



# Estimation of the position error in GPS receivers

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## Sammanfattning

GPS (Global Positioning System)-mottagare kan tillhandahålla hög noggrannhet under många förhållanden. Däremot är GPS-signaler väldigt svaga vid jordens yta och kan lätt bli utstörda. I stadsmiljöer är GPS-signalerna utsatta för reflektioner och dämpning, och dessa effekter kan bidra till betydande positionsfel. Därför är positionsnoggrannheten och tillgängligheten ofta inte tillräcklig i stadsmiljöer, speciellt vid operationer inomhus.

Huvudfokus för detta arbete har varit att kunna avgöra om positionen från en GPS-mottagare är tillförlitlig. Det innebär att vi vid varje tidpunkt vill bestämma om positionsfelet i GPS-mottagaren är acceptabelt eller inte, baserat på den information som kan antas finnas tillgänglig i de flesta handhållna GPS-mottagare. Avsikten med detta arbete är att i varje tidpunkt kunna bestämma 1) om en GPS-position är tillförlitlig som en enskild navigeringslösning, och 2) om en GPS-mottagare bör användas i en kombinerad navigeringslösning baserad på multipla sensorer eller inte.

Fältförsök har utförts för att samla in data för att 1) avgöra vilka mätvärden som kan användas för att estimerar positionsfelet och därmed kunna bestämma dess tillförlitlighet, och 2) använda som tränings- och utvärderingsdata för estimerings- och klassificeringsprestanda.

Standardavvikelsen i latitud och longitud fås genom standardmeddelandet NMEA GST och det kan användas som en uppskattning av positionsfelet. Positionsfelet uppskattas väl av denna standardavvikelse under förhållanden där signalkvaliteten är antingen bra eller dålig (t.ex. inomhus). Däremot är positionsfelet svårare att estimerar i områden med kraftigt varierande vågutbredningsförhållanden och flervägsutbredning, t.ex. i stadsmiljöer.

Vi har också visat att det finns en stark korrelation mellan positionsfelet och flertalet andra mått, och att dessa mått också kan användas för att uppskatta positionsfelet. Däremot har vi inte lyckats utnyttja dessa andra mått för att tydligt förbättra estimerings- och klassificeringsprestanda, jämfört med att enbart använda standardavvikelsen från NMEA GST.

Den huvudsakliga svårigheten med att utnyttja andra mått för estimering och klassificering är p.g.a. en okänd fördröjning mellan tidpunkten då ett mått ändras till dess att positionsfelet börjar öka, och från tidpunkten då måttet återhämtar sig tills positionsnoggrannheten är återvunnen. Denna fördröjning beror på intern filtrering av positionslösningen i GPS-mottagaren. Fördröjningen är kraftigt beroende av typen av GPS-mottagare och i vilken tillämpning den används (främst påverkat av plattformens rörelse). Dessutom är påverkan av varje mått på positionsfelet beroende av typ av mottagare, så att t.ex. tröskeln för klassificering behöver anpassas noggrant.

Nyckelord: GPS, GNSS, positionsfel, estimering, klassificering

## Summary

GPS receivers can provide sufficient accuracy in many environments. However, the GPS signals are very weak at the surface of the earth and they can easily be jammed. The GPS signal experiences reflection, scattering and attenuation in urban environments, and these effects may cause large position errors. Thus, the position accuracy and availability is often insufficient in urban environments, especially considering indoor operations.

The main goal of this work was to determine whether the position solution as delivered by a GPS receiver is reliable enough. That is, for each point in time we wish to decide whether the position error in the GPS solution is acceptable or not, based on information that is available in a standard hand-held GPS receiver. Whether a position solution is useful or not depends on the application at hand. The intention with this work is that it should be possible to determine both 1) whether a GPS position is reliable enough as a stand-alone navigation solution or not and 2) whether the GPS receiver should be used in a multisensor navigation solution or be left out.

Field trials were performed to collect data to 1) determine which metrics that could be used to estimate the position error and determine the position reliability, and 2) use for training and evaluation of estimation and classification performance.

The standard deviation of latitude and longitude delivered by the NMEA GST message can be used as an estimate of the position error for any GPS receiver that supports the standard GST message. The position error is estimated quite well, using the NMEA GST standard deviations, in good and poor conditions (such as indoors). However, in areas with heavily varying channel conditions and multipath propagation, such as in urban areas, the estimation becomes more difficult.

We have also shown that there is a strong correlation between the position error and numerous other metrics, and that these metrics also can be used to estimate the position error. However, we have not been able to exploit these other measures to make a significant improvement in terms of the estimation and classification performance, as compared to using the NMEA GST standard deviations only.

The main difficulty in exploiting other metrics for estimation and classification was due to an unknown delay from the point in time when the parameter changes until the position error starts drifting away, and from when the metrics recover until the position accuracy is regained. The delay is caused by internal filtering of the position solution in the GPS receiver. This delay depends heavily on the type of receiver and on the application (e.g. movements of the platform). Furthermore, the dependence of each metric with the position error is dependent on the type of receiver so that the classification thresholds need to be adopted carefully.

Keywords: GPS, GNSS, position error, estimation, classification

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# 1 Introduction

The main application for the work herein is a soldier (or first responder) positioning system, with a focus on urban operations which also includes transitions between outdoor and indoor environments. Currently, such systems are mainly based upon stand-alone GPS receivers. However, soldier positioning systems based on multisensor approaches are under development and such systems are expected to become available on the market within a few years.

This work is relevant also for the development of future positioning systems for small unmanned aerial/ground vehicles (UAVs/UGVs) and vehicles operating in urban environments.

## 1.1 Project description

Accurate and reliable localization of soldiers in all environments, including urban and indoor operations, is a challenge that has not yet been solved. Existing systems are not able to provide sufficient accuracy (approximately 3 meters) while simultaneously fulfilling the stringent size, weight and cost requirements.

GPS (Global Positioning System) receivers can provide sufficient position accuracies in many environments. However, the GPS signals are very weak at the surface of the earth and they can easily be jammed. The GPS signal experiences reflection, scattering and attenuation in urban environments, and these effects may cause large position errors. Thus, the position accuracy and availability is often insufficient in urban environments, especially considering indoor operations.

By integrating the GPS receiver with additional positioning sensors, e.g. inertial sensors, magnetometers, barometric and/or imaging sensors, it is believed that the problem with providing an accurate soldier positioning system can be solved. Similar technologies can also be used in future positioning systems for small unmanned aerial/ground vehicles (UAVs/UGVs) and other vehicles operating in urban environments.

The project *Robust positioning for efficient C2* is a R&D project financed by the Swedish Armed Forces. The project started in 2011 and it ends in December 2013. The overall focus of the project has been to address research problems that are considered to be key areas for increasing the robustness and accuracy for military positioning systems. The aim is to demonstrate new technologies and possibilities for localizing soldiers and vehicles, with a particular emphasis on soldier positioning systems in urban environments.

## 1.2 Description of work

When performing sensor fusion it is crucial that accurate estimates of the errors in the different sensors/sub-systems are available. Otherwise the errors from one sensor or sub-system may cause a large position error for the complete multisensor positioning system. Hence, the large position errors that are caused by multipath and signal attenuation (or blockage) that typically occur in urban and indoor environments must be estimated. However, typically used performance metrics (e.g. the Dilution of precision, DoP, metrics) does not capture these effects.

The main goal of this work is to determine whether the position solution as delivered by a GPS receiver is reliable enough. That is, for each point in time we wish to decide whether the position error in the GPS solution is acceptable or not, based on information that is available in a standard hand-held GPS receiver. Whether a position solution is useful depends on the application at hand. The intention with this work is that it should be possible to determine both (a) whether a GPS position is reliable enough as a stand-alone

navigation solution or not and (b) whether the GPS receiver should be used in a multisensor navigation solution or be left out.

Field trials were performed to collect data to 1) determine which metrics that could be used to estimate the position error and determine the position reliability, and 2) use for training and evaluation of estimation and classification performance.

## 2 Description of field trials

We have used two Ublox 6T GPS receivers [1] and one military GPS receiver (DAGR<sup>1</sup>) to collect GPS data, and a total station (see Figure 3 and Figure 4) as a reference to log the correct position. All GPS receivers were connected to the same antenna via a splitter, as shown in Figure 1. The GPS receivers were connected to a laptop via a USB cable. The data that were stored from the Ublox receivers are shown in Table 1 in the appendix. The total station's position was established by the use of reference points measured with a geodetic GPS receiver. The total station uses a prism to track the position with an accuracy of less than 5 cm. The prism and the GPS antenna were mounted on top of the rack that held the box with the GPS receivers.

During the trial, two slightly different scenarios were tested as shown in Figure 2. The first scenario (green path) was performed twice, and the second scenario was performed three times. The starting point, the total station and part of the garage building are shown in Figure 3. Figure 4 shows a photo of the total station and part of the garage building, taken from the starting point. For both scenarios, the walk started in the most eastern point in a straight line to a point approximately 10 meters in front of the garage, followed by a left turn. In the first scenario (green), the walk continued in a straight line into the garage and to an inner wall approximately 10 meters from the opening of the garage. Then, the walk continued approximately the same way back to the starting point. In the second scenario (red), the walk continued along the garage building followed by a 90 degree turn. Then, the walk continued straightforward, followed by a 180 degree turn only 1-2 meters from the outer wall of the building.



