions for the planning are proposed, starting with

$$C(x, p(t)) = 1 - p_x(t),$$

i.e. the cost of moving to cell x is the negative of the probability of the evader being there. Another option is

$$C(x, p(t)) = \frac{D(x, X_i^P(t))}{p_x(t)}$$

the distance to a cell $D(x, X_i^P(t))$ divided by the probability of the evader being there. This is the cost heuristics applied in [33]. The authors then go on to propose a Entropy type cost function

$$C_x(p(t)) = -\Sigma_n p_n(t) \log p_n(t),$$

where x denotes the implicit dependence of p on the pursuer positions x. The motivation for this choice is that in a low entropy situation you have a good idea of where the evader is. In order to enable planning for a number of steps, they let the cost of a planned path be

$$C(path) = \Sigma_x C_x(p(t)).$$

Given this path cost, they search all, say 5, step paths, using a search tree and breath first search over the different choices, and implement the best one. This planning is then reiterated each time step, in a receding horizon control fashion.

Both coupled (centralized) and decoupled planning is simulated and evaluated. The decoupled planning is done by assuming the all other pursuers remain stationary while the centralized plans for all pursuers. The simulations show that the entropy cost function performed best, in terms of average capture time, and that there was a small advantage of centralized one-step planning over decentralized one-step. However decentralized 5 step planning was better than both of the above.

3.3.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper is clearly written and the ideas are easily accessible, the paper further more contains a lot of interesting references.

3.4 Distributed deployment of asynchronous guards in art galleries, by Ganguli, Cortes and Bullo

This paper can be found in reference [21].

3.4.1 Problem Formulation

The problem addressed is the one of *distributing* the guards in an a priori *unknown* nonconvex polygon. The guards vision and communication capabilities are formalized and thoroughly treated.

3.4.2 Relation to the Camera Coverage Problem

Being a variation of the Art Gallery problem it is relevant. Perhaps not so much for the feedback exploration part, but more for the construction of a partitioning of the search area, and the corresponding data structure (the vertex induced tree).

3.4.3 Proposed Solution Method and Mathematical Tools Used

The authors propose a way to partition the polygon into a set of star shaped polygons $P_Q(s)_i$ and corresponding points $N_Q(s)$ such that for any $p_i \in N_Q(s)$, we have that $P_Q(s)_i \subset S(p_i)$, the region visible from p_i, *i.e.* p_i is in the *kernel* of $P_Q(s)_i$.

These nodes are then connected in a so-called *vertex induced tree.* Local algorithms are then proposed to move the guards between nodes of this tree, without knowing the whole search area, and thus not the whole tree. Thus the problem is transformed into a graph exploration problem.

A lot of care is taken to model the asynchronous communication part of the problem, an area that is not the focus of this survey.

3.4.4 Personal Comments, Pros and Cons, assessment of paper quality

It is a well written paper with formally proved results. The partitioning $P_Q(s)_i$ is perhaps not ideal for most other guarding applications.

3.5 Visibility-based pursuit-evasion with limited field of view, by Gerkey, Thrun and Gordon

This paper can be found in reference [22].

3.5.1 Problem Formulation

The problem addressed is *pursuit-evasion* with capture guarantee, using so-called ϕ-searchers, guards that have a field of view (FOV) limited by an angle ϕ.

3.5.2 Relation to the Camera Coverage Problem

Although the paper treats moving guards, as opposed to static ones, some of the concepts are relevant.

3.5.3 Proposed Solution Method and Mathematical Tools Used

The proposed approach builds on the following lemma:

Lemma 3.1 *Given a single ϕ-searcher in a polygonal free space F, there can be a change in the topology of the contaminated space in F* only if *there is a change in the set of vertices of F that lie in V.*

Above, V is the visible set at each time instant. Given the lemma, the trick is to create a cell-decomposition of the searchers configurations space ($\mathbb{R}^2 \times \mathbb{S}$) such that the topology of the contaminated space only changes on the boundaries. Using the lemma above they proceed in three steps to find a decomposition, first checking visibility, then checking ϕ-visibility (see below) and finally including the orientation in the decomposition.

Definiton 3.1 (ϕ-visible) *A pair of points p, q is ϕ-visible from some point s if and only if there exists a rotation θ such that both p and q lie within $V_\phi(s, \theta)$*

Definiton 3.2 (Visibility Curve) *Given two points v_1 and v_2 in the plane and a sensor field ϕ, consider the set of points p such that the pair (v_1, v_2) is ϕ-visible from p. This set includes its boundary, witch consists of circular arcs that connect v_1 and v_2 and is called the ϕ-visibility curve of v_1 and v_2, denoted $C_\phi(v_1, v_2)$.*

In this fashion, an *information graph* G_I, is created with $\mathcal{O}(n^2)$ nodes. Later, a graph search is performed on G_I to find the path of the ϕ-searcher.

