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Sammanfattning

Högprecisionsnavigering är en nyckelförmåga för både militära och civila tillämpningar. Noggrannheter på ett fåtal meter och även centimeter är önskvärda för, t.ex. navigering i trånga utrymmen och då rörelser ska spåras. De flesta tillämpningar löser detta genom att utnyttja GPS. För mer krävande applikationer kan GPS kombineras med andra sensordata för att förbättra positioneringen och få ett mer robust navigationssystem. Gemensamt för de flesta navigationssystemen är att de förlitar sig på externa signaler som t.ex. GPS-satellitsignaler, kameror eller radar vilket gör dem sårbara. Dessa signaler kan i många fall vara utom räckhåll, inaktiva eller störas ut. En metod för att minimera användningen av externa signaler är att navigera med hjälp av tröghetssensorer. Genom att använda tröghetssensorer för att mäta accelerationer och rotationshastigheter är det möjligt att följa förändringar i position och orientering. Fördelen med denna metod är att i princip behöver bara startpositionen vara känd. Alla andra positioner beräknas sedan från accelerometer och gyroskop data. Dessutom kan sensorerna hållas väl isolerad och i säkert förvar mot yttrestörningar. Få accelerometrar och gyroskop är tillräckligt noggranna för att på ett tillförlitligt sätt navigera över en längre tid. De bästa systemen finns idag i ubåtar och interkontinentala ballistiska missiler som kan nå en precision på ca 100 m efter att ha navigerat i en timme. Däremot, är tröghetsnavigering med prisvärda och mindre sensorer fortfarande beroende av GPS-signaler för att få mellanlägesuppdateringar. Utan dessa positionsuppdateringar kommer navigationsfelet att växa snabbt och resultera i fel på många tusentals meter.

Högprecisionsaccelerometrar och -gyroskop är dyra och stora. Likväl är många tillämpningar i behov av prisvärda, små och högpresterande sensorer. Idag har inte många teknologier potentialen att nå dessa specifikationer och det är nödvändigt att blicka mot nya framväxande teknologier. En kandidat som kan uppfylla alla tre specifikationerna är atominterferometrar i chipformat. Förväntat är att denna teknologi kommer att kunna demonstrera portföljstora sensorsystem med potentialen att tröghetsnavigera i en timme med en felmarginal på bara 2-5 meter. Denna otroliga precision är bortom alla nuvarande tröghetsnavigeringssystem och är nära gränsen för vad som kan uppnås med tröghetsnavigering. Den höga noggrannhet har attraherat nya koncept där det har föreslagits att dessa sensorer också kan användas för ökad situationsmedvetenhet där t.ex. en varning produceras om en stor massa närmar sig.

Atominterferometrar på chip är än i sin linda. Däremot, har större atominterferometrar för gravitationsmätningar nått en hög mognadsgrad och det finns även koncept-experiment som strävar mot tröghetsnavigering. Den höga mognadsgraden inom gravitationsmätningar har producerat minst två spin-off-företag. Dessutom finns det intresse från flera större aktörer som följer teknikutvecklingen och tar patent.

I denna undersökning studerar vi den aktuella statusen för atominterferometertekniken, förväntad noggrannhet, brusfaktorer och sensorutvecklingen. Vårt mål är att ge en överblick över de möjligheter som atominterferometri ger och presentera dess noggrannhet jämfört med andra tekniker.

Nyckelord: Tröghetsnavigering, atominterferometri, nya teknologier

Summary

High precision navigation is a key ability both in military and civilian applications. Precisions of a few meters and even centimetres are desirable when, e.g., tracking movements and to avoid obstacles. For most applications this is solved by using GPS signals. For more demanding applications a GPS signal combined with other sensor data enhances the positioning and makes it more robust. However, because most navigation systems rely on external signals like camera, radar or GPS satellite signals, they become vulnerable. These signals can in many circumstances be out of reach, disabled or jammed. A way to minimize the use of external signals is to navigate by inertial sensors. By using inertial sensors for measuring acceleration and rotation rates it is possible to keep track of the changes in position and orientation. The benefit of this method is that in principle only the starting position is needed to be known. All other positions are then calculated through accelerometer and gyroscope sensor data. These sensors can be kept well isolated and secure. Very few accelerometers and gyroscopes are sufficiently accurate to reliably navigate with high precision for a longer time. Best systems are found in submarines and intercontinental ballistic missiles that can reach a precision of about 100 m after navigating for one hour. More affordable and smaller sensors for inertial navigation still depends on GPS signals to have intermediate position updates, else the error will grow fast often reaching many thousands of meters.

High precision accelerometers and gyroscopes are expensive and large. However, many applications would benefit of affordable, small size and high precision sensors. Not many technologies have the potential to reach these specifications and it is necessary to look at new emerging technologies. A technology that is on the uprise and has all qualifications for reaching all three specifications is chip size atom interferometers. It is predicted that the technology can deliver briefcase size sensors with the potential of inertial navigation with an error of only 2-5 meters after one hour of navigation. This incredible precision is beyond any current inertial navigation system and at the limit of what can be achieved with inertial navigation. Because of the high accuracy it has been suggested that these sensors can also be used for enhanced situation awareness where a warning is produced if a large mass is approaching.

Atom chip interferometer technology is in a very early technological stage. However, table-top atom interferometers are reaching maturity in the field of gravity measurements and there are proof-of-concept experiments striving for inertial navigation. The maturity of atom based gravity measurements has produced at least two spin-off companies. In addition, several other companies are closely surveying the atom interferometer development and taking patents.

In this survey we investigate the current status of atom interferometer technology, expected accuracies, noise factors and sensor trends. Our objective is to give an overview of the potential of atom interferometry and pinpoint its relation in accuracy compared to other technologies.

Keywords: inertial navigation, new technologies, interferometers, atom optics

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1 Introduction

The possibility of measuring acceleration and rotation has lately become of big interest since compact and cheap micro electromechanical systems (MEMS) has been incorporated into cell phones, cars and other systems that do not require high precision measurements. However, the limited precision of these affordable sensors implies that they cannot be used for navigation purposes on their own. If, on the other hand, the inertial sensor data is assisted by external signals, e.g., GPS, the drift can be controlled and reliable navigation can be realized.

For demanding applications where external signals cannot be incorporated into the system, other technologies are better suited. On ships, aircrafts, guided missiles, submarines and spacecraft's, accelerometers and gyroscopes are often based on lasers or on large mechanical systems. The most accurate systems are found in military applications in, e.g., submarines and intercontinental ballistic missiles. These bulky and expensive navigation systems can reach a precision down to 100 m over a one hour mission without using any external signals [1], [2]. As will be discussed in greater detail, this precision is truly impressive and close to the limit of what is possible when navigating by dead reckoning. Further improvements of this class of systems are primarily related to size and cost reduction. On the commercial side, there exist navigation systems that can reach precisions down to an error of a few hundred meters after one hour. However, to enhance the performance, those systems require the use of external signals to continually correct the sensor data.

The road ahead is to develop new commercial high precision technologies that can be miniaturized, cost efficient, highly accurate and stable. The vision is to bring high precision navigation to new GPS denied scenarios where small size and high accuracy are needed. The most promising technology today is based on the wave properties of atoms. The wave property of an atom is a pure quantum mechanical effect which opens the possibility of interfering atoms much like light. Using the wave properties of atoms it is predicted that an inertial navigation system (INS) will be able to navigate with an error of 2-5 m/hr without any external signals [2], [3]. This is an incredibly small error. Furthermore, atom interferometry also offers the potential for miniaturization down to chip size. The long term vision is thus to have affordable briefcase size or even smaller INS that will not deviate with more than 5 m during a one hour mission. This incredible precision will certainly open up new opportunities for navigational applications.

1.1 Purpose

At FOI several studies and projects have been carried out to investigate and acquire knowledge about different navigation scenarios and techniques. These studies are ranging from guided weapons [4], jamming of GPS [5] and robust navigation [6]. For the interested reader there are several reports concerned with integrating and evaluation of small inertial sensors and their performance [7], [8], [9], [10]. Most studies are mainly oriented toward GPS technologies where inertial sensors are used as a support system to enhance the GPS positioning. In such a system the inertial sensors need only be accurate during short time periods, e.g., a few seconds between the GPS updates. Besides GPS there are more local navigation solutions where cameras, radar or lidar are used for identifying landmarks that can pinpoint the position relative to these [11], [12]. All these solutions are based on signals from the exterior of the sensor platform. An external signal can be lost or jammed, making the INS inaccurate.

In contrast to the previous studies, the ambition in this report is different – we aim to investigate inertial sensors based on atom interferometry. This technology is predicted to be a strong candidate for future small high precision inertial sensors for navigation without any external signals. There are many situations where GPS signals are not available. Some common examples are underwater, dense forest, indoor, tunnels and mines. However, it is often possible to rely on other external signals for navigation but there are situation where this is not an option. Such situations might be underwater navigation in high turbidity waters when sonars are prohibited or indoor and underground harsh environments. For example, a fire producing large quantity of soot smoke and heat can limit the range or even completely disable the sensors looking outwards.

The purpose of this study is two-fold. First it is to increase the knowledge of the possibilities, limits and consequences of high precision inertial sensing. Second, we investigate the expected performance and future possibilities that sensors based on atom interferometry might offer. We will discuss the technological potential and limitations for future applications. In the study we do frequently relate to other technologies to get a perspective of the maturity and potential of atom interferometry technology.

1.2 Scope and boundaries

The main objective of this investigation is to study atom interferometry and to understand its limitations and benefits for high precision navigation in GPS denied environment. This includes current state of the art experiments, estimations of what can be expected, technological trends, atom interferometry relation to other technologies, and the technological maturity. To get a detailed overview it is unavoidable that the report will at parts get technical. However, for the reader that is less interested in the technical parts we have included a short summary of the content at the end of each chapter.

The technical parts mostly concern the noise aspect and the physics behind atom interferometry. The noise parameters discussed can be applied to any inertial sensor and gives a measure of the sensor performance. These parameters are important as they are used to classify the application grades of the technologies. There is a range of technologies for inertial navigation and we will not go through these. Only basic properties of some key technologies will be discussed to be able to compare atom interferometry to these.

We have structured the report as follows: To put atom interferometers in an application context we start by giving an overview of the problems involving navigation by dead reckoning in chapter 2. Here, we discuss available technologies, present the technology trends and how atom interferometers fit into the diversity of available sensor technologies. In chapter 3 we present the relevant noise contributions and analysis tools for characterising the sensor signals. In chapter 4 we focus on the physics side of atom interferometers. We go through the basic principles of the technology together with the toolbox of techniques available. All the time we try to relate to light based technologies since these two are very much related. We end chapter 4 with two sections 4.3 and 4.4 on the current status of the research efforts, research programs, patent applications and companies working with atom interferometers for inertial force measurements. Chapter 5 conclude by discussing topics of interest for the military and give some recommendations of future needs.

2 Navigation

Navigation is the art of following a path on a map by determining your own position and direction. The first navigation systems are thought not too rely on scientific instruments and methods but more on memory and observation. It is believed that Polynesian navigation (around 1000 BC) used motions of stars, weather, position of certain wildlife species and the size of waves to find their path between islands. In modern times dead reckoning system using inertial sensors have made it possible to navigate, to some extent, without any external signals. The development of such systems can be traced back to the early 1940's when Germany developed their V-2 rocket (left picture in Figure 1) which carried a simple electro-mechanical gyroscope. In 1953 the Massachusetts Institute of Technology (MIT) Instrumentation Laboratory (now Draper Laboratory) demonstrated the feasibility of autonomous all-inertial navigation for aircrafts with their Space Inertial Reference Equipment (SIRE) [3].

Figure 1: Left V-2 rocket which had a simple electromechanical device for inertial navigation [13]. Right, early nuclear fast attack submarine inertial navigation system, the gyroscope (left box in picture) is spinning at 20000 rpm and remains perpendicular to the axis of earth [14] because of the gimbal suspension and the law of conservation of angular momentum. Measuring the deviation of the submarine compared to the gyroscope orientation gives the submarine orientation compared to earth.

Modern navigation exploits many different techniques; see Figure 2 for an overview of the most important technologies available during the past century [3]. For the past 20 years the maybe most important one is the global positioning system (GPS) which uses synchronized clocks in satellites to estimate the position by trilateration. A drawback of the GPS system is that navigation will depend on external signals from satellites. These signals can easily be lost, jammed or inaccessible for the problem in hand. The benefit with GPS is that it is cost efficient and for civilian application accuracy down to about 20 m for daily use are available [3], [15], [16], [17]. This accuracy can be further improved down to a few meters and even centimetres when used with inertial sensors and Kalman filters to combine signals [3].

One of the most accurate technologies available today is systems based on laser interferometers. For navigation the technology is used in high precision gyroscopes like the ring laser gyroscope (RLG) and the fibre optical gyroscope (FOG/IFOG). These systems have demonstrated excellent stability together with a high dynamic range. Unfortunately price of a unit is relative high and there is a size limitation which causes a trade-off between size and accuracy. The precision of the laser based gyroscope is limited by the wavelength of light where a short wavelength is desirable. The potential to use atoms instead of laser light comes from the fact that atoms, when manipulated in the right way, possess a much shorter wavelength then light. Interferometers based on atom interferometry have the potential of producing inertial measurement sensors that surpass the precision of all technologies used today. Accuracies that allow for GPS impeded navigation with an error of about 2-5 m during one hour are predicted [3], [2]. This incredible precision can be compared with today's inertial navigation systems (INS) where high-end commercial [18] systems have an error of about 1.5 km during one hour. Military INS in submarines and intercontinental ballistic missiles can reach errors of about 100 m after one hour of navigation [2], [19], [20]. High precision sensors are with current technology rather large and miniaturization is often related to a sensitivity decrease. However, size is a very important parameter and if reduced it will open for completely new applications. That size is of importance is seen in current MEMS explosion where sensors are incorporated and available in almost any electronic device. For atom interferometry the long term vision is thus to have affordable briefcase size or even smaller INS that will not accumulate more than 2-5 m of navigation errors during a one hour mission.

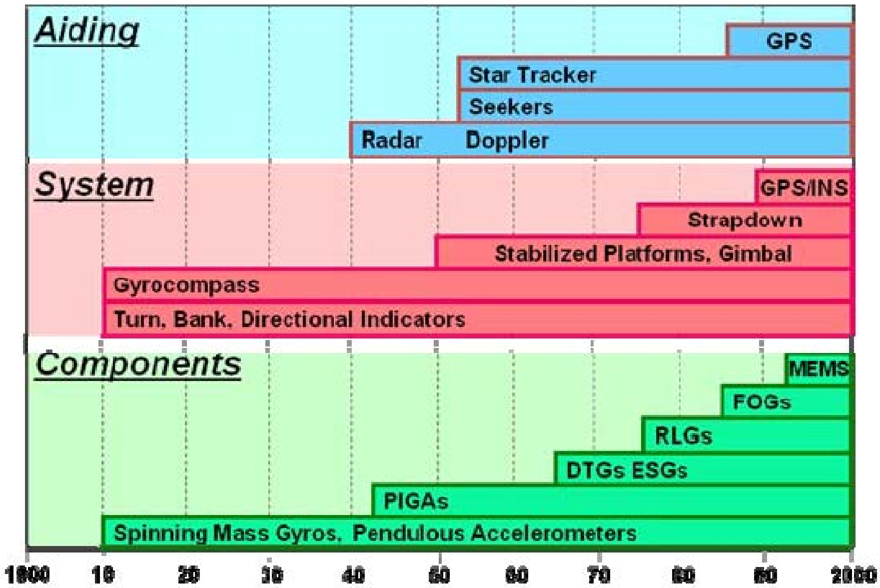


Figure 2: A historical overview of navigation technologies over the past century [3]. Abbreviations in the figure are: Pendulous Integrating Gyroscopic Accelerometer (PIGA), Dynamically Tuned Gyroscope (DTG), Ring Laser Gyroscope (RLG), and Fibre Optical Gyroscope (FOG). Picture is from [3] and used with permission of Neil Barbour [21].

2.1 Navigation by dead reckoning

The principle of navigation by dead reckoning can be summarized as the art of following the course on a map by knowing only a few basic parameters like speed and direction. There are many ways to obtain the relevant parameters for dead reckoning. A particular interesting way is through inertial sensors which basically measures acceleration and rotation rate. The name inertial refers to inertia which is the inherited property of mass to resist changes in its motion. This property is used in accelerometers and gyroscopes to acquire acceleration and rotation rate data.

Dead reckoning is of special interest since it relies only on local measurements. The method only requires data from accelerometers and gyroscopes that can be isolated in a box somewhere on the vehicle where no external signal can disturb it. This means that in principle precise navigation can be accomplish without any external signals like GPS, cameras, magnetometers, star trackers and radar. Thus with proper isolation and containment it will not be possible to disrupt the navigation system since no external signals are used.

For dead reckoning [22] inertial guidance systems the position is estimated from the previous position and velocity estimates together with data provided by accelerometers and gyroscopes. The origin is from classical mechanics and relativity that states that by knowing the local fictitious forces acting on a system (those generated by rotations, acceleration and gravity) the trajectory can be calculated and updated as changes occurs. The parameters required for dead reckoning can be measured by gyroscopes and accelerometers [23]. By keeping track of the acceleration vector $\boldsymbol{a}(t)$ the velocity and the position can be determined by integration

$$\boldsymbol{v}(t) = \boldsymbol{v}_0 + \int_0^t \boldsymbol{a}(\tau)d\tau \tag{1}$$

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + \int_0^t \boldsymbol{v}(\tau)d\tau = \boldsymbol{x}_0 + \boldsymbol{v}_0 t + \int_0^t \left(\int_0^\xi \boldsymbol{a}(\tau)d\tau \right) d\xi \tag{2}$$

where $\boldsymbol{x}_0$ and $\boldsymbol{v}_0$ are the initial values of the position and velocity respectively. The initial values of the position and velocity are usually updated by external signals which reset or enhance the calculated position. In this way dead reckoning is only needed for interpolation between updates. For long term high precision navigation by dead reckoning the initialization is performed only in the beginning of the journey. There are methods which reset some of the initial values during mission operation; an example is zero velocity updates.