Figure 1. Two Ublox 6 GPS receivers (top left), a DAGR (top right), an antenna splitter (middle of box), USB hub (rainbow colored) connecting all receivers to a laptop.

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<sup>1</sup> Defense Advance GPS Receiver.

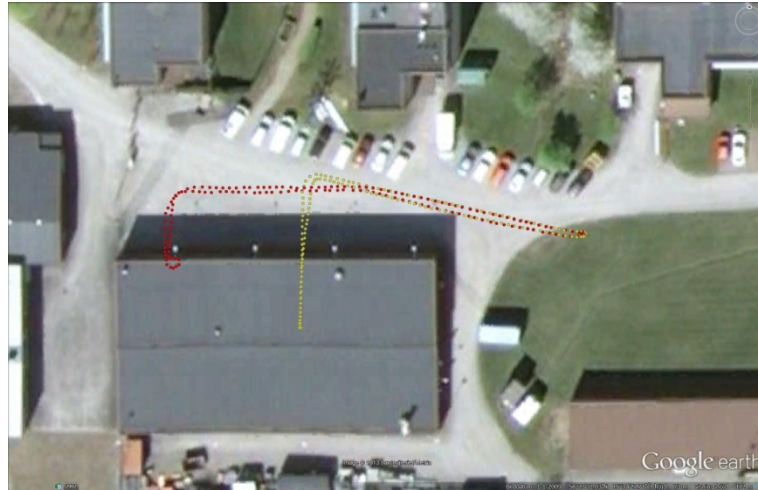


Figure 2. Paths for the two rounds (round three yellow, round six red). Map from Google earth.



Figure 3. Photo of the total station and the starting position of each round (at the bottom peak of the triangle). The garage building is visible to the right.

Another test, similar to the ones described above, has been performed previously. During these measurements, only the standard NMEA messages (see Table 1 in the appendix) were stored from a Ublox 6T GPS receiver, and the same total station was used to log the reference positions. The paths for the two measurements from this test, as stored from the total station, are shown in Figure 5. The aim with this test was to simulate movement in an urban environment. Hence, the path goes partially very close to buildings, and the GPS receivers suffer from multipath effects and signal attenuation caused by the buildings. The focus of this report will be on the measurements for the two scenarios shown in Figure 2, since a lot more parameters have been stored. The previous trial, shown in Figure 5, will be used only in the evaluation of position error classification shown in Section 3.3.



Figure 4. Photo taken from the starting position. The total station is marked with a white ellipse, and the garage building is visible to the left.



Figure 5. Paths for the measurements from the previous trial.



### 3 Study of correlation between selected metrics and the position error

In the following sections, we will show examples of how the metrics and the position error varied during the field trial.

#### 3.1 Potential metrics

In the following, we will show how the metrics varied for the Ublox receiver during the two scenarios. In this section, we show data using the last measurement, from one of the Ublox receivers, for each of the two scenarios. The other tests show similar behavior for the corresponding scenarios (as well as for the two Ublox receivers). Some parameters, such as the different dilution of precision metrics, AGC monitor, satellite elevation and azimuth, are omitted in this report because they did not show any visible variations related to the position error.

Figure 6 and Figure 7 show the positions and heights for the first and second scenarios, respectively. Both the estimates from the Ublox GPS receiver and from the reference positions from the total station are shown. It should be noted that the position and height are shown in meters relative to the minimum values during each round, in order to make the paths more visible. This is the reason that, for example, the reference height fictitiously changes with a few meters between the different scenarios.

Figure 8 shows the position error during the two scenarios. It is evident that the position error increases for a while, after entering the building, and then decreases again during the walk back to the starting point. Before entering the building, the horizontal error is in the order of one meter, and the height error is around five decimeters. After entering the building, the horizontal error is 5-15 meters, and the height error is two to six meters in these tests. We have not observed any significant systematic differences in the behavior of horizontal and height errors, in terms of how fast the errors change once the signal is lost or regained.

Figure 9-Figure 21 show numerous interesting metrics that seem to be affected approximately during the same time as the position error increases, for these two scenarios. Figure 9-Figure 12 show metrics for all satellites. The intention with these figures is to show the general trends, and not all details of each individual curve.

Figure 9 shows the estimated  $C/N_0$  for all used satellites<sup>2</sup>. It is clear that the signal quality is degraded inside of the garage, which is approximately during the same period of time as the increased position error.

The variation of the Doppler spread between different satellites is much greater than the Doppler spread variation over time for a single satellite. Moreover, the time variation seems to be very close to a straight line during these measurements (4-5 minutes). Therefore, Figure 10 shows the deviation from a straight line approximation of the Doppler spread (RXM-RAW) of each individual satellite. The absolute value of the Doppler spread deviation seems to increase in a region close to the increased position error.

Figure 11 shows the deviation from a straight line approximation of the pseudo ranges, in a similar manner as for the Doppler spread, for all used satellites. This measure also seems to be affected approximately during the time of increased position error. This is probably due to multipath effects inside of the garage. However, it is not obvious how to exploit this measure, because the variations are quite large even when the position error is small, and it differs between the measurement rounds.

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<sup>2</sup> Obtained from the Ublox proprietary message RXM-RAW.

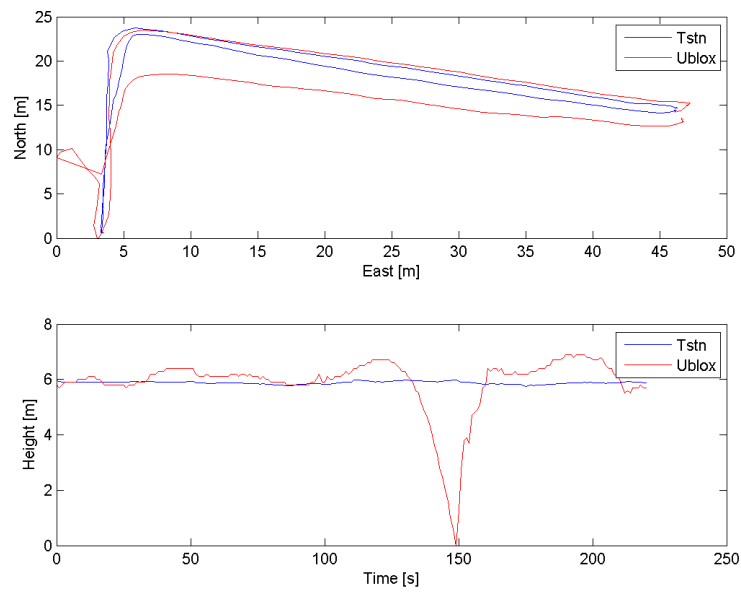


Figure 6. First scenario, GPS receiver and reference positions.

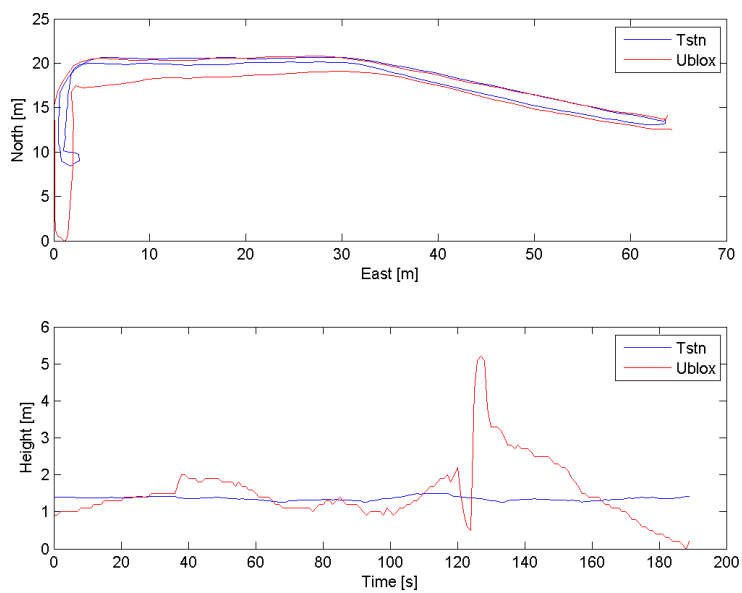


Figure 7. Second scenario, GPS receiver and reference positions.

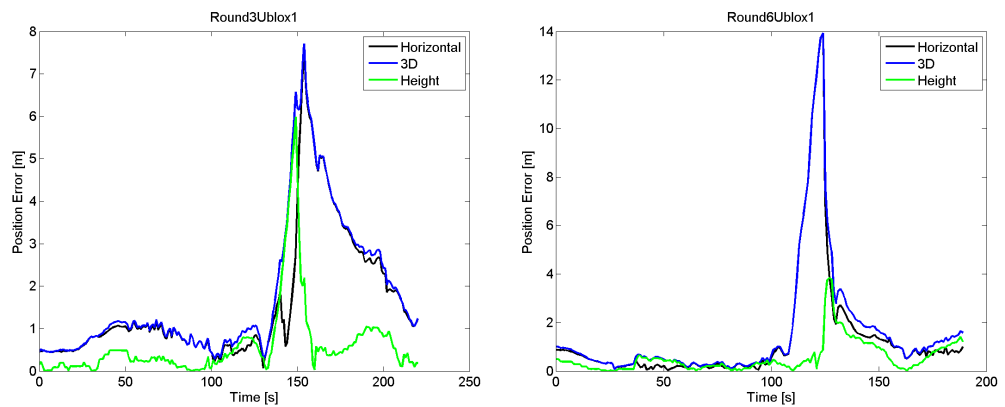


Figure 8. Position errors during scenarios one (left) and two (right).