3.5.4 Personal Comments, Pros and Cons, assessment of paper quality

It is a well written paper with interesting ideas.

3.6 Distributed surveillance and reconnaissance using multiple autonomous ATVs:CyberScout, by Saptharishi *et al.*

This paper can be found in reference [32].

3.6.1 Problem Formulation

The problem addressed is a general one on building a set of collaborating surveillance and reconnaissance UGVs. The paper focuses on both vision aspects, and path planning. In particular the problem of avoiding collisions with moving obstacles.

3.6.2 Relation to the Camera Coverage Problem

The relation is mainly in terms of hardware, and in the references to other similar projects, such as the SARGE, the MDARS-E, and the ARSKA, see references in [32].

3.6.3 Proposed Solution Method and Mathematical Tools Used

The main idea of the path planning is to identify possible conflict areas, and then prioritize the different moving entities within them to avoid collisions.

3.6.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper presents a very high level system description, with a lot of focus on the vision part of the problem.

3.7 A system to place observers on a polyhedral terrain in polynomial time, by Marengoni, Draper, Hanson and Sitaraman

This paper can be found in reference [30].

3.7.1 Problem Formulation

The problem addressed is the 3d-version of the Art Gallery Problem, *i.e.* given a 3 dimensional terrain, how many guards are needed to cover all of the terrain and where should they be placed.

3.7.2 Relation to the Camera Coverage Problem

The problem is very relevant to the guarding problem in 3 dimensions.

3.7.3 Proposed Solution Method and Mathematical Tools Used

The proposed solution uses three steps: First create a hierarchy of triangulations of the environment. Then use graph coloring to reduce the number of potential positions. Finally compute a visibility map and use *greedy set coverage* to find the best guarding positions.

The solution is an approximation, since the set covering algorithm is approximate, and it only guarantees coverage in the triangulation approximation of the environment, not the original digital elevation map (DEM).

The time complexity is $\mathcal{O}(n^3)$ in the number n of vertices of the triangulation.

3.7.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper is easy to read and presents the proposed approach in a clear way. It might be very useful.

3.8 Siting observers on terrain, by Franklin

This paper can be found in reference [19].

3.8.1 Problem Formulation

The problem treated is the 3d-version of the Art Gallery Problem, but on a digital elevation map (DEM), and not a triangulation, as in [30].

3.8.2 Relation to the Camera Coverage Problem

The problem is very much related to the guarding problem in 3d.

3.8.3 Proposed Solution Method and Mathematical Tools Used

The proposed solution is composed of four steps

1. Calculate an *approximate visibility index* of each point in the terrain, *i.e.* how large percentage of the area inside a circle of given radius R that is visible from the spot.

2. Select a number of tentative observers from the spots with high approximate visibility index. Some care must be taken to avoid having clustered point on *e.g.* a flat terrain segment.

3. Calculate the *viewshed* of each selected point, *i.e.* the area that is visible inside the R radius.

4. Given a list of viewsheds, find a quasi-minimal set that covers the whole area. This is done by greedily choosing the observer whose viewshed will increase the cumulative viewshed by the largest amount.

The result of the algorithm are as follows, first the area grows linearly with the number of observers, since R is smaller than the total region. then a total cover is slowly approached, in an example, 90 observers covered 98% and 180 observers covered 99.9% of the area.

The main focus of the paper was speed, not theoretical depth.

3.8.4 Personal Comments, Pros and Cons, assessment of paper quality

The proposed solution is very straightforward in concept, but perhaps more elaborate in the detail implementation of the different steps. The paper is well written.

3.9 Applications of Computational Geometry to Geographic Information Systems, by De Floriani, Magillo and Puppo

This paper can be found in reference [8].

3.9.1 Problem Formulation

The paper is a general survey of algorithms and problems related to Geographic Information Systems (GIS). As such, there is a chapter on Terrain Analysis, and a section on Visibility.

3.9.2 Relation to the Camera Coverage Problem

Computation of Visibility is important to the Guarding Problem.

3.9.3 Proposed Solution Method and Mathematical Tools Used

The basic tools of terrain representation and visibility computations are reviewed and/or referenced.

3.9.4 Personal Comments, Pros and Cons, assessment of paper quality

It's a well written survey.

3.10 Higher isn't Necessarily Better: Visibility Algorithms and Experiments, by Franklin and Ray

This paper can be found in reference [20].

3.10.1 Problem Formulation

The problem addressed is that of computing viewsheds from a Digital Elevation Model (DEM).

3.10.2 Relation to the Camera Coverage Problem

This is very relevant to the 3D-terrain guarding problem.

3.10.3 Proposed Solution Method and Mathematical Tools Used

A number of different viewshed computation algorithms are presented.