In principle only a set of accelerometers are needed for full navigation but these need to be separated quite far from each other to obtain reasonable angular sensitivity. A more accurate and compact method is to use gyroscopes to measure angular velocity or directly measure the angular displacement. Integrating the angular velocity or a direct readout of the angular displacement gives the pitch, roll and yaw.
It should be noted that the gravitational acceleration component will be incorporated in the acceleration measurement $\boldsymbol{a}(t)$. An accelerometer cannot be distinguished between gravitational acceleration from other acceleration components. This means that a gravitational effect will be included in the sensor data and needs to be compensated for else it will offset the data and be added as a noise factor. In most local cases a simple model where it can be considered as a fixed bias that needs to be removed from the sensor readings. However, when precision navigation is required small gravitational changes need to be accounted for. There are ways to acquire the gravity component during operation by having periods of zero acceleration or by simply relying on gravitational maps. These maps are used to compensate depending on the position but they are rough and only cover the surface of the earth with varying precision [3].

An important part of any type of navigation system is time keeping. Inertial navigation is intimately related to time as seen by its presents in the integration step of (1) and (2). Having accurate time stamps for each measurement is crucial [1], [24]. We will not further discuss this issue and assume that there are technologies which can handle this problem. An example that might be useful for timing reference and is relates to atom optics is the newly released chip size atom clock [25].

A complete INS constitutes of three accelerometers and three gyros together with a computer. In early systems a gimbal was used to keep the orientation of the sensors fixed. In Figure 3 a gimbal with sensors are illustrated. The advances of small and powerful computational devices have set a trend in inertial navigation systems towards strap-down systems where the gimbal has been replaced by a computer. The gimbal mechanism is mimicked by a computed coordinate transformation acquired from the gyroscope data and the full motion is captured by calculating the change in the system coordinate frame compared to an earth inertial coordinate frame or other preferred inertial coordinate frame [3], [26].

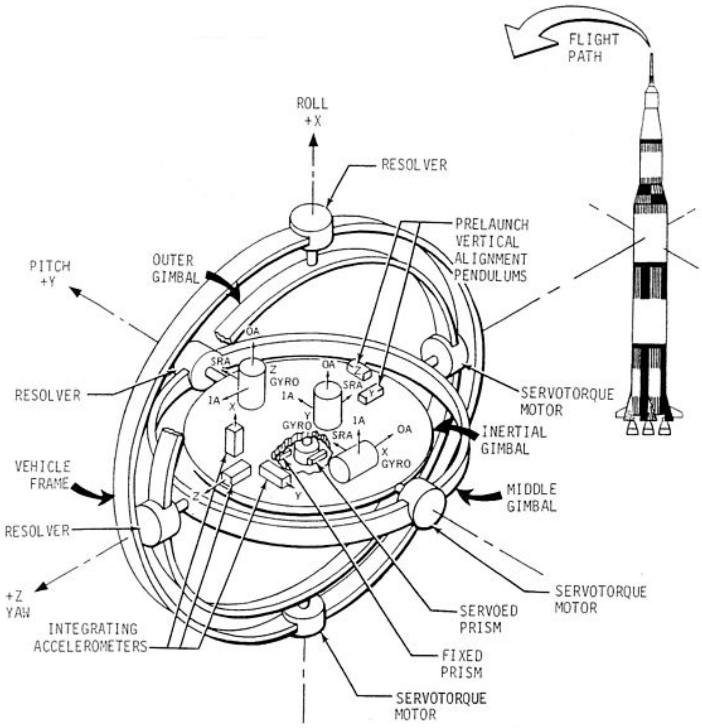


Figure 3: Components needed for realizing an inertial navigation system [27]. The gimbal structure is in today's systems replaced by a computer that keeps track of the orientation instead of having a feedback system to the gimbal that keeps the platform orientation fixed.

Our earth is not an inertial frame since it rotates around its own axis with 15 deg/hr (earth rotation around the sun can be ignored). Depending on the navigation requirements different inertial frames are used. For a global scale "fixed stars" are convenient to use as base for the inertial frame. In a more local level it is in an earth north, east and down none rotating inertial coordinate frame that the integration is most often performed. Earth rotation is then compensated to obtain the true coordinates. A diagram describing an algorithm for a strap-down dead-reckoning navigation is presented in Figure 4. The basic steps are: first integrate the angular rate $\vec{\omega}$, with the initial values from previous run, to obtain the transformation C_I^B between coordinate frames. This is used to transform the accelerometer values $\vec{f}$ to the inertial coordinate frame. From a gravity model the gravity is subtracted from $\vec{f}'$, this gives the acceleration $\vec{a}$ which can be integrated with input of previous runs. From this the velocity and position can be obtained but these are not in our normal rotating earth frame and thus need to be transformed by integrating earth rotation $\vec{\Omega}$. Our discussion only covers the beginning of the schematic shown in Figure 4. The intention is not to give a full account on all detail but to shows that precise inertial navigation requires an intricate mathematics and data analysis. For further details on the navigation models we refer to [28].

The basic principle of inertial navigation systems is captured by (1) and (2) but for a real world implementation of a none external signal aided INS for precise long term navigation one need to consider noise models of the sensor (bias stability, random walk noise, scale factor error, nonlinearities, dead zones) and compensation algorithms for effects such as vibration compensation, temperature, incorrect system initialization, imperfect gravitational model used in the computation and earth rotation. Precise navigation depends today on external signals to compensate and minimize these effects else errors will accumulate and result in a position error of many thousands of meters per hour [29]. The effect of an unknown gravitational component can by itself cause a deviation of several hundreds of meters per hour [29]. Since the INS accuracy sets an upper bound on the mission accuracy various adding systems has been tied into the INS for performance augmentation, this add-ons can be GPS, cameras, velocity meters, seekers, star trackers, magnetometers and lidar [3], see Figure 2 for a historical overview.

We will not go deeper into the subject of inertial navigation by dead reckoning and refers the reader to the comprehensive introductions of the equation of motion, coordinate transformation and algorithms in [1], [26], [28] and [30] for further information. The noise characteristics that will be introduced are, in our context, used to compare different technology performance. These can, later on, be used to estimate the navigation performance. Our goal will be to deepen our knowledge on the technological side of accelerometers, gravimeters and gyros with special focus on atom interferometers.

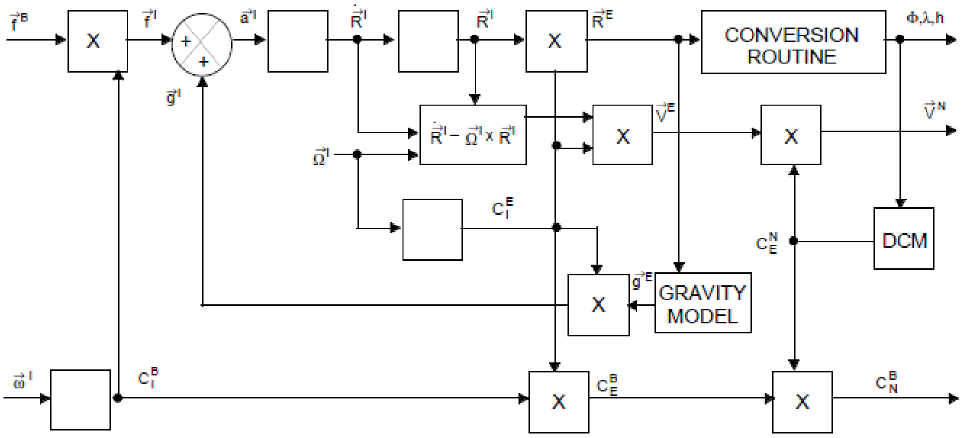


Figure 4: Rotating spherical earth navigation [27]. Here we depict an algorithm for strap-down dead reckoning navigation. It is assumed that we have a rotating spherical earth. Acceleration $\vec{f}$, the gyroscopic angular rate $\vec{\omega}$ and earth rotation $\vec{\Omega}$ are inputs to the model [28]. The outputs are from the top latitude, longitude, altitude (ϕ, λ, h), navigational velocity $\vec{v}^N$ and the north-east-down to body transformation C_N^B. The white boxes are integrations with the initial values given by the previous measurement. Boxes with X are rotations to convert from a non-rotating earth model to a rotating one. The “conversion routine” is to transform positioning coordinates to a more human readable format as latitude, longitude and altitude.

2.2 Gyros and Accelerometers technology trends

Before discussing technology trends it is necessary to introduce two important parameters which are used when comparing technologies. For actual applications other noise factor and performance parameters need to be considered, but the bias stability together with the scale factor stability gives a first estimate. Here we give a rudimentary introduction of these parameters. In chapter 3 a more thorough discussion is presented.

- **Bias stability:** This parameter is important and it is often specified by the manufactures. Basically it tells you how much the output of the sensor can drift when lying still.
- **Scale factor stability:** This parameter quantifies the average error between input and output. It is the amount the sensor deviates from the true value and is usually specified in parts per million.

There is a wide range of accuracies available for both the bias stability and scale factor stability and for that purpose a grading system is often used to classify performance to an application grade [3], [26], [31], see Table 1. The different application grades are generally strongly related to the price and technology used in the INS. A rough unit cost estimation for a commercial graded INS is <7000 SEK (<1 k$), for tactical grade is around 30 k-100 k SEK (4-15 k$) and navigation/strategic graded systems are around 350-670 k SEK (50-90 k$) and above [26].

Table 1: Application grades for inertial sensors [3], [30]. Acceleration is specified in units of the gravitational constant (1g=9.80665 m/s^2).

Application Grade	Gyro Performance		Accelerometer Performance	
	deg/hr	PPM	µg	PPM
Consumer/Commercial	>100	>100	>50000	>100
Tactical	~1	~100	~1000	~100
Navigation	~0.01	~50	~50	~50
Strategic	~0.001	<50	~1	~2

In 2001 the IEEE released an overview of inertial sensor trends which was in 2011 updated and used as material for an ORT and NATO lecturer series [3], [32]. There the authors present predictions on the gyro and accelerometer performance for several groups of technologies. The report emphasizes that electro mechanical system (MEMS) will dominate the consumer market for cheap and small INS. With gyro and accelerometer sensors in most cell phones and cars, this is not a surprising result. However, this shows that there is a big demand for small and affordable sensors.

The far term predictions on the performance of the expected dominant technologies are shown in Figure 5 and Figure 6. It is expected that high precision, with bias stability below 15 µdeg/hr, mechanical systems will be replaced by atom interferometers and that most application using a ring laser gyro (RLG) will be replaced by an interferometric fibre optic gyro (IFOG) [3].

Since 2001 the MEMS based INS systems have increased their performance and it is expected that they will reach the tactical grade. The limiting factor is currently the poor gyro performance which is around 5-30 deg/h. This should be compared to the MEMS accelerometers which have demonstrated tens of µg [3], [33]. For accelerometers the MEMS technology is soon reaching high accuracy and it is expected to compete with mechanical, silicon resonators and quartz resonators technologies in the far term, see Figure 6.

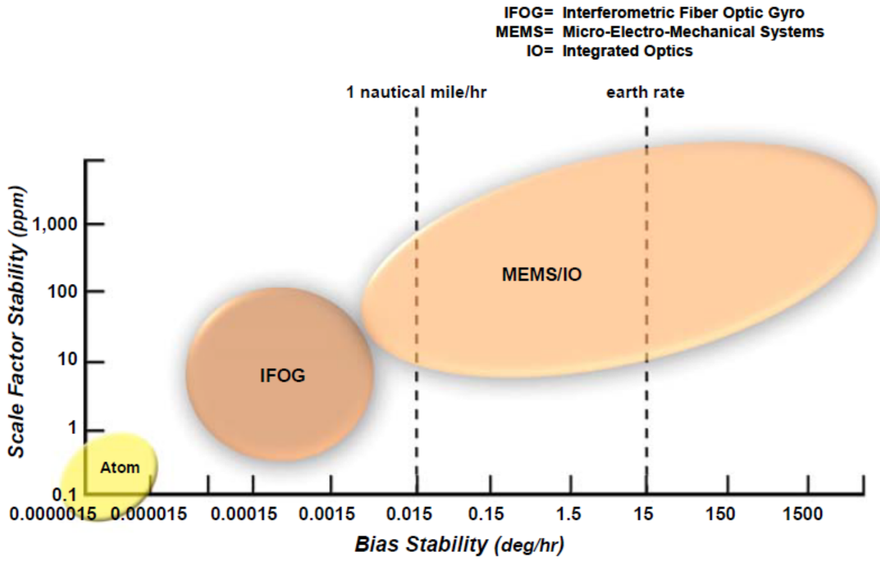


Figure 5: Prediction on far-term gyroscope technology [3]. The technologies are Interferometric fibre Optic Gyro (IFOG), Micro Electro Mechanical Systems (MEMS), Integrated Optics (IO) and Atom interferometer based. As a reference it is good to remember that earth rotation rate is about 15 deg/hr. Used with permission of Dr. George Schmidt.

But reduction in size of the MEMS sensing elements does create challenges because of decreased sensitivity and driving force with an increase in scale factor noise [3]. According to the authors of [3] it is unlikely that the MEMS gyroscope technology based on the principle of Coriolis vibratory (most of the MEMS gyros use this principle) will never be better than tactical-graded. On the other hand MEMS accelerometers (based on resonant accelerometer technology where a frequency shift is measured) have in laboratory demonstrations shown capabilities of 1 µg and 1 ppm under independent testing [3].

High precision mechanical accelerometers are commercially used in gravity measurement application. Gravimeters based on falling corner-cubes and superconducting technology have demonstrated precision down to a few nano g. Commercial atom based gravimeters are now being developed and are soon reaching these high precisions. As of today, atom interferometers cannot yet match falling corner-cubes and superconducting technologies in there long terms stability.

Regarding other technologies there are not too many new sensor technologies that are on the near horizon. Most of the INS sensors are in some stage of the production line, lower right corner of Figure 7 and thus for the next few years it is expected that performance improvements for MEMS technology will dominate. For new technologies that are on its way, upper left part of Figure 7, it is expected that these will meet there production performance goal for a prototype system within 10 years [3]. Most new MEMS based technologies will enhance current gyroscope sensitivity down to 0.1 deg/hr and these are expected within 5 years. Technologies like light force accelerometer and cold atom are expected to take longer time to develop and are closer to 10 year mark. For a full analysis of the market and trends see [3].

The expected technology that will give high precision for both gyroscopes and accelerometers is based on interferometers where the wave nature of matter is utilised. This technology is in its early development stage but is expected to have grate potential both in precision, miniaturization and low maintenance. There is much interest in the technology especially from US and France which conduct research from Yale, Stanford, MIT, Draper Laboratory, U. Arizona, Lockheed Martin, ONERA and CNRS. Both efforts of the US and France have resulted in companies dedicated to atom based gravimeters, µQuanS in France and AOSense in the US. The gravimeter developed by AOSense and Lockheed Martin is hoped to aid with real-time gravity compensation for future submarine platforms [34].

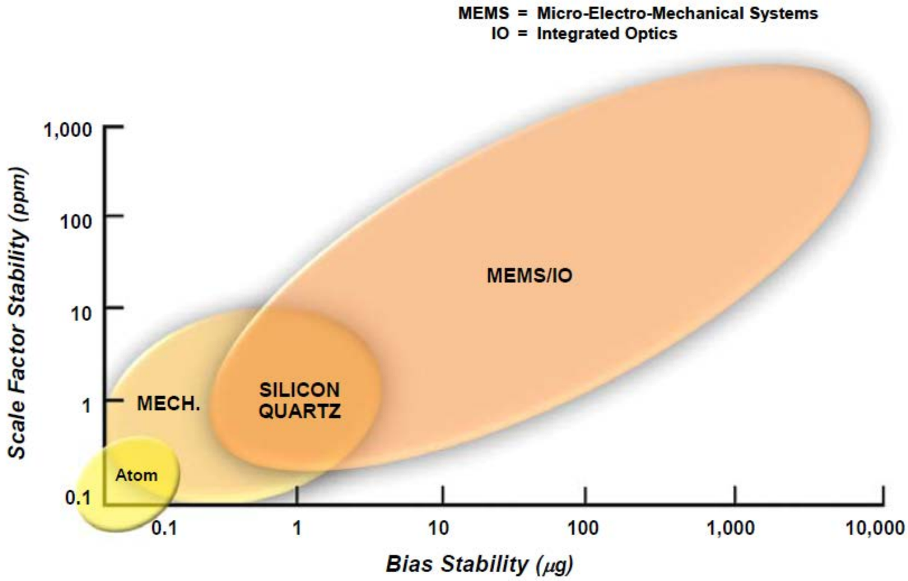


Figure 6: Prediction on far-term accelerometer technology [3]. The technologies are Mechanical (MECH.), Silicon and Quartz technologies, Micro Electro Mechanical Systems (MEMS), Integrated Optics (IO) and Atom interferometer based. Used with permission of Dr. George Schmidt.