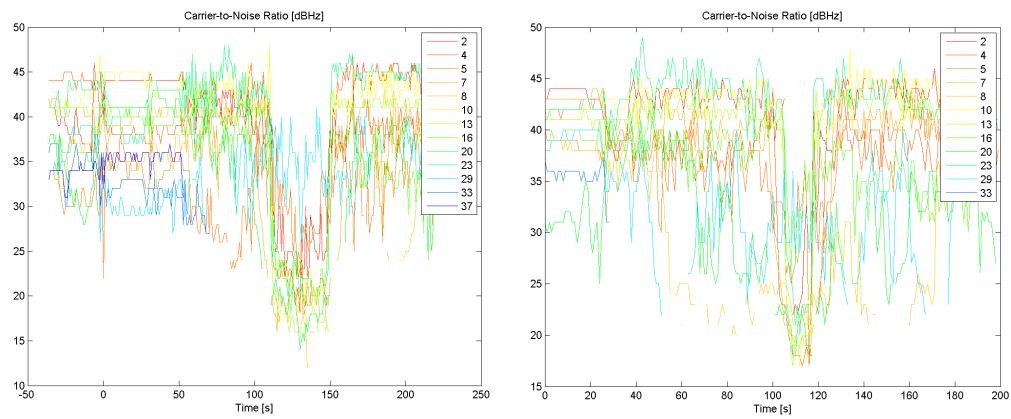


Figure 9. Estimated C/N<sub>0</sub> for used satellites (scenario one left and scenario two right).

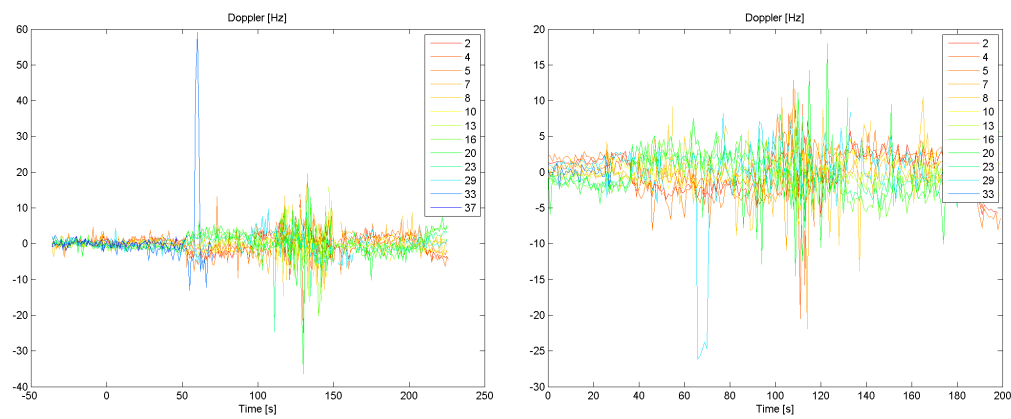


Figure 10. Doppler spread deviation, from a straight line approximation, for used satellites (scenario one left and scenario two right).

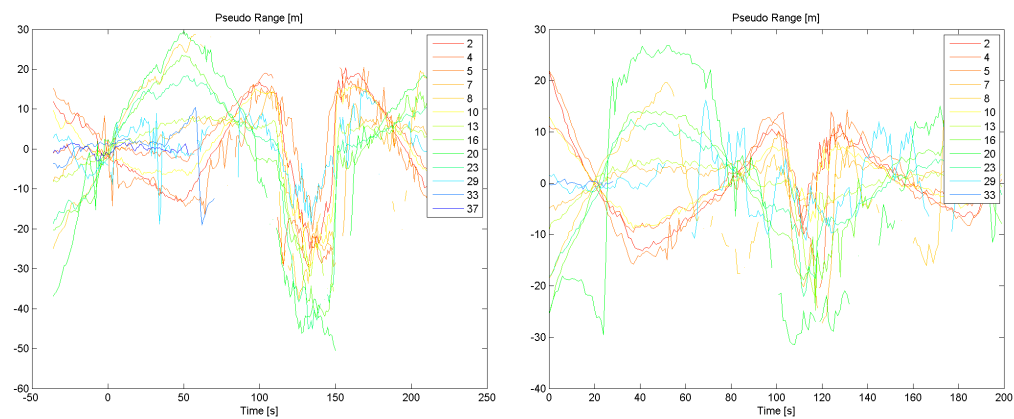


Figure 11. Pseudo range deviation from a straight line approximation, for used satellites (scenario one left and scenario two right).

Figure 12 shows the range residuals for all used satellites. The range residuals are the differences between the measured pseudo ranges and the distances from the satellites to the position solution calculated by the receiver. The deviation from zero increases in the neighborhood of the larger position error. Therefore, we have computed the standard deviation of the range residuals in each time instance, shown in Figure 13 together with the actual position error as estimated from the total station.

The NMEA GST message contains data described as the standard deviation (in meters) of the latitude, longitude, and altitude errors. These standard deviation measures are probably estimates of the variance from the internal (Kalman or similar) filter in the GPS receiver. That is, it is an estimate of the error of the actual filtered position solution in each direction (latitude, longitude, altitude). Since we are mainly interested in the horizontal error, we have computed the horizontal position standard deviation as  $\sqrt{\sigma_N^2 + \sigma_E^2}$ , where  $\sigma_N$  and  $\sigma_E$  are the standard deviations of the latitude and longitude respectively. The horizontal standard deviation is shown in Figure 14, together with the actual position error.

We noted in Figure 9 that the estimated  $C/N_0$  decreased for all used satellites when entering the garage. Figure 15 shows the average  $C/N_0$  for all used satellites and the position error. A clear correlation is seen between the average  $C/N_0$  and the position error, although a bit delayed.

It was observed in Figure 10 that the deviation from the straight line approximation of the Doppler spread seemed to have larger variations when the position error increased. Therefore, we have computed the standard deviation, shown in Figure 16, of the Doppler deviations shown in Figure 10. Figure 16 shows a possible correlation between the standard deviation and the position error, but it is not obvious.

Figure 17 shows the estimated magnitude of the I component of the signal (the magnitude of the Q part is also available but omitted here). Since the GPS signal uses a constant envelope modulation, the magnitude should be constant in good conditions. However, a possible correlation with the position error is observed in Figure 17. The I/Q imbalance is really a receiver property, and not a property of the received signal. The reason that this measure is correlated with the position error is probably because the estimate of the I/Q imbalance is affected by the weakened signal strength.

Figure 18 shows the mean carrier lock time, i.e. the amount of time that the satellite signals have been tracked on the average. Clearly the mean carrier lock time decreases when the signal quality degrades and the position error increases.

Figure 19, Figure 20 and Figure 21 show the number of used, tracked and excluded satellites respectively. There is a possible correlation between the number of used and excluded satellites, and the position error. However, the correlation does not seem to be very strong. What satellites that are excluded have not been evaluated, whether they are close to the horizon or far up on the sky. Satellites close to the horizon could potentially improve the position solution in an indoor scenario, since the signals could reach through a single wall (or even an open door) rather than passing through several floors of a building. The number of tracked satellites varies very little, or not at all, so that the correlation is negligible. A probable reason is that the Ublox is a high sensitivity receiver and keeps tracking the satellites even at a rather low GPS signal strength (such as inside of the garage). It is hard to tell whether the larger position error inside of the garage is caused mainly by decreased signal strength, or if the receiver is also affected by multipath effects and actually tracks the delayed and attenuated multipath components that leaks through the open garage door. Other receivers will probably work differently.

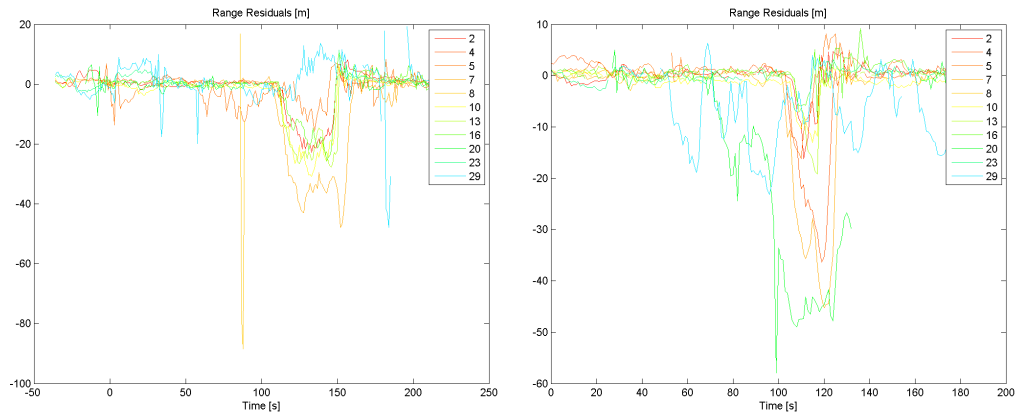


Figure 12. Range residuals for used satellites (scenario one left and scenario two right).