- Xdraw grows a line of sight (LOS)-ring from the observer outward while calculating what points are visible. It is approximate, but easy to implement and runs in $\mathcal{O}(r^2)$, where r is the radius of the circle being investigated.

- R3 is the classical algorithm, computing LOSs to each point of interest and checking if it is intersected by the terrain.

- R2 is an improved version of R3. The full LOS computation is done for the perimeter of the given range r. Then a computation is done along each LOS, as in the one-dimensional version of Xdraw above. R2 runs in $\mathcal{O}(r^2)$, but with a larger constant than Xdraw.

All three options are different tradeoffs on the accuracy/speed scale. It was noted that Xdraw is fast but rough, while R3 is exact. R2 is reported to be almost as good as R3, but much faster.

Another important point of the paper is that when a visibility index is to be calculated, *i.e.* how large percentage of the area within range that is actually visible. For a single position, a very good approximation can be found by sampling only a few rays. It was noted that 32 rays were almost as good as 128 in this respect. This is due to the law of large numbers.

3.10.4 Personal Comments, Pros and Cons, assessment of paper quality

It's a nice paper, with a compact description of some LOS algorithms for terrain occlusion.

3.11 Approximation algorithms for terrain guarding, by Eidenbenz

This paper can be found in reference [13].

3.11.1 Problem Formulation

The problem addressed is the three dimensional terrain guarding problem, formally defined as

Problem 3.1 (Minimum point guard on terrain) *Let T be a terrain. The problem* minimum point guard on terrain *is the problem of finding a minimum set of points S, on T, such that every point on T is visible from a point in S. The points in S are called* guard points.

The author goes on to define a *triangle restriction* to denote the case when triangles that are only partly visible from an observer are considered not visible by that observer.

3.11.2 Relation to the Camera Coverage Problem

This is one of the problems in the focus of the survey.

3.11.3 Proposed Solution Method and Mathematical Tools Used

The authors propose approximate solutions that first transform the guarding problems to the following problem.

Problem 3.2 (Minimum set cover) *Let $E = \{e_1, \ldots, e_n\}$ be a finite set of elements, and let $S = \{s_1, \ldots, s_m\}$ be a collection of subsets of E,* i.e. *$s_j \subseteq E$. The problem* minimum set cover *is the problem of finding a minimum subset $S' \subseteq S$ such that every elements $e_i \in E$ belongs to at least one subset in S'.*

It is then noted that a standard greedy solution to this problem is known to achieve approximation ratio of $\mathcal{O}(log(n+1))$. The algorithm simply adds the set containing the maximum number of yet uncovered elements.

Furthermore, these approximations are *optimal up to a constant factor*, due to theoretical results on the complexity of the guarding problems.

In detail, a partition of 3d-space is found from all planes intersecting an edge (v_i, v_j) and some other vertex v_k from the triangulation. A minimum set cover problem is then created by letting the elements in E be the triangles, and the sets in S the visible triangles from each cell of the partition.

3.11.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper is very well written and presents powerful and clear results.

3.12 An approximation algorithm for minimum convex cover with logarithmic performance guarantee, by Eidenbenz and Widmayer

This paper can be found in reference [15].

3.12.1 Problem Formulation

The problem studied is the *Minimum convex cover* problem, where a polygon with or without holes is to be covered by a collection of convex sets. This collection is furthermore to be as small, in cardinality, as possible.

3.12.2 Relation to the Camera Coverage Problem

The Minimum convex cover problem is very relevant to the guarding problem since once a convex cover is found, stationing a guard in each convex set guarantees that the whole polygon is guarded.

3.12.3 Proposed Solution Method and Mathematical Tools Used

The proposed solution uses something called a *quasi-grid*. By that is meant a set constructed by intersecting lines through all pairs of vertices of the polygon. The authors then study the *restricted minimum convex cover* problem, where all relevant points are restricted to lie on the quasi-grid.

A scheme using dynamic programming is then used to solve the *restricted minimum convex cover*, which is shown to differ by less than a factor 3 from the original *minimum convex cover* problem.

3.12.4 Personal Comments, Pros and Cons, assessment of paper quality

It's a well written paper.

3.13 Guarding galleries and terrains, by Efrat and Har-Peled

This paper can be found in reference [10].

3.13.1 Problem Formulation

The paper studies approximation algorithms for the guarding problem of polygons with or without holes, as well as for terrains. The guards are restricted to lie on the vertices of an arbitrarily dense grid, *i.e.* an approximation to no restriction at all.

3.13.2 Relation to the Camera Coverage Problem

It's very relevant.

3.13.3 Proposed Solution Method and Mathematical Tools Used

Instead of explicitly solving a minimum set cover problem, the following algorithm is proposed to decide if a cover of $k \log k$ guards exists. The algorithm is then called iteratively to find the minimum number of guards.