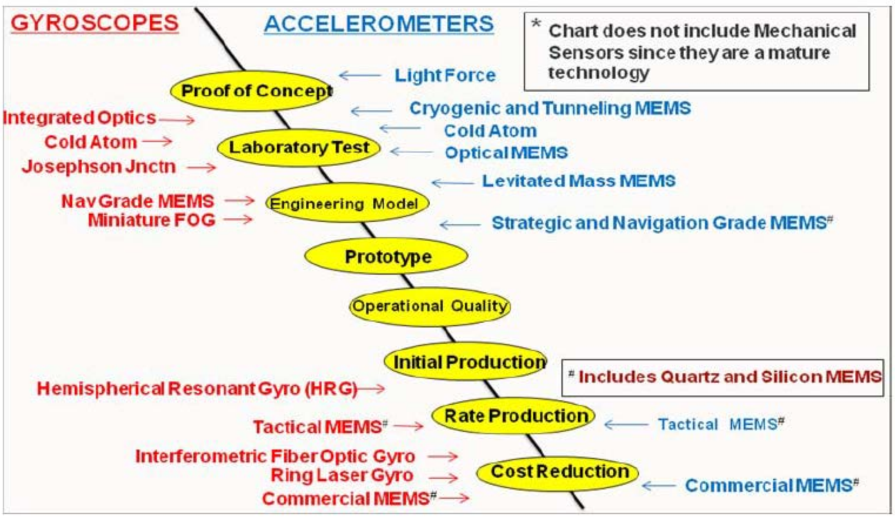


Figure 7: Sensor maturity of several gyroscope and accelerometer technologies. Picture from [3] and used with permission of Neil Barbour [21].

2.3 Summary

In this chapter we have addressed the basic principles of navigation by dead reckoning and technological trend for gyroscopes and accelerometers. In situations when navigation without any external signals is required it is possible to track the path on a map by only relying on accelerations and rotation measurements. Since the technique relies on the inertial properties of mass it is called inertial navigation. This technique is very powerful since if implemented properly it is not possible to disrupt the navigation system with external signals.

There are two ways of implementing an inertial navigation system (INS) and both require three accelerometers and three gyroscopes. Either the accelerometers and gyros are placed on a gimbal that keeps the orientation of the sensors fixed compare to earth, see Figure 3, or the sensors are placed in a strap-down configuration. The strap-down configuration has become more common with the emergence of small and powerful computational devices. In the strap-down configuration a small computer recalculates the sensor data to mimic the gimbal mechanism. For a real implementation there are many noise parameters to consider and these can easily cause navigation errors of many hundreds of meters per hour if not compensated properly.
Two important noise parameters for comparing sensor technologies are the bias stability and the scale factor stability.

- **Bias stability**: tells you how much the output of the sensor can drift when lying still. This parameter is especially notable in MEMS systems where very large errors can be caused by bias fluctuations.
- **Scale factor stability**: is the average error between input and output. It is the amount the sensor deviates from the true value and is usually specified in parts per million.

Because there is a vast amount of sensors a grading system is often used to reflect the application grad intended for the sensor. Table 1 present the grading system. The grading system gives a rough idea of the application and together with Figure 5 and Figure 6 we can get an understanding of the expected future performance for different technologies. Figure 5 and Figure 6 can be interpreted as the expected performance within 10 years. As can be seen MEMS are mostly covering commercial and tactical grade. For accelerometers MEMS are expected to reach navigational grade. For both gyroscopes and accelerometers it is atom interferometry technology that is expected to give super high precision. Not only high accuracy is expected from atom interferometry but also a miniaturization possibility is expected. The long term vision is thus to have affordable briefcase size or even smaller INS that will not deviate with more than 5 m during a one hour mission.

There is much interest in atomic interferometer technology especially from US and France which conduct research from Yale, Stanford, MIT, Draper Laboratory, U. Arizona, Lockheed Martin, ONERA and CNRS. Both efforts of the US and France has resulted in companies dedicated to atom based gravimeters, µQuanS in France and AOSense in the US. The gravimeters developed by AOSense and Lockheed Martin are hoped to aid with real-time gravity compensation for future submarine platforms [34].

3 Noise parameters for gyroscopes and accelerometers

As mentioned in previous chapter noise and drift in the sensor data can easily cause navigational error of several hundred and even thousands of meters if not properly constrained. We will in this chapter introduce the most relevant noise parameters for characterizing gyroscope and accelerometer sensors. Through some examples we hope to give some understanding of how to estimate the navigational error contributions. We begin with some discussion and examples on how errors evolve with time.

3.1 Examples of navigation errors

In the following we demonstrate how errors in the inertial sensor signal and alignment can cause navigational errors. For navigation it is required to integrate the sensor signal. Because of the integration over time the navigational error will be strongly time dependent.

3.1.1 Angular rate error

The sensitivity of an inertial navigation system needs to be very accurate for reliable navigation. For example, a small bias error in the measured angular rate $\Delta\Omega$ will affect the estimated angle θ. This will introduce an error that grows proportionally to the integration time $\Delta\theta = \Delta\Omega \cdot \mathrm{T}$. A positioning error Δx caused by the angular error can be estimated by assuming that the vehicle is traveling in a straight path with a fixed velocity v. The estimated positioning error is in this case roughly $\Delta x = \Delta\Omega \cdot v \cdot \mathrm{T}^2$ [35]. Because the time is squared, the positioning error will grow very fast with a small angular bias error.

3.1.2 Accelerometer error

The same principle as previous example applies for a bias in the acceleration measurement. Integrating twice the acceleration error $\Delta\mathrm{a}$ gives a position error that is about $\Delta x = \Delta\mathrm{a} \cdot \mathrm{T}^2/2$. Thus because of the integration over time the position error will grow as the square of the elapsed time.

In Table 2 we give some rough numbers of the navigational error for four different categories of accelerometers. The four categories represent the groups of intended application that accelerometer can be used for. These estimations are for an unaided INS without a support system such as GPS. If a re-initialization is obtained with for example GPS support then this can keep the error in the cm regime if updates are fast enough. Without a re-initialization procedure the error will grow proportional to at least T^2. The best commercial graded gyroscopes and accelerometers will in matter of a few seconds deviate and accumulate errors of several tenths of meters and degrees. For navigation graded systems the error will be lower and deviate less than a meter after a minute and 1.6 km after one hour.

3.1.3 Error caused by accelerometer misalignment

Another type of error that can occur is a cross-coupling between accelerometer channels. Since the gyroscope data is used to calculate the orientation compared to an inertial frame a misalignment will cause a cross-coupling between accelerometers. This means that a small angle rate error will add to an error in accelerometer data. The cross-coupling positioning error between two accelerometers oriented in the x and y directions can be estimated to be $\Delta x = \mathrm{a_y} \cdot \Delta\Omega \cdot \mathrm{T}^3/6$, where $\mathrm{a_y}$ is the acceleration in y-direction and Δx is the positioning error in x direction caused by the cross-coupling [1]. In this case the error grows as the cube and is directly dependent on the acceleration in the y direction.

Table 2: Overview of typical error of a well calibrated unaided INS [36]. The data follows illustrate equation: $\Delta x = \Delta \mathrm{a} \cdot \mathrm{T}^2/2$. Commercial grade is here divided in two entries industrial and automotive.

	Accelerometer Bias Error	Horizontal Position Error [m]			
Grad	[mg]	1s	10s	60s	1hr
Strategic	0.005	0.025 mm	2.5 mm	90 mm	325 m
Navigation	0.025	0.13 mm	12 mm	0.44 m	1.6 km
Tactical	0.3	1.5 mm	150 mm	5.3 m	19 km
Industrial	3	15 mm	1.5 m	53 m	190 km
Automotive	125	620 mm	60 m	2.2 km	7900 km

3.1.4 High precision inertial navigational errors

Our goal is to investigate atom interferometers that according to the predictions will offer ultra-high acceleration and rotation rate readings. If we look at Figure 6 we see that accelerometer with accuracies down to ng. There are commercial measurement equipment based on falling corner-cubes and superconducting technologies that already exhibit ng accuracy but these are slow systems and require some integration time. Therefore, they are not suitable for navigational purposes.

On the other hand atom interferometry might bridge this gap and offer in the far term ng accuracy for navigation purposes. To put the accuracy that is expected from atom interferometers in a context consider Table 3 where the change in the gravitational acceleration are presented for a variety of masses. From the list we see that a heavy truck 30 m away will change the gravitational acceleration with 2 ng and that only changing the height with 1 cm will cause a change of 3 ng. Thus a passing truck and a slight shift in height give about the same change in gravitational acceleration. Because of this it is fair to assume that there will always be a noise contribution in the ng regime. If we assume that future state of the art INS have precisions in the ng regime then an hour of navigation will result in an error of at least 1m.

For high precision navigation with errors below 1 m or time span longer than 1 hour it will be needed to compensate for tides and other environmental effects which are within the measurable tolerance. To extract acceleration that is not caused by gravity it is necessary to compensate sensor data by using a gravity map that is measured beforehand or by a direct measurement during a zero acceleration period. However, as seen from Table 3 even a passing truck or cargo ship will influence the data. Because of this an accuracy of 2-5 m/hr for inertial navigation might be the limit in a noisy environment like a city. Precise navigation in an environment where heavy objects like trucks or cargo ships move around will need external signals since these object will cause error in the sensor data. On the other hand high accurate accelerometer might find new application area such as in enhanced situation awareness.

For quiet sites where all massive objects are stationary or have predictable trajectories higher accuracy inertial navigation is possible. Submarines, spacecraft's and intercontinental ballistic missiles might candidate for higher accuracy since space and ocean environment is less noisy. Regarding enhanced situation awareness it has been proposed for submarine platforms that by monitoring the gravity and gravity gradient it will be possible to produce a warning of an unexpected mass [37]. From sensor data it is possible to extract magnitude and direction. But before reaching this high accuracy for a complete atom based system it has been considered to use atom interferometers to correct in real time the intrinsic drift of mechanical accelerometers. Such a hybrid sensor can enhance other accelerometer technologies that often suffer from long term bias drifts. According to [38] numerical simulations have been carried out which shows a 25 time enhancement for such a hybrid system.

Table 3: Change in the gravitational acceleration because of surrounding effects [39], (1g=9.80665 m/s^2)

Gravitation source	**Magnitude**
Tides at Stanford	$2 \cdot 10^{-7}$ g
1000 kg, 1.5 m away	$3 \cdot 10^{-9}$ g
Loaded truck 30 m away	$2 \cdot 10^{-9}$ g
Elevation variation of 1 cm	$3 \cdot 10^{-9}$ g
Groundwater fluctuation of 1 m	$5 \cdot 10^{-9}$ g
10^8 kg of displacing salt at 1km	$5 \cdot 10^{-7}$ g

3.2 Allan variance

A powerful method for characterizing some of the error contributions in accelerometers and gyroscopes is the Allan variance [35], [40], [41], [42], [43]. It has gained importance because of its computational simplicity and diversity to characterize the noise of a variety of sensors. This method was developed by David Allan at NIST to characterize the noise and stability in clock systems and in particular to determining the ultimate accuracy of the GPS system. The basic principle is to take a long data sequence and divide the sequence into time bins of size τ. Then take each bin and calculate the average $y(\tau)$ and then calculate

$$\sigma_{\mathrm{A}}^2(\tau) = \frac{1}{2 \cdot (n-1)} \sum_i (y(\tau)_{i+1} - y(\tau)_i)^2 \tag{3}$$

where i is the bin index and *n* is the total number of bins. Repeating this for many τ a log-log plot of σ_{A}vs. τ can be generated, see Figure 8. The output of the sensor is affected by several types of noise where the four most common contributions are from random walk, bias stability, rate random walk and rate ramp [44]. If each noise source is statistically independent of the other then the sum of squares of these four will constitute the Allan variance $\sigma_{\mathrm{A}}^2(\tau)$. In a log-log plot each of the four noise terms have different slopes and can thus be extracted, see Figure 8. Two important parameters that are often discussed are the scale factor stability together with the bias stability. These are the most important noise contributions and are used to classify different technologies. In the following subsections we will present the major noise contributions and their relation to the Allan variance.

In an application there might be other sources of errors that might be of importance. Examples are temperature, calibration, vibration and shock resistance. These parameters are outside of the scope of this report but need to be considered depending on the application.

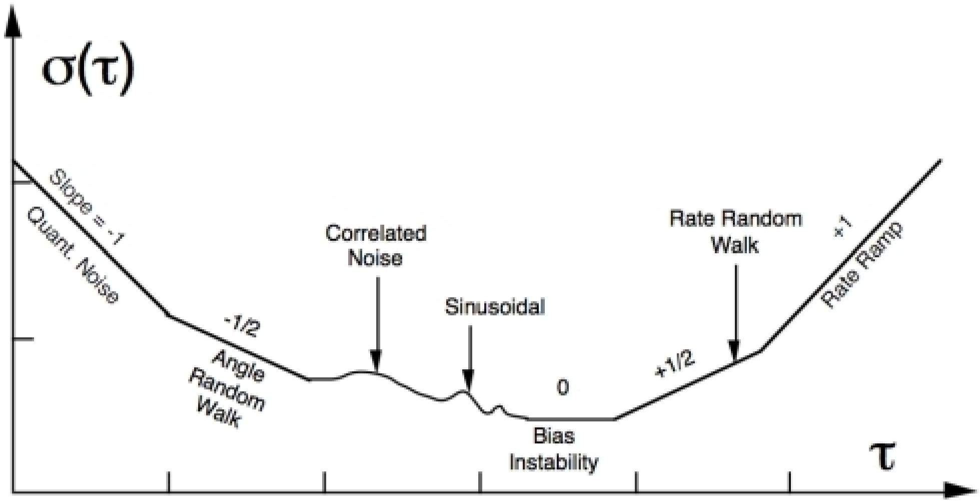


Figure 8: Characteristic behaviour of the Allan variance [45], [13].

3.3 Random walk

For short averaging times τ the Allan Variance is dominated by the noise of the sensor. This has the characteristics of white noise. The slope of the first part of the log-log graph of Figure 8 is directly related to the standard deviation of the short term noise of the sensor. This is also directly related to the angular random walk (σ_{ARW}) which is a parameter for gyroscopes that is often specified by manufacturers. To extract the σ_{ARW} from the Allan variance one can fit a straight line with a slope of -1/2 to the first part of the plot or read out the Allan standard deviation $\sigma_{\mathrm{A}}(\tau)$ at $\tau = 1$ to obtain the σ_{ARW}. The relation to the Allan variance is

$$\sigma_{\mathrm{A}}^2(\tau) = \frac{{\sigma_{ARW}}^2}{\tau} \tag{4}$$

There are several conventions to specify the short term accuracy of an angular rate sensor. Some common specifications are defined through the power spectral density σ_{PSD} in units of $[(deg/hr)^2/Hz]$ or by the fast Fourier transform σ_{FFT} in units of $[(deg/hr)/\sqrt{Hz}]$ and angle random walk σ_{ARW} in units of $\left[deg/\sqrt{hr}\right]$. The relations between these units are [46], [42], [35]

$$\sigma_{ARW} = \frac{1}{60}\sqrt{\sigma_{PSD}} \tag{5}$$

$$\sigma_{ARW} = \frac{1}{60}\sigma_{FFT} \tag{6}$$

The unit $deg/\sqrt{hr}$ for the angle random walk is typically used in navigation and gives an estimate of the angle error after integrating the angle rate signal [42], [35]. Up until now we have only discussed the white noise with respect to gyroscopes but all apply equally well to accelerometers where the unit is often $g/\sqrt{Hz}$ with g being the gravitational constant 9.80665 m/s^2.

As an example of how to use the σ_{ARW} let's consider a commercial graded MEMS (Honeywell GG5300) based gyroscope. This particular gyroscope has an σ_{ARW} of around 0.2 $deg/\sqrt{hr}$. This means that after 1 hour a standard deviation of 0.2 deg of the orientation error is obtained in the angle estimation. After 2 hours it will have grown to

$\sqrt{2} \cdot 0.2 = 0.28$ deg. This increase in the uncertainty will be the same whether the sensor is rotating or standing still. If a gyro is in an airplane moving with a speed of 1000 km/hr in a supposedly straight line, disregarding earth curvature, then after 1 minute the position estimation will have a standard deviation of 8 m and after 1 hour it will have increased to at least 3.5 km.

3.4 Bias stability

As the averaging time τ increases the $\sigma_A(\tau)$ will change until it hits a zero slope part, see Figure 8. This zero slope part is defined by manufacturers to be the bias stability and corresponds to the long term drift of the bias and is given in units of deg/hr. The value of the bias stability describes how the bias may change over a longer period of time [35] [42] [47] and is the best stability achievable with the proper compensation [48]. The bias stability B is related to the Allan variance at the zero slope part of the log-log plot by

$$\sigma_A^2(\tau) \approx \frac{2B^2 \ln(2)}{\pi} \tag{7}$$

Looking at the same gyroscope as previous (Honeywell GG5300) the data sheet specifies a bias stability of < 70 deg/hr. This means that in worst case the bias might fluctuate during the course of time with a standard deviation of 70 deg/hr. The same airplane example as before gives a position error of 340 m after one minute and over 1000 km after one hour. It is common in MEMS systems that the bias stability is greater than the random walk part. Previously in 3.1.2 we did estimate navigation errors caused by the bias stability. In Table 2 we presented examples for the different application grades.

3.5 Scale factor stability

An important parameter that cannot be derived from the Allan variance is the scale factor stability. This quantity is the ratio between the inertial sensor change of the input and output and can be viewed as an overall calibration performance of the sensor. Most often the input is directly proportional to the output. By varying the input over a specified range a linear relation can be obtained. The scale factor is defined as the best slope when varying the input over a specified range. The slope deviation from a straight line is the scale factor stability and is usually specified as parts per million of the best estimated slope [1], [35]. In Table 4 we give a list of the errors acquired for different operational grades of a gyroscope after a 360 deg rotation.

A good gyroscope has a scale factor stability of about $\mathrm{S} = 100$ ppm. If we consider an airplane circling over an area with a rotation rate of $\Omega = 1$ deg/s then the heading error because of the scale factor stability is $S \cdot \Omega = 0.36$ deg/hr. If we assume the airplane is circulating with a radius of 10 km then the error accumulated because of scale factor stability will be 40 m after one revolution. This is a very low number but normally other contributions like the angular random walk will contribute. If we look at our previous example with Honeywell GG5300 the data sheet specify 5 % in scale factor stability which is 500 times higher compared to our example.