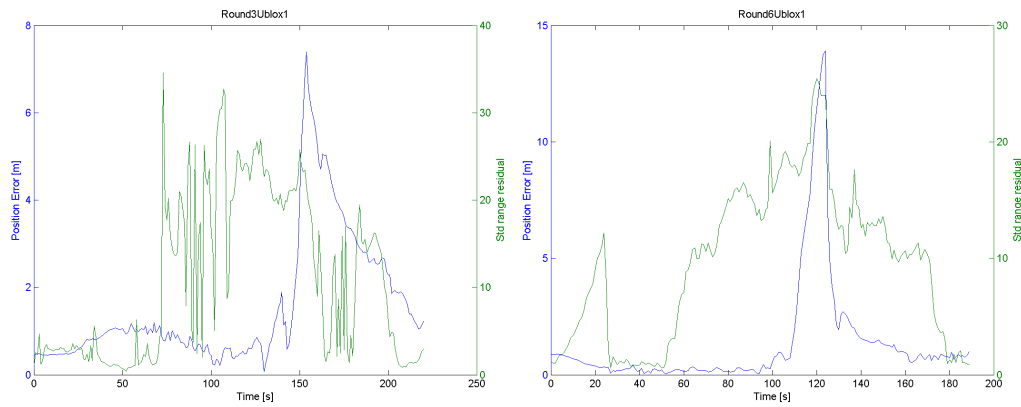


Figure 13. Standard deviation of range residuals and horizontal position error.

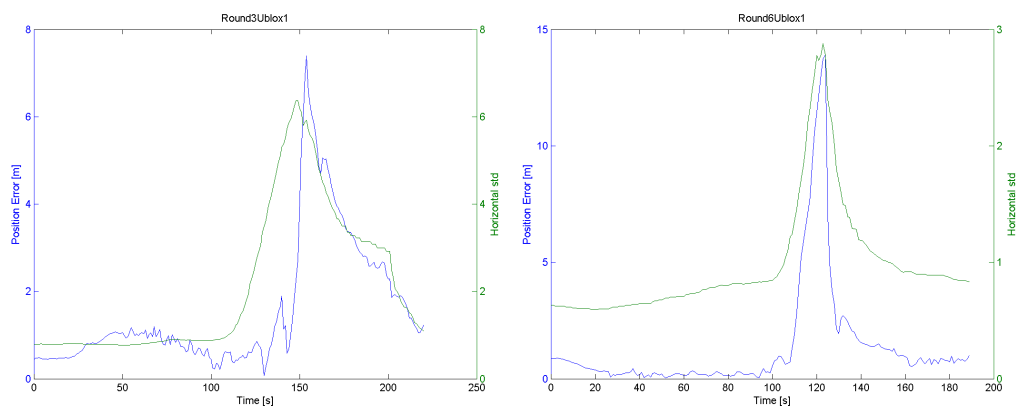


Figure 14. Standard deviation of horizontal position and horizontal position error.

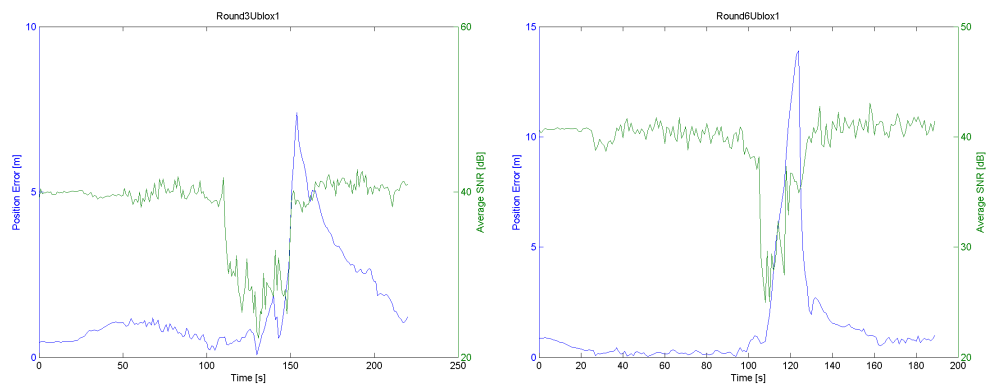
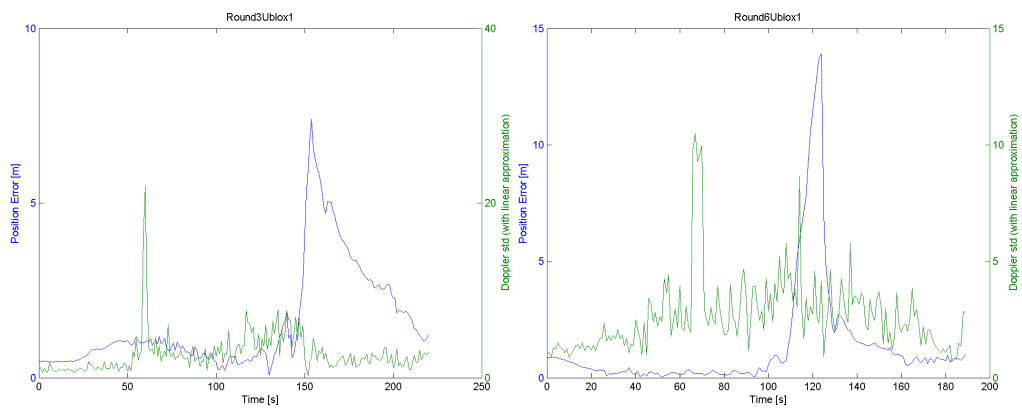
Figure 15. Average  $C/N_0$  and horizontal position error.

Figure 16. Standard deviation of Doppler spread after a straight line approximation, and horizontal position error.

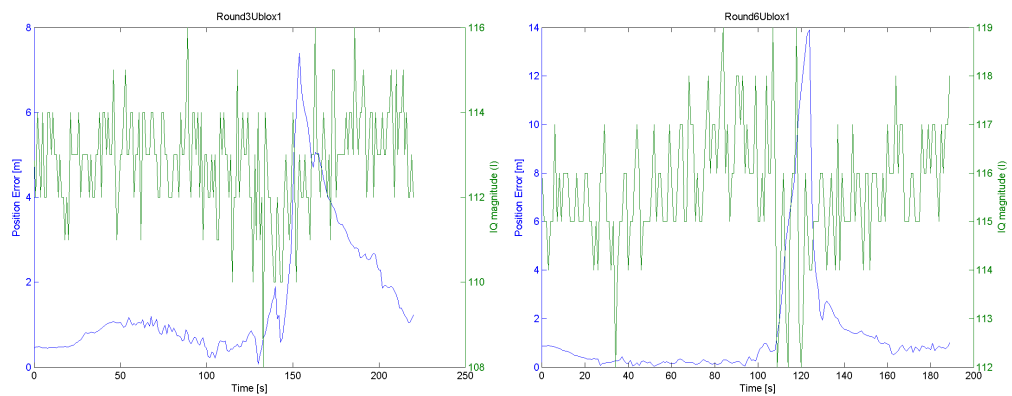


Figure 17. I/Q magnitude (I part) and horizontal position error.

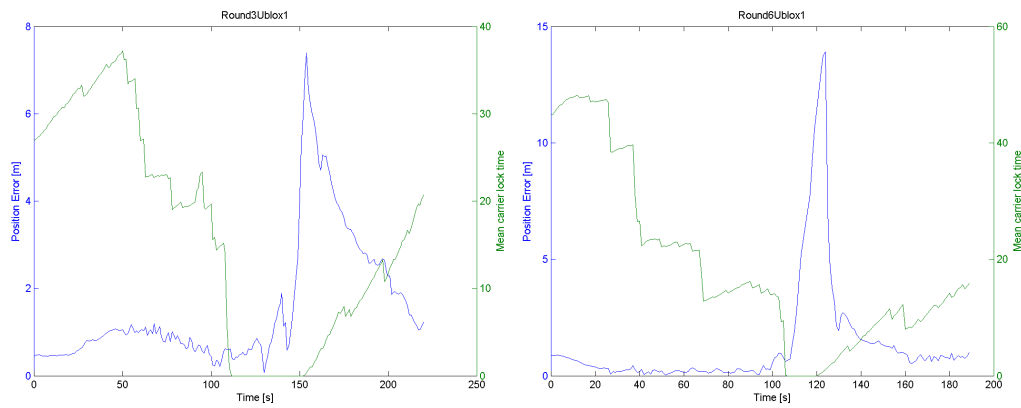


Figure 18. Mean carrier lock time and horizontal position error.