- Assign weight 1 to each element in V, the set of vertices of P

- For $i := 1$ to $\mathcal{O}(k \log(n/k))$ do:
  - Randomly pick as set S of $\mathcal{O}(k \log k)$ vertices, according to the weights.
  - Check if S solves the problem, if so return S and terminate.
  - Else, find $q \in P$ not visible from S and compute $Vis(q)$
  - If the sum of weights in $Vis(q) \cap V$ times $2k$ is smaller that the sum of all all weights in P, then double the weights in $Vis(q) \cap V$.
- Failure, no set of guards were found.

It is proposed to compute $Vis(q)$ using a standard line-sweeping procedure.

3.13.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper is very thorough, and all results are formally argued for. However, some terms, such as *arrangement* and *segment*, are not defined.

3.14 A multidisciplinary survey of visibility, by Durand

This paper can be found in reference [9].

3.14.1 Problem Formulation

The paper is a 146 page survey on visibility, taken from a PhD-thesis. Thus a wide range of problems are treated.

3.14.2 Relation to the Camera Coverage Problem

Since visibility is a central part of the guarding problem, the survey is important, in particular algorithms used to calculate visibility, such as the sub-routine *hidden part removal*, are of interest.

3.14.3 Proposed Solution Method and Mathematical Tools Used

Many methods and tools are presented.

3.14.4 Personal Comments, Pros and Cons, assessment of paper quality

The survey is a delight to read with many illustrative high quality figures.

3.15 The art gallery theorem for polygons with holes, by Hoffmann, Kaufmann and Kriegel

This paper can be found in reference [25].

3.15.1 Problem Formulation

The problem addressed and solved is that of placing point guards in a polygon with holes. The authors prove that any polygon, possibly with holes, can be guarded by at most $\lfloor \frac{n+h}{3} \rfloor$ point guards.

3.15.2 Relation to the Camera Coverage Problem

It's very relevant for urban or indoor areas.

3.15.3 Proposed Solution Method and Mathematical Tools Used

The proposed solution is inspired by earlier work on rectilinear polygons, where a set of rooms are identified and guards stationed in the doorways (intersections) of the rooms.

In this paper, a visibility structure is created by edge prolongation, similarly to [13], instead of a triangulation.

The authors note that there is in principal only 3 types of local configurations. This observation leads to a way to transform any polygon with holes into *standard form*, for which a guarding can be found. The standard form polygons are unions of standard form convex regions, so-called "rooms". These rooms are then connected to other rooms in a way captured by a *hypergraph*, a graph where the edges can connect more than two vertices.

3.15.4 Personal Comments, Pros and Cons, assessment of paper quality

It is a very interesting idea that seem somewhat cumbersome to implement.

3.16 Allocating vertex π-guards in Simple polygons via pseudo-triangulation, by Speckmann and Csaba

This paper can be found in reference [35].

3.16.1 Problem Formulation

The problem addressed is the one of finding a set of vertex guards for a polygon, but with the added constraint that the field of view is only 180 degrees (or π rad.).

3.16.2 Relation to the Camera Coverage Problem

It is very relevant. The π-constraint is unusual and interesting.

3.16.3 Proposed Solution Method and Mathematical Tools Used

The proposed solution first computes a minimum so-called *pseudo triangulation* of the polygon. A pseudo triangle is a polygon with exactly three convex vertices. Given the pseudo triangulation the guards are then allocated using the *dual graph* of this triangulation.

The main result is the following: any simple polygon with n vertices can be monitored by at most $\lfloor n/2 \rfloor$ general vertex π-guards. This bound is tight up to an additive constant of 1.

3.16.4 Personal Comments, Pros and Cons, assessment of paper quality

It's a well written paper.

3.17 A Combinatorial Theorem in Plane Geometry, by V. Chvátal

This paper can be found in reference [7].

3.17.1 Problem Formulation

The problem is to find the smallest number of guards in a polygon with n edges such that every point of the polygon is visible from at least one guard. The problem was stated by Victor Klee in 1973 and has later been called *The Art Gallery Problem.*

3.17.2 Relation to the Camera Coverage Problem

The problem is extremely relevant and Chvátal's solution and proof is a corner stone in the research area and is called *the Art Gallery Theorem* or *Watchman Theorem.*

3.17.3 Proposed Solution Method and Mathematical Tools Used

The polygon is first partitioned into triangles. A *fan* is defined as a specific triangulation where one vertex meets all of its inner edges. The proof is based on induction from the trivial cases with $n = 3, 4, 5$, for which all triangulations are fans. One major key to Chvátal's proof was the insight that for all triangulations of a polygon, there always exist a diagonal that cuts off 4,5 or 6 edges of the polygon.