Table 4: Angle error for the different grades after rotation a gyroscope 360 deg [36].

Grade	Gyro Scale Factor Error [ppm]	Angle Error [deg]
Navigation	5	0.0018
Tactical	100	0.036
Industrial	500	0.18
Automotive	60000	22

3.6 Summary

To understand performance, accuracy and limitations of gyroscope and accelerometer technologies it is important to understand the noise contributions. In this chapter we began with giving example on how the noise is affecting the navigational performance. The examples show that there are several possible noise contributors and that the error grows as the square or even cube of the elapsed time. In Table 2 we present an example where errors are estimated for the different application grades. There we see how big the navigation error will be after different time steps.

Examples presented are applicable to any accelerometer or gyroscope. However, when considering very high accuracy there will be error contributions from new sources. One contribution that might become of importance for the high accuracy offered by atom interferometers is the change in the gravitational acceleration because of massive objects. This problem comes about because accelerometers cannot distinguish between gravity and the vehicle acceleration. Therefore when very high accuracies are considered there will be a sensor reading contribution from massive objects. In Table 3 we present the change in the gravitational acceleration caused by a variety of masses. A large mass like a heavily loaded truck 30 m away will for example slightly change a very accurate accelerometer. This will limit the navigational performance but can open the possibility of using accelerometer for enhanced situation awareness. It has been proposed for submarine platforms that by monitoring the gravity and gravity gradient it will be possible to produce a warning of an unexpected mass [37]. From sensor data it is possible to extract magnitude and direction which can aid other sensors to identify the object.

We continue the chapter with a detailed discussion of the three relevant noise contributions together with their relation to the Allan variance. The Allan variance is a method for characterising the noise in gyroscopes and accelerometers. The noise parameters introduced are:

- **Random walk:** is the short term noise that sets a fundamental limit on the sensor accuracy. It is normally specified in $[deg/\sqrt{hr}]$ or $[g/\sqrt{Hz}]$ for gyroscopes and accelerometer respectively where g is the gravitational constant. Random walk is mostly used for gyroscopes and gives directly a measure on the angular error. This error grows as the square root of time.

- **Bias stability:** corresponds to the long term drift of the bias and is given in units of deg/hr or g for gyroscopes and accelerometer respectively. The bias stability indicates how much the sensor data can drift over time. In an application this is usually the biggest error contribution and sets the performance limit.

- **Scale factor stability**: is the average error between input and output. It is the amount the sensor deviates from the true value and is usually specified in parts per million.

4 Atom optics

According to quantum mechanics both matter and light have wave and particle properties. It was early predicted that this not only will have impact on science but also a technology impact would come. Some examples of the success because of quantum mechanics are the development of technologies like lasers and nuclear magnetic resonance imaging. With this ever increasing control over particle manipulation we have been able to witness the emergence of new and powerful techniques that offers new opportunities for sensor development. The field of coherent manipulation of atoms is extensive with the potential of being applied in many areas such as precision measurements, atom interferometry and quantum information processing. The later has become the Holy Grail and represents in some sense the ultimate control over individual atoms.

In precision measurements the most notable technology is in time keeping. Atom clocks have shown an extraordinary precision where the best systems today deviate with about 1 second per 10 billion years. This precision assumes that measurements are conducted in a low noise environment like a laboratory. Because of the high accuracy the clock can be used to measure many other things such as (to mention a few) gravity, magnetic fields, electric fields, force, motion and temperature. Techniques used in atom interferometry are therefore intimately related to the atom clock technology. This includes the trapping and cooling of atoms down to few parts per millions above absolute zero and to use the high precision offered by atoms as a sensor for measuring forces.

Inertial navigation requires sensors that detect rotation rate and acceleration caused by the vehicle movement, earth rotation and gravitational pull. Atom interferometers for applications towards sensing inertial forces have the advantage of having no moving parts except for the atom cloud. This provides the potential to low cost and low maintenance [37]. The atomic proof mass ensures identical material properties between distinct sensors and with the progress of atom chips it might be possible to reduce current large setups to small briefcase size systems [39]. As an example, a chip scale atomic clock [25] has been released that is only 4x4x1 cm in size. Since atomic clock technology is very much related to atom interferometry it is expected that the latter may undergo a similar size reduction as the former. Other applications relate to high precision acceleration measurements are for accurate gravity maps [49]. These maps are of importance for inertial navigation but also can be used for geological mapping, determining the height over sea level and in explorations for minerals, oil and gas pockets.

At the present day, most experiments in atom interferometry are oriented towards demonstrating the basic principles which results in rather bulky setups. However, the expected high sensitivity of these systems opens the possibility for new small and better sensors for gyroscopes and accelerometers. This potential has attracted several companies and today they are trying to establish themselves in a new market where the wave properties of atoms are harnessed for high precision sensors. Two companies visible in their efforts are Lockheed Martin and Honeywell. Both have started to invest in the development of both the bulky setups as well as the next logical technological step, i.e., atom chip for miniaturization.

The field of atom optic has grown to be a rich branch of modern physics combining atom physics and quantum optics. The field started from diffraction experiments on crystal surfaces, [50], [51]. Interference patterns from atomic beams have been observed as early as the 1930 and have then grown to a vast field that includes several Noble prices. Two of the more recent prices include the development of atom cooling in 1997 [52] and the discovery of Bose-Einstein condensate 2001 [53] [54].

In the early 20th century atomic beams were developed to isolate atoms from their environment. These techniques are of great importance to the field of atom optics since any interaction with the surrounding will cause a reduction in coherence and will result in diminishing the wave signature of the atoms. The radio frequency (rf) techniques developed by Rabi in 1938 was a big step forward [39], [55]. By addressing internal quantum states of the atom, by means of applying rf pulses, it was possible to create long lived coherent states [39], [55], [56]. These techniques have created or advanced many fields like precise frequency standards, nuclear magnetic resonance spectroscopy and quantum information gates.

Experiments in atom interferometry has been carried out with a long list of different atoms, molecules and even some biomolecules [39], [57], [58], [59]. The list of atoms and molecules used for atom interferometry is still growing. Even though many atom species has been used to demonstrate interference, one need to choose from a limited set of atoms when considering more demanding application. For laser cooling only a hand full of atom can be used, often caesium (Cs) or rubidium (Rb) atoms is chosen.

4.1 Wave properties of atoms

In 1924 Louis de Broglie proposed in his PhD thesis that matter can exhibit wave like properties [60]. He suggested that, like light, atoms possess wave properties, i.e., wavelength and frequency. According to de Broglie theory a particle has a wavelength that is inversely proportional to the momentum p of the particle and its frequency is directly proportional to its total energy E. The two relations proposed in the thesis were

$$\lambda = \frac{h}{p} = \frac{h}{m \cdot v} \tag{8}$$

$$f = \frac{E}{h} \tag{9}$$

here $h = 6.626 \times 10^{-34}$ Js is Planck's constant. These, at the time radical, ideas of de Broglie aided the development of Quantum mechanics in 1925. The first demonstration of atom interferometers where in experiments with electrons and neutrons during the 1950's and 1970's [61], [62], [63]. The choice of using lightweight particles like electrons and neutrons were mostly because of the fact that the de Broglie wavelength scales as one over the mass of the particle used. By these lightweight particles it was possible to obtain wavelengths long enough to observe interference patterns. Low atomic mass was crucial in these first experiments since laser cooling technics where not available to significantly decrease the particle velocity.

4.1.1 Atom wave properties in an ideal gas

We do not observe atom interference in our everyday life. One reason for this is that particles are mostly accessible at relatively high temperature which implies that atoms have very short wavelength. To observe interference it is necessary to cool the atoms to very low temperatures [38].

As an example let us consider an ideal gas. For an ideal gas, the atom speed distribution is described by the Maxwell-Boltzmann distribution. From this distribution one obtains that the mean speed of particles in thermal equilibrium is

$$v_B = \sqrt{\frac{8 \cdot k \cdot T}{\pi \cdot m}} \tag{10}$$

where $k = 1.3806488 \cdot 10^{-23}$ J/K is Boltzmann constant, T is the temperature in kelvin and m is the mass of the atoms. The mean velocity has a standard deviation of

$$\sigma_{v_B} = \sqrt{\frac{(3\pi - 8) \cdot k \cdot T}{\pi \cdot m}} \tag{11}$$

For room temperature single atom gases the mean velocity is around a few hundred meters per second. Thus for most situations the de Broglie wavelength is much less than a nanometre. In other words, the matter wavelength is thousands of times shorter than visible light.

Consider an atom gas of rubidium (^{87}Rb) in room temperature 20 C^o inside a container which is at thermal equilibrium. The atoms constituting the gas will have a mean de Broglie wavelength of about 20 pm with a wavelength spread of 8 pm according to the Maxwell-Boltzmann distribution.

If the gas is cooled down to -273 C^o (0.16 C^o over absolute zero) the mean de Broglie wavelength will be about 735 pm according to (10). Thus a cooled ideal gas has a longer wavelength then a warm ideal gas.

The short wavelength of particles and the fact that in everyday life we are surrounded by a vast variety of particle energies and different particle flavours makes it hard to observe the wave properties of matter. In experiments the velocity and its spread is reduced by cooling procedures that removes high speed atoms or redistribute them to another orientation. In this way the coherence length can be increased and high fringe contrast can be obtained.

4.2 Interferometers

Realizing an atom interferometer requires techniques to coherently localize, split and transfer the atoms between two locations. This is straight forward for photons where lasers, mirrors and semi-transparent surfaces are readily available but for atoms completely new methods are needed.

If we regard atoms as waves then the same interferometry schemes used in light interferometer can be applied to atoms. As for an optical interferometer the following steps needs to be realized by suitable components for an atom interferometer [39] [38], see Figure 9:

1. **State selection.** This will localize the initial state.
2. **Coherent splitting.** Two localized maxima with a well-defined relative phase are created.
3. **Free propagation.** The particles propagate though the interferometer simultaneously as an interaction can be applied to one “arm”.
4. **Coherent recombination.** The relative phase between the arms is converted into state populations though interference.
5. **Detection.** The phase information related to the applied interaction is encoded in the relative particle population between the output arms which is detected.

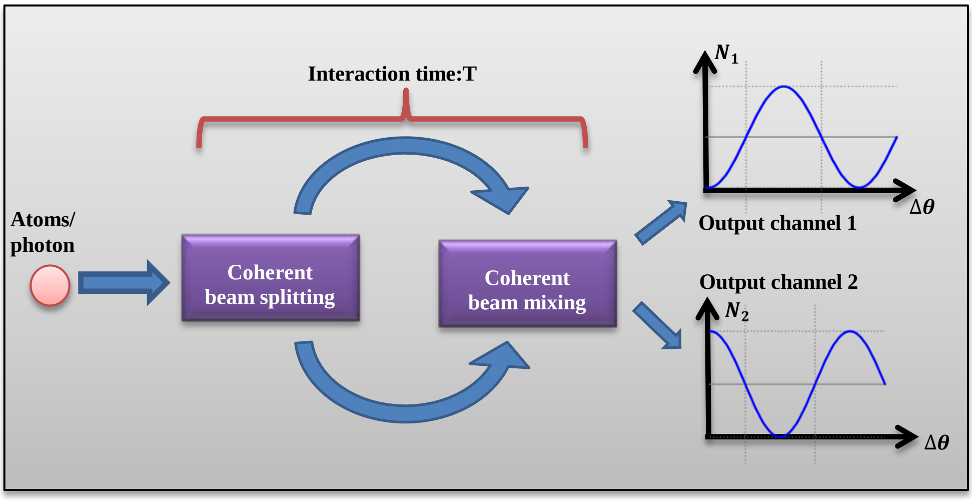


Figure 9: Principle of an interferometer [38]. Particles encounter a beam splitter process that creates superposition between the two interferometer arms. While propagating through the interferometer the two arms can be subjected to various effects which alter the path length between the two arms. After the interaction time T a beam splitter process is again applied which recombines the two arms. The particle population of the output will depend on the path difference of the interferometer arms. Varying the path difference will give rise to a periodic intensity fluctuation in the outputs where the two outputs are each other's conjugate.

4.2.1 Atom and light interferometer comparison

To get a feeling for the accuracy offered by atom interferometers consider a normal Mach-Zehnder interferometer, Figure 10. The illustration in Figure 10 shows a light based interferometer. We are interested in the atom version where instead of a light bulb an atom source is used and all mirrors, beam splitters and detectors are replaced by appropriate components. For now we do not concern our self of how to implement these components. For the interested reader who likes to know the inner workings and physics involved in the atom interferometer see appendix.

Let us assume that we use sodium atoms (Na) in the interferometer. Following the example in [39] assume that our sodium atoms entering the interferometer traveling with a speed of $v = 1000$ m/s. The mass of one sodium atom is $m = 22.99/N_a = 3.82 \cdot 10^{-26}$ kg where $N_a = 6.022 \cdot 10^{23}\text{mol}^{-1}$ is Avogadro's constant. This gives a de Broglie wavelength of $\lambda_{DB} = h/p = 17.4$ pm where h is Planks constant and $p = mv$ is the momentum.

This results in a de Broglie wavelength in the pico metre regime. A wavelength of 17 pm is extremely short compared to visible light which lies in a wavelength regime of 400 nm to 800 nm. Compared to the wavelength of a green laser, which has a wavelength of 532 nm, we see that sodium atoms have a wavelength 30000 times shorter. If we redo the exercise with caesium (Cs) atoms with a velocity of 230 m/s we have an enhancement of 40000 and a de Broglie wavelength of only 13 pm.

In an interferometer it is the relative path length difference between the two possible paths that is measured. To be able to investigate the phase sensitivity, let us consider a shift of 1 rad in an atom interferometer. This requires a path difference between the two interferometer arms of only 2.77 pm. If the path length of each arm is of the order of 10 cm then an interaction that is to induce a phase shift has to occur during 10^{-4}s, which is the time the sodium atoms are inside the interferometer.

Comparing the atom interferometer with the optical interferometer we note two substantial differences. Firstly, the photons propagate through the interferometer very fast; on the order of $3 \cdot 10^{-10}$s. Second, an optical wavelength shift of 1 rad corresponds to a much larger path difference of 84 nm [39]. From this analysis we see that atom interferometer both offers a much higher accuracy and a longer interaction time. Both properties will together enhance the reading of a fixed phase shift.

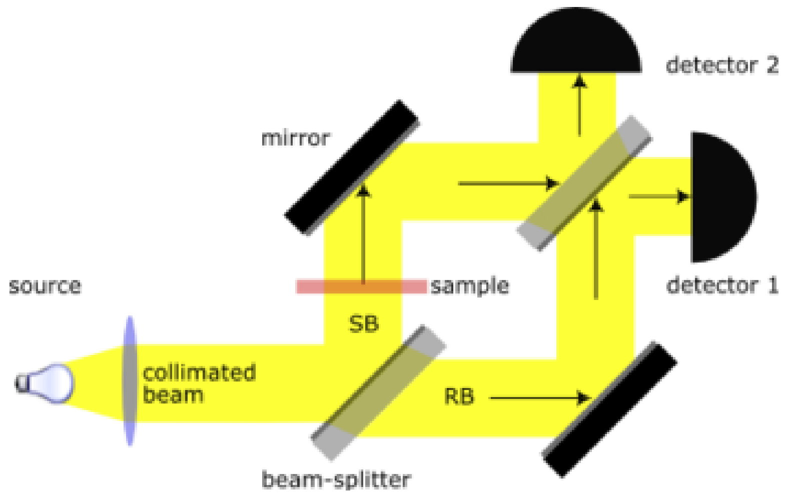


Figure 10: Mach-Zehnder interferometer for light [13]. Light from a source is split by a beam splitter. The two beams are redirected by mirrors and recombined by yet another beam splitter. The two outputs are then directed to detectors where a light intensity variation can be observed because of the path difference of the two interferometer arms.

4.2.2 Atom interferometers

Many different configurations and method are employed to realize the five steps mentioned in section 4.2. The choice of which technique to use is deeply rooted in the application at hand. Many techniques are available for realizing atom sources, mirrors, lenses, gratings, beam splitters, wavelength modulators, waveguides, detectors and amplifiers. For a comparison of light optics components and their atom optic equivalence see Table 9 in appendix 6.2.

A large variety of atom interferometers has been build. We will only mention a few to illustrate the many interferometer possibilities available. There are Mach-Zehnder, Talbot-Lau, optical Ramsey-Bordé, stimulated Raman transition interferometry, longitudinal Stern-Gerlach and longitudinal radio frequency interferometry. A broad categorization of the different interferometer types illustrates well the variety of interferometer configurations available [39]:

1. **Internal state interferometers.** A beam splitter process changes an atom's internal state, analogous to an interferometer based on light polarization states in light optics. Here a common method is to use stimulated Raman transitions to place the atoms in a superposition of different momentum-spin states.
2. **Time domain vs. space domain interferometers.** Beam splitter and mirror operations can be time dependent since they can be realized by laser light.
3. **Near-field (Talbot-Lau) and far-field (Mach-Zehnder) diffractive interferometer.** Near-field diffraction patterns can be obtained by poorly collimated beams but then grater care must be taken in design the gratings.
4. **Separated path interferometers.** Interferometers that have sufficient spatial separation between the arms where an interaction can be applied to only one of the arms.
5. **Freely propagating cold atoms.** These interferometers either drops or pushes out a cold atom cloud with a low velocity which results in a long propagation time in the interferometer.

6. **Trapped cold atom interferometers.** The advances of cold atom on a chip open the possibility of guided cold atom clouds moving on circuits.
7. **Single atom interferometers.** Atoms do react with each other and some nonlinear effects can be observed when tightly confined. Experiments with Bose-Einstein condensate have shown effects such as phase noise and number squeezing.