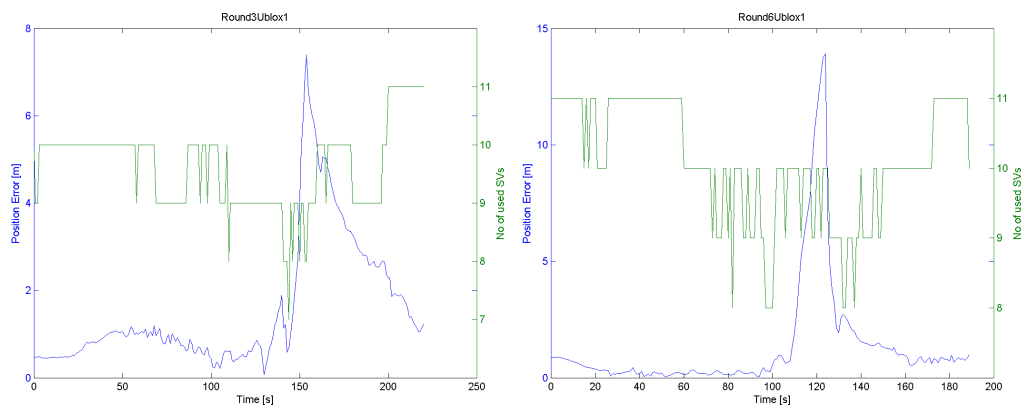


Figure 19. Number of used satellites and horizontal position error.

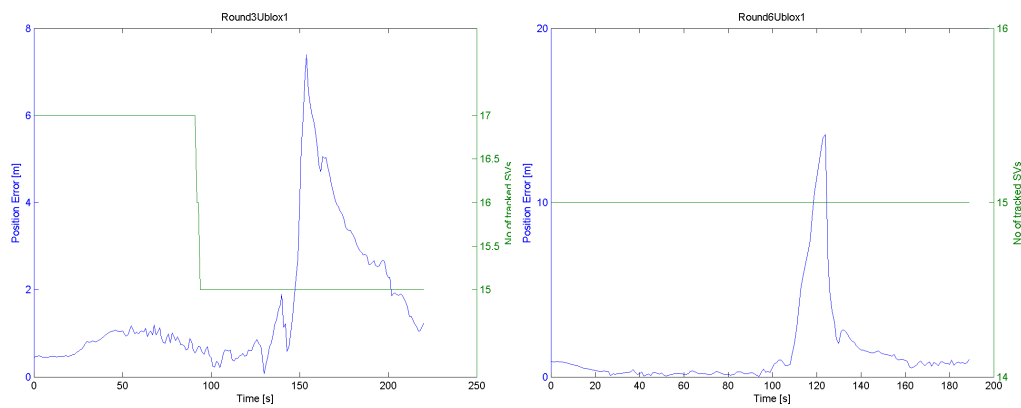


Figure 20. Number of tracked satellites and horizontal position error.

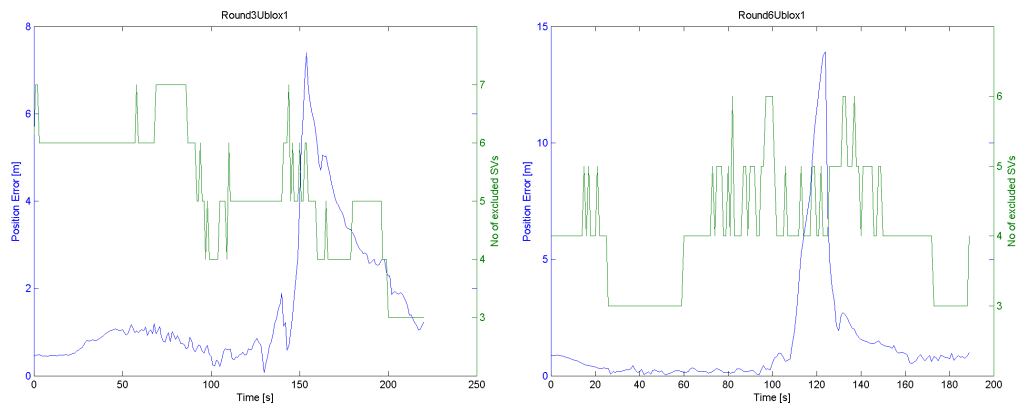


Figure 21. Number of excluded satellites and horizontal position.

### 3.2 Chosen metrics and their dependencies

It can be seen from the figures shown in Section 3.1 that for most of the measured parameters there is a delay of approximately 5-30 seconds, depending on the scenario and parameter, from the point in time when the parameter changes until the position error starts drifting away, and from when the metrics recover until the position error decreases again. The reason is that a GPS receiver uses some sort of (Kalman or similar) filtering of the position solutions, which is observed more clearly when the satellite signals are weak or no longer tracked. The tracking of the satellites was never lost completely during these trials, even when walking indoors, but the signal quality was degraded and affected by multipath propagation. The accuracy of this filtered position solution in bad signal conditions is expected to depend on the trajectory of the test subject.

Since the position is correctly delivered, from the Ublox receiver with the current settings, for a few seconds even though the signal quality degrades, it is hard to estimate the momentary position error based on the measurements at the same time. Rather, the current signal quality affects the position error with a few seconds delay. The behavior of a GPS receiver in terms of the position solution it delivers, in particular when the signal quality is poor or the satellites are lost, depends very much on the implementation by the manufacturers. How the position error is affected when the signal quality starts to degrade will also depend on the actual movement when this occurs. During our trials, after entering the garage, the path continued straight for about 10 meters followed by a 180 degree turn during scenario one and continued straight for only 1-2 meters followed by a 180 degree turn during scenario two. On the one hand, it is evident that the GPS was not able to deliver a correct position when the 180 degree turn appeared shortly after the signal quality deteriorated during scenario two. On the other hand, when the 180 degree turn appeared after approximately 10 meters, the GPS position solution is retained close to the reference position, and later starts drifting away and keeps doing so even when the signal quality is regained.

To summarize, there is a delay of approximately 5-30 seconds from the point in time when many of the parameters change until the position solution starts drifting away. This delay is dependent both on the actual receiver (hardware, filtering etc.) and on the application or scenario (e.g. movement after signal is lost), and has to be taken into account when designing position error estimators for a specific receiver. A potential solution to this problem, in the integration of a GPS receiver with other navigation sensors, is to turn off the internal filtering and use the momentary GPS position solution for post processing. This way, the solution delivered by the GPS receiver would react instantaneously on poor signal quality so that it is easier to decide whether the solution is acceptable, and the position filtering can be performed jointly with the other sensors in the integration.

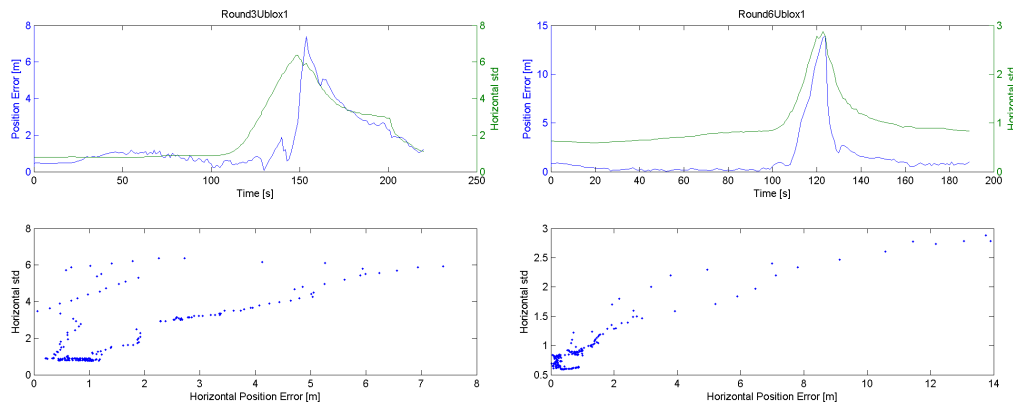


Figure 22. Horizontal position error dependence of the standard deviation of horizontal position.

In most commercial GPS receivers, there is no possibility to turn off the filtering. In some special cases, such as for the Ublox, it is possible to choose the motion profile that sets how the position solution is filtered. That is, for most commercial off-the-shelf products, the filtering has to be taken into account when deciding whether the position solution is acceptable or not.

Note, however, that the standard deviation of the position error, shown in Figure 14 is not delayed but follows the reference position error quite well. Therefore, this measure can be used to estimate the momentary position error without any delay. The reason that there is no delay in this case is probably because the standard deviation is estimated based on the already filtered solution, so that it shows the deviation of the actual position as delivered by the GPS receiver. By contrast, the signal quality metrics, such as the  $C/N_0$  and Doppler spread, affect the input parameters and are therefore independent of the actual filtering solution.

Figure 22 shows the standard deviation of the horizontal position, which was the metric that was found to be the most correlated with the position error during these trials, without any delay. The topmost plot show the standard deviation and the position error as a function of time, and the bottom plots show the standard deviation as a function of the position error. That is, the bottom plot of Figure 22 shows that the measure is correlated with the position error.

Because of the delay discussed above, we have made an attempt to estimate and classify the position error based on delayed metrics. The correlation of numerous metrics with the position error, with a 15 seconds delay, is shown in the appendix. The appendix also shows estimation and classification results, where the delayed metrics have been used in the position error estimation and classification. However, since the delay is unknown and dependent on the scenario, it is generally not a good method to work with a fixed delay. The results in the appendix only show that it could be a possible way forward, but the delay should also be estimated for each GNSS receiver and for the scenarios of interest.