3.17.4 Personal Comments, Pros and Cons, assessment of paper quality

The solution is limited to placing guards on the vertices of a polygon. The actual locations of the guards is not generally unique and not specified by the theorem, only a bound on the number of guards which is always sufficient and sometimes necessary. Later, Steve Fisk came up with a simpler proof using a 3-color argument [17]. Although Fisk's proof is easier to grasp, Chvátal's proof can be generalized in greater extent, see [31] who also give a thorough explanation on Chvátal's proof.

3.18 Recent Results in Art Galleries, by T.C. Shermer

This paper can be found in reference [34].

3.18.1 Problem Formulation

The paper attempts to collect results up to 1992. No particular problem is formulated and solved, but interesting problems (unsolved at that time) are presented briefly. Specific topics looked into concern different types of guards, covering the inside (art gallery problem), the outside (fortress problem) and both inside and outside (prison yard problem) of polygons, visibility graphs and problems closely related to the art gallery problem.

3.18.2 Relation to the Camera Coverage Problem

The paper is relevant to our problem, although much has happened in the research area over the past 15 years since the paper was written.

3.18.3 Proposed Solution Method and Mathematical Tools Used

The solutions to the various problems are generally not presented at all.

3.18.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper gives a good introduction to the art gallery problem by presenting necessary definitions and explaining terms, but does not dig very deep into the different subtopics. Various results are presented but not proved and selected problems, solutions and exceptions are illustrated.

3.19 How to Place Efficiently Guards and Paintings in an Art Gallery, by c. Fragoudakis, E. Markou and S. Zachos

This paper can be found in reference [18]. Further work by partly the same authors are found in [16].

3.19.1 Problem Formulation

The problem addressed is how to place a given number of guards and paintings (each with a length and a value) in an art gallery so that the total value of the guarded paintings is as large as possible. The problem is investigated for vertex and edge guards, for polygons with or without holes and for cases when the paintings must be overseen (every point of the paintings are seen) or just watched (at least one point of the painting is seen). All cases are proven to be NP-hard

3.19.2 Relation to the Camera Coverage Problem

The problem is very relevant to the CCP, both due to the limited number of guards and for watching valued items. The valued items can in our case be potentially dangerous spots in outdoor urban environment, such as windows and door ways where snipers might appear.

3.19.3 Proposed Solution Method and Mathematical Tools Used

A boundary partition that the authors call *Finest Visibility Segmentation* (FVS) is introduced. It is a discretization based on visibility of the boundary from vertex or edge guards.

An algorithm is proposed where the FVS points are computed. These are points on the boundary edges where two visibility segments join, which include the polygon vertices. One guard at a FVS point is determined at a time in a greedy fashion based on the multiple knapsack problem. The algorithm runs in polynomial time.

It is unclear from the paper if the FVS points, or just the polygon vertices which are a subset of the FVS points, are guard candidates. But if only the polygon vertices are guard candidates, much of the idea by the FVS seem to be a waste.

A similar algorithm for edge guards is presented and it is shown that both algorithms achieves an approximation of at most *1.58 times* the optimal. The same approximation result hold for polygons with holes.

3.19.4 Personal Comments, Pros and Cons, assessment of paper quality

This is a nice paper presenting an interesting problem and a neat solution. The only unclear part is if the FVS points are guard candidates or not. It would be interesting to compare the vertex guard algorithm with respect to computation time and result quality using the FVS points and using the polygon vertices as guard candidates.

3.20 Maximizing the Guarded Interior of an Art Gallery, by I.Z. Emiris, C. Fragoudakis and E. Markou

This paper can be found in reference [16]. It is recommended to read [18] first.

3.20.1 Problem Formulation

The problem investigated is how to place a fixed number of guards on the vertices or the edges of a simple polygon so that the total guarded area inside the polygon is maximized. This optimization problem proves to be APX-hard. The problem is also extended to the case where the guards need to see valued items inside the polygon.

A few interesting open problems are presented in the end of the paper.

Two open problems are presented in the end of the paper which are natural further work. The first step is to simultaneously determine the locations of valued subpolygons inside the polygon as well as vertex or edge guards so that a maximum value is guarded (see similarities to [18]). The next step would be to place point guards in the interior of the polygon for the above problems.

3.20.2 Relation to the Camera Coverage Problem

The problem of having a limited number of guard is interesting and relevant. In practice, there are usually a limited number of guards available.

3.20.3 Proposed Solution Method and Mathematical Tools Used

The authors introduce *the Finest Visibility Subdivision* (FVS) which is a descritization with respect to visibility of the interior of a polygon. They show that each region of the descritization can not be only partly visible from a vertex or an edge. An approximation algorithm that run in polynomial time is proposed and showed to have a constant approximation ratio. FVS regions are computed and the guards are placed one at a time so that the area coverage increase is maximized for each guard. The algorithm is applicable to polygons with holes as well.