For inertial sensing the preferred interferometer design in most experiments is a rhombus shaped Mach-Zehnder with beam splitters and mirror components realized by standing waves of laser light, see Figure 11. Caesium (Cs) or rubidium (Rb) atoms are commonly used. This configuration uses a three pulse setup where each pulse implements one component in the interferometer. Depending on the pulse length different splitting ratios of an atomic beam can be obtained. Thus, it is possible by changing the laser pulse length to alternate from mirror to beam splitter. This interpolation is associated with a quantum mechanical phase shift that is often used for indicating the component. For example a mirror is referred to as a π pulse and a beam splitter as a $\pi/2$ pulse. Because of this, the interferometer in Figure 11 is associated with the sequence $\pi/2 - \pi - \pi/2$ and it is commonly referred to according to the sequence. Note also that the time between laser pulses is T and thus is symmetric between beam splitters and mirrors.

Each interferometer design is associated with a specific phase relation that couples to the gyroscopic and acceleration effects. Depending on the design, different relations can be obtained and interactions can be isolated to obtain a sensor that only targets a specific quantity. Hence, different mirror and beam splitter configurations have been tested. An example of a modification of Figure 11 is the butterfly configuration. This configuration is obtained when changing the pulse sequence from $\pi/2 - \pi - \pi/2$ to the sequence $\pi/2 - \pi - \pi - \pi/2$. In this configuration the time between pulses is $\mathrm{T} - 2T - T$. In essence, this means that the atom path in Figure 11 is copied ones to create a second loop in the rhombus shaped interferometer. For the interested reader who likes further information on the inner workings of an atom interferometer and its components please see appendix 6.1.

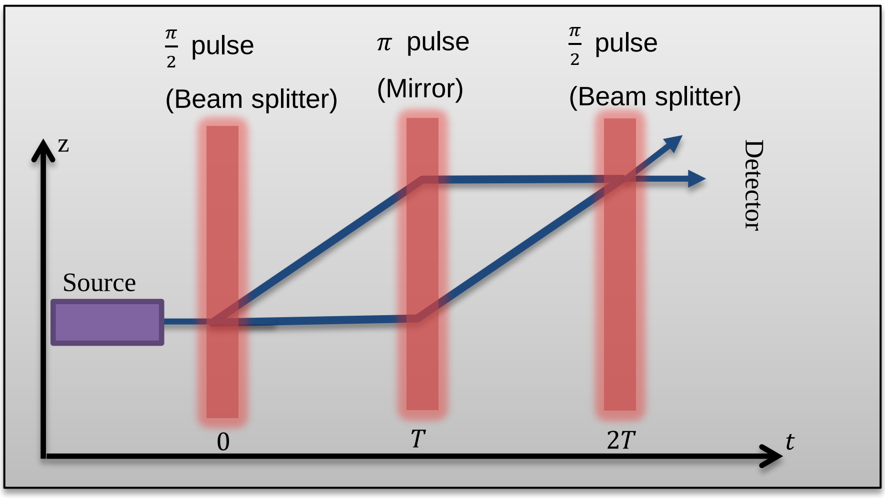
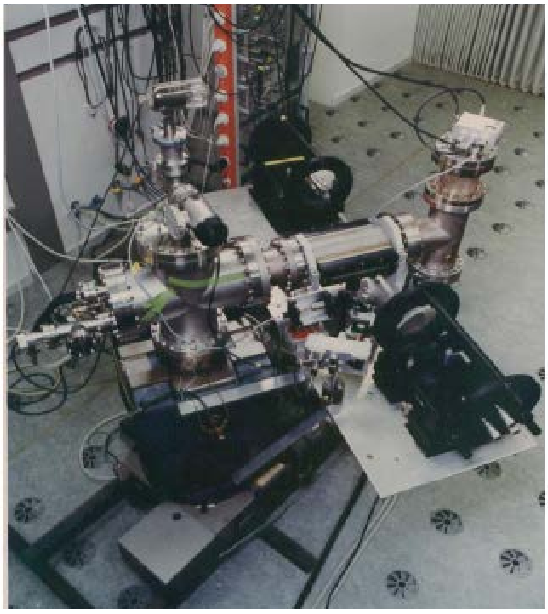


Figure 11: A three grating Mach-Zehnder atom interferometer. An atom beam enters from the left where it encounters laser beams that work as mirrors and beam splitters. The mirrors and beam splitters are commonly based on counter-propagating laser beams where photon momentum recoil is used to split and redirect the atomic beams, see section 6.1.3 for further information.

The first demonstration of an atom based gyroscope was presented in 1991 [64]. In that experiment they used a calcium (^{40}Ca) atomic beam where the atom mean velocity was 700 m/s. Mirrors and beam splitters where constructed by standing waves of laser light. The laser was targeting the transition $^3P_1 - {}^1S_0$ which corresponds to the wavelength $\lambda = 657.46$ nm. This transition has a long lifetime, $\sim$0.4 ms, which is a requirement to have a long interaction time inside the interferometer. In Figure 12 the experimental setup is shown. Note that the interferometer resides inside a vacuum system. The ultra-high vacuum is necessary and it removes the random phase contribution from atom collisions caused by background atoms.
Detection is made by observing, with a photomultiplier, the fluorescence emitted from the excited atoms when decaying back to the ground state. The interference fringe pattern can be observed by sweeping the laser frequency while monitoring the fluorescence intensity, see Figure 12 right picture.

The interferometer comprise of an oven which contains calcium. Through a small hole in the oven the atoms can effuse out creating an atomic beam. The oven is on the left side of Figure 12. Atoms that have effused out from the oven propagate down the vacuum system until they reach the section between the two opposing mirror in black holders. Here the atoms encounter the laser beams that split, redirect and recombine the atomic beam according to the scheme of Figure 11.

The configuration just described, is used in many experiments for studying gyroscopic effects with atom optics. The more straight forward experiments use internal state interferometers where laser light constitute the interferometer components and when applied not only redirect the atoms but also changes there internal state. Detection is often made through fluorescence detection where a laser beam pumps the atoms of one of the output interferometer channels to an energy level that decays by emitting a photon. These photons are detected by a standard pin-diode or camera.



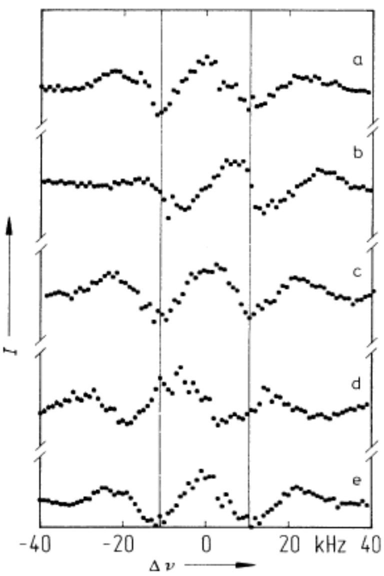


Figure 12: On the left is the experimental setup of the first demonstration of an atom gyroscope [64]. On the right we present phase shift data. Curves a, c, and e correspond to the apparatus is standing still. Curve b is the result when the apparatus is rotating clockwise (-5.1 deg/s). Curve d corresponds to the apparatus rotating counter clockwise (5.1 deg/s). Picture is from [64], [65] and used with permission of Christian J. Bordé.

Experiments have not yet demonstrated high dynamic range and are far from a complete usable atomic INS with many hurdles to overcome. Because of this, most experiments are benchmarking their systems by measuring stable signals like earth rotation for gyroscopes and the gravitational acceleration for accelerometers. There are exceptions where atom interferometer tests have for example been carried out in a 0 g airplane [66]. The closest to a full INS based on atom techniques is the experiment carried out in [67] where they demonstrate a six axis system where all rotational and acceleration components can be extracted, see Figure 13.

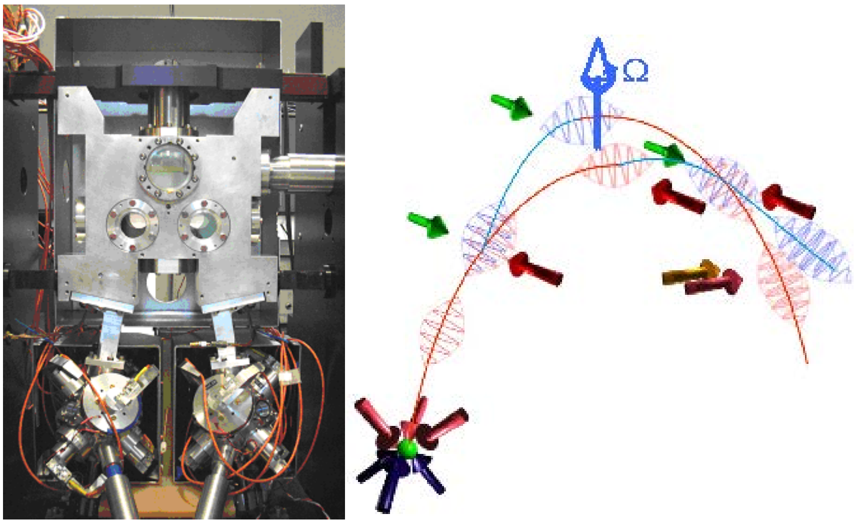


Figure 13: Six axes atom interferometer for full inertial measurement. Left is a photo of the setup. In each container where the orange fibres enter is a magneto optical trap (MOT) that creates a small atom cloud that is launched upwards in a ballistic trajectory. The trajectory and the laser that split, redirect and recombine the atom cloud is shown in the right picture. Pictures used with permission of Arnaud Landragin [68].

4.2.3 Sensing inertial forces with atoms interferometry

In section 2.1 we did discuss that the laws of classical mechanics and relativity limit the local observable fictitious forces to those produced by rotation, acceleration and gravity. These can be measured by gyros and accelerometers. To extract from sensor data the different force contributions of the measured phase it is necessary to have a model of the phase relation. The phase relation will depend on the interferometer configuration. Experiments in inertial sensing with atom optics normally use an interferometer configuration similar to the one illustrated in Figure 11. We will for that reason present the simplest model related to the interferometer configuration of Figure 11. Such a rhombus shaped Mach-Zehnder interferometer has a phase relation that couples to the inertial quantities by [69] , [35]

$$\delta\theta_{atom} = \frac{4\pi m}{h}\mathbf{\Omega}\cdot\mathbf{A} + \mathbf{k}\cdot\mathbf{g}\,\mathrm{T}^2 - \mathbf{k}\cdot(\mathbf{\Omega}\times\mathbf{g})\,\mathrm{T}^3 \tag{12}$$

where $\mathbf{k}$ is the laser light propagation vector, $\mathbf{A}$ is the vector normal to the enclosed area, $\mathbf{g}$ are all acceleration components including the gravitational acceleration, $\mathbf{\Omega}$ is the angular rate and T is the atom propagation time between the interferometer components, i.e., the time between the laser pulses that creates the beam splitters and mirrors. The magnitude of $|\mathbf{k}|$ is deduced from the effective wavelength which is describe in appendix 6.1.3 where we discuss mirror and beam splitter based on laser diffraction.

The first term of (12) is directly related to the angular rate and the second is related to acceleration while the last term is a cross coupling between rotation rate and acceleration.

As seen from (12) the phase shift depends on the orientation of the interferometer. This enables the possibility of measuring all three rotations and all three accelerations for a full inertial base with only one interferometer. The only requirement is that the lasers need to be oriented in the proper direction [67]. The left picture of Figure 13 shows an implementation of such a system. Another consequence is that a proper orientation of the interferometer compared to earth rotation and gravitational acceleration vector can isolate the different parts of (12). In this way the two first terms can be studied separately. In the following sections we study the terms of (12) one by one.

4.2.4 Gyroscope

Here, we will study the first term in (12). This term can be interpreted as the gyroscopic part of the atom interferometer. As seen in (12) this part directly related to rotation rate and is independent of T. Hence, the transit time of the atoms through the interferometer is not important, it is only the area covered between the interferometer arms that is of significance.

It is possible with clever interferometry setups to completely eliminate the second term. The third term will always be present and can only be removed by either have the rotation vector parallel to the acceleration vector or by orienting the interferometer such that $\mathbf{k}$ is orthogonal to the cross product. Experiments usually measure rotations that have a rotation vector parallel to the gravitational component and $\mathbf{k}$ orthogonal to this vector. In this way neither the second nor third term of (12) does contribute. High precision rotation rate measurements can be conducted in this configuration. However, if the setup is oriented in a different way the gravity part will contribute to the phase shift. To compensate for this contribution an accurate gravity measurement is required else it will limit the gyroscope accuracy.
The interferometer configuration used in experiments targeting the first term in (12) are usually quite elongated, 1-2 m long, to have as large area coverage as possible. Since the transit time is not present in the first term of (12) it is possible to have relatively fast moving atoms to gain in signal to noise ratio. A typical atom velocity is around 200 m/s. Best result is obtained with such a configuration, nevertheless, it is desirable to shrink such a setup or combine it with acceleration measurement as in Figure 13.

4.2.4.1 Atom vs light gyroscope

Light interferometers are readily used for high precision rotation sensing. Since atom and light interferometers are very much alike it is fairly straight forward to compare their sensitivity to each other especially in the case of gyroscopes.

Gyroscopes based on laser Sagnac interferometers have demonstrated excellent stability, down to 0.001 deg/hr, and with high dynamic range, as large as 10^6 [31]. These systems rely on a ring configuration where two counter-propagating light fields are interfered with each other, see Figure 14. The two most common configurations are the free space ring laser gyroscope (RLG) and the interferometric fibre optical gyroscope (IFOG) [70]. Both versions are widely used both in civilian and military applications where precision is more important than the relatively high cost of such a system. These devices have proven to be reliable and accurate since there are no moving parts in the design. However, there are limitations to the achievable reduction of cost and size. The size problem comes about since the phase shift of a Sagnac interferometer is proportional to the enclosed area of the interferometer. The phase shift is described by

$$\delta\phi_{gyro}^{L} = \frac{8\pi\mathbf{\Omega}\cdot\mathbf{A}}{\lambda c} \tag{13}$$

where λ is the laser wavelength, c is the speed of light and the other parameters as before. Thus monitoring the phase change of a Sagnac interferometer directly gives a measure of the rotational velocity.

Since (13) is directly proportional the covered area high sensitivity can be obtained by covering a large area. In IFOG's large areas are created by winding up optical fibres. The IFOG size limitation comes about because of the optical fibre bending properties. Common bending radius is on the order of 2.5 cm which sets the lower size limit on IFOG sensors.

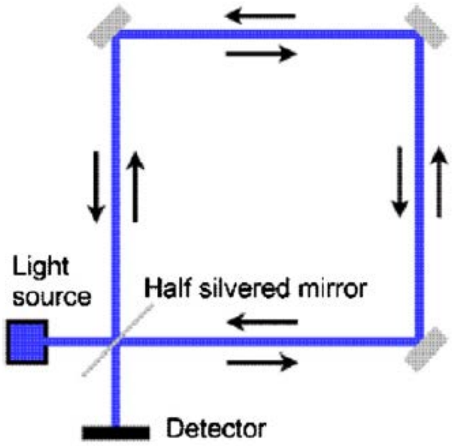


Figure 14: Schematic of the light propagation in a Sagnac ring.

Now, it is of interest to compare a light and atomic interferometer. Since IFOG and RLG are using Sagnac loops we like to have the atom equivalence. A simple way of converting the Sagnac phase relation (13) into its matter equivalent is by a simple substitution of the wavelength λ and light speed c to its matter counterpart. By using the de Broglie wavelength definition (8) and the momentum relation for mass and speed we get [35]

$$\delta\phi^{M}_{gyro} = \frac{8\pi\mathbf{\Omega}\cdot\mathbf{A}}{\lambda_{DB}v} = \frac{4\,m\,\boldsymbol{\Omega}\cdot\boldsymbol{A}}{\hbar} \qquad (14)$$

where $\hbar$ is Plank's constant divided by 2π and the other quantities are defined above. As can be seen in (14) this is the same as the first part of (12) up to a factor of 2 which is missing since the geometry of the two is slightly different. In the Sagnac loop the full phase is given between the clockwise and counter clockwise beams in the loop while in Figure 15 the two beams are recombined at half way of a full loop. With the butterfly laser arrangement $\pi/2\ -\ \pi\ -\pi\ -\ \pi/2$ it is possible to modify Figure 15 to perform almost as the optical Sagnac.

Taking the ratio between the matter-wave phase shift (14) and the optical phase shift (13) we obtain [23]

$$R_{gyro} = \frac{\lambda\cdot c}{\lambda_{DB}\cdot v} = \frac{m\lambda c}{h} \qquad (15)$$

The equation only holds for interferometers of the same area. Inserting typical wavelength for optical gyros ($\lambda\approx 800$nm) and the mass of caesium atoms ($m\approx 2.2\cdot 10^{-25}$kg) we get a phase enhancement of $5\cdot 10^{10}$ for matter-wave gyroscopes. This number suggests an enormous sensitivity increase compare to light interferometers like IFOG or RLG. This enormous expected sensitivity increase is yet not met with today's technologies because of shot noise and area coverage.

The comparison between atom and laser based gyroscopes is not completely fair since for a light based Sagnac the area covered can be made very large by winding up a long fibre. Typically 100 m to 3 km optical fibre is on a spool with a spool diameter of 5-7.5 cm, this gives an effective area of around 30 m^2 compared to the 20 mm^2 areas obtained for the largest atom gyroscopes, thus the IFOG's has around 10^6 larger area. This still implies that matter-wave interferometers have the possibility of being orders of magnitude more precise even if only a small area is covered. Enlarging the area of an atom interferometer is not possible with current techniques, but the progress of atom chip technology may open up this possibility.