### 3.3 Evaluation of classification performance

In the following, we evaluate the classification performance of the chosen parameters. The parameter that was found to be highly correlated with the position error, without introducing a scenario and manufacturer dependent delay, was the position standard deviation. The standard deviation of the horizontal position, i.e.  $\sqrt{\sigma_N^2 + \sigma_E^2}$ , where  $\sigma_N^2$  and  $\sigma_E^2$  are the standard deviations of the latitude and longitude respectively given by the NMEA GST message, is already an estimate of the position error. The classification is done by simple thresholding of the standard deviation, i.e. the position is classified as

unreliable if the standard deviation is above a predetermined threshold. As a comparison, we have also included thresholding of the average  $C/N_0$ , and the combination (using the OR rule) of the two metrics.

Figure 23 shows the classification results using thresholds 1, 2, 3, and 4 for the standard deviation and for the true position error, and 41, 39, 37 and 35 dB for the average  $C/N_0$ . The results are shown for the first measurement in scenario one, the first and second measurement in scenario two, and for the two measurements from the previously performed trial (shown in Figure 5). The horizontal dashed lines show the transition from one measurement to the next. Note that these are just examples, and the combinations of thresholds are not necessarily the optimal ones for each level of the true position reliability classification.

The levels of the standard deviation and average  $C/N_0$  are expected to vary between different types of GNSS receivers, and the thresholds have to be adapted accordingly. How to actually set the thresholds is also a matter of what application it is to be used for. In particular, it depends on whether one wants to be certain that the position is reliable, at the cost of throwing away some good position data that is erroneously classified, or whether one wants to discard only position data that are certainly unreliable, at the cost of keeping some erroneously classified bad position data.

The results in Figure 23 show that the position standard deviation can be a rather good metric. Possibly, including the average  $C/N_0$  could add some useful information for a more defensive strategy when one wishes to keep only the certainly reliable position data. However, it should be noted that these results show the behavior of one type of GPS receiver only. Other types of receivers are expected to behave slightly differently, so that these results should be taken as simple guidelines only.

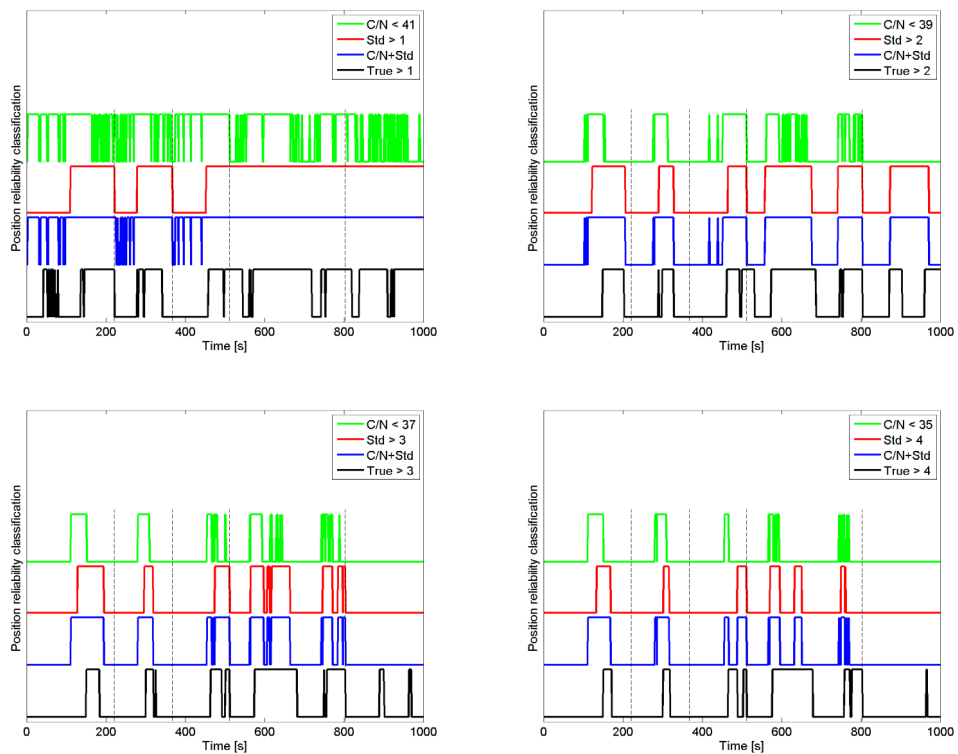


Figure 23. Position error classification based on thresholding of the standard deviation of the horizontal position and the average  $C/N_0$ , and the (OR) combination of the two for different levels of the thresholds.

## 4 Conclusions

The standard deviation of latitude and longitude delivered by the NMEA GST message can be used as an estimate of the position error for any GPS receiver that supports the standard GST message. We have also shown (in the appendix) that there is a strong correlation between the position error and numerous other parameters, and that these also can be used to estimate the position error. However, we have not been able to exploit these to make a significant improvement in terms of the estimation and classification performance, as compared to using the NMEA GST standard deviations only. More advanced estimation and classification methods could potentially better exploit the parameter correlations, and could be interesting for future studies.

The position error estimates are in the order of one meter in good conditions, and very seldom above five meters for the Ublox receiver. Therefore, it is not useful to use a classification threshold outside of the interval [1, 5] meters, to decide whether the accuracy of the GPS position solution is acceptable or not.

The position error is estimated quite well, using the NMEA GST standard deviations, in good and poor conditions (such as indoors). However, in areas with heavily varying channel conditions and multipath propagation, such as in urban areas, the estimation becomes more difficult. This was observed when including the measurements from the previous trial which included, for example, walking close to buildings and under a metal sun roof. The position error during these measurements was more difficult to estimate with the examined metrics.

The conclusions of this work are that the GPS position reliability can be determined in many cases, but it can be very difficult when the receiver is affected by strong multipath effects (such as in urban environments). The recommendation from this study, for integration of a GPS receiver with other sensors, is to use the NMEA GST standard deviations when available, and adjust the classification threshold properly. It could also be useful to use simple thresholding of signal quality measures such as the average  $C/N_0$  and number of used satellites. The actual dependence of all metrics is dependent on each individual receiver, or at least each type of receiver, so that the classification thresholds need to be adopted carefully. Thorough testing is strongly recommended to evaluate the performance of the integrated system, both in troublesome environments with strong multipath effects but also under 'easier' conditions with good or very bad signal quality.



## 5 References

- [1] *u-blox 6 Receiver Description*, 2011.
- [2] T. Hastie, T. Robert and F. Jerome, *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*, Springer, 2009.



## Appendix A – Stored parameters

The messages that were stored from the Ublox receivers are shown in Table 1 (for reference, see [1]).

Table 1. Messages stored from the Ublox 6 receiver.

Data type	Ublox message	Description	Included parameters (select sub-set)
Standard NMEA	GGA	Position data	Position data
	GRS	Range residuals	Range residuals for used SVs
	GSA	DOP and active satellites	Dilution of precision, fix status
	GST	Pseudo range error statistics	Standard deviation of lat, lon och alt
	GSV	Satellites in view	C/N <sub>0</sub> , elevation, azimuth, number of satellites in view and satellite PRN (ID).
	VTG	Course over ground and ground speed	Course over ground and ground speed
Ublox specific NMEA	UBX00	Lat/Lon position data	Position error (receiver estimate), speed over ground, course over ground, HDOP, VDOP, TDOP.
	UBX03	Satellite status	Satellite status (not used, used, ephemeris available but not used), satellite PRN, azimuth, elevation, C/N <sub>0</sub> , carrier lock time.
Ublox proprietary data	RXM-RAW	Raw measurement data	Carrier phase, Doppler, pseudorange, C/N <sub>0</sub> , GPS time (ms time of week and week number), # of satellites.
	MON-HW	Hardware status	Jamming indicator, noise level, AGC monitor
	MON-HW2	Extended hardware status	I/Q imbalance and magnitude



## Appendix B – Position error estimation

The position error will be estimated and classified using linear regression and k-nearest neighbors estimation applied to the chosen metrics from the conducted field trials. It will be shown that the position error can be quite well estimated based on various combinations of signal quality metrics. However, these results will also show that an estimate based on the standard deviation of the horizontal position performs as good, or even better, than these estimators for the Ublox receiver in the tested scenarios.

### B.1 Model and problem formulation

Let  $\mathbf{y}_n$  and  $\mathbf{x}_n$  denote the target variable and the measurement, respectively, at time  $n$ . Furthermore, let

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_{-T+1}^T \\ \vdots \\ \mathbf{x}_0^T \end{bmatrix},$$

and  $\mathbf{Y} = [\mathbf{y}_{-T+1}, \dots, \mathbf{y}_0]$  denote the measured training data and the target training data respectively, during a time period  $T$ .

At this point, the aim is to decide whether the position solution delivered by a GPS receiver is reliable enough and should be used in the position solution of an integrated multisensor system. This can be done by classification of the target data, based on some measure of reliability. One way to classify the reliability is to estimate the position error and then decide whether the estimated position error is good enough, based on a predetermined threshold. Then, the aim is to estimate the current target variable  $\mathbf{y}_n$  at time  $n > 1$  from the measurement  $\mathbf{x}_n$  based on the previous training data  $\mathbf{X}$  and  $\mathbf{Y}$ . In our application, the target variable that we wish to estimate is the GPS position error.