3.20.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper is rather short and concise, but well written and hence not to difficult to grasp. It is recommended, though, to read [18] first where a similar problem is studied by partly the same authors. Here both the guards and valued items are to be placed on the boundary of the polygon. They introduce and use the finest visibility segmentation, see Section 3.19. The finest visibility subdivision is a straight forward extension of the finest visibility segmentation.

3.21 Locating Guards for Visibility Coverage of Polygons, by Y. Amit, J.S.B. Mitchell and E. Packer

This paper can be found in reference [2].

3.21.1 Problem Formulation

The authors investigates the art gallery problem from an experimental point of view. A number of heuristics for visibility coverage is presented and evaluated experimentally. The results are compared to obtain the best heuristics with respect to the resulting number of guards, computation time and memory space.

3.21.2 Relation to the Camera Coverage Problem

The paper has much relevance and a few heuristics seem to be very useful in practice. Different approaches, although some are quite similar, are compared to each other. One of the most interesting parts is the construction of visibility extensions and the resulting guard candidate points.

3.21.3 Proposed Solution Method and Mathematical Tools Used

Thirteen of the fourteen heuristics or algorithms choose guards from a candidate set built up by vertex guards and/or guards placed in the mass center of the convex polygons obtained by constructing edge extensions or visibility extensions. A guard is chosen based on a score or randomly from the candidates which can have different weights. In the fourteenth algorithm the polygon is partitioned into star shaped pieces and the guards are placed in the kernel of each piece. When the area is covered by any of the fourteen algorithms, any redundant guards are removed.

A lower bound on the optimal number of guards is presented based on visibility-independent witness points. Some investigations are performed for a few algorithms were fewer guards are available than stated by the lower optimal bound.

3.21.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper present some interesting heuristics for visibility coverage in a practical fashion. The construction of visibility extensions for creating a candidate guard set is interesting. This construction could probably be useful in the algorithm proposed in [23] where the candidate guard positions are obtained by random.

The Figures presenting the number of guards and computational time seem to be based on only one polygon for each value of vertices, and it is not clear what shape the polygons has on which the algorithms are tested. Some of the figures are hard to read and interpret, but besides that, the paper is fairly well written.

The three algorithms chosen to yield best results for reasonable effort could be interesting for further study and investigation from a more theoretical point of view.

3.22 On Finding a Guard that Sees Most and a Shop that Sells Most, by O. Cheong, A. Efrat and S. Har-Peled

This paper can be found in reference [6].

3.22.1 Problem Formulation

Two problems are treated. The first is to find a point in a polygon where the covered area is the largest (a guard that sees most). The other is to find a point such that the Voronoi region of that point is as large as possible (a shop that maximizes its costumer area).

3.22.2 Relation to the Camera Coverage Problem

Especially the first of the two problems is relevant, where a point in a polygon is to be determined from where the visible area is maximized.

3.22.3 Proposed Solution Method and Mathematical Tools Used

The interior of the polygon is sampled uniformly and the area cover from a point is estimated from how many sampled points are visible

The paper point out the similarity to the problem of matching two planar shapes.

3.22.4 Personal Comments, Pros and Cons, assessment of paper quality

The proposed approach is quite technical and not well suited for implementation.

3.23 A Randomized Art-Gallery Algorithm for Sensor Placement, by H. González-Banos and J. Latombe

This paper can be found in reference [23].

3.23.1 Problem Formulation

The paper describes a placement strategy of guards where the visual sensing will be most effective. Given a polygonal map, possibly with holes, the task is to compute locations in the interior of the polygon where expensive 3D image acquisition can be performed, these are preferably as few as possible. The algorithm takes some limitations of physical sensors into account, like constraints on range and incidence angle to walls. The authors call this an extended art gallery problem considering the sensor constraints. Examples are given on polygons which cannot be completely covered by the proposed solution since some cases require an infinite number of guards.

3.23.2 Relation to the Camera Coverage Problem

The problem and proposed solution is very relevant, although only the edges of polygons are considered to be guarded. The consideration of sensor constraints is especially appealing.

3.23.3 Proposed Solution Method and Mathematical Tools Used

The interior of the polygon is sampled at random to construct a candidate set. For every candidate the portions of the edges visible are computed, and for all candidates this results in a decomposition of the boundary and an additional set family containing information about which sections of the boundary is visible from each candidate guard. In this manner a set system is created. Is is showed that a greedy algorithm does not fully exploit the structure of the set system. Instead a dual set system is created and the smallest hitting set of the dual system is equivalent of finding the optimal set cover for the original set. Since an optimal solution generally is elastic, the probability is large that the optimal solution is actually obtained if the sampling is dense enough.

3.23.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper give a very clear presentation of the problem and its proposed solution is both elegant and simple.

There are some resemblance to [2] where visibility extensions are constructed and a guard candidate is located in the mass center of each convex polygon achieved. A combination of this candidate set and the algorithm proposed by González-Banos and Latombe would be interesting to investigate.