In current bulky setups, like the ones in Figure 12, there are several additional restrictions that limit their accuracy. In a Sagnac loop the phase shift is independent of the speed that atoms traverse the loop. Because of this it might seem beneficial to use fast moving heavy atoms to increase the phase shift. However, photon momentum recoils is often used to generate spatial separation between the two paths for the rhombus shaped interferometers. Therefore, the area is roughly inversely proportional to the atomic mass and this makes (14) independent of the atomic mass [35]. Further because of the photon momentum recoil the area of the interferometer, for a fixed length, will decrease with greater atom velocity [35]. Thus by having slow moving atoms a larger area can be covered. But slow moving atoms will have a negative effect on the atom flux and the signal to noise ratio.

Finally one needs to consider the atom vs photon flux passing the interferometer which gives a fundamental limit for the signal to noise ratio, this will be covered in the next section. The result of all dependencies is that the best results for gyros are those with high speed atoms in meter long interferometers.

4.2.4.2 Shot noise limit of gyroscope interferometer

For a laser based Sagnac interferometer, the photon detector is fundamentally limited by the shot noise. For a laser the shot noise is given by $\sqrt{N}$ where N is the average number of photons detected in the optical mode. Together with the number-phase uncertainty relation $\Delta\phi \cdot \Delta N \geq 1/2$ we get a phase uncertainty for a coherent state of the light

$$\Delta\phi \approx \frac{1}{2\sqrt{N}} \tag{16}$$

From (16) and (14) it is possible to deduce the uncertainty in the rotation rate Ω. This uncertainty is given by

$$\delta\Omega^{\mathrm{L}} = \frac{\lambda \mathrm{c}}{16\pi A\sqrt{N}} \tag{17}$$

where N is the total amount of photons detected after the interferometer. Since the atom interferometer obeys the same principles it is easy to convert (17) to the atom version by exchanging the wavelength λ and the speed c to the matter-wave equivalent

$$\delta\Omega^{\mathrm{M}} = \frac{\lambda_{\mathrm{DB}}\mathrm{v}}{16\pi A\sqrt{N}} = \frac{\hbar}{8mA\sqrt{N}} \tag{18}$$

These two equations, (17) and (18) are correct as long as the interferometer axis is aligned with the rotation axis [71]. Applying the formula for typical experimental numbers of 10^6 rubidium (^{87}Rb) atoms in an interferometer with an enclosed area of 8 mm^2 gives an angular error of $2.5 \cdot 10^{-3}$deg/hr which qualifies it to be a strategic graded system but do not beat the best IFOG systems. To beat the best IFOG systems it is necessary to increase the flux with at least two orders of magnitude. To compare with a light based Sagnac consider the photon flux from a 1 mW laser. The amount of photons per second will be in the order of 10^{15}. If a 1km fibre is winded up on a spool of radius of 5 cm then the rotation rate uncertainty will be around $1 \cdot 10^{-3}$deg/hr after one second of integration. As seen this two are comparable even though there area coverage and particle flux are very different.

A large area will not only be beneficial for the phase shift but also for the noise as seen in (18). For that reason the most precise atom gyroscopes are meter long interferometers. An example is the work of [72] where they let the atoms propagate trough a 2 m long pipe. Another example is the work presented in [35]. The results presented is for an atom interferometer gyroscope which can reaches a long term stability of $6.6\text{x}10^{-3}$ deg/hr with an angular random walk of 8.1x10^{-3} deg/$\sqrt{hr}$ with about 10^8 caesium atoms per run. In Table 5 we compare different gyroscope technologies and present the short term and long term stabilities of these. As can be seen from the list one of the better experiments where produced in 2006 [72]. The authors present a very stable and precise gyroscope which according to them might enable navigation with a system drift of less than 1 km/hr. This is still far from what is expected both in size and accuracy but as we have seen there is much potential.

Table 5: List of short term and long term stability for various gyroscope technologies. Gravity Probe B (GPB) is a mechanical angle sensor for low g- environments, Magnetic Hydrodynamic gyro (MHD) is based on a the movement of electrons in a conducting liquid with a magnetic field applied to it, comagnetometer works as a conventional gyroscope but uses atom nucleus as flywheel, superfluid sensors monitor the flow of the superfluid with a Josephson junction. The six last entries are all atom interferometer either with atom beams or atom clouds that are thrown in a ballistic trajectory.

	Short term sensitivity deg/$\sqrt{hr}$	**Stability** deg/hr
On the market MEMS	0.2	31
MHD (ATA)	8.7x10^{-4}	2.1x10^{-4}
GPB	-	10^{-11}
Superfluid	4.8x10^{-4}	1.2x10^{-4} after 1 day
Comagnetometer	1.7x10^{-3}	4x10^{-2}
Fibre optic gyro (Honeywell) [73]	6x10^{-6}	3x10^{-4}
Large ring laser gyro (367.5m^2)	4.8x10^{-8}	7.2x10^{-7} after 10^3 s
Atomic beam	2.1x10^{-6}	7x10^{-5} after 2.7 hr
Stanford 2008 [35]	8.1x10^{-3}	<6.6x10^{-3}
Stanford 1997 [74]	6.8x10^{-5}	-
Yale 2000 [75]	2x10^{-6}	-
CNRS 2006 [67], six axis INS	1.3x10^{-4}	3x10^{-3} after 10 min
Yale 2006 [72] <5ppm	3x10^{-6}	6x10^{-5} after 4.7 hr

4.2.5 Accelerometer

In this section we consider the middle term of (12). In a simplified setup, this term can be interpreted as the accelerometer part of the atom interferometer. From (12) we see that there are two terms related to acceleration. The simplest phase relation is obtained when no rotation is present. In this case the third term cancels and only the second term remains. If the laser propagation direction is parallel to the acceleration then Figure 15 illustrates the curved path through the interferometer. This will create a phase shift

$$\delta\theta_{atom,Acc} = \mathbf{k} \cdot \mathbf{g}\, \mathrm{T}^2 \qquad (19)$$

where **k** gives the direction of the laser based beam splitter and mirrors, **g** is the sum of all acceleration components and T is the time between the laser pulses which corresponds to the components in the interferometer. The presence of T^2 indicates a strong dependents in the atom transit time. For that reason most accelerometer experiments are using slow moving atoms to increase the transit time and obtain a large phase shift [39].

Equation (19) is fairly intuitive if we consider a falling object when only gravity is involved. A falling object will only be affected by a constant acceleration towards ground. The travelled distance from its drop point will be given by $g \cdot T^2/2$. To convert the travel distance to radians of wavelength we only need to multiply by $|\mathbf{k}| = 2 \cdot \pi/\lambda$.

For acceleration measurements slow moving atoms gives the best results since the phase is proportional to the atom transit time through the interferometer [39]. Most interferometer for accelerometers constitutes of a cold atom cloud that is launched, with a velocity of a few m/s, in a ballistic trajectory that is around 30 cm high.

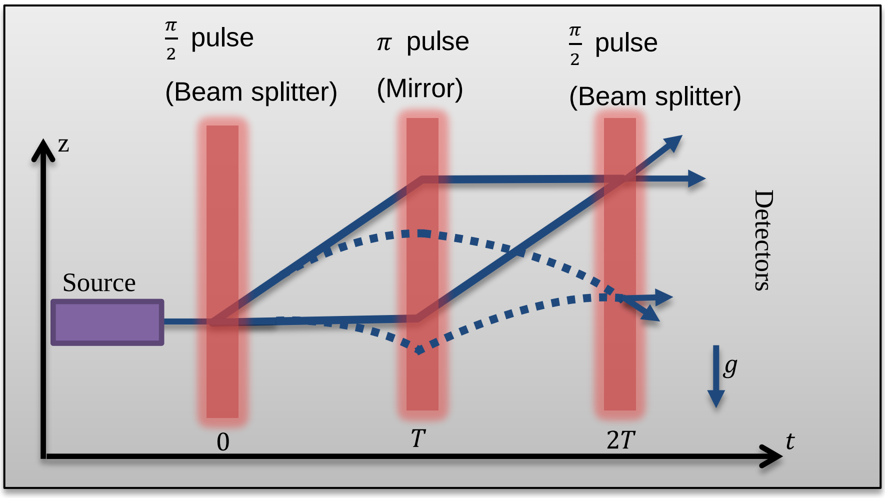


Figure 15: Atom interferometer. Sharp lines indicate the atom path when no inertial forces are acting on the atoms. Dotted lines indicate when for example gravitational forces g alter the atom path which results in a path difference between the two arms.

These slow moving atoms are created by trapping atoms in a magneto optic trap (MOT). In the MOT a small cloud of very cold atoms is created, see section 6.1.2.2 for more details. Temperatures of a few µK down to nK are reached by laser cooling. The cloud is stationary and is held in place by laser light and magnets inside a vacuum chamber. Figure 16 illustrated the laser configurations together with the magnets. By turning off the magnets and changing the frequency of the laser it is possible to launch the cloud in a ballistic trajectory with a speed of a few m/s. It is also possible to let it free fall for gravity measurements. The atom trajectory will be the interferometer. During its flight a sequence of beam splitter operations and mirror operations are applied to the ballistic cloud. These will produce the interferometer in Figure 15.

There are some drawbacks of trapping atoms in a MOT. Two of them are that there is a loading time of typically a few hundred milliseconds and because of the cooling process the amount of atoms trapped will be limited to about 10^6. This will have an impact on the possible output data speed and signal to noise ratio. On the other hand cold atoms offer long interaction times and better interference fringe contrast compared to atom beams.

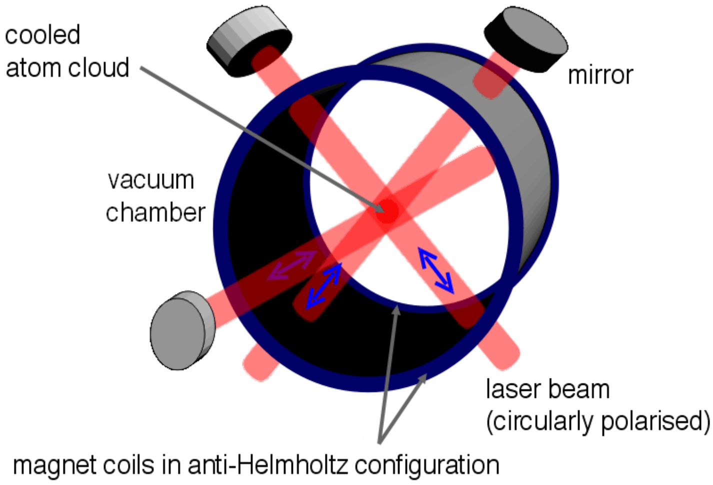


Figure 16: The laser and magnetic coil configuration of a magneto optical trap (MOT).

4.2.5.1 Atom accelerometer vs optical accelerometer

Comparing atom based and optical based accelerometer can be done in several ways. A big difference between atom and light based interferometer are that atoms fall in a gravitational field while photons which are the light particles do not. This means that atom interferometers are directly susceptible to acceleration. Equation (19) does not include any atom wave properties like in the gyroscope case making it harder to convert in a similar way. The simplest comparison is the ratio between the optical wave length and the atomic wavelength. We have already seen that this gives an enhancement in the order of 40000 times. This unfortunately does not show the enhancement of a specific experimental configuration.

A comparison where more components are present has been made by John Clauser but the setup that he describes is not used in inertial force experiments [23]. We present here his result mostly to show that a similar enhancement as for the gyroscope case is expected for accelerometers. According to [23] the ratio between an optical and an atom accelerometer is given by

$$R_{accel} = \frac{2m\lambda c^2}{vh} = \frac{2 \cdot \lambda \cdot c^2}{\lambda_{DB} \cdot v^2} \tag{20}$$

where λ and λ_{DB} is the photon and matter-wavelength respectively, c and v is the speed of light and the particle speed respectively. In (19) there is a T^2 dependence which is here converted to a v^2 dependence.

Equation (20) capture the sensitivity ratio between a laser based and an atom based accelerometer. Inserting typical laser wavelength ($\lambda \approx 800\text{nm}$) and the mass of caesium atoms ($m \approx 2.2 \cdot 10^{-25}\text{kg}$) with the assumption that the atom cloud has a speed of 100 m/s we get a phase enhancement of 10^{17}. This is an enormous enhancement but as for the gyroscope case this is compared to an equivalent laser interferometry system. As before there are many factors that limit the atom interferometer to reach this enormous enhancement. In Table 6 we present accuracies reached with different technologies. From Table 6 we note that commercial systems for gravity measurement can reach below ng. The limitation to gravity measurement is mostly because of the long integration time that is often in minutes.

Table 6: Below is a list of technologies for precise gravitational measurements. Spring/mass system uses a well characterized spring that support a test mass against gravity. Cryogenic system uses a superconducting sphere that is levitated in a magnetic field; a change in the field is counter balanced by force feedback. Falling corner cube is a system where a free falling corner cube reflector is monitored by a laser interferometer. µQuans and AO sense are two companies which are selling atom interferometer based gravimeters; see section 4.4 for further information. The last seven are experiments with atom interferometers.

	Short term sensitivity $g/\sqrt{Hz}$	**Stability** g
Spring/ mass system [76]	$1 \cdot 10^{-10}$	-
Cryogenic gravimeter [77]	$0.3 \cdot 10^{-9}$	$0.05 \cdot 10^{-9}$ after 1 minute
Falling corner-cube [78]	$15 \cdot 10^{-9}$	$2 \cdot 10^{-9}$
µQuans	$30 \cdot 10^{-9}$	$1 \cdot 10^{-9}$ after 15 minutes
AO sense	$1 \cdot 10^{-6}$	$< 0.1 \cdot 10^{-6}$
Stanford 2001 [76], [79]	$2.3 \cdot 10^{-8}$	$3 \cdot 10^{-9}$ after 1min
Yale 2002 [80]	$4 \cdot 10^{-9}$	-
CNRS 2010 [81]	$1.7 \cdot 10^{-7}$	$< 5 \cdot 10^{-9}$
ONERA 2013 [82]	$1.2 \cdot 10^{-7}$	$2.5 \cdot 10^{-8}$
CNRS 2006 [67], six axis INS	$4.7 \cdot 10^{-7}$	6.4$\cdot$ 10^{-8} after 10min

4.2.5.2 Shot noise limit of accelerometer interferometer

Here we do the same analysis as for the gyroscope case where the number-phase uncertainty relation was used to couple the phase uncertainty with the number of atoms, see equation (16). Given (19) the shot noise is

$$\Delta g = \frac{1}{2\sqrt{N} \cdot \cos(\theta) \cdot k \cdot T^2} \tag{21}$$

Here θ is the angle between $\mathbf{k}$ and $\boldsymbol{g}$. For a typical experiment the MOT can contain about 10^6 atoms, T is in the tenth of millisecond regime and k is about $6 \cdot 10^9$. Assuming that $\theta = 0$ then Δg is about 0.1 ng which is not too far from the best experiments, see Table 6. The expected far term accuracy of atom interferometers according to Figure 6 is about 50 ng and according to the company µQuans their next generation gravimeter will strive for an accuracy of 0.1 ng. However, these systems are rather slow and take several minutes to reach these high accuracies. The challenge for future experiments is to increase the bandwidth and shrinking their setups.

4.2.6 Atom chip technology

Up to now we have reviewed the most mature technologies for inertial sensing with atom interferometers. They are not mature for commercial use and have many unsolved problems before they can start competing with commercial INS products. These techniques are today not compact (meter long interferometers is customary) systems but there are promising techniques that are under development.

The atom chip has the potential to make the bulky free propagating atom technology more compact, see Figure 17. An example is the work by Wu [83] which demonstrates an atom chip where the atom cloud is guided in a figure 8 path of only 0.2 mm^2. By gaining control of the number of possible loops and speed, larger areas can be covered without losing signal to noise ratio.

Most of the research is still oriented towards the more basic properties of the atom chip. Before implementing a gyroscope or accelerometer questions about how to guide the atoms, keep the coherence, create atom beam splitters, atom mirror, compact laser system and cooling system need to be solved. An example of work in this direction is the work of a group in Vienna 2013 that demonstrate a full Mach-Zehnder sequence [84].

Several groups are striving for the development of a compact and accurate atom interferometer [71], [84], [85], [86]. A proposal, by a French team at CNRS [87], suggests constructing an atom chip with a Sagnac atom interferometer that is only 500 µm in radius. Their design will be able to store an atom cloud in the Sagnac ring for as long as 1s. With such a long interaction time a sensitivity of $6 \cdot 10^{-7}$ rad /$\sqrt{s}$ is expected with an interferometer area of only 1.1 mm^2. The potential for the atom chip is vast but isolating and manipulating the atom cloud has shown to be demanding and many hurdles need to be solved.

Despite the many technological problems and difficulties, the potential of high accuracy chip size INS has attracted several companies. Both Lockheed Martin [86] and Honeywell are active in the field and are taking patents on both atom chip technology as well as more bulky systems.