The position error that we wish to estimate could be, for example, the individual errors in latitude, longitude and altitude or the total two (or three) dimensional error combined. That is, the target variable  $\mathbf{y}_n$  that we wish to estimate is a vector in general. The measurement  $\mathbf{x}_n$  is comprised of a set of parameters that can be measured by a GPS receiver and are correlated with the position error, such as the estimated  $C/N_0$ , pseudo range statistics, number of used satellites, Doppler spread, etc.

In this work, we have chosen a few simple methods for estimation and classification. For position error estimation, we have used two methods; linear regression with a least squares estimated regressor matrix, and k-nearest neighbors. The classification has been done by simply setting a threshold on the maximum allowed position error, based on the estimate. The main focus has been on finding appropriate measures that affect the position error and use them in a simple way, and not on finding an optimal estimator or classifier.

#### B.1.1 Linear regression

The first approach that we consider is linear regression (cf. [2]). That is, the target variables are modeled as being linearly dependent on the measurement data, or (potentially non-linear) functions thereof. For simplicity of notation, assume that  $\mathbf{x}_n$  could be the measured data directly or any function thereof. That is,

$$\mathbf{y}_n = \mathbf{x}_n^T \mathbf{W} + \mathbf{e}_n,$$

where  $\mathbf{W}$  is a regressor matrix and  $\mathbf{e}_n$  is an error term, induced by measurement noise and model errors. In addition to functions of the measured parameters, a constant term (one) is usually also included in  $\mathbf{x}_n$ .

The regressor matrix  $\mathbf{W}$  is unknown and will be estimated from the training data using the same linear model. That is,

$$\mathbf{Y} = \mathbf{X}\mathbf{W} + \mathbf{E},$$

where  $\mathbf{E}$  is the noise during the training period. The estimated regressor matrix, denoted by  $\widehat{\mathbf{W}}$ , is then used to estimate the target variable as

$$\widehat{\mathbf{y}}_n = \mathbf{x}_n^T \widehat{\mathbf{W}}.$$

A standard approach to estimating the regressor matrix is to use the least squares estimate. This is equivalent to the maximum likelihood estimate if the noise  $\mathbf{E}$  would be assumed to be zero-mean white Gaussian. The noise  $\mathbf{E}$  consists of both systematic model errors and measurement noise, and there are really no strong arguments to believe that this would be white Gaussian. The consequence of the noise being non-white and non-Gaussian is only that the least squares estimate is not necessarily equivalent to the maximum likelihood estimate. The least squares estimate is

$$\widehat{\mathbf{W}}_{LS} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}$$

There are other types of least squares estimators, for example weighted least squares and regularized least squares such as Ridge regression and the Lasso (cf. [2]). If the weighting matrix is the inverse covariance matrix of the measurement noise, then weighted least squares is equivalent to performing least squares on the decorrelated data. However, the noise covariance is unknown in general, and has to be replaced by an estimated covariance matrix. The idea of regularized least squares is to add a regularization term to an error function to control over-fitting of the parameters. Regularized least squares moves the problem of finding appropriate parameters to another problem of finding a suitable regularization term. In this work, however, we will use the standard least squares estimate only.

### B.1.2 k-Nearest Neighbors

The k-nearest neighbors estimate is formed by averaging the  $k$  target training variables whose measurement points are closest to the current measurement  $\mathbf{x}_n$  [2]. That is,

$$\widehat{\mathbf{y}}_n = \frac{1}{k} \sum_{i \in \mathcal{N}_k(\mathbf{x}_n)} \mathbf{y}_i,$$

where  $\mathcal{N}_k(\mathbf{x}_n)$  is the neighborhood of  $\mathbf{x}_n$  defined by the set of closest points  $\mathbf{x}_i$  in the training data. Different measures can be used to define the distance in the measurement space. For simplicity, we will use the Euclidean distance.

## B.2 Estimation with delayed metrics

During these trials, the delay between the signal quality parameters and the position error is approximately in the order of 20 seconds during the rounds for scenario one and 5-10 seconds during the rounds for scenario two. Therefore, we choose to estimate the position error 15 seconds ahead of the current measures. In the following we show plots of the chosen measures as a function of the position error 15 seconds later.

Figure 24-Figure 28 show the measures that were found to be the most correlated with the position error during these trials, apart from the standard deviation shown in Figure 22. The topmost plots show the measure (with delay) and the position error as a function of time, and the bottom plots show the measure (with delay) as a function of the position error. That is, the bottom plots of Figure 24-Figure 28 show that these measures are correlated with the position error.

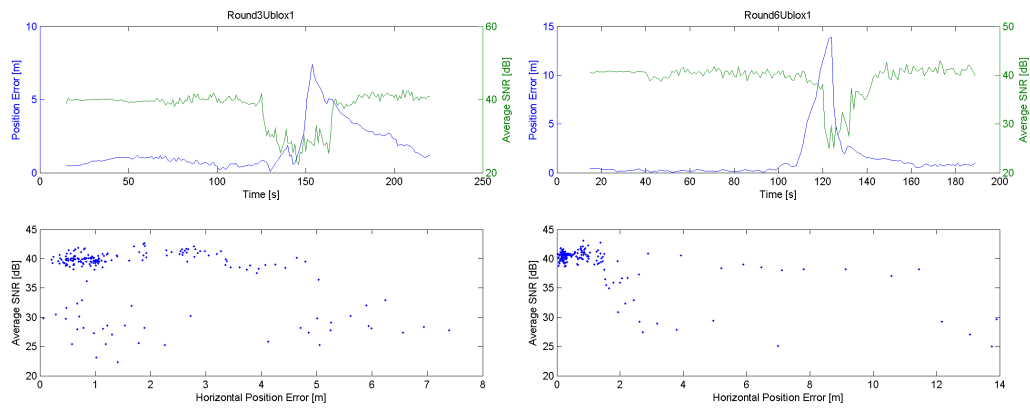


Figure 24. Horizontal position error dependence of average  $C/N_0$  with 15 seconds delay.

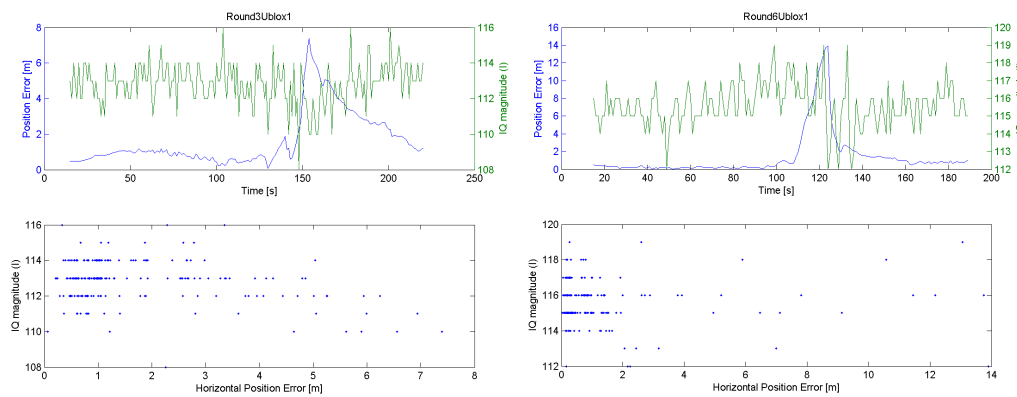


Figure 25. Horizontal position error dependence of I/Q magnitude (I part) with 15 seconds delay.

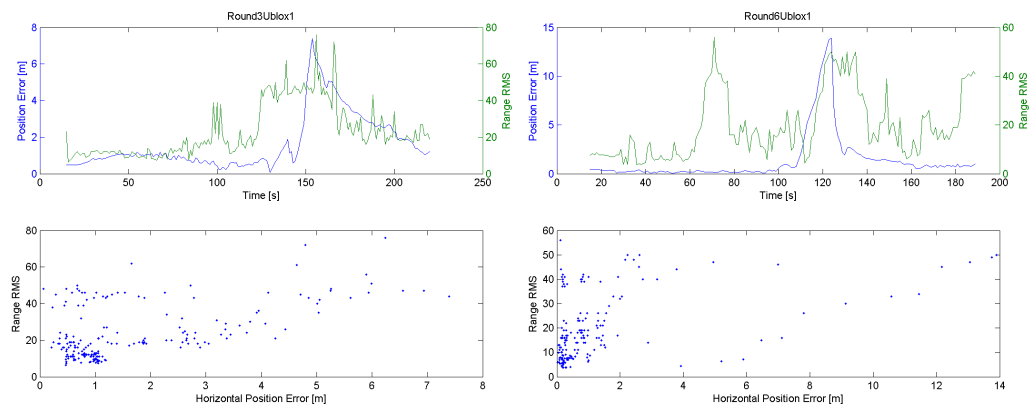


Figure 26. Horizontal position error dependence of the root-mean-square (RMS) value of the ranges with 15 seconds delay.