3.24 The Art Gallery Theorem for Rectilinear Polygons with Holes

This paper can be found in reference [24].

3.24.1 Problem Formulation

The necessary number of point guards and their locations in a rectilinear polygon with holes are investigated. Both the boundaries and the interior of the polygon should be visible.

3.24.2 Relation to the Camera Coverage Problem

Since most buildings and urban environments are more or less rectilinear, the paper is very relevant.

3.24.3 Proposed Solution Method and Mathematical Tools Used

To determine the location of the point guards the polygon is partitioned into rectilinear star shaped polygons. When a star shaped polygon is identified it is removed. Hence the problem is reduced and the smaller polygon or polygons are partitioned further. For each star shaped polygon removed, it is verified that the number of edges for the remaining polygons is reduced appropriately. If a polygon is reduced, but still not empty, it is shown that the polygon can be represented as a corridor graph.

The proofs is based on investigation of the possible cases that can be obtained.

As a remark, a simple example is given that disproves Conjecture 5.3 in [31], saying that $\lfloor 3n/11 \rfloor$ vertex guards are sufficient to cover any orthogonal polygon with any number of holes. Instead Hoffmann conjectures that $\lfloor 2n/7 \rfloor$ vertex guards is an optimal bound.

3.24.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper is very clear and well presented. Much of the notation is unusually well chosen which improves readability. A large part of the paper is devoted to proving that various cases are reducible. The illustrations are very helpful.

3.25 Inapproximability Results for Guarding Polygons without Holes, by S. Eidenbenz

This paper can be found in reference [12].

3.25.1 Problem Formulation

Given a polygon without holes, find a minimum set of guards (vertices, edges or interior points) such that every point on the boundary of the polygon can be seen from at least one guard. The paper investigates existence of polynomial time algorithms.

3.25.2 Relation to the Camera Coverage Problem

The results of the paper is considered relevant.

3.25.3 Proposed Solution Method and Mathematical Tools Used

The results are proved by describing a reduction from 5-Occurrence-3-Sat, a version of a standard problem in complexity theory.

The result of the paper is that vertex guards, edge guards and point guards for polygons without holes are APX-hard. There exist a positive constant ϵ such that no polynomial time algorithm can guarantee an approximation ratio $1 + \epsilon$, unless $P = NP$.

3.25.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper is somewhat technical and time consuming to read.

3.26 Approximation Algorithms for Two Optimal Location Problems, by A. Efrat, S. Har-Peled and J.S.B Mitchell

This paper can be found in reference [11].

3.26.1 Problem Formulation

Two problems are treated in the paper. The first one addresses where to locate a base station with respect to a number of sensors distributed in an area. The sensors transmit information, either directly to the base station or via other sensors. There are energy penalties for the sensors in transmitting and receiving information dependent on the amount of information and the distance. The objective is to find the optimal base station position and the transmission scheme that yield the longest lifespan of the network considering the available sensor battery energy.

The second problem deals with robust visibility coverage. Two definitions of robust visibility is posed. A point p is said to be *2-guarded at an angle* α by sensors g_1 and g_2 if p is visible from g_1 and g_2 and the angle $\angle g_1 p g_2$ is in the range $[\alpha, \pi - \alpha]$. A point p is said to be *triangle guarded* by g_1, g_2 and g_3 if p is contained in the triangle $g_1g_2g_3$. The region Q is to be robustly covered by sensors in P where $Q \subseteq P$.

3.26.2 Relation to the Camera Coverage Problem

The treated problems are very interesting although less relevant to the CCP.

3.26.3 Proposed Solution Method and Mathematical Tools Used

In the problem of positioning a base station, the sensors are restricted to transmit with a small number of different energy levels. These different levels will represent disks with different radii around the sensors. This in turn will yield a limited number of intersections of the disk perimeters which define the possible base station positions. An alternative discretization is to place equally spaced points on the perimeters, which is claimed to yield a faster algorithm. The transmission scheme, and there by the lifespan, is computed using linear programming for each position candidate.

For the 2-guarding problem a two-phase algorithm is proposed. First, a set $G_1 \subset P$ is found that cover Q in the regular visibility sense. Then another set $G_2 \subset P$ is found such that $G_1 \cup G_2$ 2-guards Q. A lower bound on the distance from the optimum is given.

The triangle guarding can be obtained from the 2-guards arrangement. The algorithm is based on ray-shooting from each sensor through each vertex. The intersection of the rays and the boundary ∂Q is computed and form a candidate set for additional guards.

3.26.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper is fairly well written and the problems are interesting, but less relevant to our area. The robust sensor arrangement is more relevant for applications where the location of a specific object are to be determined, or if a large number of cheap sensors are used and the guarding task must still be carried out in cases of sensor failure.