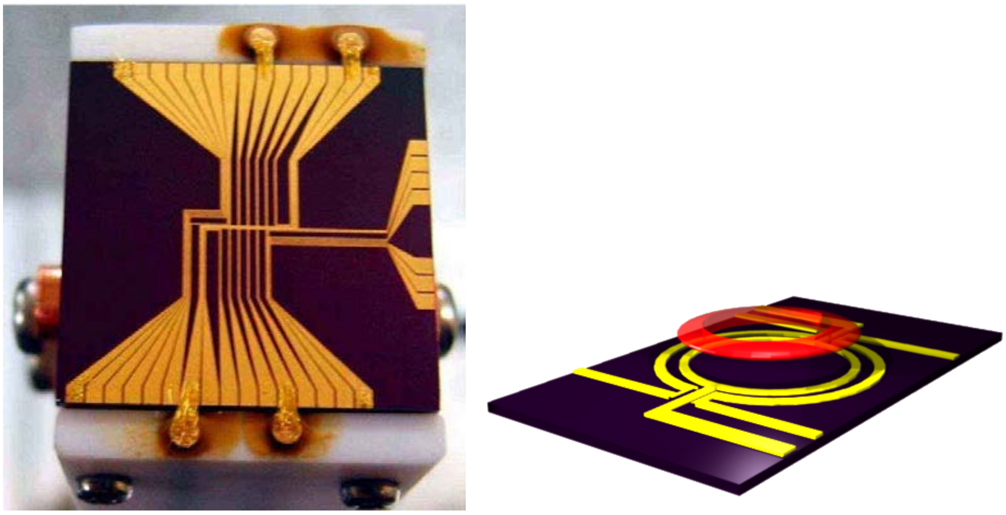


Figure 17: Left atom chip circuit which can hold an atom cloud [88]. Right an illustration of an atom chip under development by a CNRS team [87]. The red ring-like structure indicates trapped and guided atoms that float above the surface of the chip. Pictures used with permission of Arnaud Landragin [89].

4.3 Research programs

Many countries are and have invested in the development of technologies related to atom cooling, trapping and coherent manipulation. The technology is slowly getting closer to commercial applications but still the universities and research institutes are the dominating actors.

Experimental efforts in Sweden are very low. For a short period of time Umeå University had a cold atom trap. Today as far as we know there are no research programs targeting experimental efforts towards atom interferometers, cold atoms gases or atom chips. There are some efforts in the field of slowing light and matter manipulation in bulk materials in Lund but this is more oriented towards quantum memories.

Looking beyond Sweden there are, as mentioned, many groups. France and USA are especially ambitious in their efforts for applications towards inertial navigation with atom interferometers, with several dedicated groups working in the field. The Stanford group led by Kasevich [69] is oriented towards cold atoms. The teams led by Landragin and Bouyer in France conduct research both in cold atoms and gyroscopes with atom beams [90] [91]. But the techniques are similar to those in other research groups. All fields utilising coherent manipulation of atom gases are potential candidates for applications of atom interferometers. Potentially their knowledge can be applied towards atom inertial navigation systems. Germany is very strong in cold atoms, optical lattices and atom chips. The same applies for Switzerland, Austria, UK, as well France and USA which have many groups working in atom traps for coherent atom manipulation [92].

If we look closer at USA there are several research programs targeting atom interferometers. DARPA is running their High Dynamic Range Atomic Sensors program (HiDRA) where the atom interferometry company AOSense [93] is participating. There is also Chip-Scale Combinatorial Atomic Navigation (C-SCAN) and the Quantum-Assisted Sensing and Readout program (QuASAR). Besides DARPA there are other agencies interested in this sensor technology, Air-Force, NAVY, NASA are a few. In most programs the company AOSense is involved and it seems that they are one of the driving forces in these programs.

4.4 Companies working with atom interferometry inertial systems

Already in the early stage of the scientific progress it was realized that the concept of atom interferometry had potential. This new emerging technology inspired Saul Altshuler and Lee M. Frantz in 1972 to file a patent on an atom interferometry apparatus [94]. John Clauser did during the late 1980’s investigate inertial sensing with atom interferometry [23]. He considered the patent of Saul Altshuler and Lee M. Frantz to be impractical and filed a patent of his own in 1991 [95].

It seems that during the 90’s not many patent where filed. Most likely the technology was not mature and the basic concepts where already patent. But after 2005 a new trend in patent application started to emerge. Patent for atom interferometers, cold atom traps, atom chips, atom gyros, atom accelerometers, methods for error feedback with other INS started to appear.

A recurring company in the patent search is Honeywell. The company is now taking many patents related to atom interferometry. This year (2014) a patent was filed regarding correcting MEMS systems with intermediate readings from the supposedly lower update rates but very precise reading from an atom interferometer. In 2007 they filed a patent for an atomic chip inertial sensor and in 2013 a patent for an atom interferometer was filed. The French research teams are also patenting there techniques, an example is Philippe

Bouyer who has filed several patent for gravimeters with trapped atoms. A short list of patents which are directly related to INS is presented in Table 7.

It looks like the development of atom interferometry has reached a critical stage where the maturity of the technology is high enough for commercial systems to be launched. The simplest and most mature atom interferometer technology has found applications in precise gravitational measurements. Because of this, there are at least two companies selling atom based absolute gravimeters.

Table 7: A short list of patents that are directly related to atom interferometry and INS

Year	Short description
1973	Atom interferometer (SU 3761721) Altshuler
1991	Atom interferometer (US 4992656) Clauser
1999	Atom interferometer (US6314809) Lockheed Martin
2006	Atom chip (EP1733414 A2) Anderson
2006	Atom chip (US7126112 B2) Anderson
2007	Atom chip (EP 1 865 283 A1) Honeywell
2008	Atom interferometer (US8347711 B2) Lockheed Martin
2009	Atom chip (EP2199832 A1) Thales
2009	Atom interferometer (EP2104406 A1) Bouyer
2010	Atom chip (US 20100320995 A1) Tal
2013	Ultracold-matter systems (US 8405021 B2) Anderson
2013	Error feedback (EP 2 629 053 A2) Honeywell
2013	Atom interferometer (US 8 373 112 B2) Bouyer (CNRS)
2013	Atom interferometer (US 2013/0152680 A1) Honeywell
2013	Atom chip (CN102944903 A) Chines company
2014	Optical Error feedback (EP 2 685 212 A2) Honeywell
2014	Feedback MEMS and AI (US 2014/0022534 A1) Honeywell

For example, the company µQuanS offers an absolute gravimeter [38]. The company started 2011 and is a spinoff of two French laboratories. A picture and specification of their gravimeter can be found in Figure 18 and Table 8. The company is working on a second generation which will be fully autonomous with a long term goal of a sensitivity of 1 ng within 100 s with a lower sensitivity bound of 0.3 ng [96] [97].

Figure 18: Left, µQuanS [98] commercial system that measures the absolute value of the gravity by using a cold atom interferometer [38]. Right, AOSense [99] vertical gravimeter that Lockheed Martin purchased in 2011 [37], ARW$\approx$ 1µg/$\sqrt{Hz}$, stability $<$ 0.1µg.

Table 8: Specifications on the µQuanS gravimeter system.

Typical sensitivity	$30\ ng/\sqrt{Hz}$
Ultimate sensitivity	$1\ ng$ in 15 minutes
Accuracy	$2-3\ ng$
Power consumption	$300\ W$
Wight of sensor head	$25\ kg$
Wight if electronics	$35\ kg$

Another company developing accelerometers for gravitation measurements is AOSense. The company is a spinoff from Standford University and is involved in a project with the aim of developing gravimeter for use as a submarine navigation enhancement system [37]. Their product is shown in Figure 18. This project was a continuation of the Lockheed Martin and DARPA project sponsored by the strategic system program navigation branch. The company was founded in 2004 and had (2013) a staff of 39 with a large R&D department. The Stanford team has designed several atom interferometers for gravity measurements both for the gravitational strength and gradient measurements of the field.

The two companies presented are dedicated to INS but if we widen our perspective and also look at relevant and related emerging components, there are several interesting examples. For instance a cold atom kit is available from the company ColdQuanta [100]. They also offer Atom Chips, ultracold matter cells for creating exotic atom states like a Bose-Einstein condensate. The concept ColdQuanta follows is to provide simple cold atom systems to researchers, companies and universities. With these tools the obstacle of creating vacuum chambers and trapping atoms has become easier. The availability for trapped atoms has increased enormously with ColdQuanta’s component based approaches.

Figure 19: The mini MOT Rubidium cell from ColdQuanta [100]. This cell is already pre pumped and contains the ion pump to reach ultrahigh vacuum. With proper laser equipment and cameras a rubidium cloud can be formed with a temperature of 50 µK.

4.5 Summary

In this chapter we discussed atom interferometers. We begin the chapter with a discussion of the wave properties of atoms. According to the theory of quantum mechanics there is a wavelength associated with particles. This view is radically different compared to the "standard billiard ball model" of a particle which is the intuitive and the more traditional way of thinking of objects with mass. One consequence of the fact that atoms have an associated wavelength is that the atoms can exhibit interference. This wave property is not present in our everyday life is because the wavelength is extremely short and any interference is rapidly washed out. Compared to visible light it is about 40000 times shorter.

The wave property of matter places light and matter in close relation. This means that it is sometimes possible to replace light with atoms in applications where the wave properties are used. The benefit will be that atoms offer a much higher precisions since there wavelength is shorter. Because of the short wavelength there is also a miniaturisation potential beyond what is possible with light. Also, atoms have many other properties that light do not have. Two important examples for inertial sensors are that atoms are directly influenced by gravity and that it is possible to trap atoms. These properties enable the possibility to create small atom clouds that can be thrown with a velocity of a few meters per second in ballistic trajectories. These trajectories can be converted to interferometers by applying sequences of laser pulses. The laser pulses can be thought of as the atom analogy to beam splitters and mirrors.

To better understand the accuracies possible with atom interferometers we examine experiment and theory. These are compared to their light counterpart like the fibre optical gyroscope (FOG/IFOG). We go through the basic operational principles of an interferometer, see Figure 9, and discuss current state of the art experiments. These are today bulky and are oriented towards investigating basic properties of interferometers. Figure 12 and Figure 13 shows two experimental setups. The experiment in Figure 13 is of special interest because it can measure all three acceleration and all three rotational components in one setup. Thus Figure 13 demonstrates in principle a full INS.

The accuracy of an inertial sensor based on atom interferometry can be compared to its light equivalence. In 4.2.3, 4.2.4 and 4.2.5 we examine the most widely used experimental interferometer scheme which is related to the relation described by (12). The first part is directly related to rotation rate and the second term is related to acceleration. Each of these terms is studied separately. In the following a brief account on the conclusions is presented:

- **Gyroscope:** A direct comparison where it is assumed that the atom interferometer has the same size as an IFOG gives a phase enhancement of $5 \cdot 10^{10}$. This is an enormous number and is not met by current technology because of shot noise and area coverage. Including these two parameters to make a better estimation and using data from current experiments lowers the phase enhancement until it is around the accuracy of the best IFOG systems. However, atom interferometers have further potential since the parameters can be improved. In Table 5 we present a list of accuracies for different technologies. Experiments do match the accuracy predicted by our analysis but the size of the atom interferometer gyroscopes are still meter long. Current experimental approach with free flying atomic beams is not the way towards compact systems. If shot noise, area coverage and size can be enhanced with atom chip technology then it might be possible to reach the high accuracies expected in Figure 5 and sizes competitive for a commercial market.
- **Accelerometers:** A comparison of the phase enhancement to a light based accelerometer gives a possible range from 10^4 up to 10^{17}. These enormous numbers are a direct comparison to light based systems and because of technological restrictions will never be completely fulfilled. If we instead look at

shot noise and current experiments we get a rather good prediction of an accuracy of around 0.1 ng. Table 6 lists accuracies for different technologies. In an atom based accelerometers a cold atom cloud is launched in a ballistic trajectory that is around 30 cm high. Right side of Figure 13 shows a typical atom cloud trajectory. The long loading time of about 0.5 s of the atom cloud makes current systems slow for inertial navigation applications. The technology is currently more suited for precise gravity measurements and the maturity of the technology in this field is rather high. As for gyroscopes the technology is a good proof of principle but size and atom cloud loading time need to be enhanced before application towards inertial navigation are to be considered. The technology expected to solve these problems is the atom chip which is still in its early development stage.

As concluded both gyroscopes and accelerometers have great potential but current experiments are not oriented towards technologies that can compete with commercial INS. A technology that is in its early stage and has the potential to miniaturize current bulky setups is the atom chip. Experiment with atom chips is in a very early development stage and for the moment research is focused on how to guide the atoms, keep the coherence, and create atom beam splitters, atom mirror, compact laser system and cooling systems. There are several research programs dedicated to inertial sensors based on atom chip technology but very few have demonstrated any working experimental concept. This technology has many years left before producing accelerometers or gyroscopes that can compete with commercial products.

Even with these current limitations there are companies investing in patents and collaboration with universities. Two recurring companies in a patent search are Honeywell and Lockheed Martin. There are also two spinoff companies, µQuanS and AOSense, which are mostly dedicated to gravity measurements and has developed atom based gravimeters. Figure 18 shows there products. The company AOSense has a close relation with Lockheed Martin and has also proposed radical ideas for applications of atom interferometry for enhanced situation awareness discussed earlier in 3.1.4.

5 Discussion, conclusions and remarks

Navigation is key ability for military and civilian applications. The most popular method is today based on GPS where cameras, radar, sonar, lidar and INS are used for enhancing the GPS positioning. However, these navigation systems do rely on external signals which can be a drawback in many applications. In this survey we have investigated the possibility and potential of using atom interferometer technology for high precision inertial navigation. High precision is here defined as an INS that can navigate with an error of less than 100 m after one hour without any external signals. There are not many technologies that can reach this accuracy and those available are bulky and expensive. The technological challenge is to find new technologies that can satisfy the requirements of small size, high accuracy, stability and cost efficiency.

5.1 Applications for high accurate inertial sensors

There is a strong strategic benefit with inertial navigation since no external signals are needed. This means that if accelerometers and gyroscope sensors have high enough accuracy it is possible to have robust navigation which is immune from external disturbances. The possibility of a small size, high accurate INS would open completely new possibilities for applications in GPS impeded environments or other harsh environments where signals from outside are denied.

If small inertial navigation systems with accuracy of a few meters are realized then there will be many applications that will benefit but then there will also be new threats. A possible scenario can be high precision torpedoes or missiles that can navigate through difficult terrain only through INS data. However, before reaching such uses there are new forms of applications possible for accelerometer that can reach ng accuracies. It has, e.g., been suggested that accelerometers can be used for enhanced situation awareness. High accurate accelerometers will see small variations in the gravitational acceleration when massive objects are approaching. Using this data it is possible to pinpoint direction and size of the object.

A consequence of the fact that gravity and acceleration cannot be distinguished poses some restriction on the possible accuracy for inertial navigation. The discussion in 3.1.4 concludes that for inertial navigation an accuracy of 1 m over one hour is close to the limit. The reason for this is that random events like a passing truck, a slight height change or change in the ground water level will cause fluctuation in the sensor data.

5.2 The technological trend

For the last couple of years we have witnessed an incredible technological surge where MEMS are incorporated in consumer electronics. MEMS INS’s are diverse and covers a broad range of applications where very different specifications are required. But as we describe in section 2.2 they do not qualify for long term high accurate navigation in GPS denied environments. However, they are getting more accurate but there are technological limits and shortcomings that stop them from reaching high accuracies required for strategic application grade. For example, most MEMS do have a bias stability problem that limits long term measurements. What can be concluded is that current MEMS revolution does show that there is a demand for small, high accuracy, stable and affordable INS.

High accuracy and stable INS with an error of about 100 m/hr does exist today in submarines and intercontinental ballistic missiles. These systems are expensive and bulky. Also, there navigation situation is rather special. By clever manoeuvring the navigation errors can be kept to a minimal. Submarines can, e.g., do zero velocity updates or keep constant velocity for a longer time period. Even though these systems are very precise and stable they do not have the potential to be miniaturized without a decrease in accuracy.

Atom interferometers, as discussed in great length, are supposed to be the technology which has the potential of producing small size, high accuracy, stable and affordable INS. Accuracy expected is beyond all current INS technologies with a goal of reaching errors as low as 2-5 m/hr. Because of this they will not target the same market as MEMS but instead be a technology for high accuracy accelerometers and gyroscopes.

Most interesting and relevant technology for INS in this category is the atom chip. However, the atom chip technology is still in a very early development stage and so far no demonstration of a working accelerometer or gyroscope has been presented. Experiments are still struggling with implementation of basic components. Therefore, it is doubtful that a working prototype will be presented within 10 years.

Atom interferometer technologies that have demonstrated accelerometer and gyroscope are experiments with free propagating atoms. These experiments do not confine the atoms to a chip but instead use atom beams for gyroscopes and cold atom clouds for accelerometers. These are currently not of high relevance for INS since there are better and more compact technologies available on the market. However, as seen from Table 5 and Table 6 the precision of the six axis atom interferometer reported in [67] and shown in Figure 13 do qualify as a strategic graded system. It can measure all three accelerations and all three rotations rates required for an INS but the update rate is low since it takes time to load the atom cloud. Today there are several limitations that prohibit these systems to reach the high accuracies expected and bandwidth necessary for an INS. Nevertheless, if these limitations can be solved then there is potential for INS based on free flying atom beams and cold atom interferometers. These systems will most likely be in close resemblance to the gravimeters shown in Figure 18. The advantage atom interferometers have compared to equally accurate technologies is that they might offer a more affordable sensor.

Currently the technology is applied for gravimeters. The maturity of the atom based gravimeter technology has come a long way. As discussed in 4.4 there are at least two companies and both are striving to enhance their technologies toward inertial sensors. There are several proposals and patents where atom interferometers are used as support systems for enhancing less accurate INS. The intention is to offer a more cost efficient solution for high precision navigation where cheaper and less accurate INS’s are combined with an atom interferometer that is very stable, high accurate but with a lower update rate.

That the atom technology has great potential and is of high interest is reflected by the worldwide research effort. Several companies are interested and are today following closely the development. Even if the atom chip INS technology is in its very early stage there are emerging companies dedicated to producing basic atom traps and atom chip systems. The concept that, e.g., ColdQuanta follows is to provide simple cold atom systems to researchers, companies and universities. This increased availability from a component based approaches will broaden the possible actors on the market. It can be expected that this will enable the use of cold atom technology for a wider range of engineers and researchers.