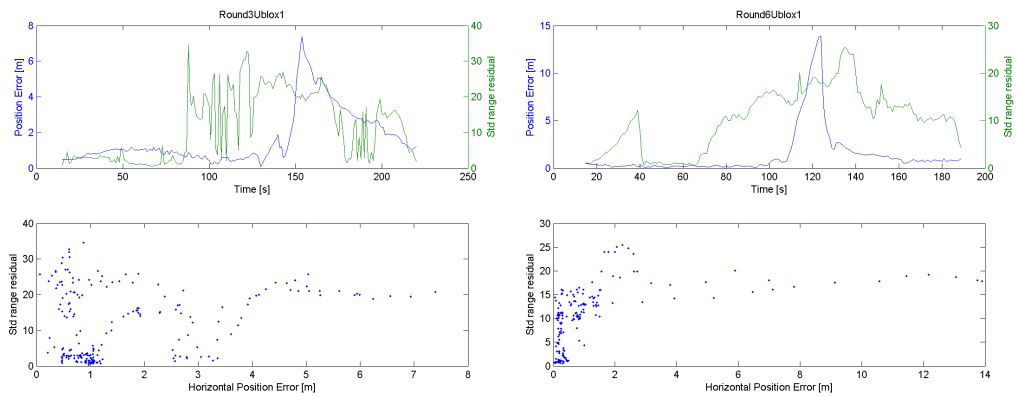


Figure 27. Horizontal position error dependence of the standard deviation of range residuals with 15 seconds delay.

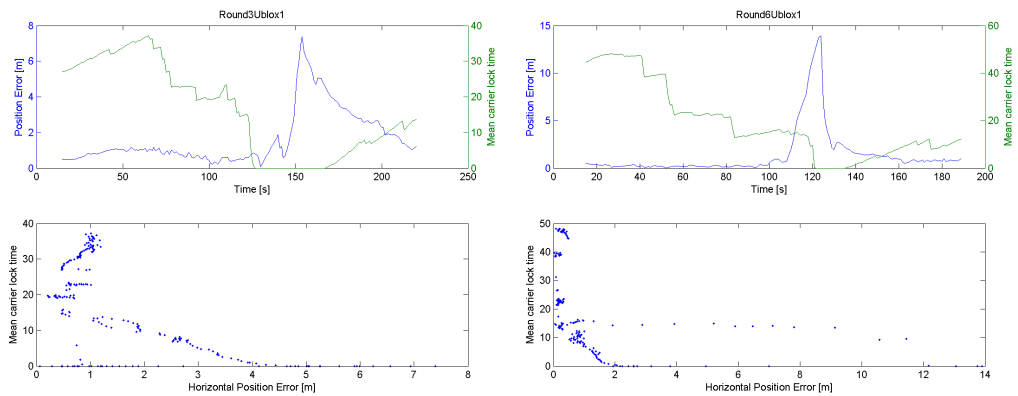


Figure 28. Horizontal position error dependence of the average carrier lock time with 15 seconds delay.

### B.2.1 Evaluation of Estimation and Classification Performance

In the following, we evaluate the performance of the chosen estimation and classification methods, using different combinations of the chosen parameters. The standard deviation of the horizontal position, i.e.  $\sqrt{\sigma_N^2 + \sigma_E^2}$ , where  $\sigma_N^2$  and  $\sigma_E^2$  are the standard deviations of the latitude and longitude respectively given by the NMEA GST message, is already an estimate of the position error. Therefore, this measure will also be used as is for comparison with the other methods. Moreover, this measure can be used in the estimation process like any other parameter.

Figure 29 and Figure 30 show the estimation and classification results when the standard deviation of the horizontal position is the only parameter used. A three meters position error threshold has been used for the classification. In these figures, the first measurement of the first scenario and the last measurement of the second scenario have been used for training data, and the position error is estimated for the remaining three measurements and for the two measurements from the previous trial (shown in Figure 5). The horizontal dashed lines show the transition from one measurement to the next. Note that classification using linear regression of a single parameter is equivalent to performing classification based on the parameter itself, with an appropriate scaling of the classification threshold. That is, instead of using the linear regression of the horizontal standard deviation, we could simply set a classification threshold for the horizontal standard deviation directly. This would of course be computationally more efficient.

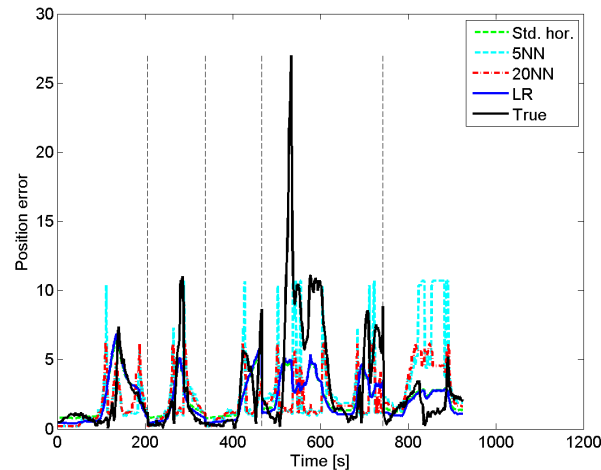


Figure 29. Position error estimates of the second measurement from the first scenario, the first two measurements of the second scenario and the measurements from the previous trial, using the standard deviation of the horizontal position as the only parameter.

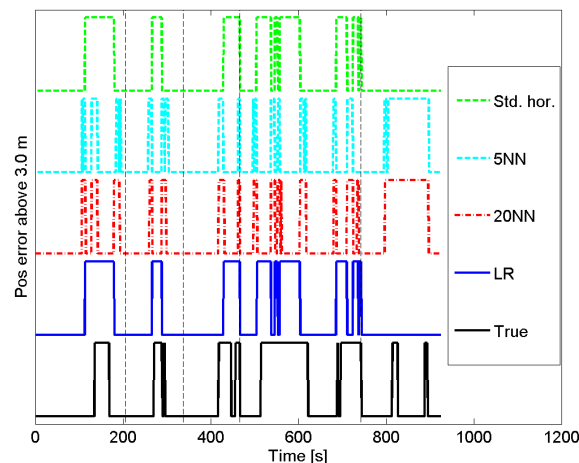


Figure 30. Position error classification using the standard deviation of the horizontal position as the only parameter, and a three meters position error threshold.

The results in Figure 29 and Figure 30 show that estimation and classification based on the horizontal standard deviation works quite well. However, the measurements from the previous trial shown in Figure 5 are more difficult to estimate. The scenario from the previous trial, with a walk close to the walls, in between buildings and under a metal sun roof, simply makes the position error harder to estimate. Unfortunately, this is the type of urban environment that we would like to classify with good precision.

Figure 31 and Figure 32 show the estimation and classification results using the average  $C/N_0$ , I/Q magnitude, range RMS value, range residual standard deviation and average carrier lock time, all with a 15 seconds delay, and the standard deviation of the horizontal position, without delay (see Figure 22-Figure 28). Again, rounds two and six have been used for training data, and the position error is estimated for rounds 3-5. The two rounds from the previous trial are not included because these metrics were not stored. A three meter position error threshold has been used for the classification. Note again that the position error is estimated quite well.

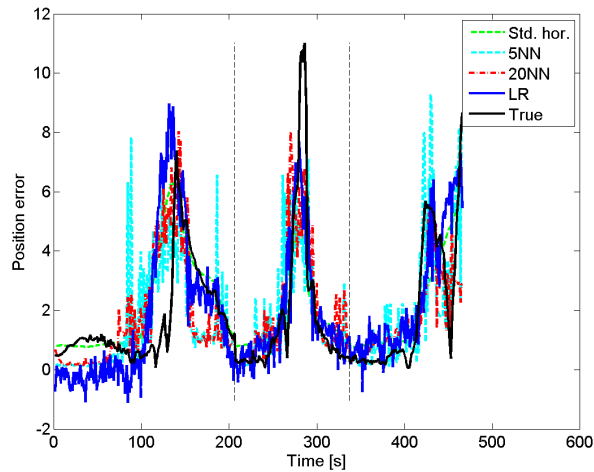


Figure 31. Position error estimates using the average  $C/N_0$ , I/Q magnitude, range RMS value, range residual standard deviation and average carrier lock time, all with a 15 seconds delay, and the standard deviation of the horizontal position, without delay (see Figure 22-Figure 28).

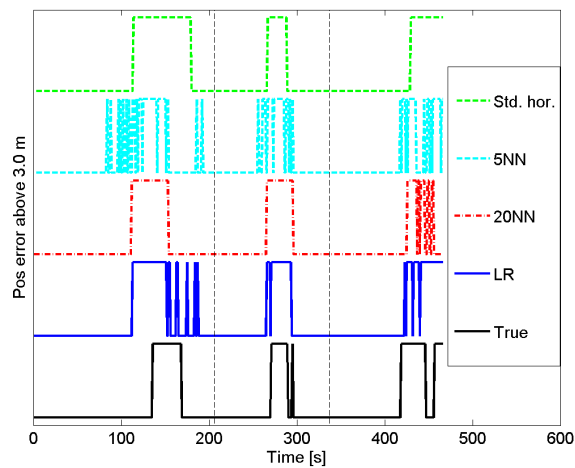


Figure 32. Position error classification using the average  $C/N_0$ , I/Q magnitude, range RMS value, range residual standard deviation and average carrier lock time, all with a 15 seconds delay, and the standard deviation of the horizontal position, without delay (see Figure 22-Figure 28), and a three meters position error threshold.

We have also tried other combinations of parameters, that are not presented here, but we have not been able to see any performance gain as compared to using the horizontal standard deviation only.

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