3.27 Art Gallery and Illumination Problems, by J. Urrutia

This paper can be found in reference [37].

3.27.1 Problem Formulation

This is a survey on art gallery problems. No specific problem is treated. Research results on the topic up to 2000 are collected.

3.27.2 Relation to the Camera Coverage Problem

This survey is very relevant to the CCP. A vast amount of theorems, conjectures and open problems for different topics are presented. Different kinds of guards, or illuminators, are discussed. Floodlights, which are lights with limited angles of illumination, are treated to some length. A floodlight is equivalent to a camera of limited field of view, and these are of particular interest to us. Other parts of special interest include guarding of treasures and avoidance of threats.

The majority of the results presented are upper bounds on the number of guards required for a certain case. Also frequently shown are the complexity of solving different problems.

3.27.3 Proposed Solution Method and Mathematical Tools Used

Solutions of the surveyed results are only discussed occasionally and briefly. For details one is directed to the various references.

3.27.4 Personal Comments, Pros and Cons, assessment of paper quality

The survey is well written and concise and is very frequently referenced in other papers. It works as a small encyclopedia for this research area. Apart from the regular art gallery problem, results on other related problems, such as the fortress, prison yard, watchman route, robbers route, safari route and the zoo-keeper's problem are presented.

3.28 Computational Complexity of Art Gallery Problems, by D.T. Lee and A.K. Lin

This paper can be found in reference [29].

3.28.1 Problem Formulation

This paper investigates the computational complexity of the minimum vertex guard, minimum edge guard and the minimum point guard problems for simple polygons without holes.

3.28.2 Relation to the Camera Coverage Problem

The treated problems are very relevant to the CCP. It is essential to know the computational complexity.

3.28.3 Proposed Solution Method and Mathematical Tools Used

The proof of the minimum vertex guard problem (which with small modifications also proves the minimum edge guard problem and the minimum point guard problem) is based on a construction in polynomial time of a polygon. It is shown that a boolean three satisfiability (3SAT) is transformable to the vertex guard problem for simply connected polygons. The polygon is coverable by a certain number of guards if and only if the instance of 3SAT is satisfiable.

As a consequence of the NP-hardness of the minimum point guard problem, the problem of decomposing a simple polygon into a minimum number of star shaped polygons such that their union constitutes the polygon is also NP-hard.

3.28.4 Personal Comments, Pros and Cons, assessment of paper quality

The proofs are quite long and tedious. The results, though, are important and the paper is frequently referenced.

3.29 Orthogonal Art Galleries With Holes: A Coloring Proof of Aggarwal's Theorem, by P. Zyliński

This paper can be found in reference [39].

3.29.1 Problem Formulation

The paper proves the old conjecture that $\lfloor (n+h)/4 \rfloor$ vertex guards are sufficient to guard the interior of a n-vertex orthogonal polygon with holes, provided that there exist a quadrilateralization whose dual graph is a cactus.

3.29.2 Relation to the Camera Coverage Problem

The proof is relevant to our problem. The conjecture was stated in 1982 and has now been proved for some different types of polygons.

3.29.3 Proposed Solution Method and Mathematical Tools Used

The proof is based on quadrilateralization, its dual graph and a 4-coloring argument. The dual graph of the quadrilateralization is obtained by putting a vertex in each 4-gon of the decomposition and connecting the vertices in adjacent 4-gons. First it is shown that the bound is valid for an orthogonal

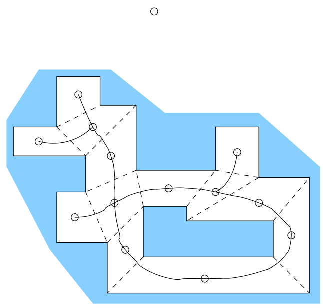

Figure 3.1: Quadrilateralization and dual graph.

polygon with one hole. For polygons with more holes than one, the polygon must satisfy the property that any two of its cycles share at most one vertex.

3.29.4 Personal Comments, Pros and Cons, assessment of paper quality

The paper is well written.

3.30 Art Gallery Theorems and Algorithms, by J. O'Rourke

This book can be found in reference [31].

3.30.1 Problem Formulation

Various problems, theorems and proofs are treated in this book.

3.30.2 Relation to the Camera Coverage Problem

This book is very relevant to our problem, although it is twenty years old. Just about all research results up to that time is collected.

3.30.3 Proposed Solution Method and Mathematical Tools Used

No specific solution is treated.

3.30.4 Personal Comments, Pros and Cons, assessment of paper quality

This is one foundation stone for the research area. It is well written and some complex results given in papers are sometimes presented more clearly, like [7] and [29].

4 Conclusions

A survey on studies relevant to the Camera Coverage Problem was presented. Even though there are many papers addressing the topic in computer science, mathematics and control theory, a lot of work remains to be done to apply and extend these results in the domain of UGV surveillance.

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