5.3 Final recommendations

Our final recommendation is to keep in mind that this technology is slowly maturing and will have an impact when commercialized. A yearly monitoring is not recommended but a new evaluation should be performed in 5 years to re-evaluate the technological progress. As discussed the technology has big potential but need time before it can be applied to a real navigation situation.

6 Appendix

6.1 Inner workings of an atom interferometer

To better understand atom interferometers we give her a brief account on the requirements of an atom interferometer and the principle behind mirrors and beam splitter based on photon momentum recoil.

6.1.1 Ultra-high vacuum

Atom optic experiments are conducted in ultra-high vacuum (from 1×10^{-9} to 1×10^{-12} Torr) chambers. This to avoid collisions with background particles that can add spurious phase shifts in the interferometer [35]. The process to obtain high vacuum is an art in its own. After the construction of the vacuum chamber, where material choices and vacuum seals need to be carefully designed, high vacuum is reached by means of a turbo molecular-pump. Helium with a helium detector is often used to find leakages. These steps is followed by a several day long thermal sequence over 100 ^{o}C to bake out any residual water from the walls of the vacuum chamber. In the compartment where the interference takes place an ion pump is used to reach ultra-high vacuum. Ion pumps have small size, long life time at ultra-high vacuum and are vibration free with no contamination. This makes them ideal for precision instruments [35]. The whole procedure needs to be redone each time the system has been opened, thus most system are carefully designed to minimize leakage to avoid contamination and sealed for long periods to avoid the baking sequence.

Also many of the atom species used in experiments are either toxic or reacts strongly with oxygen thus great care needs to be taken when handling them. Alkali metals like rubidium and caesium are routinely used in atom optics, these react violently when in contact with oxygen thus glass ampules are used for loading the atom source into the oven. These ampules are shattered when high vacuum has been reached.

6.1.2 State preparation and selection

State preparation depends on the intended application but common for gyroscope and accelerometers is that a state selection is made in momentum space. The reason for choosing momentum space compared to position space is because of Heisenberg uncertainty principle. According to Heisenberg as soon as a free atom is localized in position the momentum uncertainty will increase which will cause a back action and delocalize the atom in position as soon as it is free again. This back action is avoided by preparing the atom in momentum instead [39].

Experiments demonstrating atom interferometers for inertial sensing use several types of atom sources. Common are atom beams and trapped atoms that are launched or dropped. The atom source depends on the application intended. For instance, most gyros use relatively high speed atoms from an atom beam that is transversely cooled by laser beams before entering the interferometer, see section 6.1.2.1. Accelerometers and gravimeters instead use atom clouds that can be thrown in ballistic trajectories or let them free fall from a height [39]. These clouds are created by trapping and cooling atoms in a magnetic optical trap (MOT), see section 6.1.2.2.

6.1.2.1 Atom beam preparations

For gyroscopes the sensitivity of the interferometer is not much changed by the atom speed. However, the signal to noise ratio is affected if slow moving atoms are used thus high flux atomic beam is desirable. The best results have been obtained with atom beams with a relatively high velocity of around ≈ 220m/s [72]. One big advantage with an atomic beam is that continues measurements can be done. This is similar to monitoring the change in phase in a traditional laser interferometer where a continuous light source is used. The atom source available for such an interferometer setup is an oven with a small hole where the atoms can escape out. The large distribution of atom velocities created in such a source can be compared to a light bulb. Because of this the atom beam interferometer can be compared to a white light interferometer.

An atomic beam can be created by letting atoms out through a hole in an oven [101], [102]. The effusion process will depend on the temperature, atom weight and the size of the hole. When the atom container is heated the pressure rises and the atoms effuses out through the nozzle of the oven. This method creates a high flux of $\sim 10^9$ atoms s^{-1} but the momentum distribution will be relatively large since the kinetic energy distribution is governed by the Maxwell-Boltzmann distribution. For a discussion on the Maxwell-Boltzmann distribution see section 4.1.1.

For an atomic beam a simple way of doing a momentum-state selection is by using two slits separated by some distance in the atom propagation direction. This configuration collimates the atom beam by selecting atoms with a limited transvers momentum [39]. The first aperture defines the source and the second, a ~1 mm pinhole which is further away, filters out a small portion with a well-defined orientation. Further transvers cooling can be made by letting the beam pass a 2D optical molasses.

The 2D optical molasses reduces the transverse velocity by Doppler cooling. Not all atomic species can be cooled by this technique since this requires that there are suitable energy levels which together form a close loop. The basic principle of Doppler cooling is that an atom, when absorbing a photon, will receive a momentum kick in the propagation direction of the photon, see Figure 20. When the photon is reemitting the atom will do so in a random direction. The cooling effect is obtain when the laser is slightly red shifted to the atomic level.

The basic principle is as follows and is illustrate in Figure 20; If an atom is moving in the opposite direction of the laser then the atom will see a blue Doppler shifted laser beam and the probability of absorption will increase since the laser was red shifted from the atomic level. Absorption of a photon will give a momentum kick which slows the atom down. When reemitting the photon it will do so in a random direction which means that in average the atom will end up with a lower speed in the laser direction. Atoms which are moving in opposite direction are not affected as much since they see red shifted photons that cannot be absorbed. Thus the net average effect will be a reduction in speed in the opposite direction of the laser. By counter-propagating two laser beams the atoms moving through the light field will be cooled in the direction of both laser beams. Doing so for the two transvers directions of an atomic beam will further collimate and cool it. In the beam propagation direction the momentum distribution will still be large because no cooling has been applied in that direction. By this method the transvers beam spreading can lowered down to temperatures of mK.

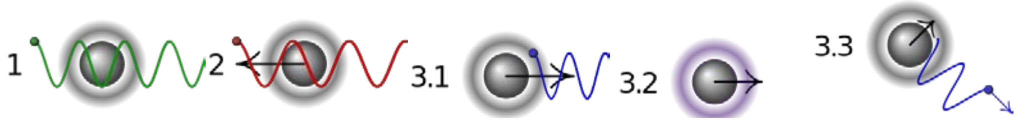


Figure 20: Laser cooling [13], 1. and 2. shows that an atom off resonance (red shifted laser) will not absorb a photon, 3.1 an atom moving towards the laser beam will see a blue shift and be closer to resonance thus the probability for absorption increases, 3.2 after absorption the atom slows down because of the momentum kick from the photon, 3.3 when the photon is reemitted it will do so in a random direction which result in an average reduction of the atom speed in the opposite direction of the laser.

6.1.2.2 Magneto-optical trap

In previous section we discussed the Doppler cooling technique for 2D cooling. By adding lasers in the third dimension 3D cooling is obtain. In the crossing of the six beams a so-called optical molasses is created. This however does not trap the atoms and they will slowly diffuse out of the cooling region. To trap the atom a magnetic field is applied which add a restoring force that is position dependent. See Figure 21 for the laser and magnetic coil configuration. This magneto-optical trap works by the principle that the magnetic field causes a position dependent Zeeman shift in the magnetic-sensitive atomic level. Further away from the centre of the trap the greater is the Zeeman shift which will lower the energy level of the atoms. The red shifted laser will then be closer to resonance because of the Zeeman shift. Thus the atom will be more likely to absorb a photon in the counter-propagating direction of the atom which will result in a momentum kick towards the centre of the trap [35]. With this technique the atom cloud can be cooled down from a few μK down to nK which will enhance the coherence and the fringe contrast of the interference.

For accelerometers and gravimeters trapped atoms are thrown with velocity of a few m/s or dropped into the interferometer. The Doppler cooling techniques discussed in the previous section are used as a precooling stage. When the atoms gas has been slowed down enough a 3D magneto-optical trap (MOT) can capture a small atom cloud. With this method an atom flux of about $\sim 10^7$ atoms s^{-1} is obtained. Compared to an atomic beam fewer atoms are available after the cooling process. Furthermore, there is a loading time of about 0.1s to 0.6 s before an atom cloud has formed. Thus, in general, the cold atoms will give a worse signal to noise ratio but with the benefit that the momentum distribution is narrower which results in higher fringe contrast and one also gets a longer interaction times in the interferometer [101].

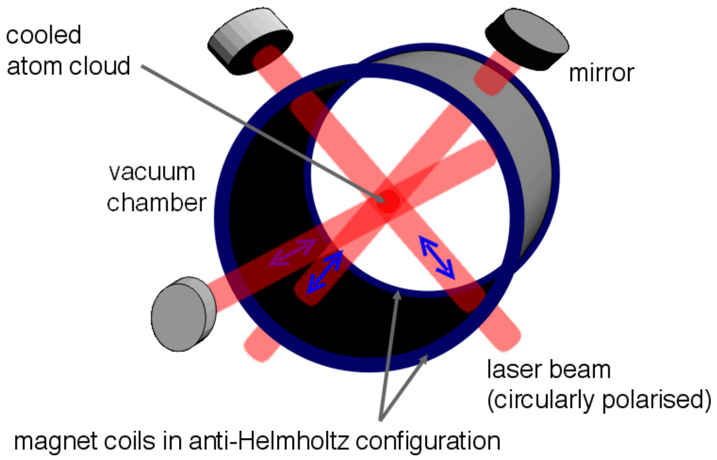


Figure 21: The laser and magnetic coil configuration of a magneto optical trap (MOT).

6.1.3 Atomic beam-splitters and mirrors

Reflections from solid surfaces are mostly inelastic with only a small probability to be elastic. This small elastic component still results in an incoherent reflection because of the rough surface structure that the short wavelength of the matter-wave encounters [57]. The first atom interferometer experiments produced by Rabi and later by Ramsey did not use any mirrors and are atomic analogies of the polarization interferometer where no separation is created between polarizations [57]. The first experiments where the atom interferometer were designed to have two spatially separated arms like the Mach-Zehnder interferometer where conducted in 1991 [103] [104] [64] [105].

There are many different approaches for manipulating atoms and the specific approach depends strongly on the target application and atom/molecules used. Coherent manipulation of matter-waves for the realization of mirrors and beam splitters are dominated by two diffraction techniques [39]. Either a material grating is used for diffracting the matter-wave, or a laser standing wave is used.

6.1.3.1 Material gratings

Material gratings have been widely used to demonstrate interference for a wide variety of atoms and molecules [57], [58], [59]. Similar to the way light diffracts on gratings, atoms diffract but because of their short wavelength it requires smaller periodicity. With modern nanotechnology it is possible to fabricate periodic structures that are much smaller than the transverse coherence in a well collimated atomic beam. With nanotechnology it has been demonstrated that single slits, double slit, diffraction gratings, zone plates, hologram masks and mirrors can be constructed for atoms [39]. The benefit of using mechanical structures is the use of feature size smaller than light wavelength, arbitrary patterns, and rugged design that affect all atom and molecule species [39]. The disadvantage of using material gratings is that they are very fragile, they block a portion of the atomic beam and for prolonged use they can clog [39], [101].

6.1.3.2 Laser gratings

For inertial measurements it is more common to use laser gratings instead of material gratings. To achieve the desired beam splitter or mirror effect stimulated Raman transitions between hyperfine ground states are used. The basic principle here is to use the conservation of momenta when a photon is absorbed or emitted by an atom.

In the absorption and emitting process the atom receives a recoil momentum kick [76]. The process is illustrated in Figure 22. Similar to the Doppler cooling, two counter-propagating laser beams illuminates the atom. These two beams are detuned to each other such that the frequency difference $\omega_{eff} = \omega_1 - \omega_2$ equals the microwave splitting ω_{hfs} between two hyperfine ground states $|a\rangle$ and $|b\rangle$ via an intermediate stat $|i\rangle$ of the atom. The atom will absorb a photon ω_1 and then emit, by stimulated emission, a photon of frequency ω_2. The intermediate state $|i\rangle$ can be suppressed by detuning the lasers by Δ [35], [38], [76].

Since the laser beams are counter-propagating, we get a total momentum transfer to the atom of $\hbar\boldsymbol{k}_{eff} = \hbar(\boldsymbol{k}_1 - \boldsymbol{k}_2)$. After the process of absorption and stimulated emission a total momentum transfer of $\hbar(|\boldsymbol{k}_1| + |\boldsymbol{k}_2|)$ will have occurred. Depending on the laser exposer time it is possible to change the transition probability between states $|a\rangle$ and $|b\rangle$. By adjusting the pulse area a mirror (unit probability to transfer state $|a\rangle$ to $|b\rangle$) or a beam splitter (50% probability to transfer state $|a\rangle$ to $|b\rangle$) can be created. This is possible because the population of the states $|a\rangle$ and $|b\rangle$ is oscillating with the laser interaction time. The oscillation is associated with the Rabi frequency Ω_{eff}. For example, the probability $P_b(t)$ of moving the state from $|a\rangle$ to $|b\rangle$ is the given by

$$P_b(t) = \frac{1 - \cos\left(\Omega_{eff} \cdot t\right)}{2} \tag{22}$$

Where t is the laser interaction time. From (22) it is noted that if $\Omega_{eff} \cdot t = \pi$ the probability P_b is one and if $\Omega_{eff} \cdot t = \pi/2$ we instead obtain a probability of 50%. Since this process is coherent and associated with a momentum kick we obtain a way of implementing mirrors and beam splitters [38], [76].

In the case of Cs atom it is possible to transfer a momentum kick orthogonal to the atom propagation direction that results in a transvers velocity of $\hbar|\boldsymbol{k}_{eff}|/m = 7.1$ mm/s. Because of the counter-propagating lasers configuration the momentum kick will strongly depend on the Doppler frequency ω_{eff}. Thus the lasers need to be locked to the relevant atom transitions and detuned with for example an acousto-optical modulator (AOM).

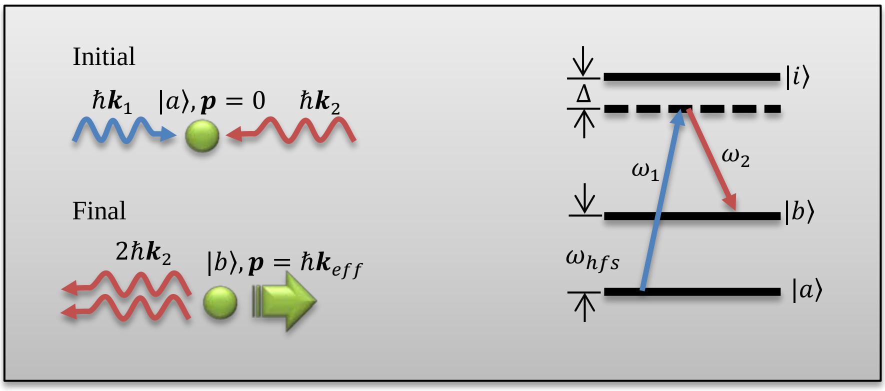


Figure 22: Raman process for creating an atom mirror or atom beam splitter.

6.1.4 Vibration compensation

The high sensitivity of an atom interferometer requires serious vibration compensation. In atom interferometers the atom beam or cloud is during the interferometer sequence flying and only interacts with the laser beams. Because the laser beams will act as mirrors and beam splitters it is important to not induce noise through them. This means that any component guiding the laser light will need vibrations compensation [35].

Several techniques to suppress mechanical mirror vibrations have been tested. Full vibration compensation by placing the complete setup in a vibration isolation table is common method. A more local method where each mirror is monitored by attaching mechanical accelerometers has been tested in [38] and [35]. From the accelerometer readings it is possible to estimate the interferometer phase contribution and in a post process subtract that contribution.

Vibrations can in some cases be self-compensated. One example is in gravitational gradiometers where the orientation of the gravitational vector is measured. In such a measurement two interferometers are used to make a differential measurement that will not be affected by vibrations.

6.2 Atom and Light optics component equivalence

Table 9: Below, we list light optics components and their atom optics analogous components [39].

	Light optics	**Atom optics**
Sources	Thermal	Supersonic beam moving molasses, launched or dropped MOT
	coherent laser	Bose-Einstein condensate
Lenses	spherical	Electrostatic quadrupole or magnetic hexapole
	cylindrical	Gaussian optical beams
	Fresnel	Nanostructure zone plates (cyl. or sph.)
	achromat	Combination zone plate and *E-M* lens
Mirrors	Mirror	(Giant) quantum reflection, helium from (bent) crystal surfaces, evanescent light waves, periodically poled magnetic domains (on curved surfaces)
Gratings	phase	Standing waves of light: Bragg or Kapitza-Dirac (pulses)
	amplitude	Nanostructure gratings, standing waves of resonant radiation
	reflection	Crystal surfaces, quantum reflection from (nanostructured) surfaces, structured evanescent light
Polarizing splitters	Polarizing splitters	Stimulated Raman transitions, optical Ramsey $\pi/2$ pulses, Stern-Gerlach magnets, optical Stern-Gerlach effect
Phase plates	glass	**E** field, **B** field, dilute gas, nearby surface
Holograms	transmission	Perforated nanostructures (with **E** and **B** fields)
	reflection	Nanostructures (with enhanced quantum reflection)
λ shifters	modulators	Amplitude modulated standing waves, gravity, bichromatic laser fields, reflection from a receding rotor
Interferometers	Young’s experiment	Micro (or nano) slits
	Mach-Zehnder	Space domain using (separated) beams, time domain, with pulsed gratings, longitudinal (rf or Stern-Gerlach beam splitters)
	Michelson	Atoms confined in a waveguide
	Fabry-Perot	Atoms confined in a three-dimensional trap
Wave guides	fibre optics	**B** fields from wires (on a chip), permanent magnets, optical dipole force, evanescent light in hollow fibre
Detectors	photon counter	Hot wire (or electron bombardment) ionizer and counter (CEM or MPC)
	state selective	Field ionization, laser ionization, metastable detection, polarization spectroscopy
	imaging	Multichannel plate for ions or (metastable) atoms (state selective) fluorescence, absorption, or phase contrast imaging
Amplifiers	stimulated emission	Four-wave mixing with BEC (nonlinear quantum optics)

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